Unified Treatment of Multi-Scale Ocean Modeling Using Finite Element Methods

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LONG-TERM GOALS

Our long-term goal is the development and validation of multi-scale ocean models for ocean research and prediction. Multi-Scale models are attractive because the grid spacing can be varied and adjusted to fit the dynamical scales of interest without undue increase in computational cost. Such a capability is very desirable for coupling coastal and basin scale models, for example.

OBJECTIVES

The main objective of our project is the development and testing of new algorithms for the Spectral Element Ocean Model (SEOM) in three dimensions, and the investigation of its benefits in multi-scale ocean applications.

APPROACHES

We have chosen the spectral element method as our basis for spatial discretization because of its accuracy, geometric flexibility, and scalability on parallel computers. Our efforts in this project have followed two parallel paths: a first centered around the implementation of an isopycnal version of SEOM, and a second directed at a continuously stratified (3D) version of SEOM in terrain-following coordinates. Aside from the PI's, Dr. Julia Levin is a major participant in this project; she brings extensive experience in implementing high-order methods to oceanic models. Byoung-Ju Choi is a graduate student working on implementing Flux corrected transport in our model; his efforts are central to the development of an out-cropping version of the isopycnal model.

WORK COMPLETED AND RESULTS

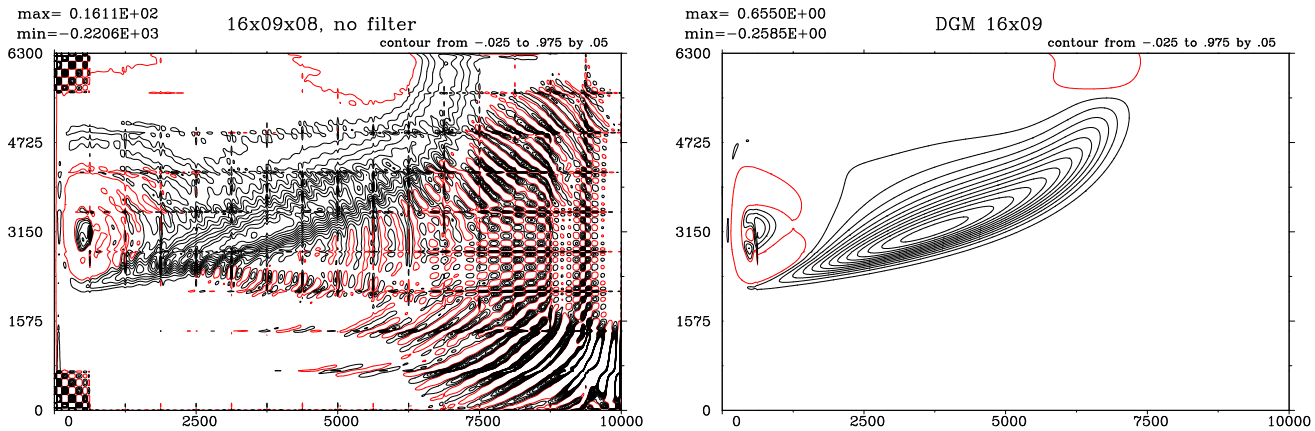

Figure 1: *Comparison of the passive tracer advection in a Stommel Gyre on 7-th order* [illegible] *element* [illegible] *lations in the contin* [illegible] *more spread o*

3D SEOM

Algorithmic improvements to SEOM-3D have centered around the advection scheme, and the redistribution of the variables within each spectral element. Our previous version of the code has relied on a classical variational form to the advection operator which re interpolation across element boundaries, the so-called C^0 [illegible]. This is problematic for advection dominated flows where very narrow fronts can develop. Furthermore, our previous model collocated all its variables at the same point. This choice is not optimal for the calculation of the tracers and the vertical velocity, particularly in the presence of short-scale topography and fronts.

The success of the Discontinuous Galerkin Method (DGM, see Cockburn 1997), in simulating advection dominated flows has prompted us to re-examine our solution algorithms. DGM's benefits include: discontinuous representation of the solution, enforcement of local conservation properties, and element-wise (independent) calculations. Each element becomes an independent entity that receives and sends information to its neighbors solely via fluxes at its boundaries. The new version of our code uses DGM for the integration of the sea surface height, and the tracers. Furthermore, our tracer and pressure points are now staggered with respect to the velocity points; the former are collocated on the Gauss roots of the Legendre polynomial while the latter are collocated on the Gauss-Lobatto roots. The staggering of the variable has required a new estimation of the pressure gradients.

We have conducted a number of simulations to assess the impact of the new formulation, and the results are very encouraging. The DGM formulation has proven to be more robust then the continuous one in all the test cases that we have done to date.

Figure 1 compares the DGM and continuous solutions of the passive tracer in a Stommel Gyre, see Hertzel *et al*, 1998). The solution remains smooth until the tracer enters the western boundary region, and is subjected to the severe shearing of its strong currents. Gibbs oscillations, symptomatic of under-resolution, appear, grow and propagate. The DGM solution exhibit small and localized spurious oscillations, whereas those of the continuous

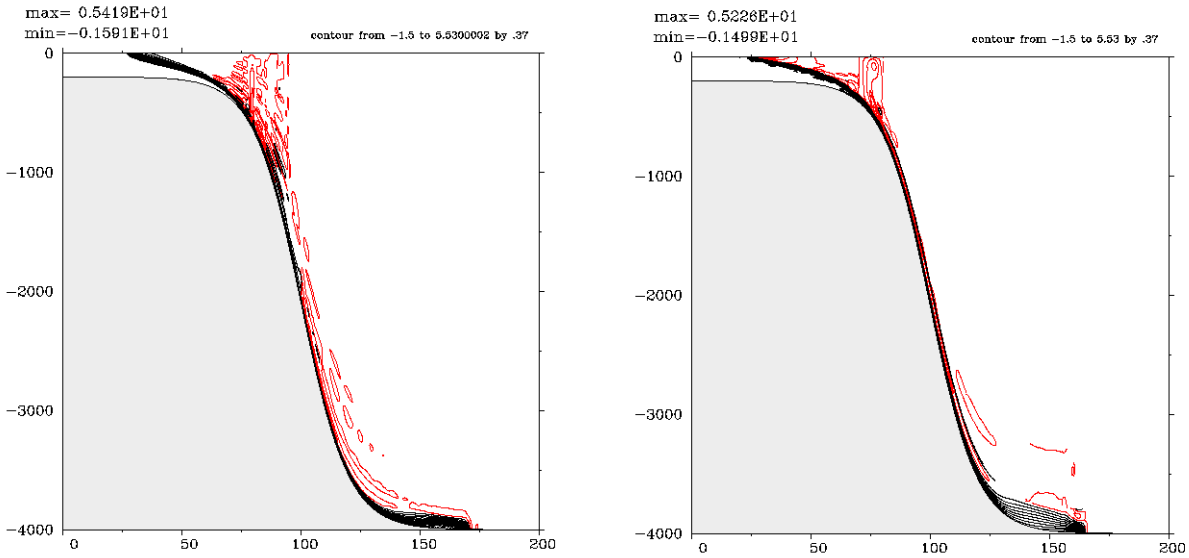

Figure 2: *[illegible] of a density front alon a [illegible] contin [illegible]) and [illegible] y on the shelf [illegible] the [illegible] the in [illegible] f and [illegible] is $\Delta\rho =$ [illegible] km, [illegible] s [illegible] in the vertical [illegible] Density [illegible]*

solution have about 100 times the amplitude and are more spread out. The continuous solution can be improved by adding dissipation and increasing resolution. For more details on comparing high order advection schemes see Iskandarani *et al.*

Figure 2 compares the temperature ditribution obtained with the continuous (unstaggered) formulation with those of the DGM (staggered) formulation. The test problem is the gravitational adjustment of a density front in the presence of steeply sloping topograph (Hidvogel, Beckmann 1 999), where baroclinicity and fronts play a prominent role in the flow dynamics. The errors in the DGM calculations are confined to the immediate vicinity of the density front, whereas they tend to spread and occupy a larger portion of the water column for the continuous formulation. The DGM results show a substantial reduction in the amplitude of the Gibbs oscillations produced near the front. Table 1 compares the extrema of density for the gravitational adjustment problem over a flat topography. The robustness of the DGM formulation is evident: reasonable results can be obtained even in the limit of very low diffusivity where the continuous formulation fails. For the full description of the test cases performed for the DGM version of SEOM see our web cite (http://marine.rutgers.edu/po)

We are now testing DGM-SEOM in realistic settings, specifically the East Sea basin with

	viscosity = 50 m^2/s				viscosity = 75 m^2/s				viscosity = 100 m^2/s			
	Version 1		Version 2		Version 1		Version 2		Version 1		Version 2	
diff	ρ_{min}	ρ_{max}	ρ_{min}	ρ_{max}	ρ_{min}	ρ_{max}	ρ_{min}	ρ_{max}	ρ_{min}	ρ_{max}	ρ_{min}	ρ_{max}
0			-1.61	6.79			-1.15	6.07			-0.82	5.71
5			-1.50	6.59			-1.16	6.04	-3.84	39.97	-0.88	5.76
10			-1.36	6.39	-3.76	40.96	-1.11	6.01	-1.03	11.95	-0.88	5.78
25	-7.05	52.3	-0.63	5.71	-1.21	10.37	-0.67	5.65	-0.46	6.79	-0.58	5.56
50	-0.55	6.99	-0.32	5.05	-0.66	5.72	-0.26	5.07	-0.36	5.31	-0.17	5.13
75	-0.12	5.32	-0.06	5.05	-0.12	5.18	-0.06	5.02	-0.08	5.09	-0.05	5.02
100	-0.02	5.04	-0.03	5.04	-0.004	5.004	-0.04	5.02	-0.003	5.004	- 0.04	5.01

Table 1: Minimum and maximum density in simulations with different viscosity and diffusivity coefficients, with and without the additional advection equation for the density difference, and with two versions of the pressure gradient calculation. The viscosity and diffusivity range from 50 to 100 m^2/s, and the density difference from 0 to 100 kg/m^3. Version 1 and Version 2 are the two versions of the pressure gradient calculation, respectively. Version 1 produces overshoots and undershoots in the density field, and Version 2 does not. The domain is 1000 km.

realistic bathymetry, and forcing (monthly mean NCEP wind stresses). We are currently developing the implicit vertical mixing scheme, required by the parametrization, in order to incoporate realistic stratification.

Spectral Finite Volume

A major drawback of unstructured finite element methods is the absence of robust and reliable limiters, a la FCT, that can prevent the generation of spurious oscillations near under-resolved features. The limiters are essential if an isopycnal model is to allow layer outcropping. This is in contrast to finite volume methods where numerous limiters are available to handle strong discontinuities. This particular advantage prompted us to develop high order, unstructured, finite volume based models for oceanic flows; the result is what we refer to as a spectral finite volume method.

We sketch this method for the simple case of a scalar advection equation $\frac{\partial T}{\partial t} + \nabla \cdot (\mathbf{u}T) = 0$. The spatial integration of this equation leads to:

$$\int_{\Omega} \frac{\partial \overline{T}}{\partial t} \, d\Omega + \oint_{\partial\Omega} T\mathbf{u} \cdot \mathbf{n} = 0, \tag{1}$$

where $\partial\Omega$ is the boundary of Ω, $\mathbf{n}$ is the outward normal, and $\overline{T}$ is the area average of T. Equation (1) is exact; the numerical approximations comes in when evaluating the boundary integral and in time-stepping the area averages. We now need to define the areas Ω and the reconstruction procedure that recovers solution values from solution averages.

The spatial discretization relies on a two-level gridding akin to the one used in spectral element method. The first level is the elemental unstructured subdivisions of the domain, and the second is the local structured $N \times N$ mesh shown in figure 3. The cells are the areas Ω in the two-level discretization. To take advantage of the interpolation properties of spectral methods, we choose the collocation points to be the Gauss points of the Legendre polynomial of degree N. The cell-averages $\overline{T}_{kl}$ can now be computed by integrating the Lagragian interpolants over the volume of each cell. This

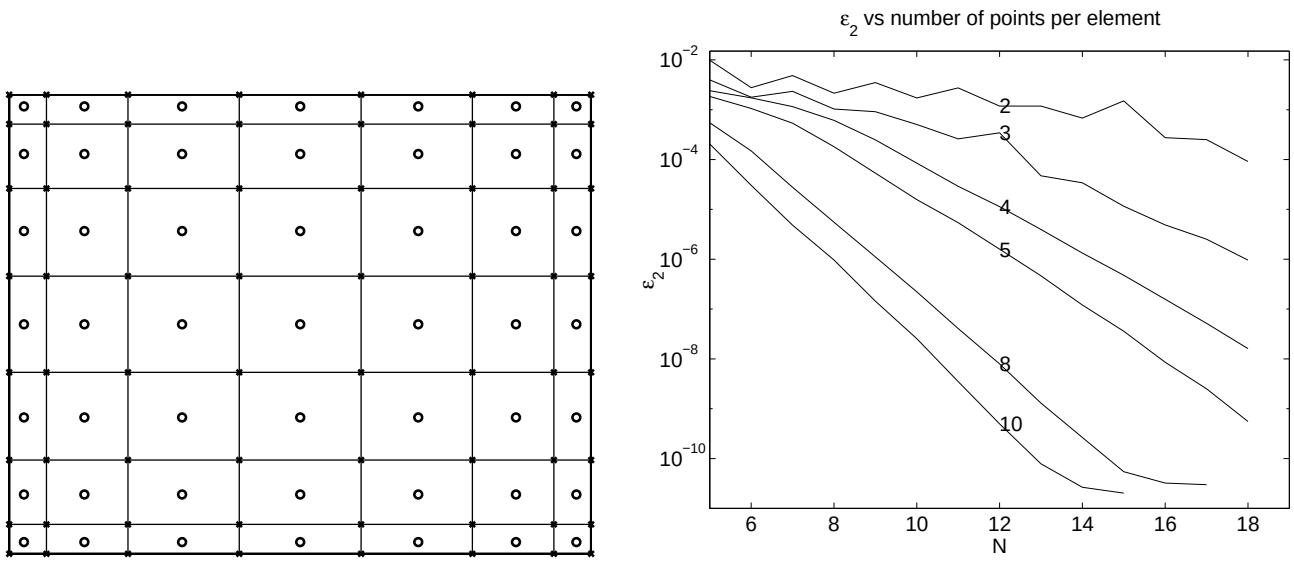

Figure 3: $N \times N$ [illegible] *elements and its* [illegible] *cells* [illegible] *in the* [illegible] *Ga* [illegible] *the* [illegible] *in each direction* [illegible] *x-axis represent the interpolation order*

operations can be cast as matrix vector product that maps the $T_{i,j}$ N^2 [illegible] N^2 [illegible] -averages $\overline{T}_{k,l}$. This matrix is inverse in the reconstuction procedure. The algorithmic steps can now be summarized as follows:

1. Given the cells averages $\overline{T}^n_{k,l}$, reconstruct the function $T^n_{i,j} = A^{-1}\overline{T}_{k,l}$.
2. Compute the boundary integrals using some form of Gaussian quadrature. These integrals require the interpolation of the function T^n on the sides of the cells. The computations of the flux on cell edges that coincide with elemental edges is problematic since two values of T^n are available. The issue is resolved by simple upwinding and propagating information along the characteristic direction.
3. Integrate the equations in time using a high order time-scheme. Kinetic AB3, RK 4, to obtain $\overline{T}^{n+1}_{k,l}$.

We have experimented with various options for the cells and collocation points within each element. The Gauss and Gauss-Lobatto points seem to be the most accurate. Figure 3 compares the convergence curves of SF and DGM: both methods can achieve spectral accuracy. The SF formulation is more expensive then DGM but has the great advantage of having several limiting procedure to prevent spurious oscillations. In fact we have implemented an FCT based procedure to go just that. A shallow water model based on the FCT Spectral Finite Volume method has been developped and tested, see B.-J. Choi et al (2002). The method is currently being extended to a multi-layer, isopycnal model.

IMPACT/APPLICATIONS

Multiscale simulations are an asset in oceanic simulations. By targetting the resolution to specific

fiently, and thus widen the range of computer resources more e
oceanic simulations possible. The flexibility afforded by unstructured grids permits the seamless transition from one resolution to the other with no increase in the complexity of the model and its code. The enhanced robustness in the underlying algorithms, particularly the advection algorithms and the variable staggering, will propell the application of these methods to a wider range of problems in oceanography. One promising area of applications of the these methods is the simulation of coupled basin-coastal circulation, and regional climate change studies.

TRANSITIONS

Our SEOM-3D code is currently undergoing tests in realistic basin geometry, bathymetry, and forcing. Once completed, high resolution simulations of the North Atlantic basin will be conducted. The extension of the spectral finite volume methodology from the shallow water equation to the three-dimensional equations in isopycnal coordinates will proceed in parallel.

The development of robust finite volume based scheme will allow the methodology developed here to be applied to other oceanic models, notably HYCOM. An unstructured-grid HYCOM will thus be able to elegantly handle the multi-scale re-quirements of the coastal zone without an excessive amount of masking and computational resources.

RELATED PROJECTS

Funds from several NSF grants were leveraged to support the present project. More specifically, fiency of the code were do improvements to our solvers and computational e -supported by NSF ACI-9814651. In addition Pacific and North Atlantic simulations were prompted by NSF-GLOBEC grant 9632809.

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