Numerical Simulation of Internal Waves in the Littoral Ocean

Robert L. Street
Department of Civil and Environmental Engineering
Stanford University
Stanford, CA 94305-4020
phone: (650) 723-4969 fax: (650) 725-39720 email: street@stanford.edu

Award Number: N00014-99-1-0413
http://www-ce.stanford.edu/faculty/street/

LONG-TERM GOAL

Our long-term goal is to employ numerical simulation to generate accurate predictions of nonhydrostatic internal-tide events, such as large internal waves and solitons, in the coastal areas of the ocean.

OBJECTIVES

Our numerical objective is to blend a proven field-scale code with large-eddy simulation [LES] and the modeling of domains with irregular boundaries. Our tool is LES in three dimensions and time. Our numerical analysis objectives include accurate representation of the flow near rough boundaries and creation of improved models for the sub-filter scale [i.e., unresolved] motions.

Our laboratory-scale simulation objective is to quantify the nature of the breaking instability as well as to study the three dimensional mechanisms of the breaking. These results will inform our field-scale efforts. This objective has been achieved by carrying out laboratory-scale simulations of breaking waves and comparing them to experiments done by others in our laboratory.

Our oceanographic-scale objective is to work collaboratively with oceanographers carrying out field-scale experiments to quantify the significant wave events triggered by internal tides, including the nonhydrostatic formation of solitons and their evolution.

APPROACH

Laboratory-scale simulation work has been carried out to examine in detail the physics of breaking internal waves and to test theory against repeatable laboratory experiments done in the Environmental Fluid Mechanics Laboratory. The primary goal of this work was to quantify the mixing efficiency of laboratory-scale breaking interfacial waves in order to parameterize the effects for our larger scale models. The code used to model these laboratory-scale waves employs many of the features of the Casulli code mentioned below. The code splits the pressure into its hydrostatic and hydrodynamic components. The hydrostatic component consists of the barotropic and the baroclinic components of the pressure, and because of the high speed of the barotropic waves, we model them implicitly with a theta-method. The hydrodynamic pressure is computed with a multigrid method.

For simulations of the coastal ocean, we using the UnTRIM field-scale code [Casulli, 1999 a & b and Casulli and Walters, 2000]. It is a finite-volume, nonhydrostatic and free-surface code that employs an unstructured, staggered-grid. This code uses cells composed of Delaunay triangles in the horizontal plane with layers of uniform [but arbitrary] thickness in the vertical. The triangles allow boundary following in plan form, with a variable grid spacing so that one can concentrate grid points over canyons, etc. The thickness of the layers varies from layer to layer. We will install our large-eddy simulation [LES] subfilter-scale [SFS] model to handle turbulence and rough wall boundaries [Katopodes, et. al, 2000 a & b and Chow & Street, 2002].

A major focus of our work is dealing with the nonhydrostatic internal-tide events leading to large nonhydrostatic internal waves and solitons. We will examine the effect of three-dimensional bathymetry on internal tide generation, propagation, and transformation. Our code treats the entire domain and so the evolution from hydrostatic to nonhydrostatic motions is seamless.

WORK COMPLETED

In the past year, we have:

1. completed direct numerical simulations of breaking internal waves at the laboratory-scale and made comparisons with laboratory experimental results.
2. made tests for internal wave simulations in Monterey Bay.
3. identified critical numerical and physical issues in simulating internal waves in the coastal domain.
4. made validation studies of the kriging method, which is required in constructing high-order advection schemes on an unstructured grid.
5. worked on further field-scale code development, with the goal to accurately simulate the internal waves of the coastal ocean. Specifically: (1) develop a high-order advection scheme for momentum, and (2) develop an accurate and conservative advection scheme for scalar transport.

RESULTS

Laboratory-scale simulations: We have developed robust techniques to generate large amplitude interfacial waves [with wave number k] in a periodic domain. We used one technique to generate breaking interfacial waves in two and three dimensions in a numerical simulation of a laboratory scale tank with length L = 0.2 m, width W = 0.2 m, and depth d = 0.3 m, which is the size of the experiments of Troy and Koseff (2000). The interface is located at mid-depth, and we vary the interface thickness δ to test its effect on the breaking dynamics. A summary of this work is presented in Troy, et al., 2002 [which was selected as the most outstanding student presentation at the 2002 AGU Ocean Sciences Meeting]. The laboratory experiments were supported by NSF Physical Oceanography Grant NSF OCE-9871808. The conclusions of this work were:

1. the instability of breaking interfacial waves is always shear-induced.
2. long waves become unstable to a modified Kelvin_Helmholtz [K-H] instability originating at the wave crests and troughs.

3. as shown in Figure 1, the Richardson number = ¼ stability boundary applies well to low $k\delta$ waves, but fails for higher $k\delta$. Here a is the interfacial wave amplitude.
4. for waves with $k\delta > 0.6$, wave breaking results from a mixed shear/convective instability and the breaking behavior is predicted according to a limiting Raleigh number argument .
5. the initial instability is always two dimensional.
6. three-dimensional longitudinal rolls (see Fig.2) extract energy from the two-dimensional K-H billows and account for 50% of the dissipation.
7. the maximum instantaneous mixing efficiency [the ratio of the rate of increase in background potential energy to the total rate of irreversible energy loss due to wave breaking] is a weak function of $k\delta$.
8. the maximum instantaneous mixing efficiency is 0.36 according to the numerical simulations.

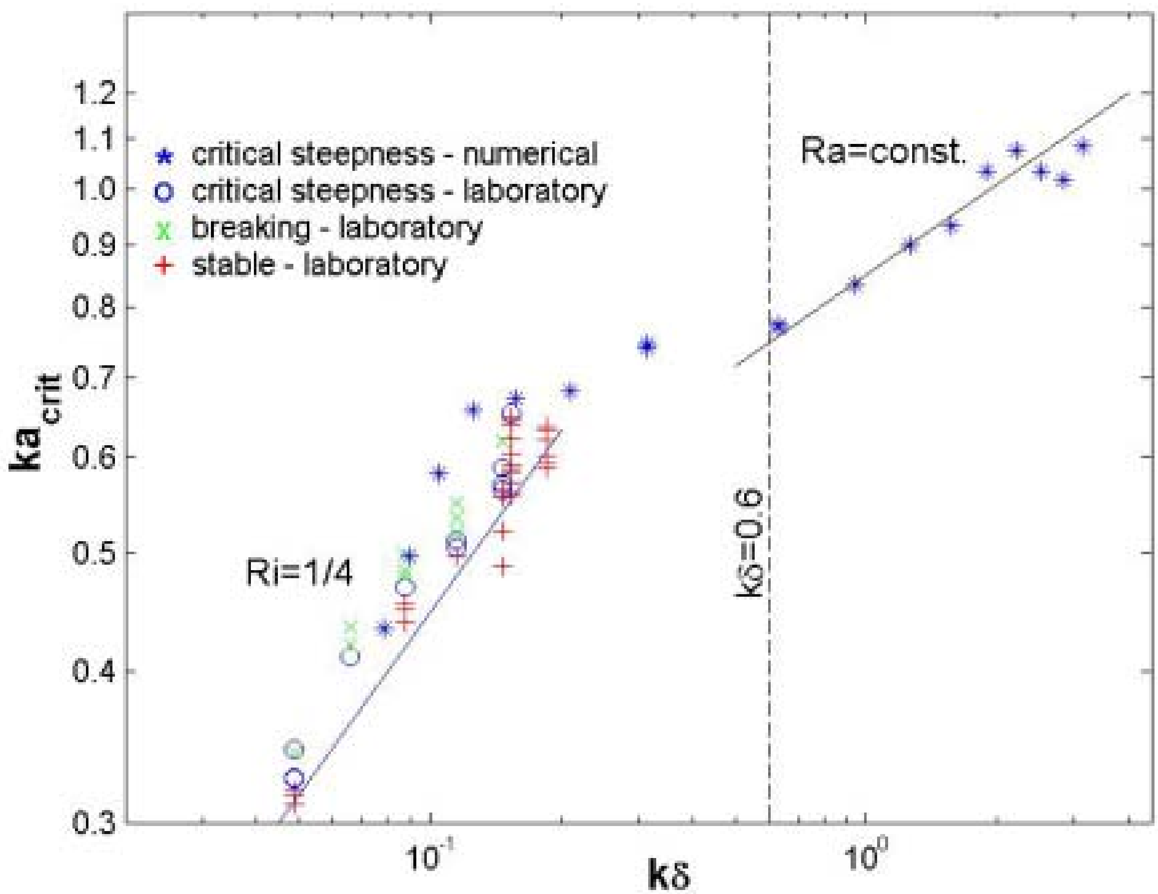

Figure 1. Critical wave steepness (ka_{break}) as a function of interfacial wave number ($k\delta$) as determined by laboratory and numerical experiments.

Field-scale simulations:

1. Internal wave simulations in Monterey Bay: We simulated the internal waves in Monterey Bay under the same condition as that used in Rosenfeld et al. (1999). The main objectives of this application were to demonstrate that the code can be predictive in a real world environment and to understand the different performance between a hydrostatic code (POM, used in Rosenfeld et al., 1999) and our nonhydrostatic code. The simulation domain is 113 km along-shore and 130 km cross-shore, with a 1 km uniform horizontal resolution and 50 vertical levels of varying thicknesses to mimic approximately the σ-coordinates used in POM. The simulation was forced by the same offshore forcing (M_2) used in

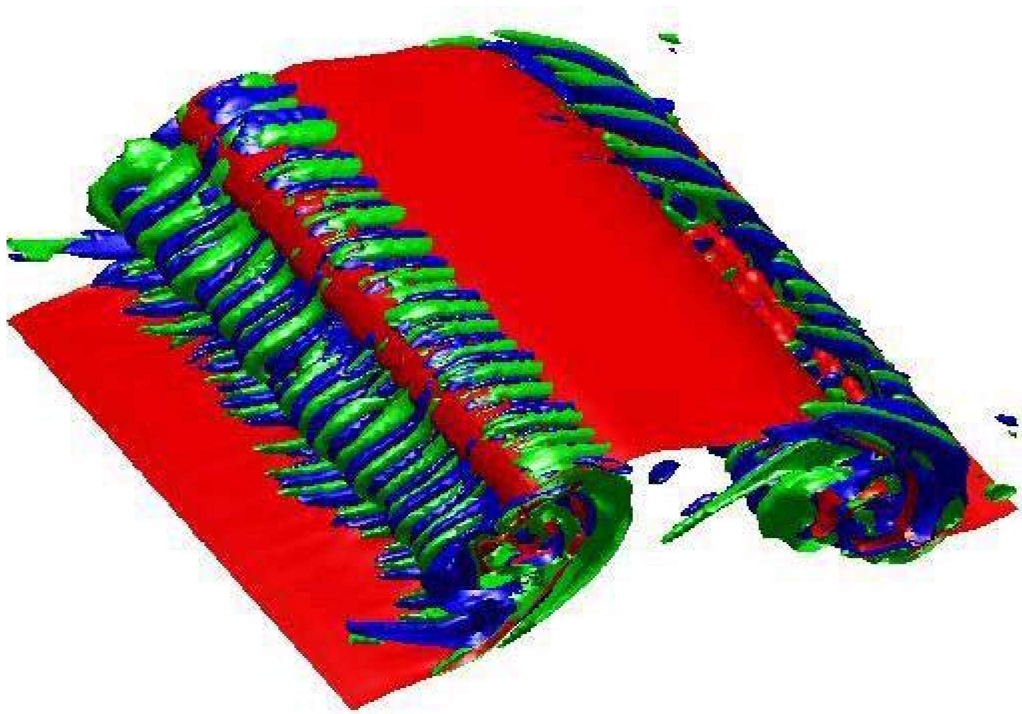

Figure 2. Positive (blue) and negative (red) isosurfces of longitudinal velocity superposed upon the mean denisty interface isosurface (red) of a breaking interfacial wave ($k\delta=\pi/10$). Here the section shown is 20 cm on a side, k is the wave number and δ is the thickness of the zone over which the density changes by 30 kg/m^3.

Rosenfeld et al. (1999). Our results show the co-oscillating tide behavior of Monterey Bay in the barotropic mode. The code also captures the internal wave behavior in the baroclinic mode. Internal wave characteristics such as frequency and isopycnal displacement are analyzed. The results show that internal tide (internal wave of tidal frequency) is the dominant constituent, which is consistent with the observation in Monterey Bay. However, the isopycnal displacements in our simulation were much smaller than those given by the POM simulation and the observations.

2. Critical numerical and physical issues in simulating littoral internal waves: Significant efforts were made to identify possible reasons for the suppression of the internal waves in our simulations. This led to our understanding of the following critical issues related to littoral internal wave simulations.

(a) Low numerical dissipation

By a separate simultion of an internal wave in a tank, we demonstrated clearly that it is the numerical dissipation in UnTRIM that significantly suppresses the internal waves. The numerical dissipation is introduced by the first-order Euler-Lagrangian method (ELM). Our work makes it clear that schemes with low numerical dissipation must be used in internal wave simulations, as confirmed by Hodges et al. (2000) and Le Roux et al. (1997 and 2000). The advection scheme used in UnTRIM needs to be modified.

(b) Accurate scalar transport scheme

We have developed a conservative scalar transport scheme by applying the finite volume method to an unstructured grid. Our results show that a conservative scheme may allow unphysical scalar values to exist (e.g., overshoots and undershoots generated from a conservative central-difference scheme). An accurate, conservative, and oscillation-free scalar transport scheme is required.

3. Validation study of the kriging method: The advection scheme requires the use of a high-order ELM, and hence a high-order interpolation scheme on an unstructured grid. We tested the kriging interpolation scheme, with a goal to verify its potential for use in our code.

Kriging is a geostatistical interpolation method formulated directly for irregularly sampled data points (Kitanidis, 1997). Kriging assumes that a physical phenomenon may be represented by a *spatially* random function. The unknown values can be estimated by a weighted linear combination of the known values at the sampled points. In kriging, the weights are chosen based on the principle of "best linear unbiased estimation" (BLUE). In addition, kriging requires the specification of a generalized covariance function, which is a measure of the spatial correlation of the random function. Depending on the properties of the phenomenon under study, suitable covariance functions can be chosen accordingly. Commonly used covariance functions include the Gaussian model, exponential model, spherical model, and polynomial model.

Our objective is to use kriging to interpolate the values of the flow variables (velocities and scalars) and their derivatives on an unstructured grid. A formulation for obtaining derivatives of the variables was derived also because that will be useful in LES formulations using sub-grid-scale models. In addition, we tested different covariance functions. Our results show that the polynomial covariance functions are more suitable for real applications due to their flexibilities, in agreement with Le Roux et al. (1997 and 2000). Overall, our validation study verified the performance of the kriging method, and it is being used in developing the high-order advection schemes.

4. Completing the coastal-ocean internal wave simulation code: Our work showed that further code development is required in order to accurately simulate internal waves in the coastal ocean. Over the next three months we will complete the following two tasks:

(1) Develop a high-order advection scheme for momentum
Working strategy: A high-order advection scheme for momentum can be developed by using a high-order ELM. The high-order ELM can be developed by using the kriging interpolation method. The resulting high-order advection scheme will be incorporated into our code.

(2) Develop an accurate and conservative advection scheme for scalar transport

Working strategy: An accurate advection scheme for scalar transport can be developed by using a high-order ELM [kriging-based] combined with flux-limiting scheme in a finite volume frame. The resulting scalar scheme will be incorporated into our code.

We will then proceed with simulation of the internal waves in Monterey Bay. We are collaborating on this phase of the work with Prof. L. Rosenfeld at the Naval Postgraduate School Oceanography Department.

RELATED PROJECTS

The laboratory experiments with which we are making comparisons are supported by NSF Physical Oceanography Grant NSF OCE-9871808, “An experiment to measure the mixing efficiency and fine-scale structure in a breaking internal wave.”

REFERENCES

Casulli, V. 1999a A semi-implicit finite difference method for non-hydrostatic, free surface flows. *I.J. Numer. Meth. Fluids*, 30, 425-440.

Casulli, V. 1999b A semi-implicit numerical method for non-hydrostatic free surface flows on unstructured grid. *Numer. Modelling of Hydrod. Sys.,* Proc. International Workshop [European Science Foundation, sponsor], 175-193.

Casulli, V., and Walters, R.A. 2000 An unstructured grid, three-dimensional model based on the shallow water equations. *I.J. Numer. Meth. Fluids*, 32, 331-348.

Chow, F. K., and Street, R. L. 2002 Modeling unresolved motions in LES of field-scale flows. *15th Symp. On Boundary Layers & Turbulence*, AMS, 432-435.

Hodges, B. R., Imberger, J., Saggio, A., and Winters, K. B. 2000 Modeling basin-scale internal waves in a stratified lake. *Limnol. Oceanogr.,* 45(7), 1603-1620.

Katopodes, F.V., Street R. L., and Ferziger, J. H. 2000a A theory for the subfilter-scale model in large-eddy simulation. *EFML Technical Report* 2000-K1, Stanford U., http://www.stanford.edu/~katopod/efmlreport2000-K1.ps.

Katopodes, F.V., Street R.L., and J.H. Ferziger 2000 Subfilter-scale scalar transport for large-eddy simulation. *14th Symposium on Boundary Layer and Turbulence*, AMS, 472-475.

Kitanidis, P. K. 1997 *Introduction to Geostatistics: Applications in Hydrogeology*. Cambridge University Press.

Le Roux, D. Y., Lin, C. A., and Staniforth, A. 1997An accurate interpolating scheme for semi-Lagrangian advection on an unstructured mesh for ocean modeling. *Tellus*, 49A, 119-138.

Le Roux, D. Y., Lin, C. A., and Staniforth, A. 2000 A semi-implicit semi-Lagrangian finite-element shallow-water ocean model. *Monthly Weather Review*, 128, 1384-1401.

Rosenfeld, L.K., Paduan, J.D., Petruncio, E.T., and Goncalves, J.E. 1999 Numerical simulations and observations of the internal tide in a submarine canyon. *Internal Wave Modeling (Eds. P. Muller and D. Henderson), Proc. Aha Huliko'a Hawaiian Winter Workshop*, 63-71.

Thorpe, S. A. 1978 On the shape and breaking of finite amplitude internal gravity waves in a shear flow. *J. Fluid Mech.,* **85**, 7-31.

Troy, C. D., and Koseff, J. R. 2000 Laboratory experiments on breaking interfacial waves. *Fifth International Symposium on Stratified Flows,* 839-844.

Troy, C. D., Fringer, O. B., Koesff, J. R., and Street, R. L. 2002 Laboratory and numerical experiments on breaking interfacial waves. *Eos*. *Trans. AGU***,** 83(4), Ocean Sciences Meeting Suppl., Abstract OS41E-68.

PUBLICATIONS

Fringer, O. B., and Street, R. L. 2001 The dynamics of breaking progressive interfacial waves, *Proceedings of the International Symposium on Environmental Hydraulics*, CD-ROM, ID # 00058.pdf.

Fringer, O. B., Armfield, S. W., and Street, R. L. (2002) A background-potential-energy-preserving scheme for interfacial waves. *International J. for Numerical Methods in Fluids*, 28 p., submitted.

Troy, C. D., Fringer, O. B., Koesff, J. R., and Street, R. L. 2002 Laboratory and numerical experiments on breaking interfacial waves. *Eos*. *Trans. AGU***,** 83(4), Ocean Sciences Meeting Suppl., Abstract OS41E-68. [Awarded an Outstanding Student Paper Award, 2002 Ocean Sciences Meeting, AGU.]