

# Air-Sea Interaction, Faraday Waves and Hydrodynamic Stability

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## LONG-RANGE OBJECTIVES

My long-range objectives are to understand (or at least contribute to our understanding of) the generation, nonlinear evolution, dissipation and diffraction of water waves over the spectrum from capillary waves to tides.

My work on air-sea interaction stems from my 1957 [113]<sup>†</sup> paper, in which the wind-to-wave energy transfer is derived from the phase shift across the critical layer (where wind speed = wave speed). That formulation omits the perturbation Reynolds stresses and gustiness, either of which may effect energy transfer comparable with that ascribed to the critical layer. The perturbation Reynolds stresses based on a viscoelastic-closure hypothesis and the effects of gustiness have been incorporated in subsequent papers.

My work on nonlinear waves stems from my 1976 paper [240] on the average-Lagrangian formulation for standing waves and my 1977 paper [248] on Hamiltonian formulations for surface waves. Of particular interest are Faraday waves, for which recent experimental work has raised significant, unanswered theoretical questions.

My work on wave dissipation goes back to my 1967 Royal Society paper [186], with special reference to capillarity and surface films. A full understanding of viscous dissipation, informed and supported by laboratory measurement, requires a more extensive knowledge of capillary dynamics than we currently possess. It is worth noting that advances in our understanding of capillary dynamics are relevant for the rational description of radar (especially X-band) scattering from the surface of the sea.

My work on diffraction goes back to my doctoral dissertation (1944) and subsequent investigations of acoustic and electromagnetic waves. My 1967 paper [188] adopted Schwinger's variational method to surface-wave scattering and laid the foundation for subsequent oceanographic investigations by myself and others.

All of my work during the past forty years has been supported in part by the Mathematical Sciences Division (Fluid Dynamics Program) and the Ocean Sciences and Technology Division (Physical Oceanography) of ONR, and, although much of it is of a rather basic character, I believe that it and its future extensions are relevant and significant for problems in oceanography and naval architecture.

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<sup>†</sup>Square-bracketed numbers in this (LONG-RANGE OBJECTIVES) section refer to the SELECTED-PUBLICATIONS list in my c.v. (see below).

## APPROACH

My primary approach is through mathematical models. Solutions ultimately are developed in both analytical and numerical form, but my primary objective is to obtain analytical results that inform phenomenological models for the prediction of physical events.

## WORK COMPLETED

The results of the investigations supported by ONR N00014-92-J-1171 and completed prior to 8/02 are reported in the thirty-nine papers listed below and are discussed in prior annual reports. [The serial numbers in the list refer to my complete list (not included here) of publications. Omitted numbers represent reviews.] Work completed in 8/02-8/03 is reported in items [1]-[4] and described in the accompanying abstracts.

## IMPACT/APPLICATIONS

The results of my research are applicable to coastal engineering and naval operational problems and to our understanding of air-sea interaction.

## RESEARCH PUBLISHED UNDER ONR GRANT N00014-92-J-1171

357. "On Rayleigh's investigation of ripples of fluid resting on a vibrating support," *J. Fluid Mech.* **244**, 645-648 (November, 1992)
358. (with J. Becker) "Progressive radial cross-waves," *J. Fluid Mech.* **245**, 29-46 (December, 1992)
359. "On surface waves with zero contact angle," *J. Fluid Mech.* **245**, 485-492 (December, 1992)
360. "On Faraday waves," *J. Fluid Mech.* **248**, 671-683 (March, 1993)
361. "The electric bell revisited," *SIAM Rev.* **35**, 289-293 (June, 1993)
362. (with Q. Zou) "Parametric excitation of a detuned spherical pendulum," *J. Sound and Vibration* **164**, 237-250 (June, 1993)
363. "Analytical integration for the exponential profile," Appendix to Morland, L. C. & Saffman, P. G. 1993 "Effect of wind profile on the instability of wind over water, *J. Fluid Mech.* **252**, 396-398 (July, 1993)
364. (with Q. Zou) "Gravity wave reflection at a discontinuity in bottom slope," *J. Phys. Oceanography* **23**, 1870-1871 (August, 1993)
365. "Surface-wave generation revisited," *J. Fluid Mech.* **256**, 427-442 (November, 1993)
366. (with J. Becker) "Resonance leads to chaos in standing radial cross-waves," *Chaos in Australia* (Edited by G. Brown & A. Opie). World Scientific (1993)
367. "Analytical solutions for the Ekman layer," *Boundary-Layer Meteorology* **67**, 1-10 (March, 1994)

369. "On transversely isotropic eddy viscosity," *J. Phys. Oceanography* **24**, 1077-1079 (May, 1994)
370. "Faraday waves: rolls versus squares," *J. Fluid Mech.* **269**, 353-371 (June, 1994)
371. "On viscoelastic eddy viscosity," *Boundary-Layer Meteorology* **70**, 435-437 (September, 1994)
372. (with D. Henderson) "Surface-wave damping in a circular cylinder with a fixed contact line," *J. Fluid Mech.* **275**, 285-299 (September, 1994)
374. "A note on modulated cross waves," *J. Fluid Mech.* **295**, 301-304 (July, 1995)
376. "On forced capillary-gravity waves in a circular cylinder," *Wave Motion* **23**, 387-391 (1996)
377. "On viscoelastic eddy viscosity for flow over a wavy surface," *Boundary Layer Met.* **79**, 177-179 (April, 1996)
378. "Surface-wave generation: a viscoelastic model," *J. Fluid Mech.* **322**, 131-145 (September, 1996)
379. "On Janssen's model for surface-wave generation by a gusty wind," *J. Phys. Oceanogr.* **27**, 592-593 (April, 1997)
380. (with Rick Salmon) "On the vorticity of long gravity waves in water of variable depth," *Wave Motion* **25**, 273-274 (1997)
381. "The generation of surface waves by wind: a retrospective", *Appl. Mech. Rev.* **50**, R5-9 (July, 1997)
383. "A note on the Burgers-Rott vortex with a free surface", *ZAMP* **48**, 162-165 (1998)
384. (with Glenn Ierley) "Surface-wave generation by gusty wind", *J. Fluid Mech.* **357**, 21-28 (February, 1998)
385. (with Peter Chamberlain) "Topographical scattering of gravity waves", *J. Fluid Mech.* **361**, 175-188 (April, 1998)
386. (with Diane Henderson) "A note on interior vs boundary-layer damping of surface waves in a circular cylinder", *J. Fluid Mech.* **364**, 319-323 (June, 1998)
387. "On gravity-wave scattering by non-secular changes in depth", *J. Fluid Mech.* **376**, 53-60 (December, 1998)
388. "The quasi-laminar model for wind-to-wave energy transfer", *Wind-Over-Wave Couplings: Perspectives and Prospects.* (Eds. Sajjadi, Hunt and Thomas), Oxford U. Press (1999)
389. "On Faraday resonance of a viscous liquid", *J. Fluid Mech.* **395**, 321-325. (September, 1999)

390. (with Diane Henderson) "Pinned-edge Faraday waves", *Fluid Dynamics at Interfaces* (edited by Wei Shyy). Cambridge University Press (1999)
391. "A note on surface-wave scattering by a small plate", *Wave Motion* **32**, 153-156 (2000)
392. (with P.G. Baines) "Topographic coupling of surface and internal tides" *Deep Sea Res. I* **47**, 2395-2403 (2000)
393. "Stability of inviscid flow over a flexible boundary" *J. Fluid Mech.* **434**, 371-378 (May, 2001)
394. (with G. Ierley) "On Townsend's model of the turbulent-wind-wave problem" *J. Fluid Mech.* **435**, 175-189 (May, 2001)
395. "Gravity waves on shear flows" *J. Fluid Mech.* **443**, 293-299 (September, 2001)
396. "A note on surface waves generated by shear-flow instability" *J. Fluid Mech.* **447**, 173-178 (November, 2001)
398. "On slow oscillations in coupled wells" *J. Fluid Mech.* **455**, 283-287 (March, 2002)
399. "On resonant reflection by a plane grating" *Wave Motion* **35**, 311-314 (April, 2002)
400. "Gravity waves in a circular well" *J. Fluid Mech.* **460**, 177-180 (June, 2002)

#### PAPERS COMPLETED SINCE 8/02 ONR ANNUAL REPORT

[1] (with A. Sneyd) The response of a floating ice sheet to an accelerating line load (*J. Fluid Mech.*, in press, August, 2003)

The two-dimensional response of a thin, floating sheet of ice to a line load that accelerates from rest at  $t = 0$  to a uniform velocity  $V$  for  $t \geq T$  is determined through an integral-transform solution of the linearized equations of motion. If  $T = 0$  -- i.e. if the load is impulsively started with velocity  $V$  -- the solution exhibits singularities at  $V = c_0$ , the shallow-water-gravity-wave speed, and  $V = c_{\min}$ , the minimum speed for transverse motion of the ice, but these singularities are avoided by the acceleration of the load through the critical speeds.

[2] (with D. Mahdmina) Capillary-gravity waves with a linear wetting condition in a circular cylinder (submitted to *J. Fluid Mech.*, May, 2003)

Surface waves of small amplitude in a circular cylinder of finite depth are studied on the assumption that the dynamical variation of the macroscopic contact angle is proportional to the contact-line velocity. Three forms of the eigenvalue equation for the complex natural frequencies (the imaginary parts of which are damping coefficients) are constructed. Approximations are determined for the limits of fixed contact line and fixed contact angle.

[3] On resonant rotation of a weakly damped pendulum (submitted to *J. Sound & Vibration*, June, 2003)

The minimum, sinusoidal drive for resonant rotation of a weakly damped pendulum and the contiguous loci of stable states in a frequency-energy plane are determined by perturbing the solution for undamped, unforced oscillations and invoking the method of harmonic balance. Instability occurs through turning-point and period-doubling bifurcations, and the resonant states are stable only in rather small frequency intervals between these bifurcations.

[4] Slow, nonlinear oscillations in a circular well (submitted to *J. Fluid Mech.*, August, 2003)

The Helmholtz mode ( $\omega^2 \ll g/a$ ) in a circular well of radius  $a$  and ambient depth  $h_0$  that is bounded above by a free surface ( $z = h$ ) and below by a semi-infinite reservoir ( $z < 0$ ) is studied. The analysis is informed by that for linear oscillations in coupled wells (Miles 2002) and that of Hirata & Craik (2003) for nonlinear oscillations in a three-armed tube. The solution for free oscillations of amplitude  $A < h_0$  ( $A > h_0$  would imply swallowing of the free surface) to an elliptic integral. The corresponding period decreases monotonically from  $T_0 = 2\pi(h_1/g)^{1/2}$ ,  $h_1 = h_0 + 0.849a$ , for  $A = 0$  to  $0.900T_0$  (which is an upper bound to  $T$  for  $A = h_0$ ) for  $A = h_1$ .