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Development of Turbulent Biological Closure Parameterizations

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LONG-TERM GOAL

The long-term goals of this project are:

(1) to develop a theoretical framework to quantify turbulence induced NPZ interactions.

(2) to apply the theory to develop parameterizations to be used in realistic environmental biodynamical coupling numerical models.

OBJECTIVES

Connect the Goodman and Robinson (2007) statistically based pdf theory to Advection Diffusion Reaction (ADR) modeling of NPZ interaction.

APPROACH

A nonlinear model for biological and physical dynamical interactions in a laminar flow field being upwelled into the mixed layer Robinson(1999) (Fig 1, below) has been extended to turbulent flow (Goodman and Robinson, 2007). The approach of the Goodman and Robinson theory has been to develop a probability density function (pdf) for the turbulent displacement field and use that to calculate the turbulence induced biological interaction (TIBI) terms, i.e. $< P_i N_j >, < P_i Z_j >, < Z_i N_j >$, where N_i, P_i, Z_i are the ith component of a field of different nutrients, phytoplankton, and zooplankton embedded in the turbulent field. Contrast the TIBI terms with the biological turbulent flux terms $< \vec{u} N_i >, < \vec{u} P_i >, < \vec{u} Z_i >$. The formalism for modeling the latter type of terms are well developed, typically involving some type of eddy diffusivity or higher order closure such as Mellor and Yamada, (1982). However, at present, no biodynamical basis for closure of the TIBI terms has been developed.

The current approach to handling the TIBI terms is either: (1) to ignore them by setting $< (..)_i (..)_j > = < (..)_i > < (..)_j >$ in an advection diffusion reaction (ADR) equation approach (Donaghay, and Osborn , 1997), or (2) to perform a numerical simulation for the turbulence displacement and

explicitly calculate the TIBI terms. The former, as we will show below, can typically result in a large overestimate of the effect of turbulence on these interactions, while the latter is very limited in its domain size, suffers from difficulty in imposing realistic boundary condition at the laminar turbulent interface, and can only reveal significant physics of the TIBI terms with a large number of repetitive runs, which prohibit the size of the computational domain to very limited environmental scenarios. In addition, turbulence numerical models such as LES and DNS are difficult to embed in larger regional scale biophysical coupling models. What is needed to be useful in the larger scale physical biological coupling models is development of a realistic parameterizations of the TIBI terms, analogously to the development of turbulent flux parameterizations used in regional and large scale ocean circulation models. In addition development of such parameterizations will lead to new physical/ biological insights into the role of the TIBI terms.

WORK COMPLETED

Since the beginning of this project, in order to develop parameterizations for the TIBI terms, we have been examining the relationship of the Goodman and Robinson pdf theory approach to that of the ensemble averaged ADR approach. We have made a major breakthrough and have obtained an exact solution to the ADR equation for realistic boundary conditions using the Goodman and Robinson pdf theory approach. With these results we are now in a position to formally examine various parameterizations of the TIBI terms. Preliminary results indicate that for the single component bilinear NP interaction problem, the potential suitability of a linear parameterization. Below we outline the results to date.

RESULTS

Consider first the Advection Diffusion (AD) equation with no biological interaction. Let $\hat{\rho}$ be some biological scalar such the total biomass density being upwelled into a turbulent optically inactive mixed layer , with $\hat{\rho} = P + N$, where P, N are the phytoplankton and nutrient mass densities, respectively. Using the simple linear upwelling field of Goodman and Robinson (2007) (Fig.1) and, by symmetry, assuming that turbulent mixing is dominated by vertical (1D) fluxes , it is straightforward to show that $\hat{\rho}$ satisfies the one dimensional AD equation

$$(1) \quad \frac{\partial \hat{\rho}}{\partial t} - \alpha z \frac{\partial \hat{\rho}}{\partial z} - \kappa \frac{\partial^2 \hat{\rho}}{\partial z^2} = 0$$

with the boundary conditions

$$(2a) \quad \frac{\partial \hat{\rho}}{\partial z} = 0 \quad \text{at } z = 0$$

$$(2b) \quad \alpha \hat{\rho} + \kappa \frac{\partial \hat{\rho}}{\partial z} = \alpha \rho_0 \quad \text{at } z = 1$$

where the depth z is normalized by the mixed layer depth, D; $\alpha = \frac{T_{uptake}}{T_{upwelling}}$ is the upwelling strain rate, $\frac{1}{T_{upwelling}}$,normalized by the nutrient uptake time, T_{uptake} ; $\kappa = \frac{\tilde{\kappa} T_{uptake}}{D^2}$, is the normalized turbulent diffusivity , $\tilde{\kappa}$, ρ_0 is the biomass density being upwelling into the turbulent mixed layer, which is located at z <1. Boundary condition (2a) ensures that no material is fluxed out of the mixed layer at the surface, while boundary condition (2b) ensures that mass flux is conserved at the bass of the mixed layer. It should be noted that in that the model pdf discussed in Goodman and Robinson (2007) once a particle entered the mixed layer it is trapped in the mixed layer. This lead to a no flux boundary condition, $\frac{\partial \hat{\rho}}{\partial z} = 0$, at the base of the mixed layer and resulted in a build up of the integrated primary production. Boundary condition (2b) is a more realistic characterization of the role of turbulence at the base of the mixed layer, i.e. transition zone of Fig1. Eq. (1) with boundary conditions (2a) and 2b) can be solved exactly where it can be shown that

$$(3) \quad \hat{\rho} = \rho_0[1 - \sum_{m=1}^{m=\infty} A_m G_m \exp[-\frac{(Pe)z^2}{4}]\exp(-\alpha\gamma_m(t - t_0)]$$

where $\gamma_m, G_m,$ are eigenvalues and eigenfunctions solutions of the equation

$$(4) \quad Pe[\gamma_m - \frac{1}{2} - Pe\frac{z^2}{4}]G_m + \frac{\partial^2 G_m}{\partial z^2} = 0$$

where Pe is the Peclet' number defined by

$$Pe = \frac{\alpha}{\kappa}$$

and measures the relative importance of advection to turbulent diffusion. The time parameter t_0 is the initial time when turbulence is turned on. From the orthonormal property of eigenfunctions, it is straightforward to show that

$$A_m = \int_0^1 dz' G_m(z')\exp(\frac{Pe}{4}z'^2)$$

Note that at $t = t_0 \quad \hat{\rho} = 0$ z<1

We can formally write (3) as

$$(5) \quad \hat{\rho} = \int_{-\infty}^{t} dt' H(t_0, t')(-\frac{\partial \hat{\rho}}{\partial t'}) = \int_{-\infty}^{t} dt' \{\rho_0 H(t_0, t')\} \{\frac{1}{\rho_0}(-\frac{\partial \hat{\rho}}{\partial t'})\}$$

with the Heaviside function defined by

$$H(t_0,t') = \begin{matrix} 1 & \qquad t' \geq t_0 \\ 0 & \qquad t' < t_0 \end{matrix}$$

Eq (5) can be interpreted as the prescription for obtaining the average density $\hat{\rho} \equiv \overline{\rho} = <\rho>$ from the initial density $\rho = \rho_0 H(t_0,t')$ using the probability density function pdf, F, namely

$$(6) \quad \overline{\rho} = \int_{-\infty}^{t} dt' \rho(t') F(t';t)$$

with

$$(7) \; F(t';t) = \frac{1}{\rho_0}(-\frac{\partial \hat{\rho}}{\partial t'}) = \alpha \sum_{m=1}^{m=\infty} \gamma_m A_m G_m \exp[-\frac{(Pe)z^2}{4}] \exp(-\alpha \gamma_m (t-t'))$$

Changing variables to $\tilde{z}$=exp(-$\alpha(t-t')$ we find that

$$(8) \quad \overline{\rho} = \int_{0}^{1} d\tilde{z} \rho(\tilde{z}) F(\tilde{z})$$

In general we find that an average value can be written as

$$<\ldots> = \int_{-\infty}^{t} dt' \{\ldots\} F(t')$$

Consider phytoplankton of density, P, and Nutrients of density, N, being upwelling into a turbulent mixed layer.

Let the interaction of P, N be a simple bilinear form

$$(9a) \quad \frac{dP}{dt} = PN$$

$$(9b) \quad \frac{dN}{dt} = -PN$$

Using the pdf F we can write

$$(10) < P >= \int_{-\infty}^{t} dt' P(t') F(t') = \int_{0}^{1} dt' P(\tilde{z}) F(\tilde{z})$$

$$(11) < PN >= \int_{-\infty}^{t} dt' P(t') N(t') F(t') = \int_{0}^{1} dt' P(\tilde{z}) N(\tilde{z}) F(\tilde{z})$$

Consider now the equation for from (9a) and (b). It follows that

$$(12) \quad P = \frac{P_0}{P_0 + N_0 \exp[-(t - t_0)\}} \; H(t, t_0)$$

which upon using in (10) and (11), noting that N = 1-P ,and with some algebra yields the differential equation

$$(13) \quad \frac{\partial \overline{P}}{\partial t} - \alpha z \frac{\partial \overline{P}}{\partial z} - \kappa \frac{\partial^2 \overline{P}}{\partial z^2} = \alpha P_0 \delta(z-1) + < PN >$$

subject to the boundary conditions

$$(14a) \quad \frac{\partial \overline{P}}{\partial z} = 0 \quad \text{at } z = 0$$

$$(14b) \quad \alpha \overline{P} + \kappa \frac{\partial \overline{P}}{\partial z} = 0 \quad \text{at } z = 1$$

Note that the first term on the RHS of (13) represents a source term at z =1 and thus $\overline{P}$ can be cast in the exact same form as equations (1) and (2) or

$$(15) \quad \frac{\partial \overline{P}}{\partial t} - \alpha z \frac{\partial \overline{P}}{\partial z} - \kappa \frac{\partial^2 \overline{P}}{\partial z^2} =< PN >$$

$$(16a) \quad \frac{\partial \overline{P}}{\partial z} = 0 \quad \text{at } z = 0$$

$$(16b) \quad \alpha \overline{P} + \kappa \frac{\partial \overline{P}}{\partial z} = \alpha \overline{P}_0 \quad \text{at } z = 1$$

This proves that using equation (7) in (10) satisfies the ADR equation, (15) subject to boundary conditions (16a) and (16b), which is the same as (2a) and (2b).

We are now in a position to use this rigorous approach to compare various parameterizations of the TIBI terms. Consider the bilinear NP interaction discussed above. The simplest parameterization is to take the relationship for induced turbulent TIBI term as $< P'N' > \; = C\overline{P}\overline{N}$, which results in the RHS of (15) as
$< PN > \; = (1-C)\overline{P}\overline{N}$. Note that the standard approach (Donaghay, and Osborn , 1997) is to set C =0 .
Table I below shows values of vertically integrated values of C, namely $< C >_s = \int_0^1 dzC$ for different values of α and Pe = 1, i.e. advection and turbulence, playing an equal role. Also shown are exact solutions for $< C >_s$ obtained with our new pdf theory, described above. Although it should be noted that we are not comparing the actual local values of C, i.e. C = C(z), the closness between theory and this parameterization approximation is very encouraging. Note as α increases the uptake time increases relative to the advection time and turbulence plays a decreasing role in TIBI interaction. Conversely

for small α and small uptake time relative to the advection time the role of turbulence on the TIBI term increases.

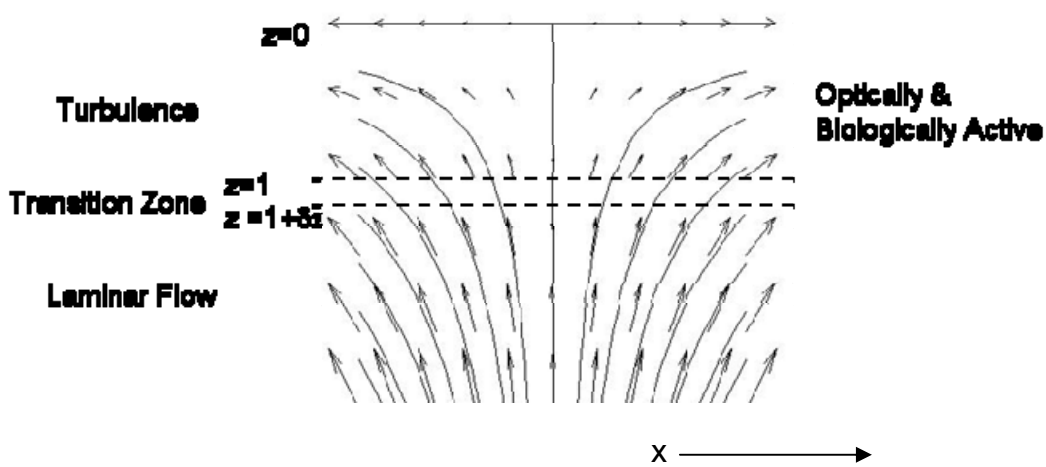

Figure 1. Upwelling flow field into an optically and biologically active mixed layer used in the Goodman and Robinson (2007) biodynamical model. Note as □□increases or as the uptake time increases relative to the advection time

Table 1.

	α	.01	.05	.1	.25	.5
Exact	C	.96	.80	.63	.35	.15
Parameterized	C	.96	.81	.64	.29	.19

Values of $C = \frac{< P'N' >}{\bar{P}\bar{N}}$ for Pe =1 and difference values of α,

IMPACT/APPLICATION

This work forms the basis for a new class of subgrid scale biodynamcial interaction for input into regional and circulariuon models.

RELATED PROJECTS

DRIs: LATMIX, LOCO

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