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DEVELOPMENT OF NUMERICAL 3-WAVE INTERACTIONS MODULE FOR  
OPERATIONAL WAVE FORECASTS IN INTERMEDIATE-DEPTH AND SHALLOW  
WATER

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## LONG-TERM GOALS

The goal of the project is to develop an effective and numerically efficient implementation of 3-wave (triad) interactions for finite-depth wave models, in coordination with the project “Nonlinear and Dissipation Characteristics of Ocean Surface Waves in Estuarine Environments“ funded by the Office of Naval Research (ONR) through the National Ocean Partnership Program (NOPP). The theoretical approach and numerical module, based on efficient stochastic formulations (e.g., Agnon and Sheremet, 1997), will be developed and tested following the same development cycle. The numerical module will be compared with accurate but computationally expensive pilot models, on a number of observational field and laboratory data sets. The module will be integrated and tested within operational model framework (e.g., WAVEWATCH III, STWAVE, SWAN, and others), in collaboration with operational code developers and other researchers working under the present NOPP program.

Due to funding delays, the project is in a very early stage of development.

## OBJECTIVES

On mildly sloping beaches and over relatively short distances,  $O(1 \text{ km})$ , 3-wave (triad) interactions have been shown to dominate wave dynamics in the shoaling zone outside the surf zone (e.g., at Duck, NC, Herbers et al., 1995a, 1995b; Sheremet et al., 2002, 2005; Henderson et al., 2006, and many others). This evolution has been successfully modeled in the spectral domain based on both phase-resolving models (Agnon et al., 1993; Kaihatu and Kirby, 1995; Agnon and Sheremet, 1997), and phase-averaged (stochastic) models Agnon and Sheremet (1997); Herbers and Burton (1997).

Historically, operational models have focused on the deep-water domain, where 3-wave nonlinearities are eliminated as irrelevant to wave dynamics (e.g., Hasselmann 1962; 1963a; 1963b; Zakharov, 1968; Zakharov et al., 1992; Krasitskii, 1994). Using an analogy from thermodynamics, deep-water wave conditions are characterized by homogeneity and evolution scales that allow for a quasi-equilibrium description, with relaxation times (e.g., life of oscillatory non-resonant 3-wave interactions), much shorter than the evolution scale. Attempts to extend deep-water models into finite-depth inevitably lead to a self-contradictory thought process, exemplified by the use of wave-number Fourier representations, resulting unrealistically slow evolution scales, and a discussion focused on direct energy cascade in the short-wave tail of the spectrum, where deep-water, rather than shallow-water dynamics dominate (e.g., recently, Onorato et al., 2009). Important physical aspects, such as infragravity wave (IG) generation, are not accounted for in the 4-wave interaction framework.

Neglecting finite-depth 3-wave interactions has led to simplified, empirical process calculations with large errors, particularly for complex wave trains with multi-modal spectra (e.g., Holthuijsen et al. 1994a; 1994b; 1995). At present, 3-wave interactions are either ignored or poorly implemented in existing operational models. For example, the LTA approach implemented in SWAN (Eldeberky and Battjes, 1995; based on a single laboratory experiment by Beji and Battjes, 1993) is typically disabled in the model (with LTA activated, results are as often worse as they are better). However, an accurate and efficient representation of triad evolution is becoming increasingly nec-

essary as the resolution of operational models in coastal waters increases. Operational models at NCEP now run with a coastal resolution of approximately 7.5 km. For coastal applications of the SWAN model (inherently designed for more regional applications) by the Naval Oceanographic Office, 3-6 km resolution is typically used. The National Weather Service envisions resolutions increasing to 5 km soon (by the end of this NOPP period), with 2.5 km not too far in the future. During hurricane/typhoon landfall, wave conditions in large areas of the grid are similar to nearshore environments. US Army Corps of Engineers nearshore applications are often run at a resolution as fine as 100-200 m. The pressure to move to higher resolution is always present, particularly as nearshore models such as Delft3D (Lesser et al., 2004) become a larger part of forecasting systems (military or otherwise). Moreover, methods for deduction of bathymetry from remote sensing (e.g., Stockdon and Holman, 2000) offer continual improvement of bathymetric estimations, provided the requisite high resolution models is available.

The goal of this project is to develop a simplified, low-dimensional, physically effective model of near-resonant 3-wave interactions, and formulate an efficient numerical implementation for operational wave models.

## **APPROACH**

The project will include a thorough investigation of the 3-wave interaction system, essentially paralleling the effort to develop efficient and accurate solutions of the 4-wave interaction equations (DIA, ?; methods based on WRT approximation, Webb, 1978; Tracy and Resio, 1982; Van Vledder, 2006; the two-scale approximation TSA, Resio and Perrie, 2008, and many others). The complexities of analytical approaches to the triad-interaction problem to be (non-resonant character, importance of phase correlations, and inhomogeneity of the finite-depth dispersion relationship – e.g., Stiassnie and Shemer, 1984; Resio et al., 2001 for 4-wave formulations) will be addressed following the same principle of a two-pronged approach (theoretical effort supported and guided by numerical studies) that is outlined in the project “Nonlinear and Dissipation Characteristics of Ocean Surface Waves in Estuarine Environments“ supported through NOPP.

We will use “exact”, high-dimension pilot models to identify efficient and effective ways to lower the dimension of the system, and to evaluate the performance of the simplified modules throughout the development cycle. For skill evaluation, we will use detailed metrics involving the evolution of wave energy (2-order statistics, power spectrum), as well as phase information (higher order statistics, bicoherence, skewness, kurtosis). At later stages, candidates for operational modules will be tested within a semi-operational environment. NCEP and USACE models that are used operationally will be set up to adequately represent the geographical areas where field data are available (e.g., Atchafalaya shelf, LA, Jaramillo et al., 2009; Duck, NC, Sheremet et al., 2002, 2005), and with laboratory data (Smith and Vincent, ?2003; and ongoing studies on a reef structure). This will allow for the development of the low-dimensional formulation to be carried out in close relationship to a practical semi-operational modeling environment.

The final numerical module will be designed for easy importing into existing operational models such as SWAN, WAVEWATCH III, and STWAVE.

## **WORK COMPLETED**

The project is in a very early stage of development. Work has started at setting up operational models (e.g., WAVEWATCH III) at UF. NCEP is providing support this effort through access to the NOAA subversion server access. We are in the process of developing the theoretical approach and strategies for numerical investigations. Unidirectional implementations of triad interaction models have been made operational on selected datasets (Atchafalaya LA; Laboratory sloping beaches). Additional numerical implementations for directional triads are under development. We are also working with other NOPP project PI's to compile an accessible set of test data for use in model development and validation.

## **RESULTS**

Figure 1 illustrates the capabilities of the existing model (unidirectional version). In this example, the stochastic model is used to simulate wave propagation over a reef. The data used for the simulations are laboratory observations collected during a wind-wave flume study conducted in August-September 2006 at the University of Michigan at Ann Arbor (Demirbilek et al., 2007).

## **IMPACT/APPLICATIONS**

The project will provide an accurate and computationally effective numerical implementation that will improve the performance of operational wave models such as WAVEWATCH III, STWAVE, SWAN, in the intermediate depth and shallow water.

## **RELATED PROJECTS**

This research benefits from, and enhances, parallel research (Sheremet) funded under Coastal Geoscience to study wave-sediment interaction in muddy environments, conducted in collaboration with the MURI wave-mud project. The scope and approach of the present research builds on the strong, ongoing collaboration between U. Florida and U. Texas illustrated by a number of papers in print and in preparation. The project is also closely aligned with the NOPP project "Nonlinear and Dissipation Characteristics of Ocean Surface Waves in Estuarine Environments" (PI: Kaihatu; Co-PI: Sheremet).

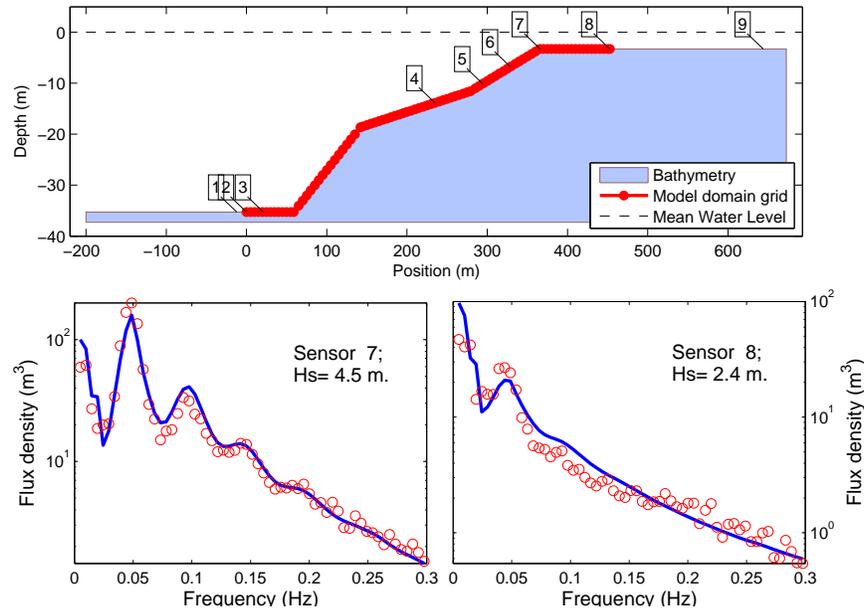


Figure 1: Comparison of observations and numerical simulations based on the stochastic model (Agnon and Sheremet, 1997), for one of the wave tested in the laboratory flume study conducted in 2006 at the University of Michigan at Ann Arbor (Demirbilek et al., 2007). Upper: bathymetry profile for the experiment (prototype scale), reproducing a fringing reef in Guam. The location of the wave gauges is marked by numbers. Lower: Spectral energy flux density (circles – observations; line – numerical simulations) at the location of sensors 7 and 8, for a JONSWAP spectrum characterized by initial peak period of 20 s and significant height of 5 m.

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