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# Propagation of Low-Frequency, Transient Acoustic Signals through a Fluctuating Ocean: Development of a 3D Scattering Theory and Comparison with NPAL Experimental Data

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Grant Number: N0001410IP20048
http://www.esrl.noaa.gov/psd

## LONG-TERM GOALS

Development of a new, 3D theory of low-frequency, long-range sound propagation through a fluctuating ocean, including both CW and transient acoustic signals.

Comparison of theoretical and numerical results with NPAL experimental data.

## OBJECTIVES

To develop a 3D, modal theory of broadband sound propagation though a fluctuating ocean, including analysis of the coherence function for transient acoustic signals and temporal coherence.

To develop computer codes for calculation of the horizontal and vertical coherence functions of transient acoustic signals and temporal coherence.

To compare theoretical and numerical results with 1998-1999 and 2004 NPAL experimental data and those which will be obtained in the 2010-2011 NPAL experiment in the Philippine Sea.

## APPROACH

Studies of statistical characteristics of low-frequency sound waves propagating through a fluctuating ocean are important for many practical applications. With the support of our previous and current ONR grants, we have been developing a 3D, modal theory of sound propagation in a fluctuating ocean [1-9,14-17]. In this theory, we have calculated and analyzed the mean sound field, the mean intensity, the

vertical and horizontal coherence functions, and other second statistical moments of narrow-band acoustic signals propagating through an ocean perturbed by linear internal waves (IWs). The theoretical results were compared with experimental data obtained in the North Pacific Acoustic laboratory (NPAL) experiments carried out in the North Pacific in 1998-1999 [10] and in 2004 [11]. Our next goals are to generalize the narrow-band, 3D, modal theory and corresponding computer codes to include broadband, transient acoustic signals propagating in a fluctuating ocean and to study temporal coherence of acoustic signals in a non-stationary medium. Theoretical and numerical results obtained will be compared with experimental data, including those obtained in the previous NPAL experiments and that will be obtained in the Philippine Sea.

Dr. A. Voronovich and Dr. V. Ostashev are PI and Co-PI in this project.

## WORK COMPLETED

During the reporting period, the following two tasks were completed:

Task 1. The narrow-band, 3D, modal theory of sound propagation in a fluctuating ocean was generalized to include broadband acoustic signals and temporal coherence.

Task 2. The developed theory was applied to the analysis of temporal coherence of narrow-band acoustic signals propagating in an ocean perturbed by linear IWs with the Garrett-Munk (GM) spectrum. It was shown that the coherence time $\tau_c$ of the signal decreases with the propagation range as $\tau_c \approx r^{-0.5}$ for a fixed frequency and decreases with frequency as $\tau_c \approx f^{-1}$ for a fixed range.

## RESULTS

The following results were obtained in FY2010:

Task 1.
Closed-form equations for the spatial-temporal coherence function of broadband acoustic signals propagating in a fluctuating ocean were derived and analyzed in detail. The analysis is done in a cylindrical coordinate system $(r,\varphi,z)$, where $r$ is range, $\varphi$ is the azimuthal angle to an observation point, and $z$ is the ocean depth. The spatial-temporal coherence function of the sound field $\tilde{p}(r,\varphi,z,t)$ is defined as

$$\left\langle \tilde{p}(r,\varphi_1,z_1,t_1)\tilde{p}^*(r,\varphi_2,z_2,t_2)\right\rangle = \iint e^{i\omega_2 t_2 - i\omega_1 t_1} Q(\omega_1)Q(\omega_2)\left\langle p(r,\varphi_1,z_1,\omega_1,t_1)p^*(r,\varphi_2,z_2,\omega_2,t_2)\right\rangle d\omega_1 d\omega_2. \quad (1)$$

Here, $(r,\varphi_1,z_1)$ and $(r,\varphi_2,z_2)$ are two points of observation at which the coherence function is calculated, $t$ is time, $\omega$ is sound frequency, the function $Q(\omega)$ characterizes a spectrum of the source, and the brackets $\langle ... \rangle$ denote averaging over an ensemble of sound-speed realizations. A second moment of the spectral amplitudes of the sound field appearing on the right-hand side of Eq. (1), is expressed in the following form

$$\left\langle p(r,\varphi_1,z_1,\omega_1,t_1)p^*(r,\varphi_2,z_2,\omega_2,t_2)\right\rangle = \frac{2}{\pi r}\sum_{n,m}\frac{\exp\left[i(\xi_n(\omega_1)-\xi_m(\omega_2))r\right]}{\sqrt{\xi_n(\omega_1)\xi_m(\omega_2)}} I_{nm}(r,\varphi_1-\varphi_2,\omega_1,\omega_2,t_1-t_2) \times u_n(z_1,\omega_1)u_m(z_2,\omega_2). \quad (2)$$

Here, $\xi_n(\omega)$ are propagation constants, $u_n(z,\omega)$ are mode profiles, $I_{nm}$ is the matrix of cross-mode coherences, and summation is over all waterborn acoustic modes. The cross-mode coherences $I_{nm}(r,\varphi,\omega_1,\omega_2,\tau)$ satisfy a set of differential equations:

$$\frac{I_{nm}(r,\varphi,\omega_1,\omega_2,\tau)}{dr} = -\sum_{n'} e^{i(\xi_{n'}(\omega_1)-\xi_n(\omega_1))r} s_{nn'}(\omega_1) I_{n'm}(r,\varphi,\omega_1,\omega_2,\tau) - \sum_{m'} e^{i(\xi_m(\omega_2)-\xi_{m'}(\omega_2))r} s^*_{mm'}(\omega_2) \times I_{nm'}(r,\varphi,\omega_1,\omega_2,\tau) + \sum_{n',m'} e^{i(\xi_{n'}(\omega_1)-\xi_n(\omega_1)+\xi_m(\omega_2)-\xi_{m'}(\omega_2))r} \sigma_{nm,n'm'}(r\varphi,\omega_1,\omega_2,\tau) I_{n'm'}(r,\varphi,\omega_1,\omega_2,\tau). \quad (3)$$

Here, $\varphi = \varphi_1 - \varphi_2$, $\tau = t_1 - t_2$ is the time lag between two observations, $s_{nm}(\omega)$ and $\sigma_{nm,n'm'}(r\varphi,\omega_1,\omega_2,\tau)$ are matrices which are expressed in terms of a spatial-temporal correlation function of sound-speed fluctuations:

$$B\left(\left|\vec{r}_1-\vec{r}_2\right|, z_1, z_2, t_1-t_2\right) = \left\langle \delta c(\vec{r}_1,z_1,t_1)\delta c(\vec{r}_2,z_2,t_2)\right\rangle, \quad (4)$$

where $\delta c(\vec{r},z,t)$ is a spatial-temporal random field of sound-speed fluctuations.

The developed theory is rather general and allows analysis of the second-order statistical characteristics of broadband acoustic signals propagating in a fluctuating ocean. For example, using Eqs. (1)-(4), the averaged amplitude and shape of an acoustic pulse propagating in a fluctuating ocean can be studied.

Task 2.
The theory developed in Task 1 was applied to studies of temporal coherence of narrow-band acoustic signals for the case of an ocean perturbed by linear IWs with the GM spectrum. The temporal coherence function $\Gamma(r,\omega,\tau)$ is a particular case of the spatial-temporal coherence function considered in Task 1 and is obtained from the latter by setting $\varphi = 0$, $z_1 = z_2$, and $\omega = \omega_1 = \omega_2$. The range and frequency dependence of the coherence time were investigated numerically and compared with experimental results and predictions of the path-integral theory.

In the numerical results presented below, the Brunt-Väisälä frequency and the sound speed profiles were chosen as the Munk canonical profiles. It was assumed that ocean depth is 3 km and the source depth is 807 m. (The source depth corresponds to the 1998-1999 NPAL experiment [10].) Calculations of the temporal coherence function $\Gamma(r,\omega,\tau)$ were done for five values of the sound frequency $f = \omega/2\pi = 12$, 25, 50, 75, and 100 Hz. For these frequencies and the considered ocean stratification, the number of waterborne acoustic modes is $N = 6$, 14, 28, 41, and 55, respectively.

The normalized temporal coherence function, $\Gamma(r,\omega,\tau)/\Gamma(r,\omega,0)$, is plotted in Fig. 1 versus the sound propagation range $r$ and time lag $\tau$ for the sound frequency $f = 50$ Hz. It follows from the figure that, with increasing propagation range, the temporal coherence decreases faster with $\tau$, as it should. Figure 2 depicts the normalized temporal coherence function for $f = 100$ Hz. This plot is similar to the previous one, except that temporal coherence decays faster than that in Fig. 1; note the difference in scales along the time axis in these figures.

The coherence time $\tau_c$ is defined as a value of $\tau$ for which the temporal coherence function $\Gamma(r,\omega,\tau)$ decreases by a factor $1/e$. In Fig. 3, $\tau_c$ is plotted versus propagation range $r$ for five frequencies $f$ ranging from 12 to 100 Hz. Apart from the left-most parts of the curves corresponding to $f = 12$ and 25 Hz, it follows from Fig. 3 that the dependence of $\tau_c$ on $r$ can be approximated as $\tau_c \sim r^{-0.5}$. This dependence is also predicted with the path-integral theory [12] and is close to that for experimental data, see Ref. [13]. Note that predictions of the path-integral theory are valid for an individual ray path, while predictions of the 3D modal theory are valid for superposition of acoustic modes, e.g., the total acoustic signal. Figure 4 depicts the coherence time $\tau_c$ versus frequency $f$ for four propagation ranges $r$. The dependence of $\tau_c$ on $f$ can be approximated with $\tau_c \sim f^{-1}$. This dependence is also predicted with the path-integral theory [12] and is consistent with the experimental data reported in Ref. [13].

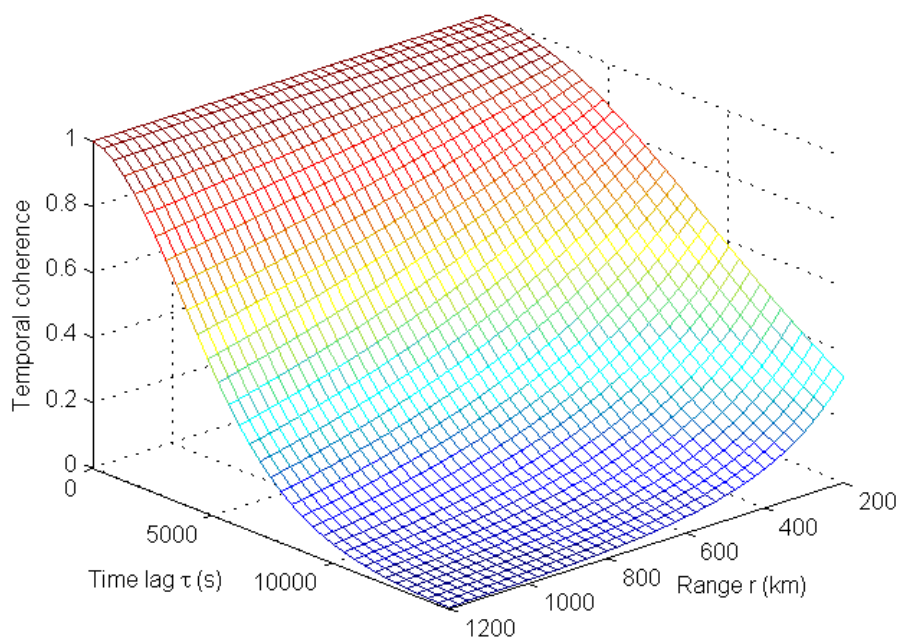


***Figure 1. Normalized temporal coherence function of a narrow-band sound signal versus propagation range and time lag for sound frequency f = 50 Hz. The plot corresponds to sound propagation through a fluctuating ocean perturbed by linear IWs with the GM spectrum.***

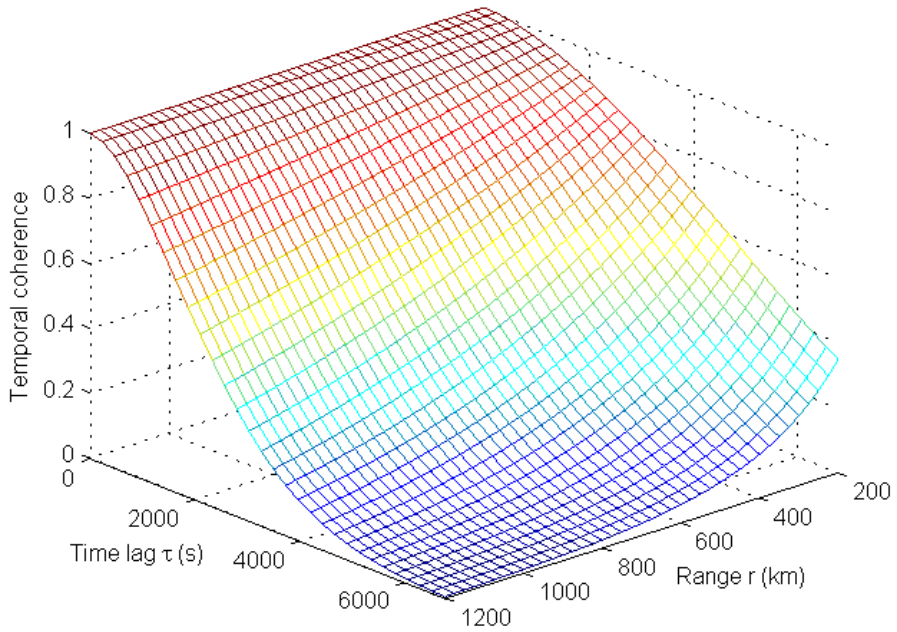


***Figure 2. The same as in Fig. 1 but for sound frequency f = 100 Hz.***

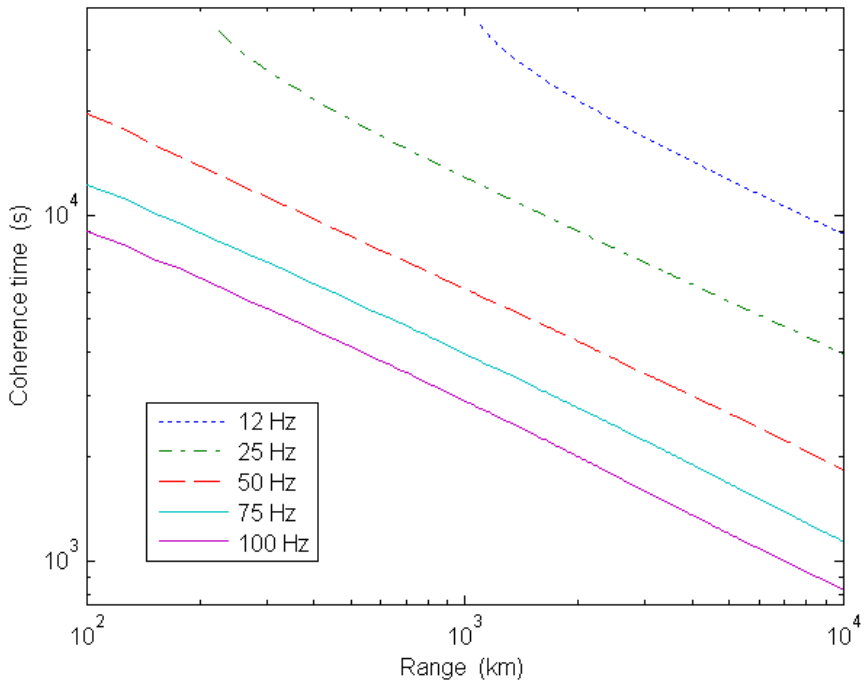


***Figure 3. Coherence time of a narrow-band sound signal versus propagation range r for different sound frequencies f.***

## IMPACT/APPLICATIONS

A 3D, modal theory of broadband sound propagation in a fluctuating ocean was developed. Based on this theory, the dependence of temporal coherence on range and frequency was studied numerically.

## RELATED PROEJCTS

(a). The 2004-2005 NPAL experiment, see Ref. [11].

(b). The 2009-2011 NPAL Philippine Sea Experiment.

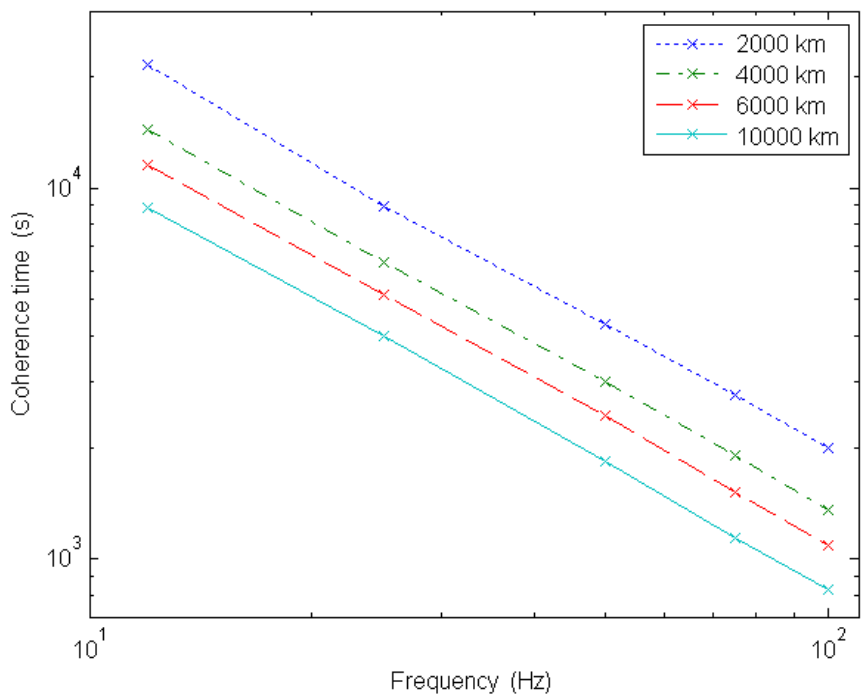


***Figure 4. Coherence time of a narrow-band sound signal versus sound frequency f for different propagation ranges r.***

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