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Modeling Wind Wave Evolution from Deep to Shallow Water

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LONG-TERM GOALS

Ocean waves are an important aspect of upper ocean dynamics, in particular on the shallow continental shelves and in coastal areas. The long-term objective of this work is to advance modeling capability in such shallow areas by improving model representations of nonlinear effects and dissipation.

OBJECTIVES

The specific objectives of the present work are 1) to develop and implement an efficient, scalable approximation for the nonlinear quadruplet source term, 2) to develop and implement a generalized nonlinear source term that is accurate in water of arbitrary depth, 3) to develop and implement an improved nonlinear closure for triad nonlinear interactions in shallow water, and 4) improve representations of dissipation by wave breaking and wave-bottom interactions.

APPROACH

Modern, third-generation (3G) wave models are based on the action balance (or radiative transfer) equation, which is a geometrical optics description of the evolution of wave energy (or action) through a slowly varying medium and time. In Lagrangian form (for convenience) this balance equation can be written as

$$\frac{dN(\mathbf{k})}{dt} = S_{in}(\mathbf{k}) + S_{ds}(\mathbf{k}) + S_{sc}(\mathbf{k}) + S_{nl}(\mathbf{k}) \qquad (1)$$

where $N(\mathbf{k})$ is the wave action at wavenumber vector $\mathbf{k}$ and t is time. The forcing terms on the right-hand side are known as source terms and account for the input of energy by the wind (S_{in}), spectral redistribution of energy through scattering by seafloor topography (S_{sc}) or through nonlinear wave-wave interactions (S_{nl}) , and dissipation of wave energy (S_{ds}) through e.g. breaking or bottom friction.

In this study we will develop and improve the source terms for nonlinear interactions S_{nl} and energy dissipation S_{ds}, to account for effects of finite depth and shallow water, and to ensure a consistent and smooth model representation of wave evolution from deep to shallow water.

Nonlinearity
We will develop an efficient method for the evaluation of the nonlinear source term, allowing for greater efficiency and accuracy in operational use. To allow modeling of wave propagation from deep to shallow water, we will modify the nonlinear source term to account for changes in relative water depth [Janssen *et al.* 2006], and develop an improved closure approximation for nearshore wave propagation [Janssen, 2006].

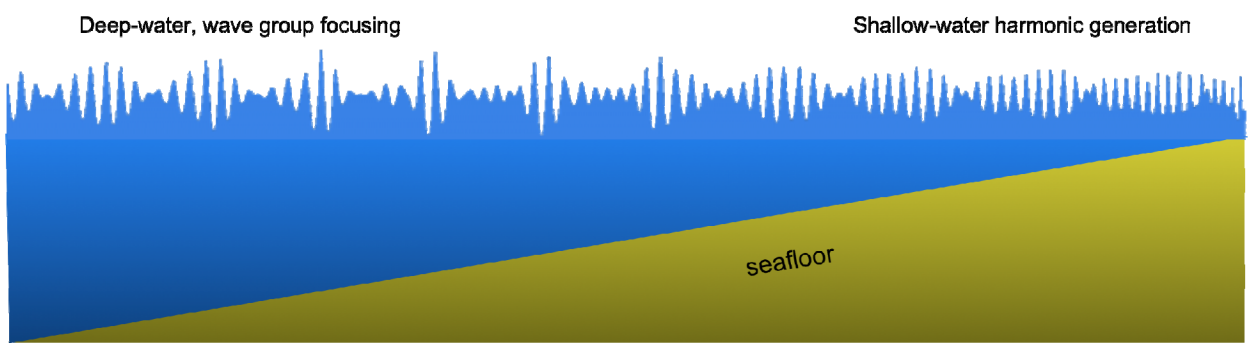

Figure 1. Illustration of changes in nonlinear regimes as ocean waves travel across the shelf into shallow water.

Dissipation
We will develop and test improvements to wave dissipation parameterizations through detailed comparisons against laboratory and field observations.

WORK COMPLETED

The completed work in this project includes:

- Analysis of dissipation in nonlinear shoaling waves.
- Development and preliminary testing of nonlinear shallow-water wave model.
- Development and preliminary testing of new scaling for breaking parameter γ .
- Completed review of formulations for nonlinear four-wave interactions in shallow water.

In the following we outline some of these efforts and their principal results.

Analysis of dissipation in nonlinear shoaling waves

To establish the spectral structure of dissipation in breaking waves, we used observations by Boers (1996) to estimate the dissipation spectrum. The analysis is based on the one-dimensional energy balance equation for stationary, unidirectional wave propagation, which can be written as

$$\frac{d\mathrm{E}_1}{dx} = -\mathrm{N}_1^{(\mathrm{dis})} + \mathrm{N}_1^{(\mathrm{nl})} \tag{1}$$

where E_1 is the wave energy flux, and the terms of the right represent (from left to right) source terms for dissipation and nonlinearity respectively.

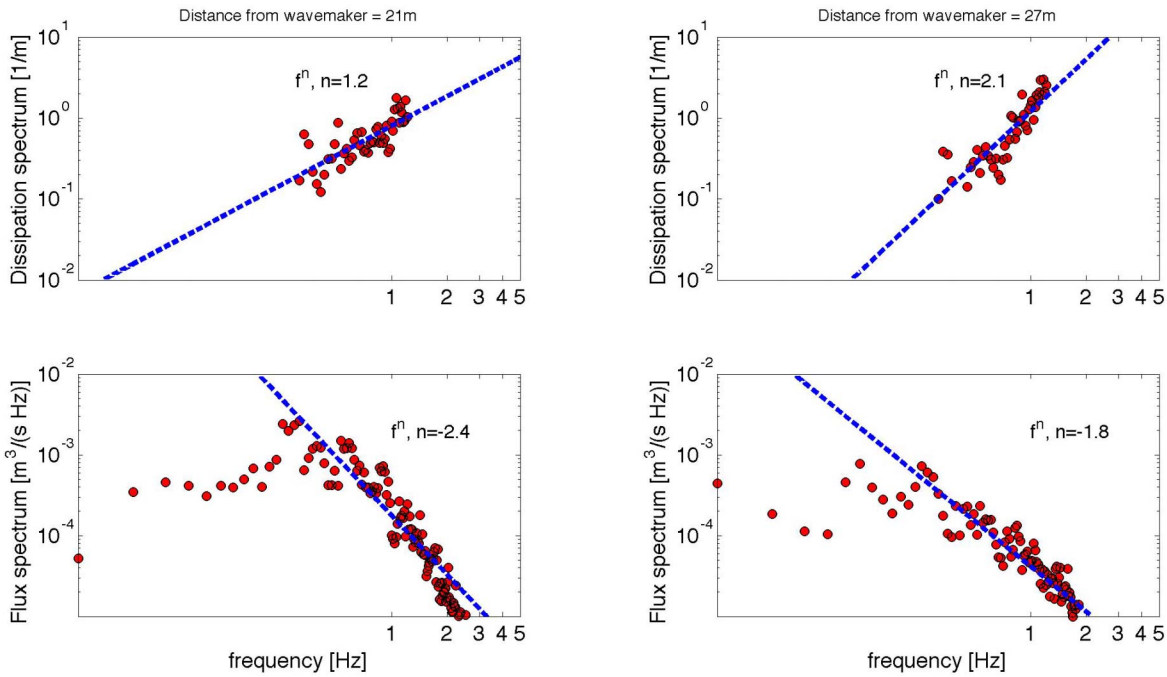

Figure 2. Regression analysis (loglog scales) of dissipation and flux spectrum.

Dissipation is estimated from (1) using bispectral analysis and finite differencing to estimate the nonlinear source term and the flux gradient from the observations. Although the exact form of the dissipation spectrum is unknown, we assume that it can be written as

$$\mathrm{N}_1^{(\mathrm{dis})} = D\, f_1^n\, \mathrm{E}_1 \tag{2}$$

where f_j is frequency. The exponent n and constant D are estimated through regression analysis of the observed dissipation spectrum (figure 2). The analysis (examples shown in figure 2) shows that the exponent n in (2) varies around a value of 2 approximately (figure 3). The approximately frequency-squared weighting is consistent with other studies and, combined with the fact that the flux spectrum tail drops off as f^{-2} in shallow water (figure 3), supports the conjecture that the nearshore dissipation spectrum is fairly white [Kaihatu *et al.* 2007]. Application of this analysis to other laboratory and field observations is ongoing.

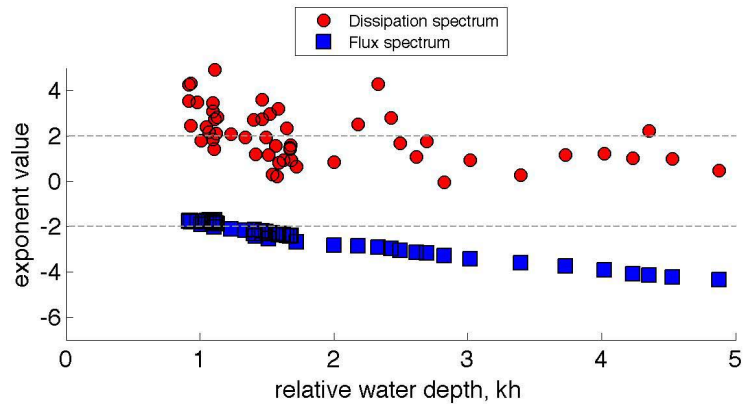

Figure 3. Exponent values from regression analysis for dissipation spectrum and flux spectrum tail as a function of relative water depth.

A nonlinear model for shallow-water wave propagation

We have developed and tested a nonlinear wave model for stationary waves, which – for unidirectional waves -- can be written as

$$\left[\frac{d}{dx}+D_1\right]\mathrm{E}_1=-4\int\int d\omega_2\, d\omega_3 \mathrm{M}_{23}(x)\delta_{23;1}\int_{-\infty}^{x} dx'\mathrm{H}_{23}(x',x)\mathrm{B}_{23}(x') \tag{3}$$

where E_1 denotes wave energy flux density at frequency ω_1, M_{23} is an interaction coefficient derived from second-order wave theory, and

$$\mathrm{B}_{23}=\mathrm{E}_{(2+3)}\left(\mathrm{M}_{(2+3)(-3)}\mathrm{E}_3+\mathrm{M}_{(2+3)(-2)}\mathrm{E}_2\right)-\mathrm{M}_{23}\mathrm{E}_2\mathrm{E}_3 \tag{4}$$

Further, in (3), we have

$$\mathrm{H}_{23}(x',x)=\exp\left[-\int_{x'}^{x}\Gamma_{23}(x'')dx''\right] \tag{5}$$

where

$$\Gamma_{23}=D_{23}-i\Lambda_{23}+\mu_{23}. \tag{6}$$

In (6), the D_{23} accounts for dissipation, Λ_{23} is dispersion (resonance mismatch), and μ_{23} is a relaxation term for which we propose

$$\mu_{12}=\alpha\left(\frac{\left|\mathrm{N}_1^{(\mathrm{dis})}+\mathrm{N}_2^{(\mathrm{dis})}+\mathrm{N}_{1+2}^{(\mathrm{dis})}\right|+\left|\mathrm{N}_1^{(\mathrm{nl})}+\mathrm{N}_2^{(\mathrm{nl})}+\mathrm{N}_{1+2}^{(\mathrm{nl})}\right|}{\mathrm{E}_1+\mathrm{E}_2+\mathrm{E}_{1+2}}\right). \tag{8}$$

Here α is an empirical constant (generally of O(1)). This relaxation term measures the effects of dissipation and nonlinearity and allows the system to reduce to a near-Gaussian state in the presence of strong dissipation and nonlinearity. The form (8) is inspired by the physics of the

problem, and not consistently derived from basic principles. Work is ongoing to improve the relaxation term using a more systematic derivation of this contribution.

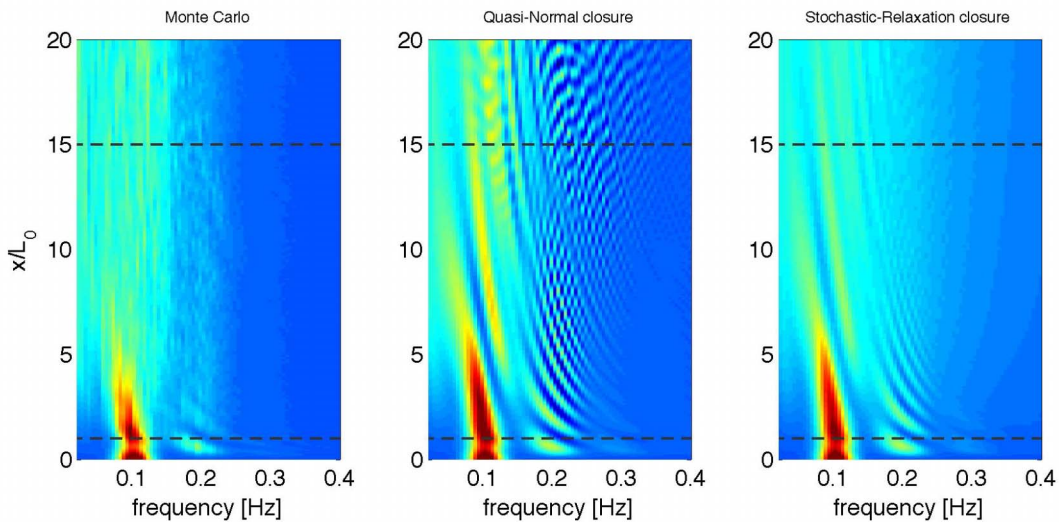

Figure 4. Spatial evolution of a freely developing spectrum (no dissipation). A narrow-band wave field is incident at x=0 at which location the nonlinearity is suddenly turned on. The evolution of the spectrum for the quasi-normal approximation (middle panel) shows rapid energy transfers, and negative energies. The stochastic model with the relaxation term (right panel) is in much better agreement with the Monte-Carlo simulation (left panel).

Without the relaxation term (8) our model would reduce to a quasi-normal closure approximation, in which case triple correlations would be over-predicted and the system retains too much information from its past states [see e.g. Orszag, 1970; Leslie, 1973; Holloway & Hendershott, 1977; McComb, 1990; Herbers *et al.*, 2003]. An example is shown in figure 4. Without relaxation, triple correlations are overestimated, resulting in rapid transfer of energy and the occurrences of negative energies (the gaps the spectra shown in figure 5). Similarly, relaxation improves estimates of higher-order correlations (not shown). However, relaxation of the system using (8) does result in a spectral tail that is too flat. The latter shortcoming we expect to improve by deriving a closure approximation from first principles.

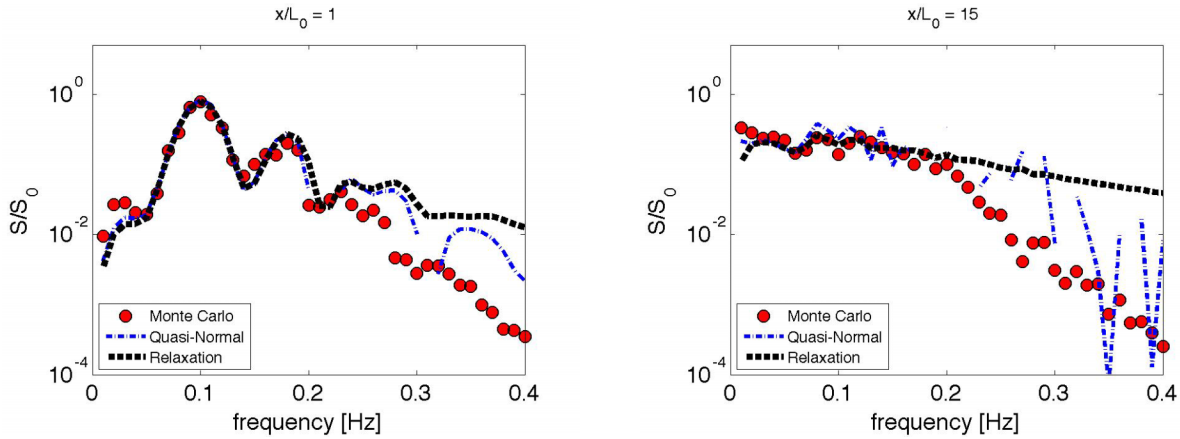

Figure 5. Comparison of spectra at locations as indicated in figure 4. The quasi-normal approximation results in negative energies (blue dashed line; the gaps in the spectrum correspond to negative values). The relaxation approximation remains well behaved but the tail of the model-predicted spectrum is less steep than seen in the Monte-Carlo simulation.

The agreement between observations and model predictions (equation (3) with frequency-squared dissipation weighting, and $\alpha = 1$ in the relaxation term) is very good, both in terms of spectral shape (figure 6), and the evolution of higher-order bulk statistics (figure 7).

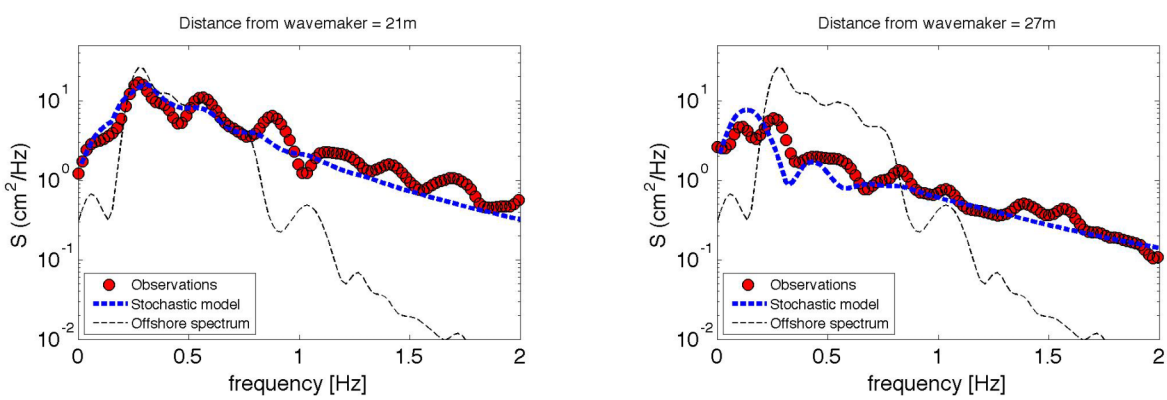

Figure 6. Comparison of modeled and observed spectra at four locations in the flume. The combination of frequency-squared weighting of the dissipation function in the QNR approximation results in an accurate representation of the nonlinear wave dynamics.

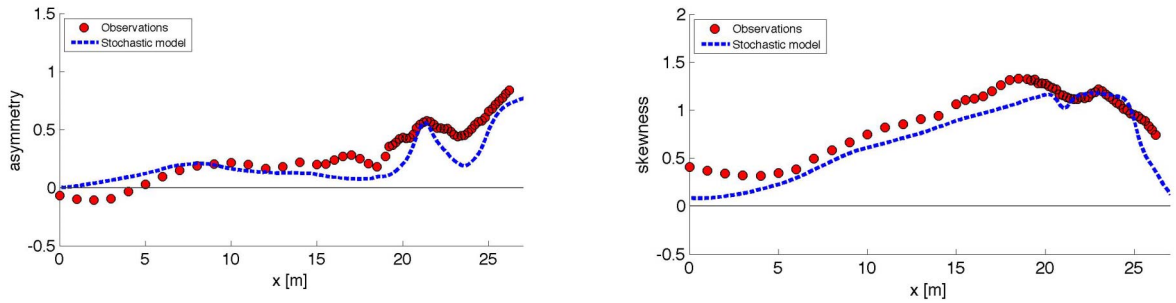

Figure 7. Evolution of third-order bulk statistics for Boers experiment. Left panel: asymmetry; right panel: skewness.

Development of a new scaling of breaking parameter γ

The breaker index γ is an important factor in widely-used wave breaking parameterizations [e.g. Janssen & Battjes, 1978; Thornton & Guza, 1983; Janssen & Battjes, 2007]. A new scaling for this index, referred to as $n-kd$ scaling [Holthuijsen & Salmon, 2010], is calibrated and tested using field and laboratory observations. The new scaling makes γ dependent on both bottom slope and relative depth and one aspect in particular that this model addresses is that it reconciles the seemingly contradictory high and low γ values in the literature for horizontal bottoms (with variations from 0.51 to over 1.0). Comparisons between model predictions and laboratory observations for both the traditional fixed-value γ and the new $n-kd$ scaling (calibrated with field and laboratory observations) are shown in figure 8. Further validation of the new scaling with field observations is ongoing.

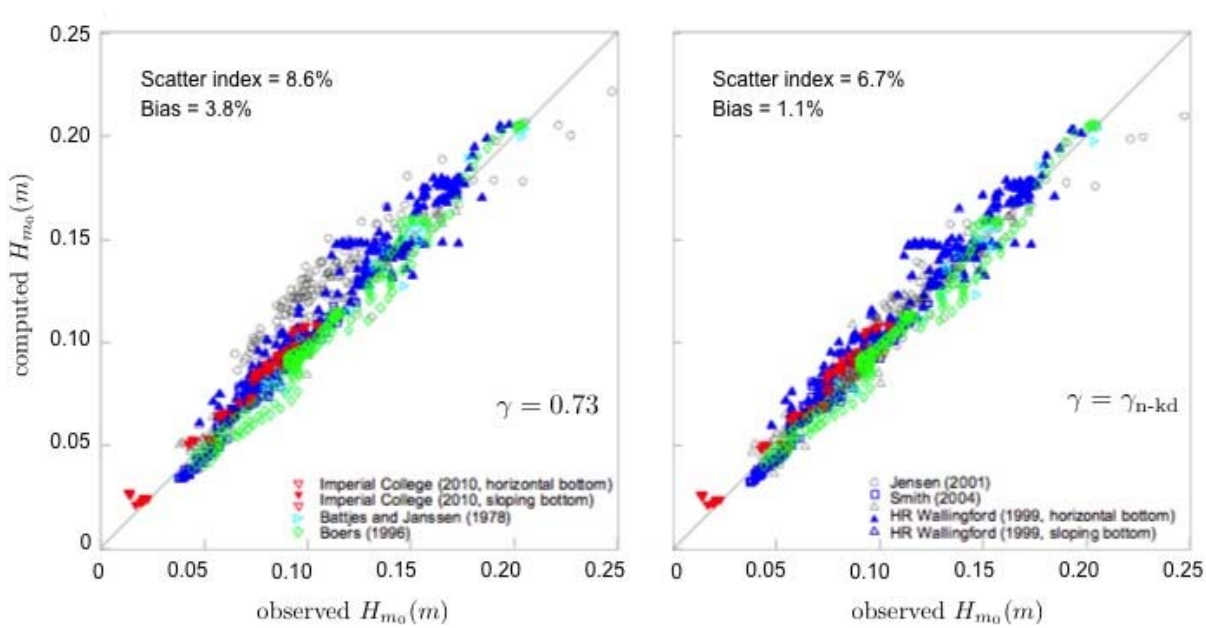

Figure 8. Scatter plots for predicted and observed significant wave height for a number of laboratory studies. Left panel: breaker index default fixed value $\gamma = 0.73$. Right panel: new $n-kd$ scaling for γ.

RESULTS

Dissipation and nonlinear effects in shallow-water waves are of great importance to coastal hydrodynamics and transport processes. The findings in this study contribute to our better understanding of the combined effects of dissipation and nonlinearity on wave propagation in shallow water, and improve modeling capability of wave statistics in nearshore areas.
In particular, the bispectral-dissipation analysis of laboratory observations suggests an approximately frequency-squared weighting of the dissipation spectrum in shallow water. The analysis results are successfully used in a numerical model to predict nonlinear wave evolution, and it supports the idea that the shallow-water wave dissipation spectrum is fairly white [see Kaihatu et al. 2007].

To allow modeling of highly nonlinear waves and strong dissipation, a relaxation term is presented to provide a closure approximation for the nonlinear system, which improves the model's performance in areas of strong nonlinearity and dissipation. Further development and refinement of the relaxation term is underway, and testing against field observations of the model is ongoing.

A new scaling for the breaker index γ is calibrated and tested against laboratory cases and shows improved model performance by including both dependencies on bottom slope and relative water depth. Validation of the scaling against field observations is ongoing.

IMPACT/APPLICATIONS

The model improvements developed and tested in this study will contribute to improvements in modeling capability of nearshore wave propagation in research and operational models. In turn, improved modeling capability of wave dissipation, spectral evolution, and higher-order statistics such as skewness and asymmetry, will contribute to improvements in research and modeling of coastal circulation and transport processes.

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