

## **Near-Bottom Turbulence and Sediment Resuspension Induced by Nonlinear Internal Waves**

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### **LONG-TERM GOALS**

The long term goal of this work is to develop a fundamental understanding and predictive capability of the underlying physics of the interaction of nonlinear internal waves (NLIWs) with the continental shelf seafloor over a broad range of environmental conditions. We are particularly interested in how such interactions impact underwater optics and acoustics and shelf energetics and ecology by stimulating enhanced bottom boundary layer (BBL) turbulence and particulate resuspension leading to benthic nepheloid layer (BNL) formation.

### **OBJECTIVES**

The specific objectives of this project are directed towards:

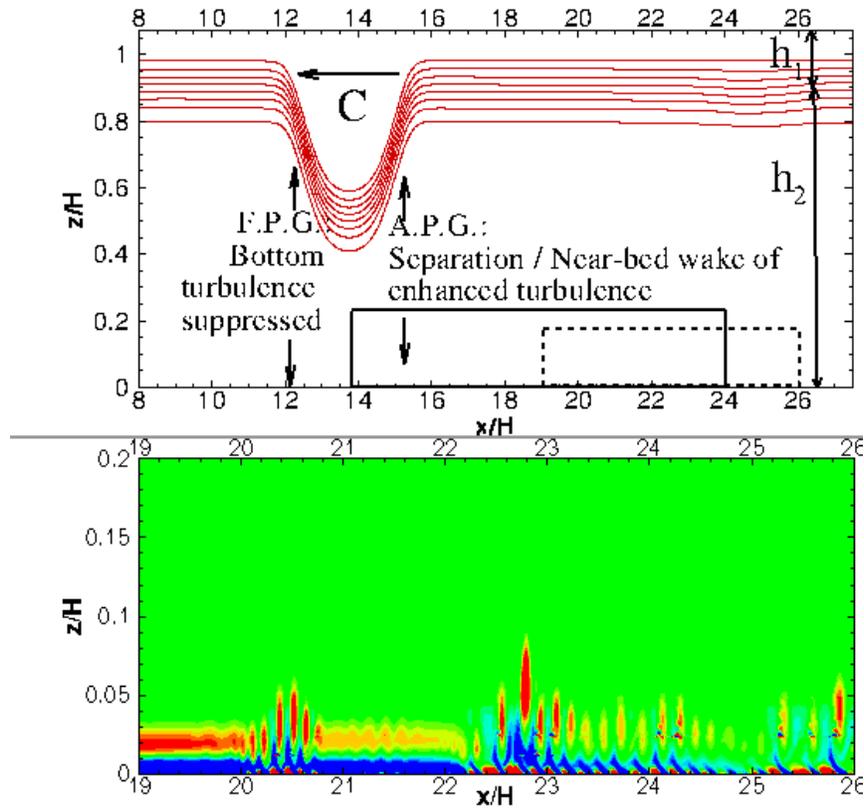
- Using Large Eddy Simulations (LES), investigate the structural transition to turbulence within the separated BBL layer under a NLIW of depression and quantify the resulting NLIW energy losses.
- By means of Lagrangian coherent structure (LCS) theory, identify mechanisms for the capturing of nearbed particles by the BBL-turbulence and their transport/deposition into BNLs.
- Analyze field observations from the New Jersey shelf to identify the applicability of hypothesized BBL physics and flesh out the underlying fluid mechanics from the field data.

## APPROACH

Our approach relies on implicit 3-D Large Eddy Simulation (LES) based on a recently developed spectral quadrilateral multidomain penalty method (SMPM) Navier-Stokes solver developed by a senior PhD student in the PI's group (Escobar-Vargas et al. 2011a, Escobar-Vargas et al. 2011b). This code is an extension of spectral multidomain penalty solver previously developed by the for the simulation of high Reynolds number incompressible flows in vertically finite and horizontally periodic domains PI (Diamessis et al. 2005). The new code maintains the advantages of high accuracy and resolution, spatial adaptivity and minimal artificial dissipation present within the singly non-periodic solver. By virtue of incorporating a Legendre-polynomial spectral-multidomain discretization in the x-horizontal direction, it enables localization of resolution in both along-wave and vertical directions and, most importantly, more general boundary conditions in the along-wave direction (see below). The latter feature allows one to focus the computational domain in the adverse pressure gradient (APG) region in the NLIW footprint, where previous 2-D numerical simulations (Diamessis and Redekopp 2006 and Diamessis and Stefanakis 2011) have demonstrated the presence of a vigorous near-bed turbulent wake (figure 1 --- bottom panel). The transverse direction is periodic and readily treated through a Fourier discretization.

Numerical stability within the code is ensured through a combination of explicit spectral filtering, penalty methods and an over-interpolation-based de-aliasing scheme (Kopriva 2009). The SMPM-discretized pressure-Poisson-equation is solved iteratively using the GMRES algorithm and a two-level preconditioner, with a block-Jacobi algorithm at the fine level and a low-order SMPM approach at the coarse level (Escobar-Vargas et al. 2011a). As a result, highly accurate Navier-Stokes (i.e. highly nonlinear and non-hydrostatic) computation of the unsteady, long time evolution of unsteady 3-D separated flows are possible enabled with a turbulent Reynolds number that falls within the range of values where oceanically relevant physics occur. The new SMPM code has already been successfully applied to a number of benchmark cases with an example of the gravity current in a canonical lock exchange flow shown in figure 2. The code is currently being parallelized and will be ready for the 3-D production runs planned for spring 2012.

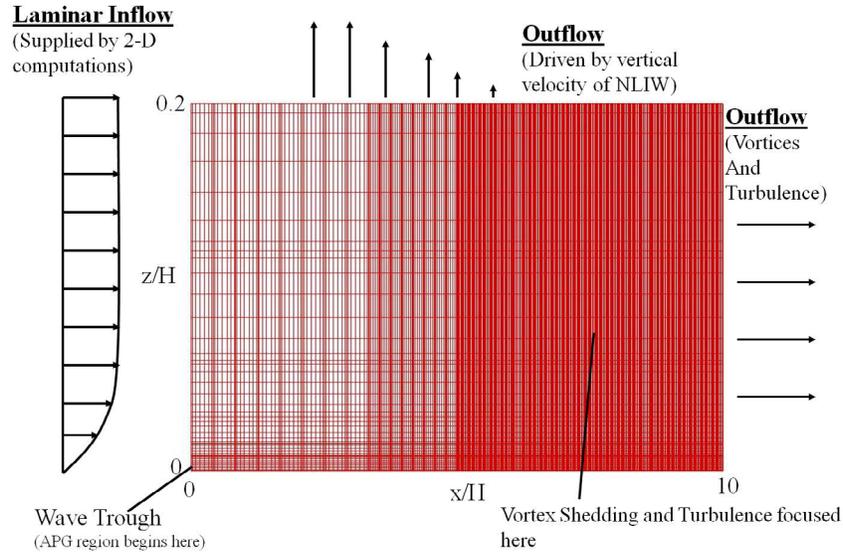
Our problem geometry considers a mode-1 wave of *depression* fixed in a frame of reference moving with the NLIW through a uniform-depth waveguide (figure 1). The background stratification across the full water column consists of a two uniform density layers separated by a finite-thickness pycnocline (figure 1 –top panel). As the wave is kept fixed in time, we solve for the perturbation to this wavefield that develops through the mismatch between the non-zero wave velocity field and no-slip condition at the bed (Diamessis and Redekopp 2006). To maximize resolution of the 3-D turbulence in the NLIW-induced BBL, our computational domain is a truncated version of the full domain shown in the top panel of figure 1, whose streamwise and vertical boundaries are delineated by the dashed line in the same figure. A detailed view of the computational domain with the appropriate boundary conditions and a sample grid is shown in Figure 2. At the top and left boundaries, inflow conditions directly linked to the velocity fields of the NLIW are imposed.



**Figure 1: Pressure gradient distribution along a NLIW, computational domain and vortex shedding from the NLIW-induced BBL.**

**[Top Panel: A model wave in a continuous two-layer stratification with top to bottom layer thickness ratio of  $h_1/h_2=1/5$ . The favorable pressure gradient (FPG) in the leading-edge of the NLIW suppresses turbulence, whereas the adverse pressure gradient (APG) in the wave's trailing edge drives BBL separation and a near-bed wake of enhanced turbulence. The black line is an outline of the computational domain for the simulations to be performed in the proposed research. Bottom Panel: Contours of transverse vorticity from previous 2-D Direct Numerical Simulations of the NLIW-driven BBL (Diamessis and Stefanakis 2011) showing vortex shedding in the lee of the wave. The dimensions of this view correspond to the dashed line in the top panel.]**

Finally, our particle-tracking tools revolve around libraries built by the co-PI based on a higher-order accuracy Eulerian-Lagrangian (EL) approach (Jacobs and Hesthaven 2006, Jacobs and Don 2009). These tools, based on a parallel, high-order accurate EL approach on multi-block, spectral and fully unstructured grids, have been used to determine particle-laden flow with relevance to liquid-fuel combustors (REF). Originally developed for a discontinuous Galerkin-based flow solver for the compressible Navier-Stokes equations, these tools are directly compatible and implementable within the incompressible spectral multidomain method that the PI has developed and will be the primary vehicle for the simulation and study of particle dispersion and Lagrangian structures in the NLIW-driven BBL.

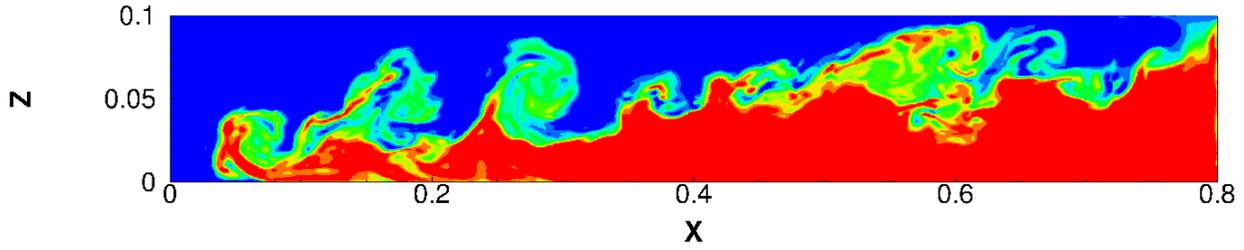


**Figure 2: Stream-depth section of the computational domain and boundary conditions for 3-D simulations (box with solid outline in figure 1-bottom panel). For the sake of illustration, a smaller number of subdomains than those actually used is shown. In the vertical, resolution is concentrated in the BBL and, in the horizontal, subdomains are clustered in the region of vortex shedding.**

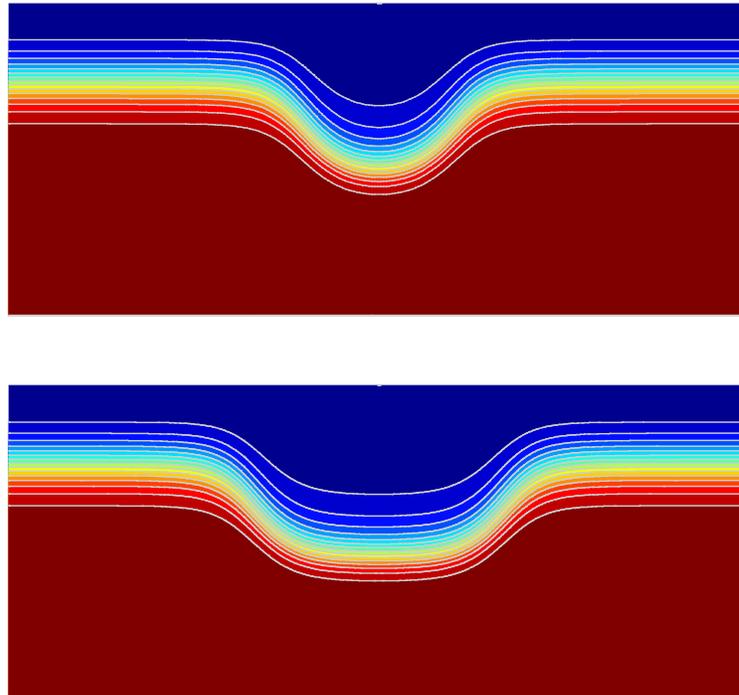
## WORK COMPLETED

Funding of the project was not officially awarded to Cornell until June 2011. The postdoctoral researcher, Dr. Takahiro Sakai did not arrive at Cornell until mid-August. He is currently becoming an informed user of the quadrilateral SMPM code.

Prof. Marek Stastna at the University of Waterloo, Canada, has generously provided us with a spectrally accurate Dubreil-Jacotin-Long (DJL) wave generation code for arbitrary stratification profiles (Dunphy et al. 2011). Dr. Sakai has become a capable user of the DJL code and has generated a number of example fully nonlinear waves such as those shown for a two-layer continuous stratification profile in figure 4 for two different values of NLIW potential energy. At the time of writing this progress report, we are currently focused on incorporating the NLIW into the quadrilateral SMPM Navier-Stokes solver.



*Figure 3: Isopycnal surfaces from 2-D simulations of the canonical lock-exchange problem with our recently developed quadrilateral spectral multidomain penalty method model. The gravity current forms in a box of dimensions  $0.1 \times 0.8 \text{ m}^2$ , with a 5% density difference across the two layers. A  $30 \times 15$  subdomain grid is used with 15 Gauss-Lobatto-Legendre grid points in each direction.*



*Figure 4: Isopycnal surfaces for the fully nonlinear internal waves used in our LES. The waves computed by solving the DJL equation for a two-layer stratification with an intermediate continuous stratification in the pycnocline. The top to bottom layer thickness ratio is  $h_1/h_2=1/3$  and the pycnocline thickness is 10% of the total water column depth. The wave in the bottom panel has double the potential energy of the one in the top and is close to the conjugate state limit.*

## RESULTS

As this is a recently initiated project, in this section we provide a timeline of planned tasks for FY12:

- **10/1 to 10/15/2011:** Complete incorporation of DJL waves as an initial condition in quadrilateral SMPM Navier-Stokes solver and ensure such waves propagate undistorted with a steady phase-speed in a uniform-depth waveguide with free-slip velocity boundary conditions at the top and bottom of the domain.
- **10/15 to 11/1/2011:** Perform 2-D simulations in a full computational domain (encompassing the entire NLIW) with both a “frozen” and naturally evolving NLIW using a no-slip bottom boundary condition. The goal is to confirm that in the latter case, the NLIW does not undergo significant deformations due to interactions with the bed. Such deformations are expected to be negligible, as the vortices shed from the near bottom wake, do not reach the pycnocline in 2-D simulations. Three-dimensionalization of the vortices should further limit their height of maximum ascent from the bed.
- **11/1/2011 to 11/20/2011:** Perform a subset of focused 2-D simulations in a full computational domain where we examine differences in results for a domain with a top boundary vs. a one where the water-column is supplemented with the hydrostatic pressure field introduced by the free-surface displacement induced by the NLIW (Moum and Smyth 2006). All existing DNS/LES of NLIW-induced BBL’s so far consider a rigid-lid top boundary and the near-bottom pressure signature of the free-surface displacement could significantly alter the near-bed along-wave distribution of NLIW-driven pressure gradient, which in turn will alter the response of the near-bed wake.
- **11/20/2011 to 2/28/2012:** Focus on truncating computational domain to match the configuration shown in figure 2. The domain truncation will be performed in a 2-D framework, as the spanwise direction is statistically homogeneous. The top boundary will be configured to have the NLIW-driven vertical velocities as inflow fluxes at the top of the domain. The left boundary will be placed either exactly at the wave trough or a little before it and the inflow condition will be supplied from the state near-bed velocity obtained in companion full domain simulations. At this point, we will check and see whether the streamline field near the boundaries matches the one observed in simulations in a full domain. As the final stage of this effort, we will implement outflow conditions and buffer zones in the right boundary of the domain. The co-PI’s experience with simulating similar separating flows in mechanical engineering applications (e.g. diffusers) will be invaluable in this regard.
- **3/1/2012 to 6/30/2012:** Perform all 3-D particle-free simulations of NLIW-induced boundary layers. We will consider two different stratification profiles and will focus on waves of different amplitudes (energy content) at sufficiently high wave-based Reynolds numbers for near-bed 3-D vortex shedding to develop. Towards the spring of 2012, the postdoc will visit the co-PI to be trained in implementing the co-PI’s particle tracking routines into the SMPM solver.
- **7/1/2012 to 9/30/2012:** Analyze results of 3-D particle free simulations.

## IMPACT/APPLICATIONS

The accurate representation of the structure and magnitude of shear stress field in the NLIW footprint and accurate estimation of the NLIW energy losses due to bottom interactions will allow the formulation of improved subgrid-scale parameterizations of energy dissipation and bottom boundary conditions for larger-scale operational forecasting models used to simulate environments with high NLIW activity. An enhanced understanding of the underlying physics of the NLIW-driven BBL also provides critical insight on how the bottom shear stress and pressure fields conspire to generate high-amplitude sandwaves, such as those observed in the South China Sea, which can pose significant challenges in efforts of acoustic bathymetry mapping. Finally, the generated resuspended particle distributions under NLIWs, a reliable proxy of BNLs, can be used to quantify the transmission or backscatter of optical/acoustic signals of importance to remote sensing efforts and near-bed SONAR operation.

## RELATED PROJECTS

Funded, by an NSF-CAREER award in Physical Oceanography, the P.I. is using the quadrilateral SMPM code to study the shoaling of NLIWs in both canonical configurations and domains with bathymetry replicating the South China Sea (SCS). The physical phenomenon of interest is the formation of trapped cores in shoaling waves which involve comparison with field data from Dr. Ren-Chieh Lien of APL-U. Washington obtained in the SCS region. A recently arrived Ph.D. student has now begun work on implementing deformed spectral subdomains in the SMPM code which will enable the simulation of shoaling bathymetries. The PI has also collaborated with Professors Leon Boegman and Kevin Lamb of Queens U. and U. of Waterloo, respectively in Canada, to develop a criterion for near-bed vortex wake formation under NLIWs as a function of quantities measurable by ADCPs and thermistor arrays in the field (Aghsaei et al. 2011). Finally, the P.I. is serving as a co-P.I. with Prof. Phil Liu (Civil and Env. Eng., Cornell) in N.S.F.-CBET project where a PhD student is modifying the spectral multidomain code originally created by the PI (with one non-periodic direction) to accommodate rippled bathymetries, where the unstable boundary layer under surface solitary waves will be examined.

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## **PUBLICATIONS**

Aghsaei, P., Boegman, L., Diamessis, P.J. and Lamb, K. G. 2011: “Boundary layer separation and vortex shedding beneath internal solitary waves”. *J. Fluid Mech.* (accepted).

*Submitted (in revision):*

Diamessis, P.J. and Stefanakis, T.S. 2011 "Near-Bottom Instabilities under Strongly Nonlinear Internal Waves of Depression", (submitted to *Phys. Fluids*)..