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Advances in Statistical and Deterministic Modeling of Wind-Driven Seas

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LONG TERM GOALS

Development of accurate and fast advanced statistical and dynamical nonlinear models of ocean surface waves, based on first physical principles, which will improve and accelerate both long term ocean surface waves forecasts and prediction of strongly coherent events, such as freak waves, tsunami and wave-breaking.

OBJECTIVES

Finding of physically correct wind input term for Hasselmann equation, understanding of the balance of source terms in Hasselmann equation, investigation of the problem of interaction of different scales on the ocean surface (sea and swell), development of new water surface analytic models and methods of their solution.

APPROACH

Advanced analytical techniques: Hamiltonian formalism, self-similar solutions, analytic solutions of integral equations; numerical methods for solution of integral and pseudo-differential equations; comparison of analytic and numerical results with experimental data

WORK COMPLETED

- Finding of the new wind input term through experimental, theoretical and numerical approaches
- Theoretical and numerical proof of nonlinear interaction term domination over wind input and dissipation terms in Hasselmann equation.
- Detection of swell feedback by sea background through theory and experimental data
- Design of new method for numerical integration of Hasselmann equation
- Derivation and numerical testing of one-dimensional version of Zakharov equation, especially convenient for theoretical and numerical study of freak waves

RESULTS

1. New wind input term through experimental, theoretical and numerical consideration

Nowadays, Hasselmann equation is widely accepted model of wind-driven seas [1]:

$$\frac{dN}{dt} = \frac{\partial N}{\partial t} + \frac{\partial \omega_k}{\partial \vec{k}} \frac{\partial N}{\partial \vec{r}} = S_{nl} + S_{in} + S_{diss} \tag{1}$$

The nonlinear interaction term S_{nl} is perfectly known. Unfortunately, the knowledge of wind input S_{in} and white-capping dissipation S_{diss} terms is poor. Creation of reliable and well justified theory of S_{in} is hindered by strong turbulence of the air boundary layer over the sea surface. As a result, there is a dozen of heuristic models of S_{in} with enormous scatter of parameters [4]: the Donelan model [2] predicts S_{in} about five times higher than Hsiao-Shemdin model [3]. Situation with S_{diss} is not better - the theory is not developed, the experimental data are far from being complete. The forms of S_{diss} used in operational models are heuristic and badly justified [18].

We managed to find the new form of the dissipation term S_{diss} through experimental, theoretical and numerical approaches. Resio and Long, 2004 [5] presented data of wind wave spectra from several experimental installations in non-traditional form, expressing coefficient

$$\beta = \frac{1}{2}\alpha_4 \left[\left(u_\lambda^2 c_p \right)^{1/3} - u_0 \right] g^{-1/2} \tag{2}$$

in Kolmogorov-Zakharov energy spectra

$$F(k) = \beta k^{-5/2} \tag{3}$$

in terms of specific combination of velocities. Here $\alpha_4 = 0.00553$, c_p - spectral peak velocity, $u_0 = 1.93$ м/сек, u_λ - wind speed at the height equal fixed portion $\lambda = 0.065$ of the spectral peak wave length $2\pi / k_p$, where k_p is the wave number of the spectral peak. It is remarkable that practically all experimental results presented in [5] exhibit portion of the spectrum obeying the law (3).

Consider Hasselmann equation (1) in "time-domain" form

$$\frac{\partial E(\omega,\theta)}{\partial t} = S_{nl} + \gamma(\omega,\theta) E(\omega,\theta) \tag{4}$$

where $E(\omega,\theta) = \frac{2\omega^4}{g} N(\vec{k},t)$, $\vec{k} = \frac{\omega^2}{g}$ is the energy spectrum. Using dimensional estimate

$$S_{nl} = \omega \left(\frac{\omega^5 E}{g} \right)^2 E$$

forcing function in the form $\gamma(\omega,\theta) = \alpha\omega^{1+s} f(\theta)$ and self-similar substitution

$$E(\omega,\theta,t) = t^{p+q} F(\omega t^{q}) \tag{5}$$

we get the indices of self-similar solution (5):

$$q = \frac{1}{s+1} \qquad p = \frac{9q-1}{2}$$

Comparison of the indices p and q with characteristics of the regression line (2) gives the index of the wind source $s = 4/3$ and parameterization of the new wind input term:

$$S_{in} \propto \omega^{7/3} \tag{6}$$

Numerical simulation of the Hasselmann equation (4) with wind input term (6), presented on ***Fig.1*** , reproduces the regression line (2) with high accuracy for the wind speed values from 2 to 10 m/sec.

We have found new wind input term through experimental, theoretical and numerical considerations, which can be used for quality prediction improvement by operational wave forecasting models.

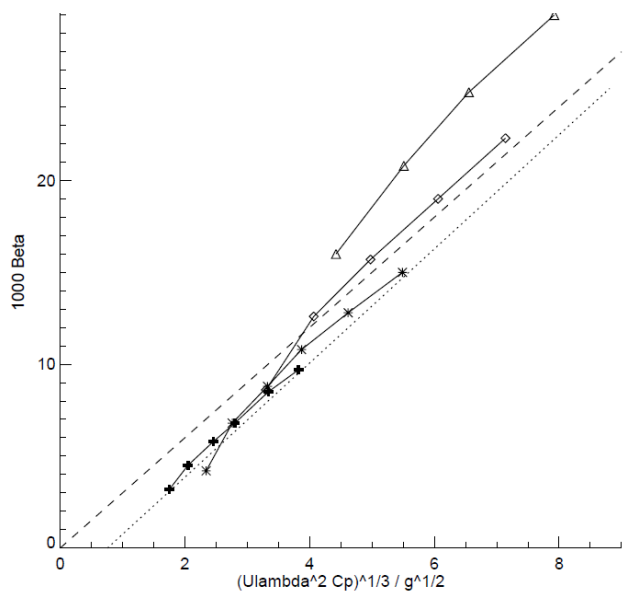

Fig. 1 Experimental, theoretical and numerical evidence, presented on the same plot, in variables 1000β ***as a function of*** $\left(u_\lambda^2 c_p\right)/g^{1/2}$***. Dotted line -- experimental regression line by Resio and Long, 2004 [5]; dashed line -- theoretical result, reproducing regression line; numerical result: crosses -- wind speed u=2.5 m/sec; stars -- wind speed u=5.0 m/sec; rectangles -- wind speed u=10.0 m/sec; triangles -- wind speed u=20.0 m/sec.***

2. *Energy balance in wind-driven seas within the Hasselmann equation*

The problem of balance of different terms in the right-hand side of the Hasselmann equation (1) is a key question for both wind-wave interaction theory and modeling. Today's mainstream emphasis is on developing new functions S_{in} and S_{diss}, but not correct and accurate calculation S_{nl}. Confusion comes from [6], where all three source terms in (1) have been compared for the case of fully-developed (mature) sea, and conclusion has been made that terms S_{in} and S_{diss} can be two-three times greater than S_{nl}. We show, in fact, that situation is opposite: S_{nl} has a leading role in balance of wind-driven seas. The analysis is based on decomposition of S_{nl} into nonlinear damping $\Gamma_k N_k$ and forcing F_k

$$S_{nl} = F_k - \Gamma_k N_k \tag{7}$$

where Γ_k – positive nonlinear damping decrement, N_k – spectral density of wave action. Our numerical and analytical results show that $\Gamma_k N_k$ and F_k surpass conventional parameterizations of input and dissipation of wind-driven waves by, at least, one order of magnitude, see ***Fig.2.***

An additional argumentation is presented in ***Fig.3,*** where nonlinear damping decrement $\Gamma_k N_k$ is compared to empirical parameterizations of wind-wave growth given by different authors and used in the today wave forecasting models as an option.

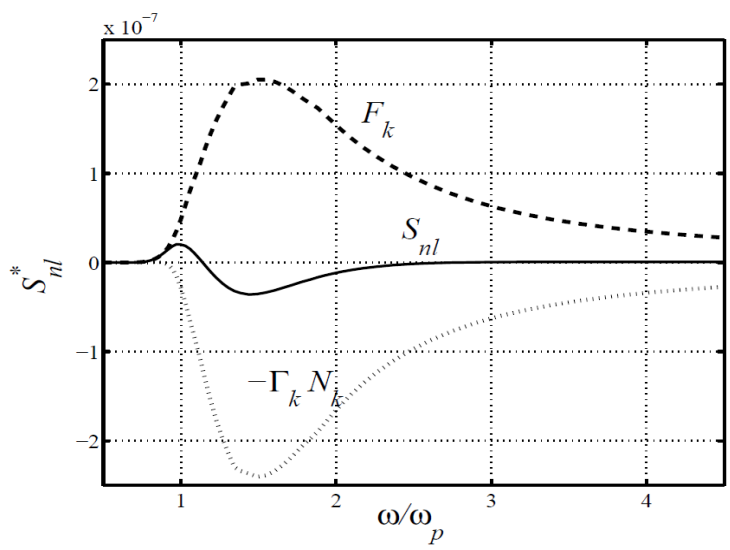

Fig.2 Decomposition of the nonlinear term S_{nl} (solid line) for the case by Komen et al. (1984) [6] into nonlinear forcing (dashed) and damping (dotted) terms (see Eq.7)

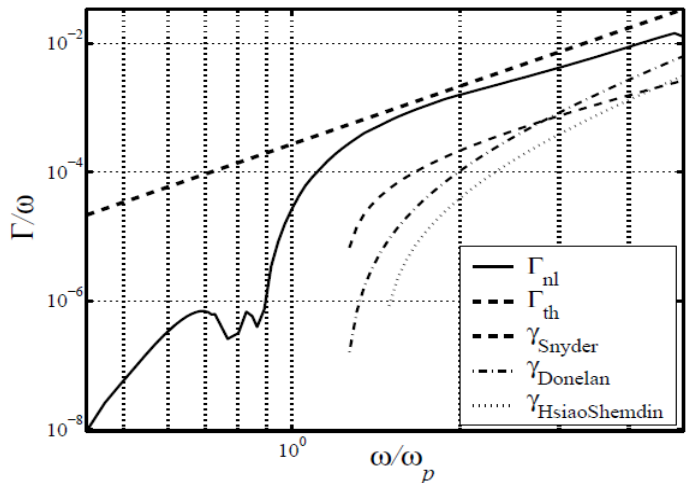

Fig.3 Nonlinear damping coefficient Γ_k given by theoretical estimate and by the numerical simulation (dashed and solid bold curves, correspondingly). Conventional dependencies of wind growth increments are shown by thin curves with authors' names in the legend

Our result on dominating effect of nonlinear interaction on wave spectra evolution should not be interpreted as a call to ignore the effects of wind input and wave dissipation. The leadership of S_{nl} does not mean that we disregard wind input and dissipation, we just put them in proper place. The strong nonlinear forcing and damping that compose the conservative term S_{nl} determine strong relaxation and a universality of spectral shaping due to inherent wave dynamics, while S_{in} and S_{diss} are responsible for growth of total energy.

Obtained result will help to make better theoretical estimates of solutions of Hasselmann equation and help to develop simplified approximations to source terms.

3. Global visual observations as a tool for discriminating swell and wind seas

Today's Voluntary Observed Ships (VOS) data are considered as a source of information on climatic features of the world ocean rather than experimental background of studies of physical mechanisms of wind-wave dynamics. We are trying to change this tradition by applying results of asymptotic weakly turbulent model of wind-driven seas [1], [4], [8] to extensive data base of the Global Atlas of Ocean Waves [9], [10].

The global visual wave observations are re-analyzed within the theoretical concept of self-similar wind-driven seas . The theoretical criteria of discriminating wind-driven and swell seas are formulated and shown to be adequate to the problem. The results are detailed for the South Pacifica, which wave climatology based on VOS data is well studied and the swell component is well pronounced. The core of the analysis are one-parametric dependencies "wave height - wave period" $H_s = CT^Z$. The reference cases [11], [12],[13] have found, correspondingly, $Z = 5/3$, $Z = 3/2$ and $Z = 4/3$. This set of exponents Z has been interpreted recently in [1], [4], [8] in terms of spectral fluxes and total wave input. An alternative reference case – sea swell gives an opposite signature of the exponent $Z = -1/2$.

This simple criterion was used and appeared to be robust for the problem of swell-wind sea discrimination.

The corresponding exponent Z appears to be slightly higher than $Z = -1/2$, that implies a pumping of sea swell by wind-driven sea background [17]. This important issue is considered both in the context of methodology of obtaining VOS data and within the physics of the mixed sea. This result contradicts to commonly accepted vision of sea swell as a neutral or slightly decaying fraction of ocean wave field.

Prospects of further study are quite promising. In particular, satellite data are seen to be used for tracking ocean swell and for studies of physical mechanisms of its evolution.

4. A new method for solving the exact S_{nl} in the energy balance equation

One of the most challenging tasks in improving the wave forecasting models is solving accurately and in a reasonable time the collision integral in the energy balance equation (1). Here we present a new method for solving the collision integral in the Hasselmann equation that can be used in arbitrary depth. The method has in principle some advantages with respect to previously developed methods such as Resio-Tracy *(RT)* [14], Lavrenov *(LAV)* [15], Masuda *(MAS)* [16]: (i) the loci are straight lines instead of eggs-shaped in *RT* method; (ii) The calculation of the loci is exact and does not require iterative procedure; (iii) it does not require interpolation of the spectrum in the frequency domain, but just in angle (the interpolation is therefore 1D with respect to 2D in *RT, MAS* and *LAV*); (iv) The *MAS* and *LAV* methods contain some singularities that have to be treated with some caution. The method presented here does not have singularities as the *RT* method. The collision integral is

$$\frac{\partial N_1}{\partial t} = 4\pi \int T^2_{1,2,3,4} N_1 N_2 N_3 N_4 \left(\frac{1}{N_1} + \frac{1}{N_2} - \frac{1}{N_3} - \frac{1}{N_4} \right) \delta(\Delta \vec{k}) \delta(\omega) d\vec{k}_{234}$$

The difference between all these methods is the way the δ-functions in frequency are removed. The methods *RT, LAV* and *MAS* remove the momentum δ-function by integration over wave vector. Then δ-function needs to be solved numerically. Here we follow different approach. First we are making transform to polar coordinates - the resulting integral is a multidimensional one - three integrals in wave numbers (modulus) and three in angles. Integration over two angles is then performed using the property of δ-unction and the resulting integral is transformed in angular frequency coordinates (three integrals in frequency and one in angle). After using the property of δ-function in frequency, the integration is removed and we come to the final result:

$$\frac{\partial N_1}{\partial t} = 4\pi \int T^2_{1,2,3,4} N_1 N_2 N_3 N_4 \left(\frac{1}{N_1} + \frac{1}{N_2} - \frac{1}{N_3} - \frac{1}{N_4} \right) k_3 \omega_2 \omega_3 \omega_4 \frac{\Theta_2 \Theta_4}{|\sin(\theta_2 - \theta_4)|} d\omega_{23} d\theta_3$$

with $\Theta_4 = \Theta\left(k_4 - |k_1 \cos(\theta_1) + k_2 \cos(\theta_2) - k_3 \cos(\theta_3)|\right)$ where Θ is the step-function. A similar definition holds for Θ_2. The numerical integration of S_{nl} consists in discretization of the above integral. With respect to the *RT* method the loci over which we make the integration are straight lines. We give a number of graphical examples comparing the loci of integration for the RT method and the method

here developed. In ***Fig.4a*** we show the classical egg-shaped loci obtained from the RT method. In ***Fig. 4b*** we show the respective loci in our domain of integration

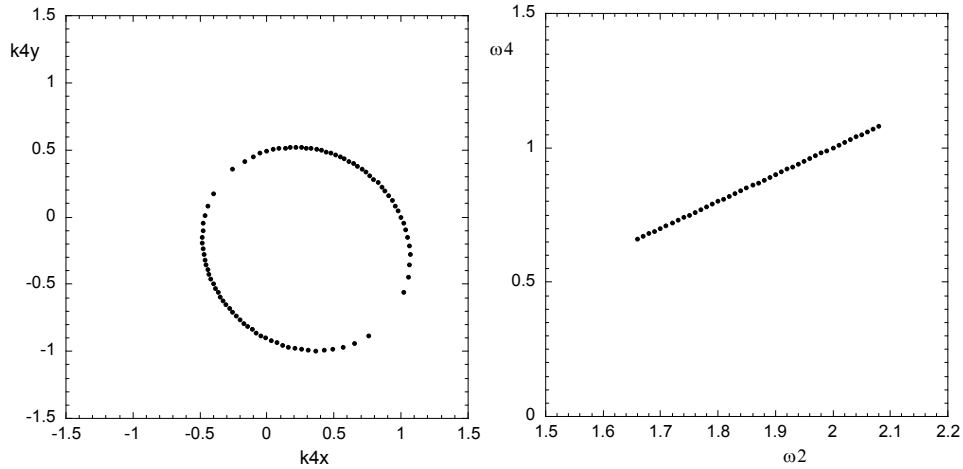

Fig.4. a) Egg-shaped loci in the RT method. b) Loci in the new numerical method

We developed new promising, in terms of accuracy and speed, method for solving Hsslmann equation nonlinear term.

5. New canonical equation for one-dimensional surface waves

We applied canonical transformation to the water wave Hamiltonian

$$H = \frac{1}{2}\int(\psi\hat{k}\psi + g\eta^2)dx + \frac{1}{2}\int\eta\{\psi_x^2 - (\hat{k}\psi)^2\}dx + \frac{1}{2}\int\eta(\hat{k}\psi)\{\hat{k}(\eta(\hat{k}\psi)) + \eta\psi_{xx}\}dx$$

where

$$\psi(x,t) = \phi(x,y,t)\big|_{y=\eta} = \phi(x,\eta(x,t),t).$$

The transformation removes not only cubic nonlinear terms, but simplifies drastically fourth order terms in the Hamiltonian:

$$H = \int b^*\hat{\omega}_k b dx + \frac{1}{4}\int|b'|^2\left[\frac{i}{2}(bb'^* - b^*b') - \hat{k}\,|b|^2\right]dx.$$

b – is new normal canonical variable. This transformation explicitly uses the fact of vanishing exact four waves interaction for water gravity waves for 2D potential fluid. After the transformation well-known but cumbersome Zakharov equation is drastically simplified and can be written in X-space in compact way. This new equation is very suitable for both analytic study and numerical simulation:

$$i\frac{\partial b}{\partial t} = \hat{\omega}_k b + \frac{i}{8}\left[b^*\frac{\partial}{\partial x}(b'^2) - \frac{\partial}{\partial x}(b^{*\prime}\frac{\partial}{\partial x}b^2)\right] - \frac{1}{4}\left[b\cdot\hat{k}(|b'|^2) - \frac{\partial}{\partial x}(b'\hat{k}(|b|^2))\right].$$

Also, this equation is convinient for experimentalists. The simplest solution of the equation is the monochromatic wave $b(x) = B_0 e^{i(k_0 x - \omega_0 t)}$ with the frequency $\omega_0 = \omega_{k_0} + \frac{1}{2} k_0^3 \mid B_0 \mid^2$. Growth-rate of modulation instability is also calculated:

$$\gamma_k^2 = \frac{1}{8} \frac{\omega_{k_0}^2}{k_0^4} (1 - 6\mu^2) k^2 \left[\mu^2 (k_0 - \frac{|k|}{2})^2 - \frac{k^2}{8} \right].$$

In the framework of this "improved" Zakharov equation we have performed numerical simulation of freak-wave formation from the initialy uniform water waves, see ***Fig.1***

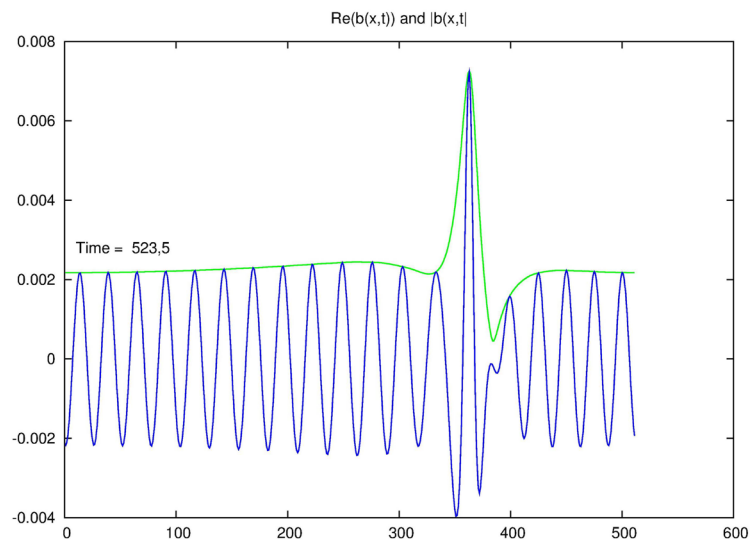

Fig.5 $|b(x)|$ ***and*** $Re(b(x))$.

We developed simple new equation for 1D nonlinear surface waves, which is convenient for both theoretical study and numerical simulation of wave propagation in narrow experimental tanks.

IMPACT/APPLICATIONS

- Obtained results are expected to have significant impact on operational wave models through improved wind input term and, as a consequence, better prediction of ocean surface wave evolution.
- Correct understanding of Hasselmann equation source terms balance will help to develop better theoretical estimates and approximations to the source terms.
- Understanding of swell - wind sea interaction has to help to develop approximations to nonlinear source term and, as a result, build simplified theoretical and fast numerical models of nonlinear interaction of surface waves.
- New method of numerical integration of Hasselmann equation will help to develop faster working operational wave prediction models.

- New one-dimensional nonlinear equation for surface waves will help to carry out more effective theoretical analysis for freak waves and faster numerical simulation of the processes of wave propagation in narrow experimental tanks.

RELATED PROJECTS

None

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