

Dynamics and Modeling of Turbulent Mixing in Oceanic Flows

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LONG-TERM GOALS

The long-term goal of this research is to refine and extend simple (existing) model parameterizations for turbulent diapycnal mixing for use in large scale numerical ocean models where such processes occur at the subgrid-scale. From a scientific view point, the goal is to obtain new insights into the dynamics of stratified turbulence that will translate into simple effective parameterizations for use in ocean models.

OBJECTIVES

The primary objective of this project is to bridge the gap between parameterizations and models for small-scale turbulent mixing developed from fundamental direct numerical simulations (DNS) and grid turbulence experiments (Venayagamoorthy and Stretch 2006, 2010, Stretch and Venayagamoorthy 2010) to geophysical (mesoscale and larger) scale models with an emphasis on making progress towards improved turbulent parameterizations in the ocean. The strategy is to use model-data comparisons in conjunction with theoretical modeling efforts to investigate and improve the efficacy of existing parameterizations for turbulent mixing.

APPROACH

This research takes a twin pronged approach involving process studies to assess existing turbulent parameterizations. First, through collaborations Dr. Louis St. Laurent (WHOI), we are looking at data possibilities that will provide high quality turbulence measurements to compare with the parameterizations/scaling results developed from previous DNS/theoretical work of Dr. Venayagamoorthy (Venayagamoorthy and Stretch 2006, 2010, Stretch and Venayagamoorthy 2010). Second, we are carrying out further theoretical studies in the context of second moment closure (SMC) schemes in a Reynolds-averaged Navier-Stokes (RANS) framework. Direct numerical simulation (DNS) data of stably stratified homogeneous turbulence are used to study the parameters in two-equation RANS turbulence models such as the buoyancy parameter $C_{\epsilon 3}$, and the turbulent Prandtl number Pr_t in the $k-\epsilon$ model.

WORK COMPLETED

In FY2011, the bulk of our effort has focused on collecting turbulence measurements in internal-wave forced flows in the ocean as well as implementation of new parameterizations for stratification in RANS models. To this end, the PI joined the scientific team on a recent IWISE expedition in Luzon Passage (in June-July 2011) led by Dr. Louis St. Laurent (WHOI). This study focused on conducting a microstructure study of the world's most energetic internal wave generation region. The PI spent nearly 3 weeks at sea as part of this expedition. We were involved in examination of the data and found that the relationship between turbulent dissipation rate and overturn length-scales (e.g. Thorpe scales) may deviate from accepted ideas in cases of extremely strong turbulent forcing, such as slope convection. We are continuing our collaboration with Dr. St. Laurent to examine the microstructure and CTD data to understand the turbulence properties and compare with scaling parameterizations.

Our previous work in FY2010 on theoretical modeling aspects in a RANS framework to evaluate different turbulent Prandtl number parameterizations in zero-equation turbulence models for stably stratified environmental flows was recently published in *Dynamics of Atmospheres and Oceans* (Elliot and Venayagamoorthy 2011a). We have since focused our efforts on developing and testing formulations for two-equation turbulence models in RANS simulations. Both the gradient Richardson number $Ri = N^2/S^2$, where N is the Brunt-Väisälä (or buoyancy) frequency and $S = d\bar{u}/dz$ is the mean shear rate, and the turbulent Froude number $Fr_k = \varepsilon/(Nk)$, where ε is the turbulent kinetic energy dissipation rate and k is the turbulent kinetic energy, are used as correlating parameters to characterize stratification in the k - ε model. We show that it may be more appropriate to use Fr_k as the parameter of choice for modeling the stratification parameters in the k - ε model since it is based on the local properties of the turbulence as opposed to Ri , which is a mean property of the flow.

The proposed modifications were implemented in a one-dimensional water column model called the General Ocean Turbulence Model (GOTM) and used to simulate stably stratified channels flows. The results from numerical simulations using the modified k - ε model were compared to stably stratified channel flow DNS data to assess their efficacy. In addition to the many well-known and tested turbulence models, GOTM also contains a number of built-in stability functions that are computed and can be used to calculate the turbulence closure parameters. Depending on the level and complexity of turbulence model used, these stability functions can either be constants, empirical functions, or functions of parameters characterizing the local state of turbulence. In the standard k - ε model, these stability functions are constants. The version of GOTM used also has a subroutine for computing stability functions according to Munk and Anderson's (1948) model for Pr_t . GOTM's open-source modular format allows stability functions to be modified and/or created to test different parameterizations of the stability functions. In order to rigorously test the effectiveness of the k - ε model in simulating stably stratified flows and propose an efficient model, different variations of the k - ε model were assessed by using different stability functions to define the turbulence parameters. The stability functions tested include the built-in standard form of the model using only the original empirical constants developed by Jones and Launder (1972), the built-in Munk and Anderson (1948) model for computing ν_t and Pr_t as functions of Ri , and finally a proposed model for stably stratified flows based on Pr_t and $C_{\varepsilon 3}$ as functions of the turbulent Froude number Fr_k (please also see Elliot and Venayagamoorthy 2011b for more details).

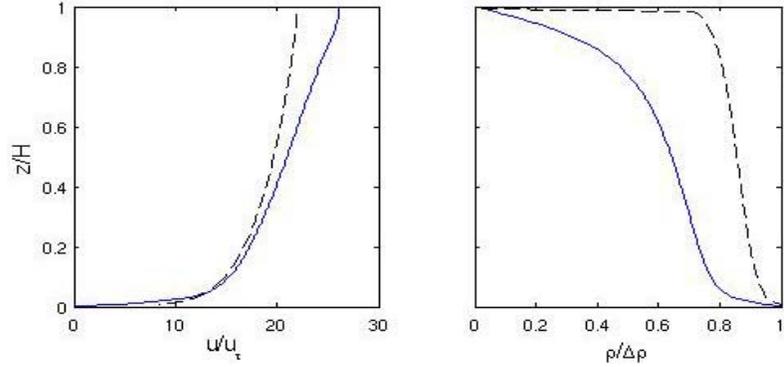
RESULTS

The test case used for the numerical simulations is that of a pressure gradient driven open-channel flow in which the density is held fixed at both the lower solid boundary and upper free surface (i.e. Dirichlet boundary conditions), similar to the simulations and experiments done by Venayagamoorthy *et al.* (2003), Komori *et al.* (1983) and recent direct numerical simulations (DNS) by García-Villalba & del Álamo (2011). This set-up is somewhat of an arbitrary test case, but enforcing Dirichlet boundary conditions allows the parameters that characterize stratification to be rigorously tested in a stratified environment since in these simulations the density was allowed to evolve and mix naturally with the flow as an active scalar between the fixed boundaries. In this way, the flow was kept under stratified conditions for the duration of each of the simulations. By fixing the density at the upper and lower boundaries, the initial density profile becomes a continuously stratified profile varying with depth in the channel. The numerical simulations were carried out using a one-dimensional water column model called General Ocean Turbulence Model (GOTM) developed by Burchard *et al.* (1999) and used for a wide range of applications in geophysical turbulence modeling.

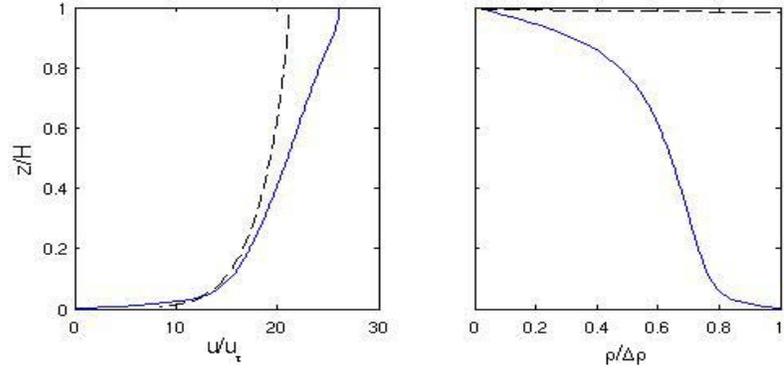
Figure 1 (top row) shows results of the k - ε model with constant stability functions for Pr_t , $C_{\varepsilon 3}$, and each of the other k - ε model parameters in contrast to DNS data (García-Villalba & del Álamo 2011). Values of $Pr_t = 0.71$ and $C_{\varepsilon 3} = 1.44$ for buoyancy effects were used in addition to the standard constant values for the other parameters in the k - ε model. The constant stability functions missed the effect of stratification on the development of the mean turbulent velocity profile u , especially near the free surface as shown in figure 1 (left panel– top row). The same flow and stratification conditions were then used to test the Munk and Anderson (1948) stability functions in conjunction with the k - ε model. This stability function computes Pr_t as a function of Ri . Figure 1 (middle row) shows the predictions of the k - ε model using the Munk and Anderson (1948) stability function versus DNS results for stably stratified turbulence. It is important to note that this formulation based on Ri did a poor job in that it over-predicted the turbulent mixing of density ρ in figure 1 (right panel – middle row), essentially washing out the stratification in the channel such that buoyancy effects as a function of Ri were not accounted for. As a consequence, the effect of stratification on the mean velocity profiles u was significantly missed near the free surface as shown in figures 1 (left panel – middle row).

Since Ri seems to be a poor parameter for characterizing the effects of stratified turbulence in dynamic two-equation models, it is not surprising that the k - ε model with constant stability functions performed better than the Munk and Anderson (1948) at predicting the turbulent velocity profile and density field. In lieu of the gradient Richardson number Ri , the turbulent Froude number Fr_k has been suggested and used by others, including Shih *et al.* (2000) and Venayagamoorthy *et al.* (2003), to describe the effects of stratification on turbulence parameters. It can be argued that Fr_k is a better parameter than Ri for characterizing stratified turbulence because it is based on local turbulence quantities k and ε , whereas Ri is more of a mean property of the flow based on global, linear quantities N (buoyancy frequency) and S (mean shear). It may be especially suitable for the k - ε model because k and ε are already computed quantities. Figure 1 (bottom row) shows results using the k - ε model with stability functions for Pr_t and $C_{\varepsilon 3}$ defined as functions of Fr_k . By defining both Pr_t and $C_{\varepsilon 3}$ as functions of the turbulent Froude number, the model was able to more closely match the mean velocity profile in figure 1 (left panel – bottom row) while providing excellent predictions of the density profile (figure 1, right panel – bottom row). Our analysis of the results show that Ri may not be the best parameter to characterize stratification in dynamic two-equation models and that Fr_k may be a more appropriate model parameter to represent turbulence in stratified flows.

(i) Top Panel



(ii) Middle Panel



(iii) Bottom Panel

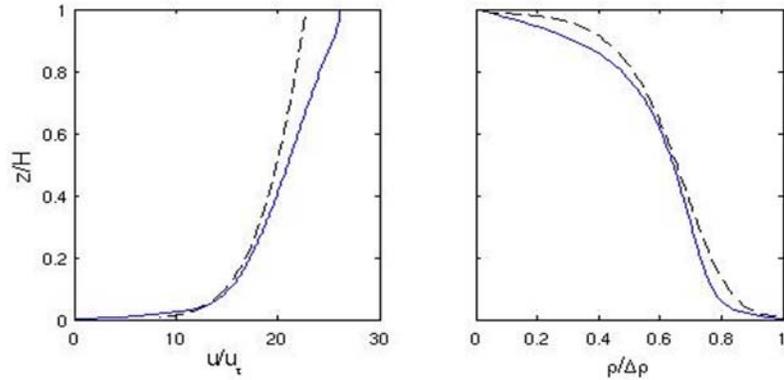


Figure 1: Distributions of mean velocity u and density ρ ; in pressure gradient driven open-channel flow at $Re_\tau = 550$ and $Ri_\tau = 60$: blue solid line, DNS data (García-Villalba & del Álamo 2011); black dashed line, GOTM results for (i) standard $k-\varepsilon$ model with constant coefficients (top row); (ii) $k-\varepsilon$ model with Munk and Anderson (1948) stability function based on Ri formulation (middle row); and (iii) $k-\varepsilon$ model with $Pr_t = f(Fr_k)$ and $C_{\varepsilon 3} = f(Fr_k)$ (bottom row).

IMPACT/APPLICATIONS

This project will contribute to an improved understanding of small-scale mixing processes and development of better parameterizations of such processes for applications in large-scale oceanic numerical simulations models where such processes are not explicitly resolved. In particular, this

research will make contributions to the strategic ongoing efforts at ONR on assessing the effects of submesoscale ocean parameterizations and to the task of linking (small-scale) turbulence models to (larger-scale) mesoscale models.

RELATED PROJECTS

Dr. Venayagamoorthy is collaborating with Dr. Amit Tandon (University of Massachusetts, Dartmouth) and Dr. Amala Mahadevan (Woods Hole Oceanographic Institution) on implementing and testing the proposed formulations for buoyancy effects in stratified flows for some oceanographically relevant flow (with coriolis and diurnal cycling included, for instance). We are currently working on comparing the results for 1-D oceanic scenarios and will then implement these schemes for sub-mesoscale flows.

Dr. Venayagamoorthy in collaboration with Dr. Lakshmi Dasi (Colorado State University) is conducting research on the the dynamics of turbulent shear flows (both free-shear and wall-bounded flows). The emphasis of this research (funded through start-up funds) is to understand the relevant length and time scales in shear flow turbulence. This ongoing effort should provide fundamental insights on turbulence and how to incorporate effects of inhomogeneity into turbulence models.

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PUBLICATIONS

Elliott, Z. A. and Venayagamoorthy, S. K. (2011), Evaluation of turbulent Prandtl (Schmidt) number parameterizations for stably stratified environmental flows, *Dynamics of Atmospheres and Oceans* **51**:137– 150.

Elliott, Z. A. and Venayagamoorthy, S. K. (2011), Implementation and testing of RANS turbulence models for stably stratified flows, *proceedings of World Environmental and Water Resources Congress*, ASCE, pp 2156-2165.

HONORS/AWARDS/PRIZES

Subhas Karan Venayagamoorthy, Colorado State University Best Teacher Award, 2011.

Subhas Karan Venayagamoorthy, Borland Chair of Hydraulics, Department of Civil and Environmental Engineering, Colorado State University, 2011.