DISTRIBUTION STATEMENT A. Approved for public release; distribution is unlimited.

Verification-Based Model Tuning

Caren Marzban
Applied Physics Laboratory, University of Washington
1013 N.E. 40th St.
Seattle, WA 98105-6698
phone: (206) 221-4361 fax: (206) 545-4404 email: marzban@stat.washington.edu

David W. Jones
Applied Physics Laboratory, University of Washington
1013 N.E. 40th St.
Seattle, WA 98105-6698
phone: (206) 543-3236 fax: (206) 545-4404 email: dwjones@apl.washington.edu

Scott A. Sandgathe
35574 Camp Creek Rd.
Springfield, OR 97478
phone: (541) 988-0289 fax: (541) 988-0289 email: sandgathe@apl.washington.edu

Award Number: N00014-12-1-0276

LONG-TERM GOALS

We put forth the idea of developing a statistical model representing the relationship between model parameters and macroscopic features in meteorological quantities, with the latter quantified in terms of measures commonly used in spatial verification methods. For example, if/when a spatial verification method suggests that frontal speed was forecast incorrectly, we would like to be able to set the model parameters (e.g., diffusion rate) to remedy that problem in some optimal sense. This primary goal requires the identification of relevant model parameters, which in turn requires performing sensitivity analysis (our secondary goal). To validate the results of the variance-based sensitivity analysis, comparisons are made with a more traditional method based on adjoints. After the "important" model parameters are identified, an emulator will be developed. An emulator consists of a statistical model which represents the relationship between model parameters and macroscopic features (e.g., spatial structure) of forecast fields. Such an emulator (and its inverse) has both scientific value and practical utility. An example of the former is the knowledge gained from identifying the statistical relations between model parameters and forecast parameters, and model-tuning is an example of the practical utility.

OBJECTIVES

1. Perform variance-based and adjoint sensitivity analysis of the Lorenz 1963 model, in preparation for application to COAMPS. The sensitivities are computed with respect to both initial conditions and model parameters, but only the latter are used as inputs to the emulator.

2. Develop emulators to represent the complex, nonlinear relationship between model parameters and forecast features arising in spatial verification techniques.

APPROACH

The specific type of sensitivity analysis we have attempted to tailor to our needs is called variance-based sensitivity analysis (Saltelli 2010). We have applied the method to both COAMPS and the Lorenz 1963 model. Applying the method to a complex and a simple model at the same time has been fruitful in explaining some of our COAMPS results.

The emulators we have developed (trained) thus far are still tentative, because the sensitivity analysis work is still in progress. We have tested polynomial regression and neural networks as emulation models, and they both seem to be adequate for the task at hand.

The variance-based sensitivity analysis approach and the emulation method are closely tied (Oakley and O'Hagan 2004, Rouigier 2005). This connection between variance-based sensitivity analysis and model emulation is the main reason behind our approach, because it is the emulator which is ultimately used for model tuning.

WORK COMPLETED

Various objectives are being addressed in parallel. Both variance-based and adjoint sensitivity analysis of Lorenz 1963 has been done, but the comparison between them is not complete. The variance-based results for the Lorenz 1963 model and COAMPS were presented at a seminar at NRL on October 10, 2011. Emulators have been trained for both Lorenz 1963 and COAMPS, but they are all in developmental stage.

We have initiated a collaboration with an expert in adjoint sensitivity analysis (Greg Hakim, University of Washington). We have discovered that in fact the variance-based and the adjoint method may be quite different in terms of their assumptions and goals, almost to the point of making a direct comparison very difficult (or nearly impossible). Through the collaboration with Greg Hakim, we hope to address this question of how to perform the comparisons. Thus far that work has focused on the Lorenz model. Although the Lorenz model is qualitatively different from COAMPS (because the Lorenz adjoint exists in analytic form, but only a numerical solutions exists for the COAMPS adjoint), the comparison

of the two sensitivity analysis methods (variance-based vs. adjoint) on the Lorenz model is important for understanding the COAMPS results.

RESULTS

The Lorenz 1963 model is well-studied: State variables X, Y, Z are coupled to one another in a first-order (in time) differential equation, involving parameters s, r, b. To perform sensitivity analysis, one considers a *reference trajectory* denoted X_R, Y_R, Z_R, with respect to which perturbations $\delta X, \delta Y, \delta Z$ are introduced. The perturbations may be caused by uncertainty in the initial conditions $(\delta X_0, \delta Y_0, \delta Z_0)$ or in the model parameters $(\delta s, \delta r, \delta b)$. One question is to assess the manner in which the perturbations $\delta X, \delta Y, \delta Z$ depend on $\delta X_0, \delta Y_0, \delta Z_0$, and $\delta s, \delta r, \delta b$. Note that we are considering not only sensitivity to **initial conditions**, but also sensitivity to the perturbations (uncertainty) in the **model parameters**.

We used a method developed by one of the pioneers of the adjoint method (Hall 1986); this formulation of the adjoint method has the virtue of allowing one to derive simple mathematical expressions which can aid in understanding the complexity brought about by the nonlinear nature of the Lorenz model. Specifically, we have derived the equations for the time-average of the perturbations. For example, the time-average of δZ, over a period from 0 to τ is

$$\begin{aligned} \overline{\delta Z} = & \int_0^\tau \; [v_1(Y_R - X_R)\, \delta s + v_2 X_R\, \delta r + v_3 Z_R \delta b\,]\; dt \\ & - \; v_1(t=0)\, \delta X_0 - v_2(t=0)\, \delta Y_0 - v_3(t=0)\, \delta Z_0 \end{aligned} \tag{1}$$

The $v_i(t)$ are solutions to

$$N^* \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \frac{1}{\tau} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \tag{2}$$

where N^* is the matrix defining the adjoint of the tangent linear model; it is well-known and can be written in analytic form (not shown here). The adjoint solutions v_i must satisfy the condition $v_i(t = \tau) = 0$.

Solving the adjoint equation (2) "backward" in time gives $v_i(t = 0)$, which then, according the equation (1) are precisely the sensitivities with respect to initial conditions. Equation 1 also gives the sensitivities with respect to model parameters, namely the coefficients of $\delta s, \delta r, \delta b$. These too involve the adjoint solutions, but integrated over time.

The simple and explicit nature of this formulation is precisely what is required for comparing it with the variance-based method. We are currently casting the variance-based method in a form that allows comparison with Eq. 1. One difference between the two methods (which makes comparisons difficult) is that the variance-based method explicitly contain interactions between the various perturbations, whereas the adjoint method (1) does not.

Turning to numerical/simulations (important for transitioning to COAMPS), Fig. 1 shows the sensitivities defined above as a function of time. Apart from the variations in

time, the relative vertical displacement between the three curves in the left panel clearly shows that the three parameters have very different effects. By contrast, the sensitivities with respect to initial conditions (right panel) are initially different, but quickly approach a common value of zero.

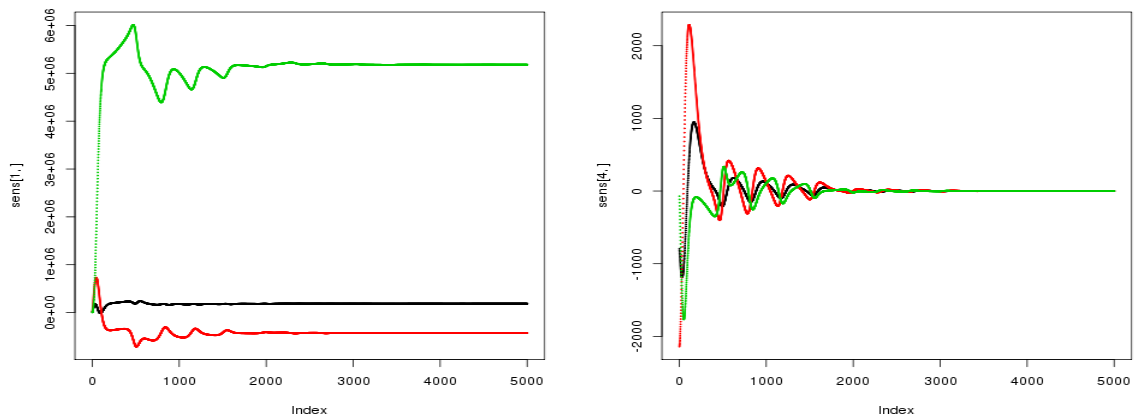

Figure 1. Left: Sensitivity of $\overline{\delta Z}$ to model parameters δs (black), δr (red), and δb (green). Right: sensitivity of $\overline{\delta Z}$ to initial conditions δX_0 (black), δY_0 (red), and δZ_0 (green). The x-axis in both plots is time.

Fig. 2 shows the same sensitivities, but as a function of the X and Z state variables of the model. One task at hand is to compare these results with those obtained via the adjoint code for the Lorenz 63 model. An example is shown in Fig. 3 (already produced by Greg Hakim as part of a class taught at the University of Washington). The sensitivity metrics used in the two analyses (the analytic method above, and that based on the adjoint code) are different because of the way the two methods compute sensitivity. For example, the metric in the latter is required to be a positive definite quantity, but no such restriction is required in the variance-based method.) We are currently working to use similar metrics in all the methods being compared.

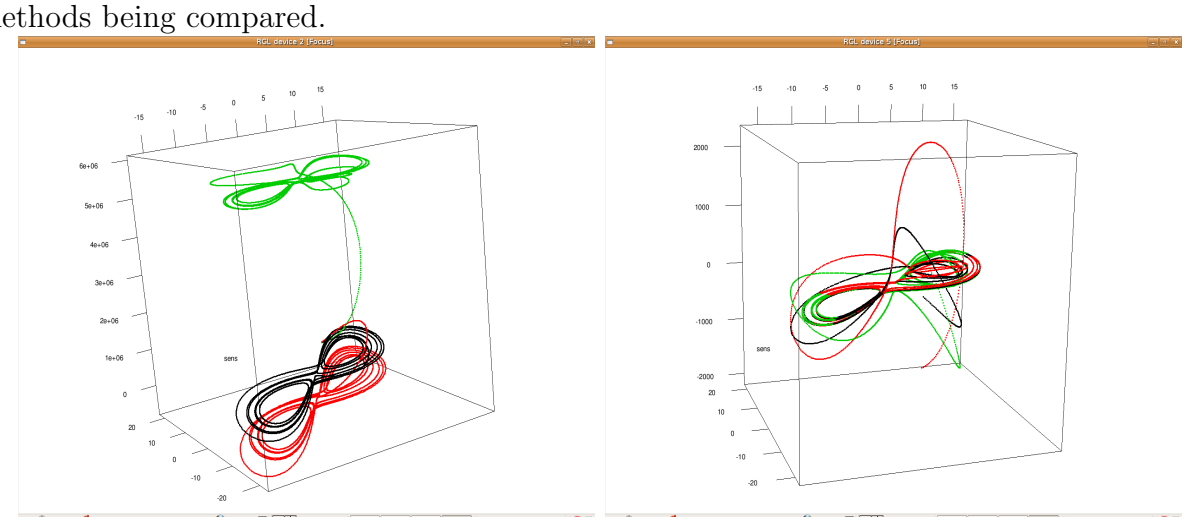

Figure 2. Same sensitivities as in Fig, 1, but as a function of the X and Z state variables.

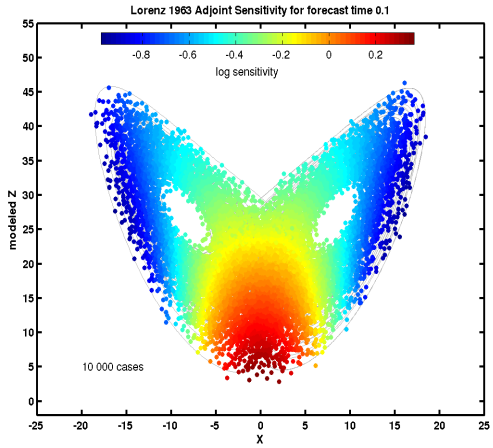

Figure 3. Sensitivity according to the adjoint code for the Lorenz 1963 model. Produced here with permission of Greg Hakim.

IMPACT/APPLICATIONS

The impact of our work on the Lorenz 1963 model is likely to be mostly of academic and scientific value. Its main purpose here is to guide the work on COAMPS. By contrast, the practical values of the sensitivity analysis of COAMPS and the emulator are likely to be significant. Together they allow one to assess the effect of model parameters on spatial forecast verification, and vice versa. As such, not only the effects of changing the parameters can be anticipated, but also desirable properties of forecasts can be assured (in some optimal sense) by tuning the parameters accordingly.

REFERENCES

Hall, M. C. G., 1986: Application of Adjoint Sensitivity Theory to an Atmospheric General Circulation Model. *Journ. Atmos. Sci.*, **43(22)**, 2644-2651.

Oakley, J. E., and A. O'Hagan, 2004: Probabilistic sensitivity analysis of complex models: a Bayesian approach. *J. R. Statist. Soc., B*, **66(3)**, 751-769.

Rouigier, J., 2005: Efficient Emulators for Multivariate Deterministic Functions. *Journal of Computational and Graphical Statistics*, **17(4)**, 827-843.

Saltelli, A., P. Annoni, I. Azzini, F. Campolongo, M. Ratto, S. Tarantola, 2010: Variance based sensitivity analysis of model output: Design and estimator for the total sensitivity index. *Computer Physics Communications*, **181**, 259270.