

## **Sequential Geoacoustic Filtering and Utilizing Ambient Noise for Geoacoustic Inversion**

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Award Numbers: N00014-11-1-0320 and N00014-11-1-0321  
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### **LONG-TERM GOALS**

The development of new geoacoustic inversion methods, their use in the analysis of shallow water experimental data, and evaluation of geoacoustic model and parameter uncertainties including the mapping of these uncertainties through to system performance uncertainties.

### **OBJECTIVES**

Analysis of geoacoustic inversion data collected from various experiments. Of specific technical interest are: (1) development of methods to track the environmental parameters using sequential filtering, (2) use of ambient noise for estimation of seafloor structure parameters, and (3) the development of new inversion methods for use into the kHz frequency regime. In an ONR Graduate Traineeship Awards we address using Random Matrix Theory in ocean acoustics.

### **APPROACH**

#### **1. Sequential filtering**

A common feature of inverse problems in ocean acoustics is that estimates of underlying physical parameters are extracted from measured acoustic data. Geoacoustic inversion has been approached in the same framework, estimating, in addition to source location, ocean environment parameters and their uncertainty. Often, those parameters evolve in time or space, with acoustic data arriving at consecutive steps. Information on parameter values and uncertainty at preceding steps can be invaluable for the determination of future estimates but is often ignored.

Sequential Bayesian filtering, tying together information on parameter evolution, a physical model relating acoustic field measurements to the unknown quantities, and a statistical model describing random perturbations in the field observations, offers a framework for the solution of such problems.

## 2. Extracting information from noise cross-correlations

We have focused extensively on extracting information from noise in ocean acoustics with both theoretical work as well as experimental work. The passive fathometer is based on relating the down- and up-going signals on the array and can be implemented in the time or frequency domain. Here, we are exploring the passive fathometer by aligning arrivals using phase information from the fathometer (currently only the magnitude is used). We have evidence that the vertical fathometer array moves with the waves on the sea surface. Thus if we can correct for this movement it will be possible to align the reflections better and then average the reflection time series with phase as opposed to just using the envelope. This should give sharper definition of the seafloor and sub bottom reflections and enables estimating environmental geoacoustic parameters in addition to depths of reflecting layers. We also are exploring accelerating convergence for the noise cross-correlation by various signal processing strategies, e.g. averaging, rejecting interference dominated time series, eigenvalue/eigenvector decomposition, and focusing on specific arrivals using beamforming.

### WORK COMPLETED

One application of passive estimation of the time-domain Green's function is in the use of cross-correlations of upward and downward pointing vertical line array beams observing ambient noise to extract seabed layer structure (i.e. a passive fathometer) [Traer et al., 2011, 2012]. This so called passive fathometer technique exploits the naturally occurring acoustic sounds generated on the sea-surface, primarily from breaking waves. The method is based on the cross-correlation of noise from the ocean surface with its echo from the seabed, which recovers travel times to significant seabed reflectors. To limit averaging time and make this practical, beamforming is used with a vertical array of hydrophones to reduce interference from horizontally propagating noise. The initial development used conventional beamforming, but significant improvements have been realized using adaptive techniques. An analytical model is presented in [Traer et al., 2011] for the passive fathometer response to ocean surface noise, interfering discrete noise sources, and locally uncorrelated noise in an ideal waveguide. The leading order term from the ocean surface noise produces the cross-correlation of vertical multipaths, yielding the depth of sub-bottom reflectors.

We have explored incorporating Kalman and particle filter tracking techniques into the geoacoustic inversion problem [Yardim 2011a, 2011b, 2012, Michalopoulou 2012]. This enables spatial and temporal tracking of environmental parameters and their underlying probability densities, making geoacoustic tracking a natural extension to geoacoustic inversion techniques.

### RESULTS

In many cases, it is of interest to estimate geoacoustic parameters over a larger spatial region rather than just the parameters characterizing propagation between a fixed source and receiver (or receiving array) location. Data might be available at a moored vertical receiving array from a towed acoustic sound source or a source might be received by a towed horizontal array. In both cases, the typical approach would be to treat each record of data independently of the others and carry out a full geoacoustic inversion for every record resulting in a sequence of geoacoustic parameter estimates and, in some cases, posteriori probability densities of the environmental parameters. The latter enables the environmental uncertainty to be projected into other waveguide characterizations such as propagation loss and its uncertainty.

In a review paper we have studied the basis and use of sequential filtering in ocean acoustics [Yardim 2011]. Sequential filtering provides a consistent framework for estimating and updating the unknown parameters of a system as data become available, see Figs. 1-2. Despite significant progress in the general theory and implementation, sequential Bayesian filters have been sparsely applied to ocean acoustics. The foundations of sequential Bayesian filtering with emphasis on practical issues are first presented covering both Kalman and particle filter approaches. Filtering becomes a powerful estimation tool, employing prediction from previous estimates and updates stemming from physical and statistical models that relate acoustic measurements to the unknown parameters. Ocean acoustic applications are then reviewed focusing on the estimation of environmental parameters evolving in time or space. Some possible scenarios for geoacoustic inversion are shown in Fig. 3.

## Random Matrix Theory

A new direction is taken in Menon [2012a, 2012b, 2012c] and Gerstoft [2012] where random matrix theory is used to analyze noise cross-spectral density matrices. Isotropic noise fields are often used to model environmental noise surrounding an array of sensors. For a line array of equidistant sensors in such a noise field, the true covariance matrix of the observations in the frequency domain is a symmetric Toeplitz sinc matrix. In Menon [2012a], we derive the eigenvalues of the true covariance matrix as the size of the matrix approaches infinity. For arrays spaced at less than half a wavelength apart, the covariance matrix is shown to be rank deficient and this has implications in techniques such as adaptive beamforming, which require the inverse covariance matrix. The zero eigenvalues are related to classical array processing concepts such as the invisible region in frequency-wavenumber space (region where there is no propagating energy, but a spectrum can be calculated). Using random matrix theory, we derive the eigenvalue density of the sample noise covariance matrix, whose knowledge is useful in reliable signal detection. An example of such processing is seen in Figure 4.

Often the ocean acoustic data sample covariance matrix (SCM, or cross-spectral matrix) is assumed to consist of a few large signal-plus-noise eigenvalues followed by a set of equal-value noise-only eigenvalues representing uncorrelated noise. However, it is well-known that the SCM from real data observations is characterized by steadily decaying noise-only eigenvalues. In array processing, a common rule of thumb is that the SCM is “well-estimated” when the number of snapshots is 2 to 3 times the array dimension. This depends on the type of noise and application under consideration. Often, the number of snapshots available for forming the SCM is less than this, especially for large arrays. Using Random Matrix Theory (RMT) to model the statistical properties of the SCM, the eigenvalue distributions are more informative than using the expectation alone. A random matrix is a matrix-valued random variable, i.e., the elements are stochastic variables. RMT can be used to study the distribution of eigenvalues under asymptotic assumptions. [Menon 2012a, 2012b, 2012c; Gerstoft 2012]. Here we will use the SCM to extract eigenvalue distributions for representative noise scenarios and this will quantify the information content in the data.

The excellent matching of SCM eigenvalues using RMT is demonstrated using data from a towed horizontal array during the long range acoustic communications (LRAC) experiment from 10:00-11:00 UTC on 16 September 2010 in the NE Pacific in 5-km water depth. Other data periods yield similar results to those shown here. The array was towed at 3.5 knots at a depth of 200 m. The data were sampled at 2000 Hz using a nested array with each configuration having 64 channels

Figure 5 shows the eigenvalues of the SCM at selected values  $\beta$ , the element spacing to wavelength ratio, for the four arrays. Due to the low sampling frequency (2000 Hz), the HF array only can be used

up to  $\beta=1/4$  (1000 Hz). All eigenvalues are based on one-hour observations, meaning that for  $M=64$  the eigenvalues are averaged over 13 SCM eigenvalues. The first few eigenvalues for each SCM likely are due to the distant transiting ships and noise from the towship, as seen in the beamform power vs angle time series. The eigenvalues drop sharply above  $2\beta(N-1)+1$  (vertical dotted line) as predicted by theory, and indicates that the coherent noise is stronger than the incoherent noise. The eigenvalues of the SCM of the LF and ULF arrays show a similar behavior as the MF and HF arrays though with less strong transition between the two eigenvalue regimes.

Figure 5 shows that the eigenvalues depend on  $\beta$ . As  $\beta$  increases, all the eigenvalue spectra become more extended and at  $\beta=0.5$  (half wavelength spacing) the SCM ideally should become diagonal with eigenvalues that approximately are all equal. Comparing the four arrays at  $\beta=1/8$ , first column in Fig. 5 shows that the higher eigenvalue numbers (containing mostly incoherent noise) are relatively larger at low frequencies. At half-wavelength spacing  $\beta=1/2$ , last column in Fig. 5, all eigenvalues remain large for the three arrays, except when using a relatively small number of snapshots ( $M=N$ ).

The observed and modeled noise eigenvalues are compared in Fig. 6. It is important to realize that there is towship radiated noise as well as broadband signatures from several distant ships arriving at the array, especially at low frequencies. These “signals” are among the largest eigenvalues extracted from the data.

## **IMPACT / APPLICATIONS**

Geoacoustic inversion techniques are of general interest for the estimation of waveguide parameters thus facilitating system performance prediction in shallow water. Natural transition paths for these results will be the PEO-C4I Battlespace Awareness and Information Operations Program Office (PMW-120) and the Naval Oceanographic Office.

## **RELATED PROJECTS**

None.

## **PUBLICATIONS**

### **2012:**

Gerstoft, Peter, Ravishankar Menon, William S. Hodgkiss, Christoph F. Mecklenbräuker, Eigenvalues of the sample covariance matrix for a towed array, *J Acoust. Soc. Am.*, 132, 2388-2396.

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Menon, Ravishankar, Peter Gerstoft, William S. Hodgkiss (2012b), Cross-correlations of diffuse noise in an ocean environment using eigenvalue based statistical inference, *J Acoust. Soc. Am.*, 132, DOI:10.1121/1.4754558, Nov 2012. [published, refereed]

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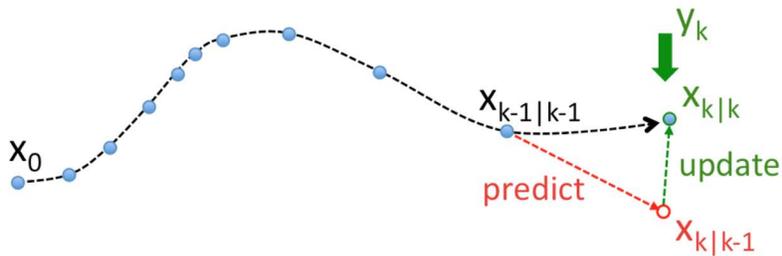
Park, Cheolsoo, Woojae Seong, Peter Gerstoft, and WS Hodgkiss (2011), Fluctuating arrivals of short-range acoustic data, J Acoust. Soc. Am., 129, 98–103, doi:10.1121/1.3514505. [published, refereed]

Traer, James, Peter Gerstoft, and W. S. Hodgkiss (2011), Ocean bottom profiling with ambient noise: a model for the passive fathometer, J. Acoust. Soc. Am., 129, 1825-1836, doi:10.1121/1.3552871, April 2011. [published, refereed]

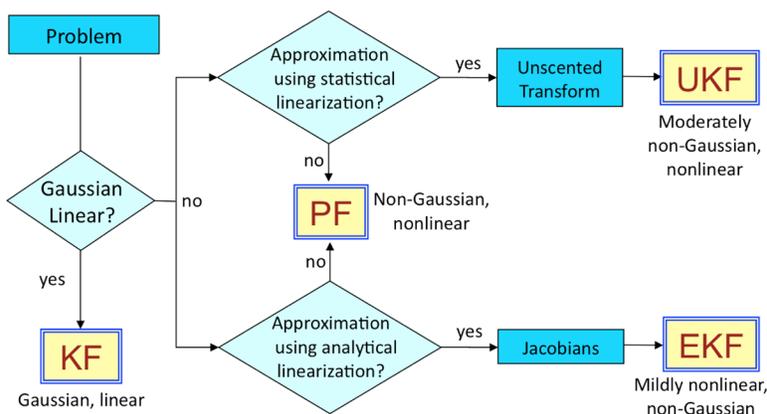
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DOI: 10.1121/1.3654026 [Published, refereed]

Yardim, Caglar, Zoi-Heleni Michalopoulou, and Peter Gerstoft (2011a), An overview of sequential Bayesian filtering in ocean acoustics, IEEE Oceanic Eng 36(1), 73-91, Doi:10.1109/JOE.2010.2098810. [published, refereed]

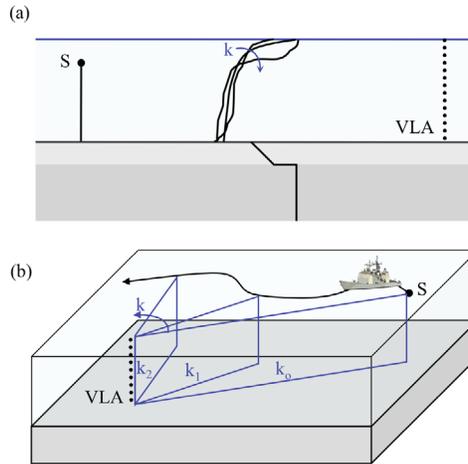
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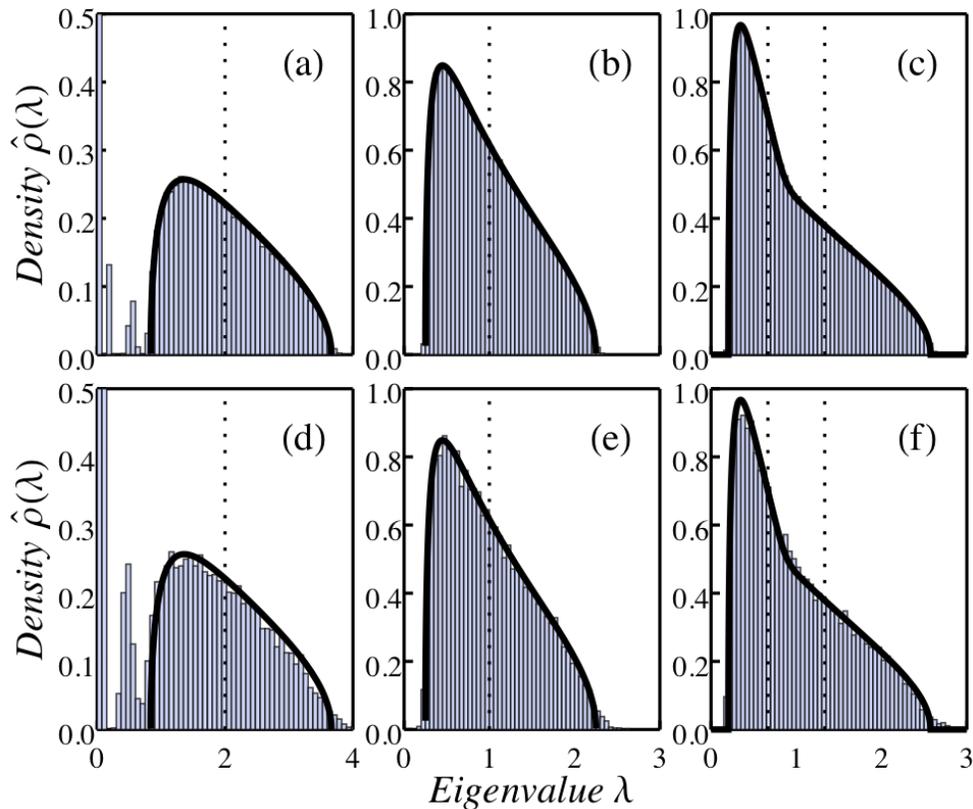
**Figure 1. Sequential Bayesian filtering.** From state  $x_{k-1}$ , state  $x_k$  is first predicted via the state equation, providing  $x_{k|k-1}$ . As data  $y_k$  becomes available, the observation equation is employed to update state  $x_{k|k-1}$ , providing  $x_{k|k}$ .



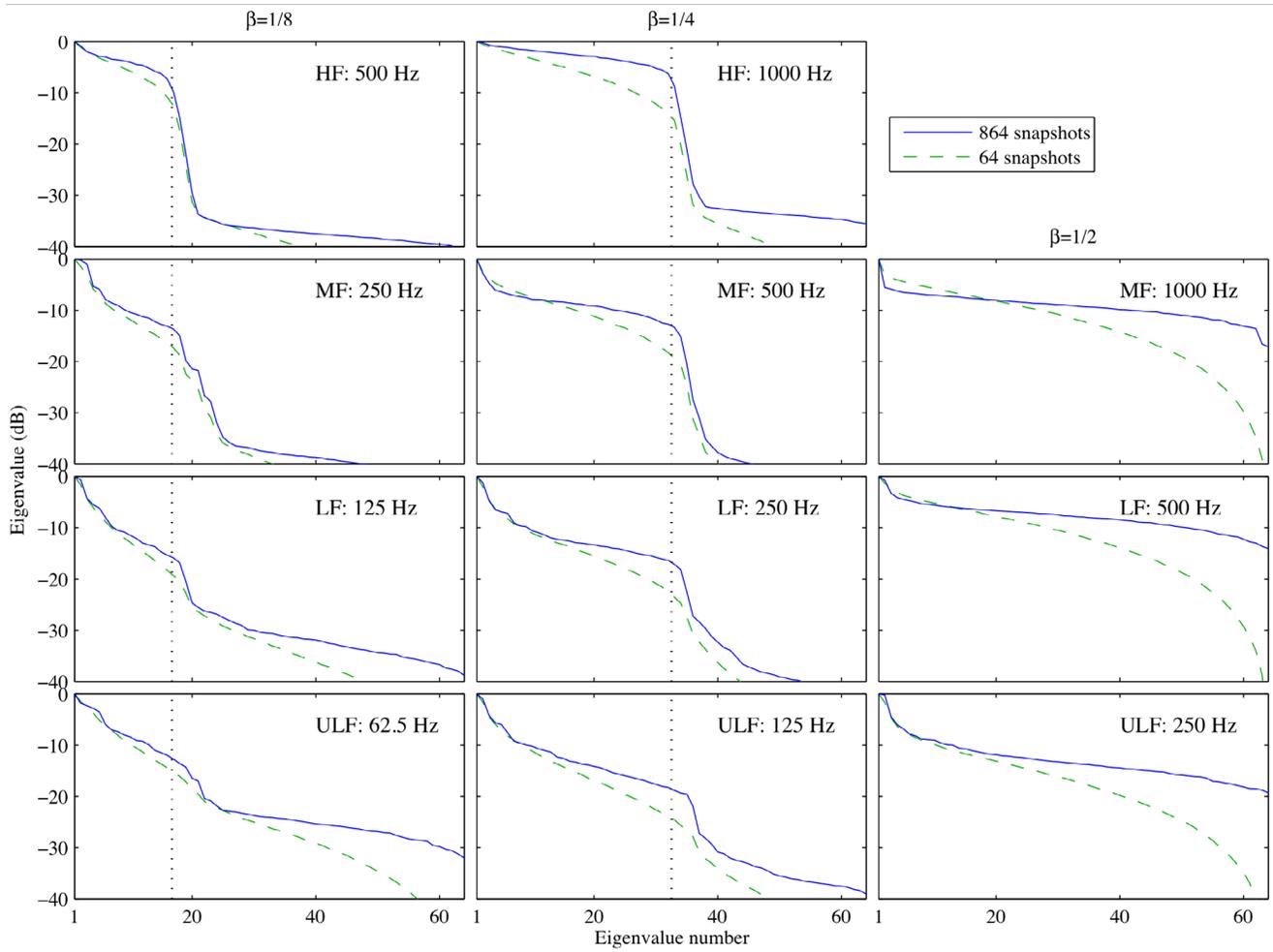
**Figure 2. A quick guide to filter selection leading to the Kalman filter (KF), extended Kalman filter (EKF), unscented Kalman filter (UKF), and particle filter (PF).**



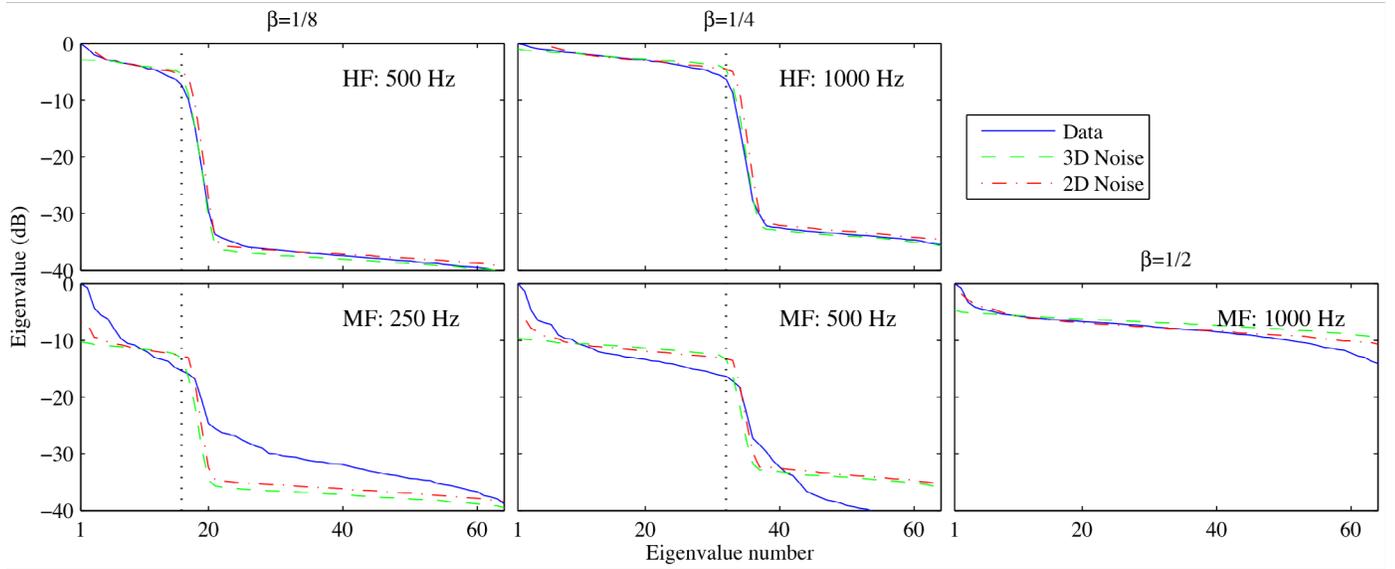
**Figure 3. Geoacoustic environmental tracking: (a) Temporal tracking of the ocean sound speed profile for a fixed-receiver and a fixed-source and (b) tracking of the changing environment between the receiver and a moving source. Here shown for a vertical line array (VLA) of receivers.**



**Figure 4. Asymptotic eigenvalue density (solid line) and the empirical eigenvalue density, with  $N = 100$  array elements,  $v = 1/4$  (ratio of number of sensors to number of snapshots) and spacing to wavelength ratios  $\beta$  of (a) 1/4 (b) 1/2 and (c) 3/4. (d,e,f): Same as in (a,b,c) except with  $N = 20$ . The dotted lines show the locations of the distinct non-zero true eigenvalues. For more information see Menon [2012a].**



**Figure 5. Eigenvalues of the towed array SCM for HF array (1st row,  $\beta=1/8, 1/4$ ), MF array (2nd row,  $\beta=1/8, 1/4, 1/2$ ), LF array (3rd row,  $\beta=1/8, 1/4, 1/2$ ), and ULF array (4th row,  $\beta=1/8, 1/4, 1/2$ ). The eigenvalues are based on 64 ( $\nu=1$ , dashed) and 864 ( $\nu=13$ , solid) snapshots. The eigenvalues are normalized with the largest eigenvalue and the vertical dotted line indicates the edge of the visible region,  $\beta$  is the element spacing to wavelength ratio. For more information see Gerstoft [2012].**



**Figure 6.** Comparison of observed and modeled eigenvalues. Eigenvalues of the towed array SCM (solid) and modeled eigenvalues 3D (dashed) and 2D (dash-dotted) for HF array (1st row,  $\beta=1/8$ ,  $1/4$ ) and LF array (2nd row,  $\beta=1/8$ ,  $1/4$ ,  $1/2$ ). The observed eigenvalues are based on 864 ( $\nu=13$ , solid) snapshots, the modeled eigenvalues use a coherent/incoherent noise ratio 700 for HF array and coherent/incoherent noise ratio 100 for LF array. The eigenvalues are normalized with the largest eigenvalue and the vertical dotted line indicates the edge of the visible region,  $\beta$  is element spacing to wavelength ratio. For more information see Gerstoft [2012].