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Dynamical Systems Theory and Lagrangian Data Assimilation in Geophysical Fluid Dynamics
FY 2013 (October 1, 2012 – September 30, 2013)

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LONG-TERM GOALS

To improve our undestanding and predictive capability for three dimensional, time-dependent ocean circulation.

OBJECTIVES

Our main objective is to develop modeling approaches, theory and observational diagnostics that will allow a better understanding of the role of processes that are intrinsically three dimensional and time varying. We hope to use this knowledge to inform parameterizations and predictions, and also to guide observational strategies, including strategies for autonomous underwater sensors and vehicles.

APPROACH

Our approach centers on the Lagrangian view of ocean circulation; that is, the study of fluid motion through tracking of fluid trajectories. The methods of analysis are based in the mathematical field of dynamical systems.

WORK COMPLETED

We have developed a number of methods for identifying and computing internal boundaries and other coherent features that drift and evolve with ocean currents and that separate qualitatively different regions. These structures provide a template for understanding fluid pathways, mixing, and transport processes in 3D. These methods have been tested using a variety of models ranging from idealized to realistic. We have also developed diagnostics that indicate when two-dimensional analysis suffices. Another major thrust involves the development of methods for assimilating Lagranigan data into ocean models.

RESULTS

We note that our program was the subject of a 3-year review at ONR in May of this year. All presentations from that meeting can be downloaded from the our MURI program website:

http://www.whoi.edu/ocean3dplus1/

under MURI meetings.

New methods for computing barriers and other Lagrangian coherent structures (LCS) include the ergodicity defect (a measure of trajectory complexity), Koopman Operator techniques (which rely on properties measured along trajectories), techniques that identify regions of high hyperbolicity (corresponding to rapid fluid stretching and folding), and methods that calculate the rate of separation of fluid from it neighboors. Examples are depicted in Figures 1-5. All have been proved capable of identifing key Lagrangian structures in the flow field and the choice of a particular method will depend on the nature of the model or observation information available We have also made progress in identifying the kind of barriers that can exist in fully 3D, time-dependent flows (Figures 5-7 and 9). Our serarch for barriers is being extended to more realistic 3D+1 simulations (various figures).

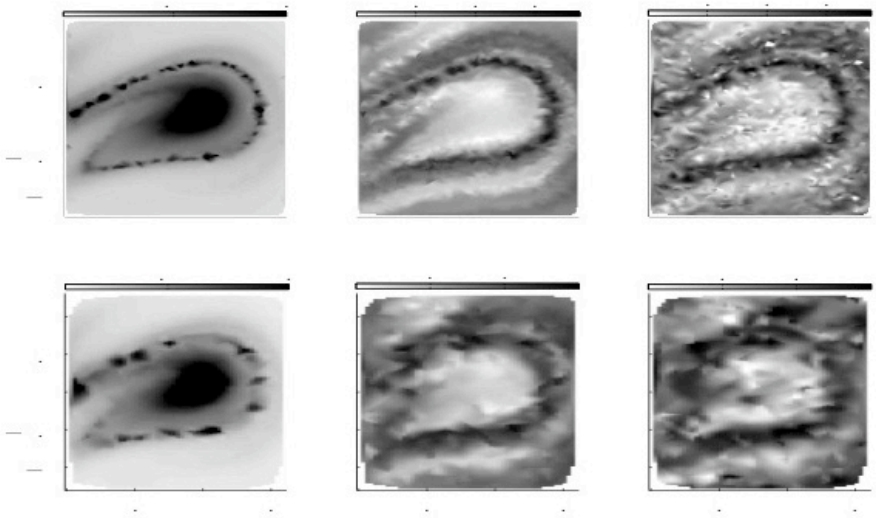

Figure 1. A method for calculation of LCS using measures of trajectory complexity: work by Rypina, Pratt, Scott, and Brown.

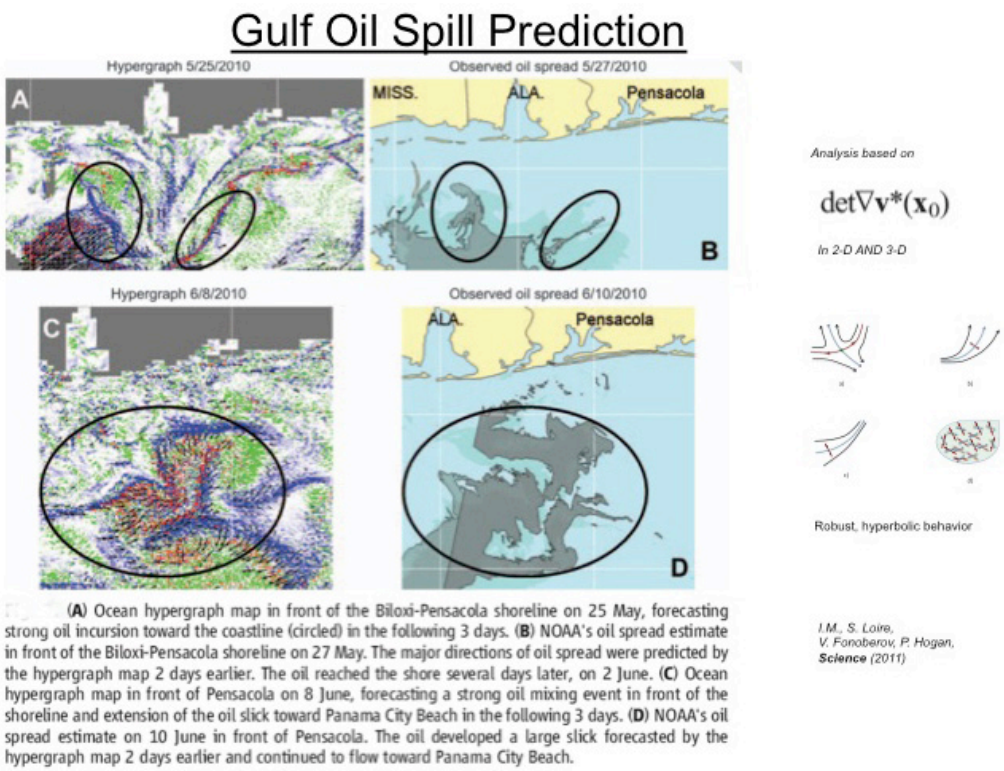

Figure 2. Mesohyperbolicity identifies areas of strong filamentation (Mezic).

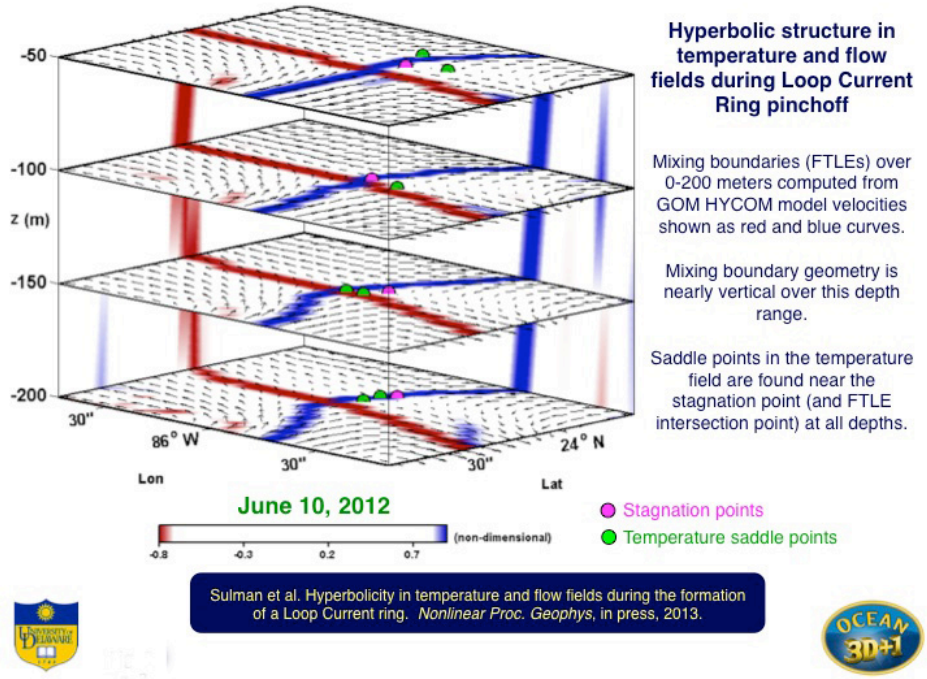

Figure 3. Use of mesohyperbolicity to identify barriers and strongly stirred regions.

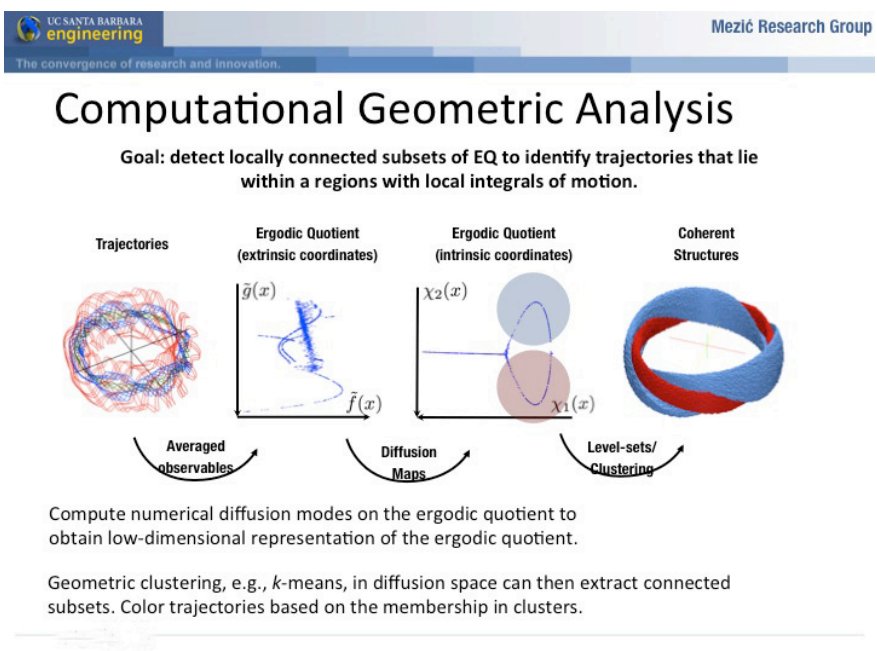

Figure 4. The Ergodic Quotient is based on Lagrangian averages of scalar quantities.

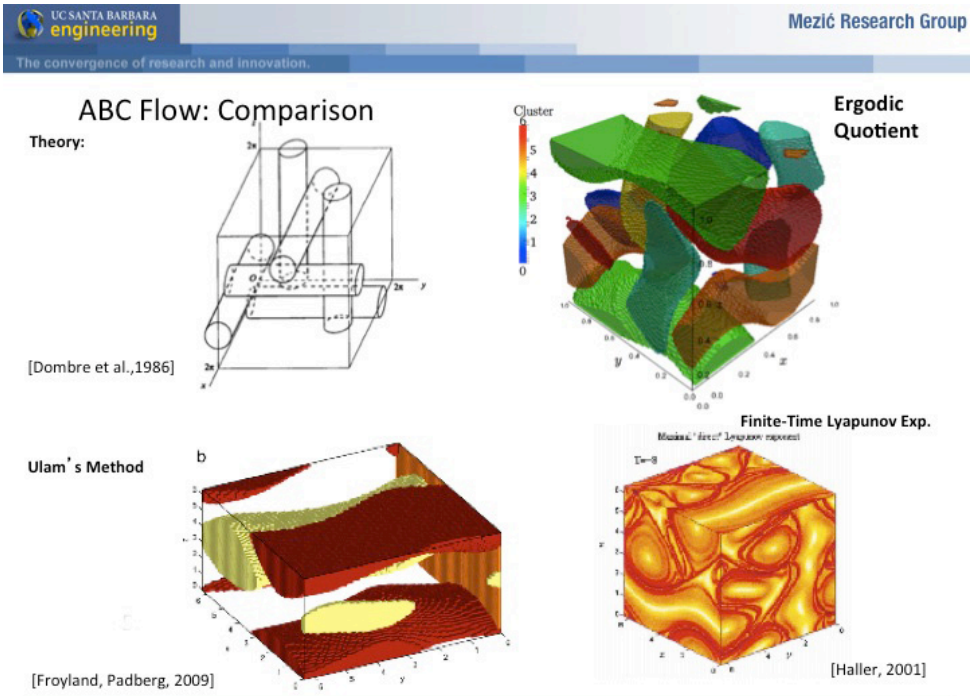

Figure 5. Comparison with known examples such as ABC flow is favorable.

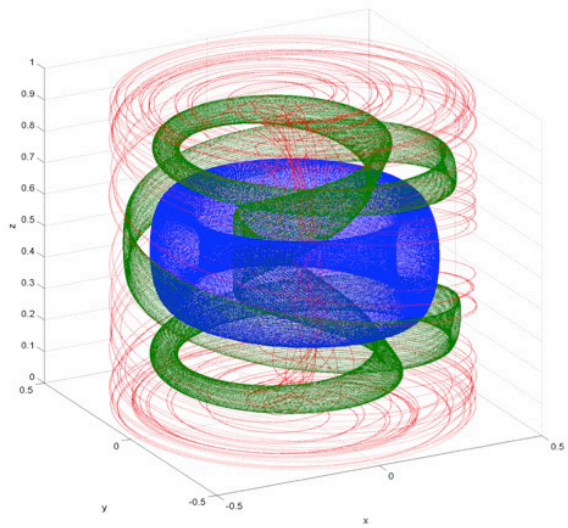

Figure 6. Idealized models of fully 3D eddy circulations tell us what Lagrangian barriers look like in 3D. In this case, the barriers consist of tori, including twisted hula hoops. Sandwiched between the surfaces are bands and islands of chaos. (From Pratt, Rypina, Ozgokmen, Wang, and Childs. JFM 2013.

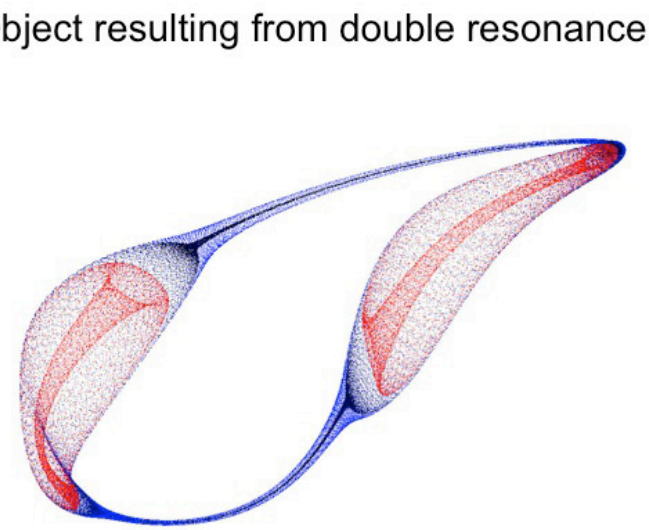

Figure 7: When the flow is fully 3D and time-dependent, the barriers become exotic. In this case the cocoon-like structure pulsates and rotates in the eddy.

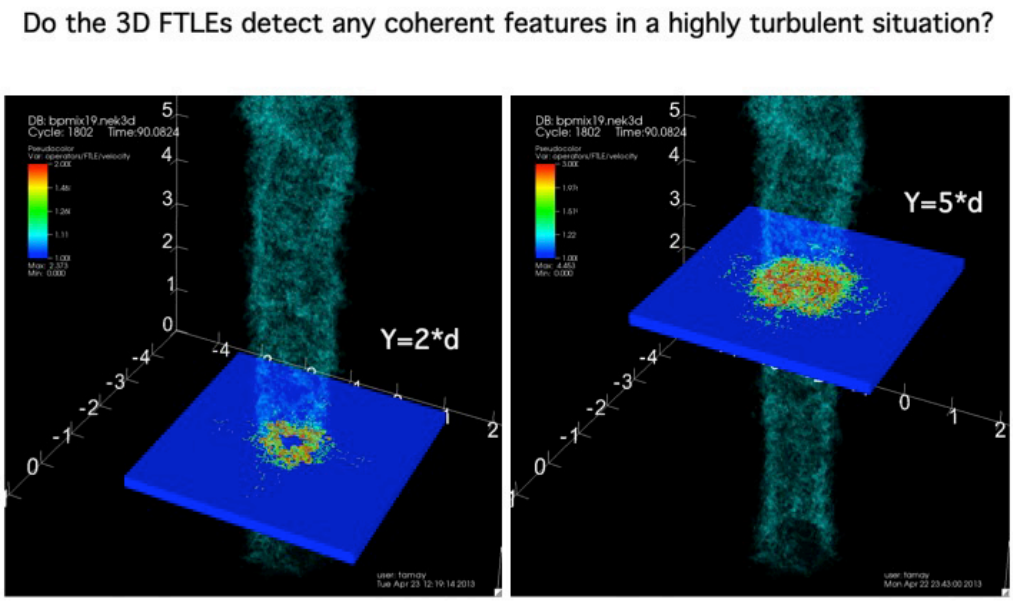

Figure 8: The methods used to compute LCS also provides information about changes in the stirring distribution in the cross section of a fully 3D turbulent rising oil plume. (Ozgokmen)

Our team is also exploring the application of LCS methodology in connection with other realistic models. This work emphsizes the role of the submesoscale (Figs. 10-14), which is commonly unresolved in ocean models.

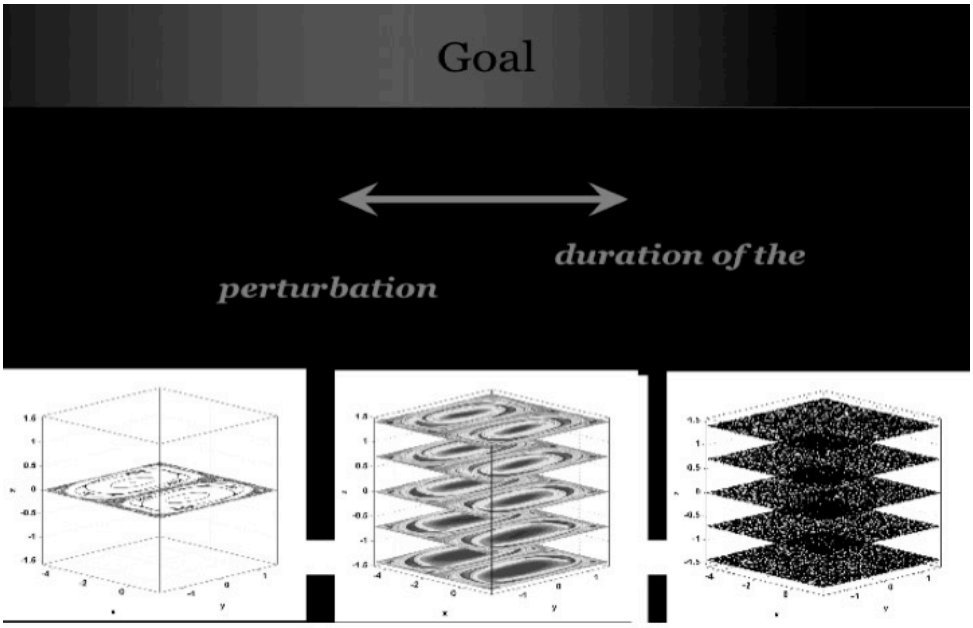

Figure 9. We can get a path of understanding into fully 3D, time dependent systems but looking a weakly 3D systems for long time intervals. (Llewellyn Smith)

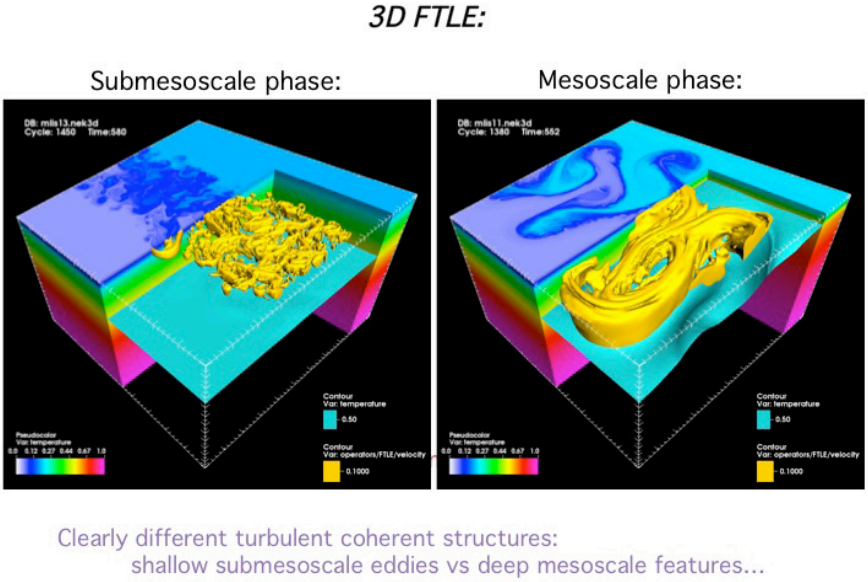

Figure 10. We are exploring the effects of submesoscale motions (left), which are unresolved in most models. (Ozgokmen)

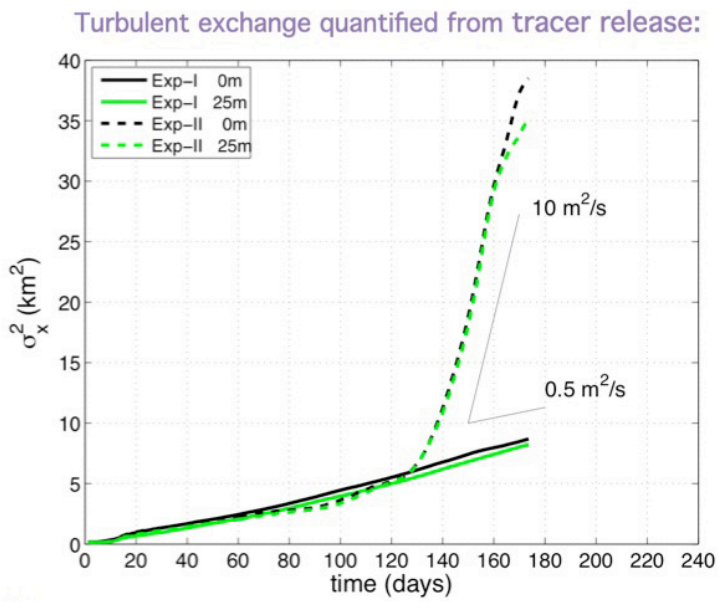

Figure 11. An order of magnitude increase in frontal turbulent diffusivity after the deep instability kicks in; submesoscale overcome by mesoscale in this case.

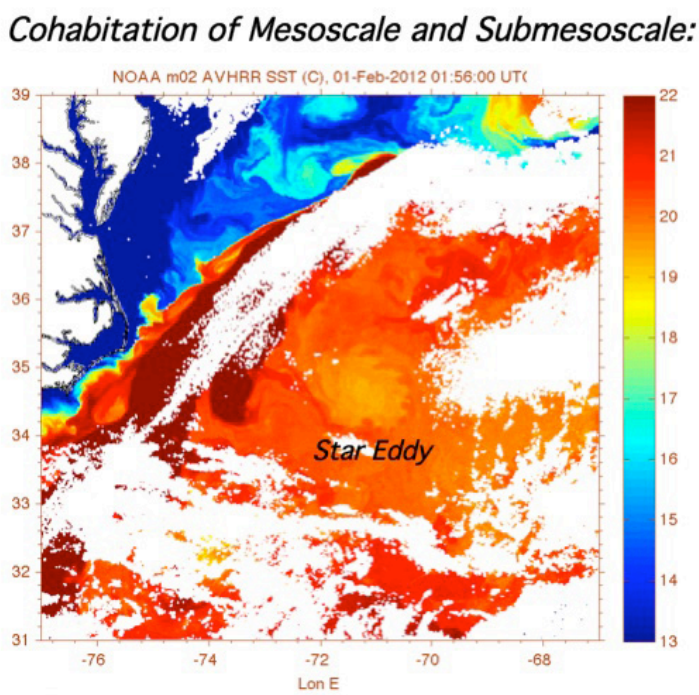

Figure 12. The instabilities that cause star eddies reside in the submesoscale (Ozgokmen).

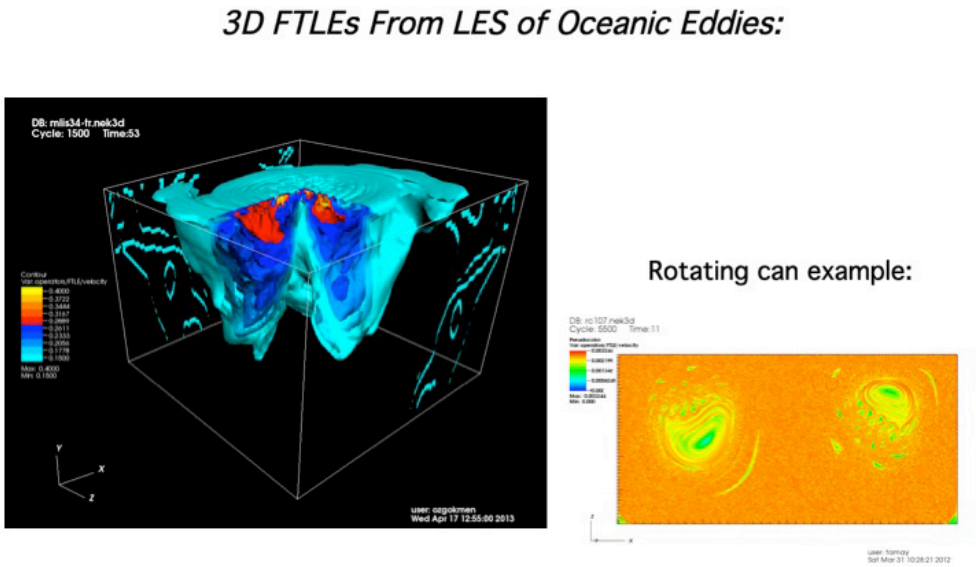

Figure 13. We are searching for barriers in realistic 3D, time-dependent eddies. Do we see generalizations of the tori found in idealized models? (Ozgokmen, Pratt, Rypina, and Wang.)

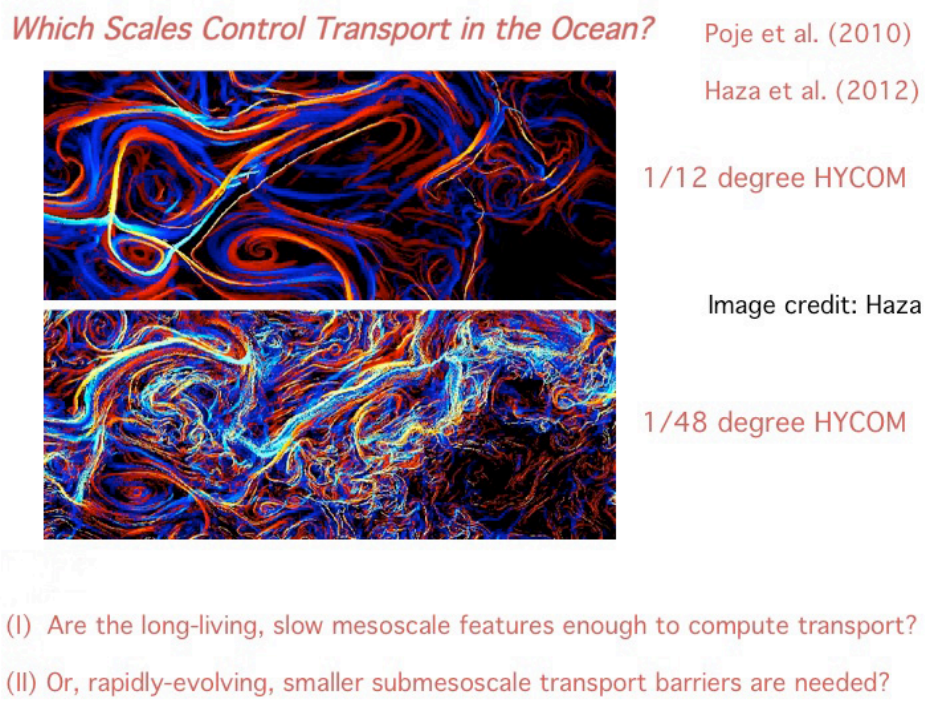

Figure 14. Comparison between LCS are different model resolution. The lower panel resolves the submesoscale.

In the area of Lagrangian Data Assimilation (Fig. 15), we are developing methods that are more efficient and accurate, and that work in 3D. This work attempts to use information obtained from LCS analysis in order to determine where to best lauch floats and drifters and to optmize navigation strategy for autonomous underwater vehicles Fig. 16.

Augmented system

Append equations for drifters (floats, gliders, AUVs)

Ide, Jones and Kuznetsov (2002)

$$\mathbf{x} = \begin{pmatrix} \mathbf{x} \\ \mathbf{x}_D \end{pmatrix} \quad \text{-- augmented state vector}$$

$$\frac{d\mathbf{x}_F^{\mathrm{f}}}{dt} = M_F(\mathbf{x}_F^{\mathrm{f}}, t) \quad \text{-- flow equations}$$

$$\frac{d\mathbf{x}_D^{\mathrm{f}}}{dt} = M_D(\mathbf{x}_D^{\mathrm{f}}, \mathbf{x}_F^{\mathrm{f}}, t) \quad \text{-- advection equation}$$

Apply filtering to augmented system:

1. Ensemble Kalman Filter
2. MCMC (Particle Filter)

Figure 15. The general ideal behind Lagrangian data assimilation work of Jones, Spiller and colleagues.

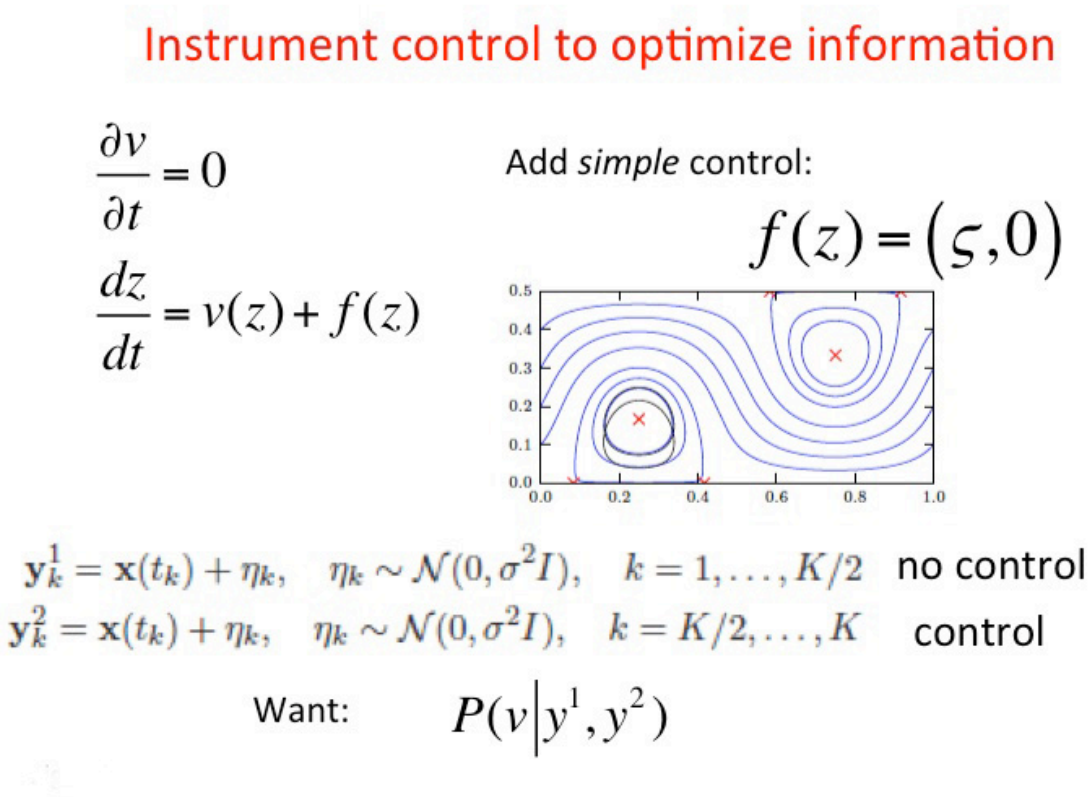

Figure 16. Is it possible to optimize navigation strategies for AUV's in order to collect the best data for LDA? (Jones, Spiller and colleagues.)

IMPACT/APPLICATIONS

The Lagrangian view offers insights into transport and mixing processes that are different from those gained in the Eulerian frame. These insights naturally lead to better physics-based parameterizations, such as turbulent eddy coefficents. (One of the criticisms of Eulerian-based parameterizations is that they depend only on the local properties of the flow and do not consider patches of turbulence that are advected into the local.) Lagrangian methods can also identify time-dependent barriers that may be important in predicting regions that polutants, including oil and radioactive material, have difficulty crossing.

The Lagrangian approach to data assimilation will, in principle, allow information that has been previoiusly discarded to be used in prediction and state estimation. In particular, previous assimilation schemes that use sequences velocity measrments from drifting sensors do not assimilate the information that each measurement comes from the same trajectory.

TRANSITIONS

The tools being developed will aid in the prediction of catestrophic release of oil, radioactive material or floating debree into the ocean.

RELATED PROJECTS

A British Petrolium-funded project on the Gulf of Mexico oil spill. Prof. Tamay Ozgokmen is involved along with several other PIs.

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