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Modeling Water Waves with Smoothed Particle Hydrodynamics

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LONG-TERM GOALS

Smoothed Particle Hydrodynamics (SPH) is a meshless numerical method that is being developed for the study of nearshore waves and other Navy needs. The Lagrangian nature of SPH allows the modeling of wave breaking, surf zones, ship waves, and wave-structure interaction, where the free surface becomes convoluted or splash occurs.

OBJECTIVES

To improve the ability of the meshfree Lagrangian numerical method Smoothed Particle Hydrodynamics (SPH) to be a useful hydrodynamics model for breaking waves and the nearshore zone, particularly for case where spray and splash are important. To utilize the massively parallel graphics processing units on computers to develop the GPU-accelerated model GPUSPH to solve a number of problems relevant to the U.S. Navy. The science objective is to be able to accurately model the complex flows associated with breaking water waves, including instantaneous motions as well as (time-averaged) wave-induced flows, such as undertow, longshore currents, and rip currents.

APPROACH

The approach is based on improving various aspects of the SPH code, including the use of a multi-machine/multi-graphics processing unit (GPU) version of the open source code (GPUSPH) to explore its ability to model phenomena in the nearshore zone; and to examine new capability of the model by applying it to different situations relevant to Navy needs.

WORK COMPLETED

- Open source version of GPUSPH 3.0 released on the web: June 3, 2014. A new website was established as well: www.gpusph.org
- The GPUSPH model is now multi-machine, multi-GPU, allowing for many millions of particles to be used in simulations for higher definition or for larger domains.

- GPUSPH has been shown to model waves in the nearshore zone, including wave refraction, diffraction, shoaling, wave-current interaction. Wave-induced currents and radiation stresses in the waves have been validated for bathymetrically-controlled nearshore circulation systems.
- Validation of tsunami propagation and breaking (solitary wave) with GPUSPH has been done.
- Turbulence coherent structures under spilling breaking solitary waves shown to be source of obliquely descending eddies.
- GPUSPH is shown to reproduce nonlinear water wave properties and exhibits nonlinear resonant interactions, such as the generation of subharmonic edge wave by an incident wave field and harmonic generation in shallow water.

RESULTS

Open source release—the GPUSPH model, version 3.0, was released as open source in June of this year after a week long programming charette with some of the code developers (Alexis Hérault, Giuseppe Bilotta, Eugenio Rustico, Arno Meyerhofer, R. Dalrymple) in Paris, France. The code, in CUDA C++, with test problems, is available at www.gpusph.org

Breaking waves—the ability of GPUSPH to model breaking waves in the surf zone is well established, but the details of breaking remain an area of research as there is a considerable amount of coherent turbulence produced that affects the flow field and the sediment transport. These coherent turbulent structures include rollers and obliquely descending eddies. The rollers and plunging breaker tubes have vorticity, with the flow rotating about a horizontal line parallel to the shore. On the other hand, an obliquely descending eddy has the vorticity oriented nearly vertically. The scientific question is how does this change in vorticity happen? To examine this, we modeled the laboratory experiments of a solitary breaking wave by Ting (2006). The advantage of a solitary wave is that there is no pre-existing turbulence in the water when the wave breaks. We replicated his wave tank and used a numerical wavemaker with a paddle motion given by Goring & Raichlen (1980) for the generation of solitary waves, just as in the laboratory. Figure 1 shows a comparison between the GPUSPH modeled wave heights versus the measured laboratory wave heights in the laboratory at different gages, showing excellent agreement.

The numerical model consisted of about 7 million SPH particles of diameter 0.007 m and the particle positions and velocities were saved at fixed time intervals and then converted from their Lagrangian positions to a fixed Eulerian grid for the purposes of spatial and temporal averaging and examining coherent structures. Using the gridded values for velocity, the lambda-2 method for detecting vortices was used. In Figure 2, the solitary wave is breaking as a spilling breaker (moving to the right) and leaving behind horseshoe vortices in the water column. Examining a plane across the wave tank that intercepts a horseshore vortex shows that the two legs of the vortex look like a counter-rotating vortex pair. With time the horseshore vortices descend into the water column towards the bottom. It appears that a horshore vortex with its two legs are responsible for what has been called to-date 'obliquely descending eddies.' In the lower panels of Figure 2, the vorticity, velocities, and turbulent energy associated with the vortex is shown.

Tsunami Wave—a canonical experiment for tsunamis was conducted at the Tsunami Basin at Oregon State University by Patrick Lynett's student, D.T. Swigler (2009). The experiment consists of generating a solitary wave that shoals on a submerged coastal promentory that also has a small island. One of the difficulties we discovered was a shortcoming of the Goring generation method for solitary waves—that is, there is a diminution of wave height with distance as the wave is not constant in form. This did not affect the turbulence modeling discussed above as both the laboratory and GPUSPH used Goring's method. To correct this problem, we use the Rayleigh solitary wave solution of Guizien and Barthélemy (2002) to generate the wave. The new method results in very little loss of amplitude in the initial stage of the wave propagation.

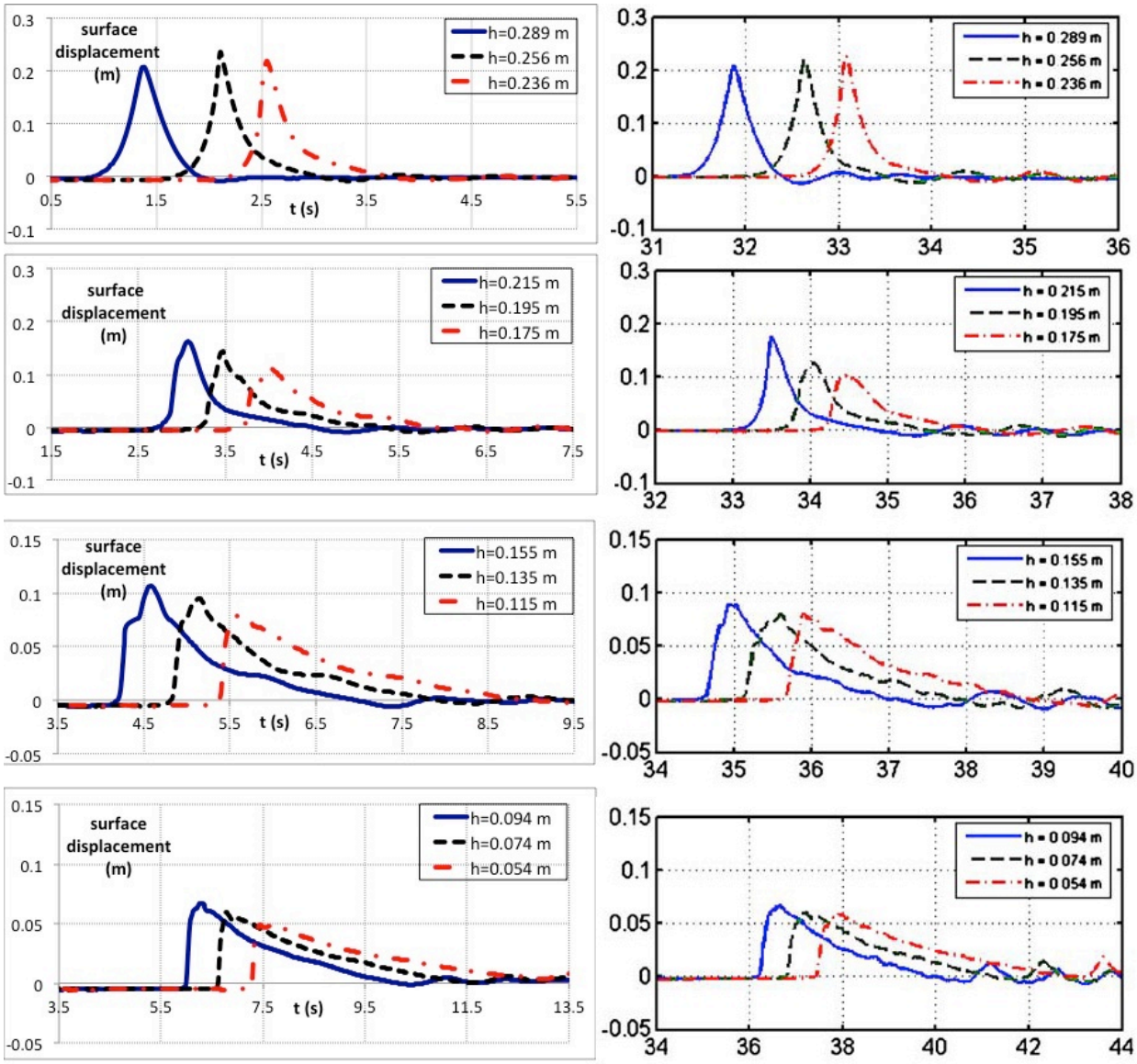

Figure 1. Measured water surface elevations with time as predicted by the GPUSPH model (left column) and as measured by Ting (2006) (right column) at different locations in the wave tank. The various wave gages are at locations with different water depths. Note there is an arbitrary 31 s time shift between the models.

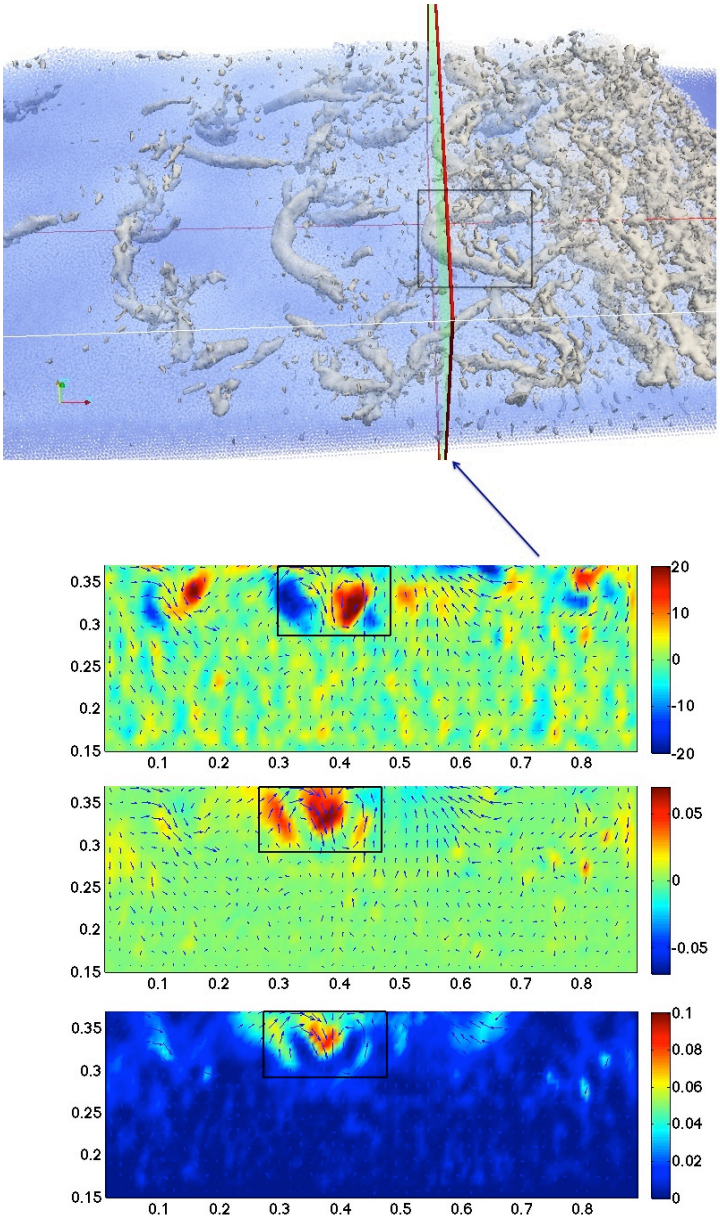

Figure 2 Top: Reversed horseshoe structures under a solitary wave at time = 4.7 s detected by λ_2; bottom: (a) streamwise vorticity profile and turbulent velocity vectors (v', w') over a vertical plane at the location of x = 8.07 m, (b) turbulent momentum flux (−u'w') over the same vertical plane, (c) turbulent kinetic energy over a vertical plane.

The top panel of Figure 3 presents the numerical tsunami wave overtopping the island. Due to shoaling there is a steep wavefront. As the wave passes by the island, it continues to propagate and then runs up the beach as shown in the bottom panel of Figure 3. Overall, this test reveals the great potential of GPUSPH to predict the behavior of tsunamis in the nearshore zone.

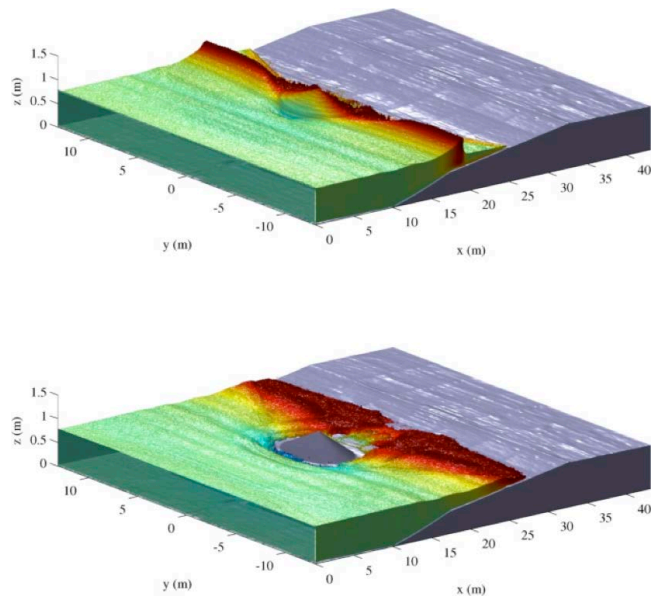

Figure 3. Two instances in time for an SPH tsunami impinging on a coastline with the conical island (Swigler bathymetry). The top frame shows runup on the island, while the bottom frame shows the wave after passing the island and running up on the shoreline.

Edge waves—since the 1970's, we have known about edge waves in the surf zone; however most analyses of edge waves are predicated on small amplitude assumptions. This year the Instituto Hidráulica Ambiental (IHC) of the University of Cantabria, Spain, carried out experiments on subharmonic generation of edge waves on a steep (1:5) beach as part of the ANIMO project, lead by Dr. Giovanni Coco. The mechanism, as elucidated by Guza and Davis (1974), involves a three-wave interaction, consisting of the incident wave with period T and two subharmonic edge waves (period 2T) that create a standing

edge wave system on the beach. The question was whether the SPH method, with its millions of particles moving according to Navier-Stokes equations would indeed model the three-wave resonance and the subsequent highly nonlinear motion that occurred in the IHC tank, which was initiated by simply generating a normally incident wave train. The edge waves spawn and grow via the instability. As shown in Figure 3, which is a top view of the tank, with the beach at the top of the figure, strong nonlinear cusp behavior occurs. Note the color denotes the magnitude of the velocity with red being the largest. This proves additional evidence that the SPH method can treat nonlinear instabilities correctly.

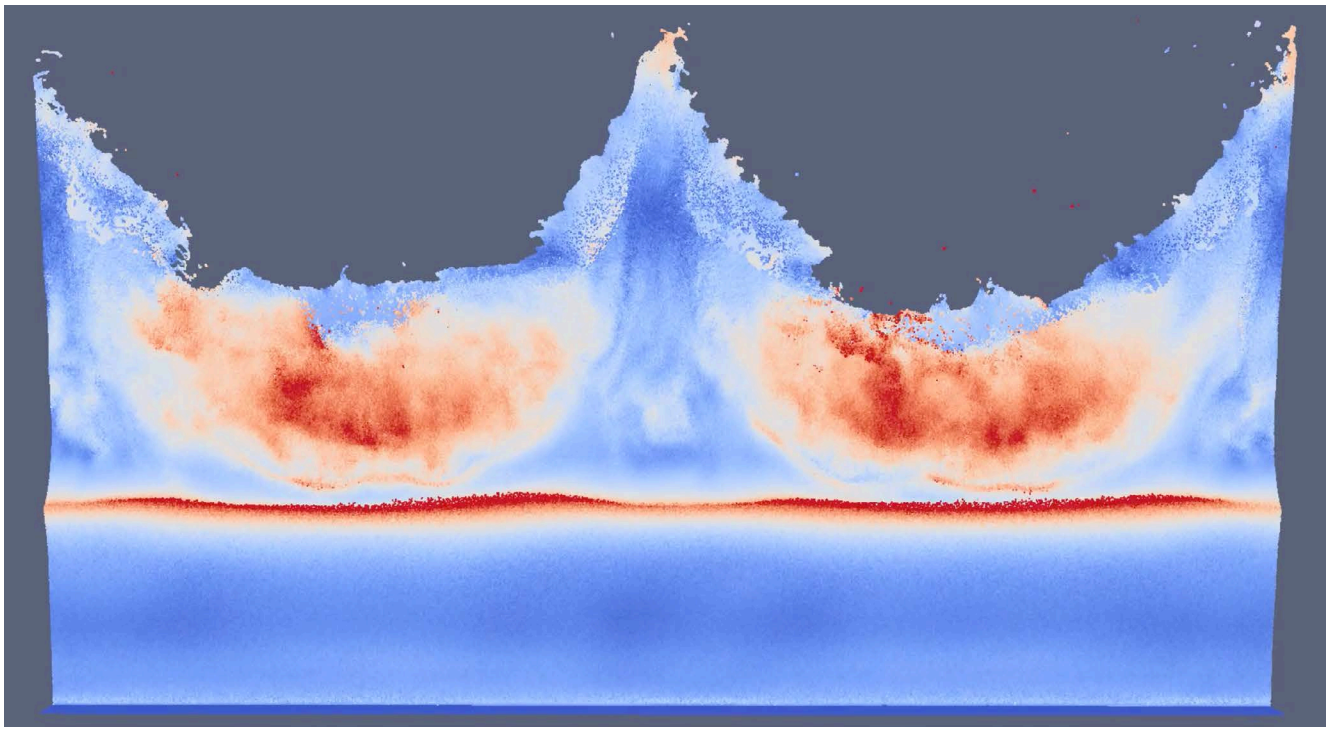

Figure 4. Nonlinear edge wave created by subharmonic resonance. Plan view: beach is to the top, wave maker to the bottom, and red denotes large velocity magnitude. The horizontal red line is an incoming wave crest.

Rip currents generated by short-crested wave breaking--Rip currents can be generated by various mechanisms, which can be grouped into two categories: wave-wave interaction and wave-structure interaction (Dalrymple, 1978). Jalali Farahani et al. (2013) showed that GPUSPH is able to predict rip currents generated by wave-structure interaction by comparing the model with experiments involving a 'sand bar/rip channel' bathymetry. Here, we apply GPUSPH to rip currents generated by wave-wave interaction. Dalrymple (1975) showed that two intersecting trains of waves of the same wave period but different directions would create nodal lines of zero wave amplitude, which do not move with time. These lines of zero wave amplitude are the location of the rip currents, which are forced by the high mean water levels that occurs where the incident waves reinforce in the surf zone.

In a numerical wave tank with a planar sloping beach, two intersecting trains of waves of the same wave period and amplitude but equal and opposite incident directions are generated offshore using a

numerical directional wavemaker comprised of numerous individual paddles. The spacing of rip currents is specified to be equal to the tank width; as a result, one nodal line is formed in the middle of the tank that is perpendicular to the beach. The incident wave height was chosen to create spilling breakers as shown in the top panel of Figure 5. In the very nearshore zone, the wave crests display a honeycomb pattern emblematic of intersecting nonlinear shallow water waves, with the major wave crests connected by short and smaller straight crest segments. The wave-induced rip current is seen in the phase-averaged velocity field shown in the bottom panel of Figure 3. Note that there is a change of width of the rip currents; it is narrow inside the surf zone and much wider outside; secondly, rotational motions are generated by short-crested wave breaking, and the momentum is transferred from incident high frequency waves to the rotational motions with a lower frequencies. This observation was also reported by other work (e.g., Clark et al., 2012). More effort to examine the nature of vortex forces and short crestedness as a mechanism for vortex generation is underway.

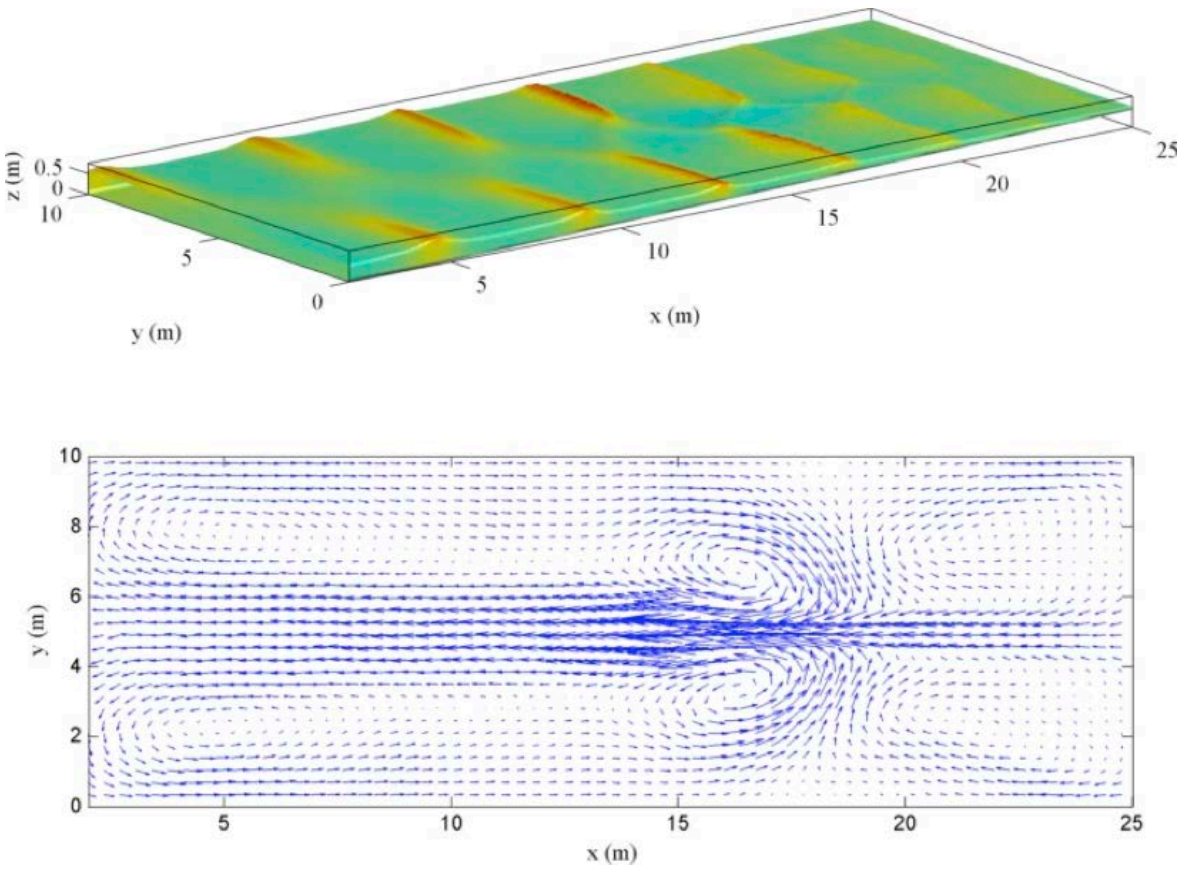

Figure 5. Numerical results of rip currents generated by short-crested wave breaking. Top panel: snapshot of wave field (Note: the waves are colored by the cross-shore velocity); bottom panel: rip current system (phase-averaged velocity) under short-crested wave breaking.

International Collaborations: We currently have a strong international collaboration for the development of GPUSPH, including the Istituto Nazionale di Geofisica e Vulcanologia (sezione di Catania), Italy; Conservatoire National des Arts et Métiers, Paris, France, for the development of GPUSPH. Dr. Hérault (CNAM and INGV), Dr. Bilotta (INGV), and Dr. Eugenio Rustico (BAW) are members of this collaboration. ATHOS, the consortium to support this GPUSPH collaboration,

established in October 2011, consists the original GPUSPH developers and EDF R&D (the research and development division of Electricité de France) and the Bundesanstalt für Wasserbau.

Improvements have been made in GPUSPH by the international collaborators this year that we are taking advantage of. Multi-GPU/multi-CPU code is now fully functional and available (Rustico et al., 2014) Numerical accuracy has been improved by utilizing a cell-based coordinate system as opposed to a single coordinate system, referred to as homogeneous accuracy—this is particulary important for problems with large aspect ratios (river flow or long wave tanks for example), Hérault et al. (2014), and finally new boundary conditions have been implemented that provide better accuracy near the boundary and improved prediction of hydrodynamic forces.

Ongoing efforts—Currently we are doing a number of validation tests with high numbers of particles to examine the range of and accuracy of the problems we can do. We need to do comparisons to the IHC edge wave data sets as well as rip current data sets that currently exist. We are continuing to determine the nonlinear effects modeled in GPUSPH; for example, we are now solving for vortex force as an alternative to radiation stresses for nearshore modeling. We are also comparing a number of experimental data sets with tsunamis to look at the longwave modeling capabilities of GPUSPH. Comparisons to field data are being planned.

IMPACT/APPLICATIONS

Smoothed Particle Hydrodynamics is proving to be a competent modeling scheme for free surface flows in two and three dimensions. As the GPU hardware improves, it is expected that the resolution of SPH will increase tremendously bringing the modeling into realistically sized domains. Further, GPUs are becoming more powerful on smaller machines, such as tablets. It is expected that this methodology will become more powerful and portability in the near future.

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