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Three-Dimensional Shallow Water Acoustics

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LONG-TERM GOALS

Both physical oceanographic processes and marine geological features in continental shelves and shelfbreak areas can cause lateral heterogeneity in medium properties, so horizontal refraction of sound can occur and produce significant three-dimensional (3-D) sound propagation effects. The long-term goals of this project are targeted on understanding the 3-D acoustic effects caused by the environmental factors existing commonly in the continental shelf and shelfbreak areas, such as slopes, submarine canyons, surface waves, internal waves and shelfbreak fronts.

OBJECTIVES

One of the research objectives in this project is to develop efficient and accurate 3D acoustics models for studying underwater sound propagation in complex ocean environments. The ultimate scientific objective is to study the underlying physics of the 3-D sound propagation effects caused jointly by physical oceanographic processes and geological features. To achieve this goal, individual environmental factor will be first studied and then considered jointly with a unified ocean, seabed and acoustic model. Another major objective is to develop a tangent linear model to predict acoustic fluctuations due to 3-D sound speed perturbation in the water column. This tangent linear model will also be used for sensitivity analyses to assess the joint ocean and seabed effects.

APPROACH

The technical approaches employed in the 3D sound propagation study include theoretical analysis, numerical computation and real data analysis. A 3-D normal mode method has been used to study canonical environmental models of shelfbreak front systems [1] and nonlinear internal wave ducts [2-3]. 3-D parabolic-equation (PE) wave propagation models with improved split-step marching algorithms [4-6] are used to study sound propagation in realistic environments. When the acoustic mode coupling can be neglected, a vertical-mode horizontal-PE model is used.

The parabolic-equation (PE) approximation method, first introduced by Tappert [1] to underwater sound propagation modeling, has long been recognized as one of the most efficient and effective numerical methods to predict sound propagation in complex environments. The advantage of this method is due to the fact that it converts the Helmholtz wave equation of elliptic type to a one-way wave equation of parabolic type. The conversion allows efficient marching solution algorithms (see Figure. 1) for solving the boundary value problem posed by the Helmholtz equation. This can reduce

significantly the requirement for computational resources, especially for modeling three-dimensional (3-D) sound propagation.

A higher order numerical algorithm has recently been proposed to split the square-root Helmholtz operator employed by the PE method for modeling sound propagation. This operator splitting method includes multidimensional cross terms to yield a more accurate approximation, and, most importantly, it still permits efficient 3-D PE numerical solvers, such as the Split-Step Fourier method [5] and the Alternative Direction Implicit (ADI) Padé method [6]. The higher order operator splitting algorithm is shown in the next equation:

$$\begin{aligned}\sqrt{k_{ref}^{-2}\nabla_{\perp}^{2}+n^{2}} &= \sqrt{1+\varepsilon+\mu} = -1+\sqrt{1+\varepsilon}+\sqrt{1+\mu}-\frac{1}{2}\left(-1+\sqrt{1+\varepsilon}\right)\left(-1+\sqrt{1+\mu}\right) \\ &\quad -\frac{1}{2}\left(-1+\sqrt{1+\mu}\right)\left(-1+\sqrt{1+\varepsilon}\right),\end{aligned} \tag{1}$$

where $k_{ref} = \omega/c_{ref}$ is the reference wavenumber, n is the index of refraction $n = c_{ref}/c = k/k_{ref}$, and $\nabla_{\perp}^{2}$ denotes the Laplacian operating on the transverse coordinates. There are two different ways to split the square-root operator in (1): in the split-step Fourier PE, they are the free-space propagator and the medium phase speed anomalies, respectively [5]; whereas the split-step Padé PE, they are the derivations in the horizontal and vertical directions [6].

Another numerical method to be used is a tangent linear model to predict acoustic fluctuations due to 3-D sound speed perturbation in the water column. This model has been developed and published [8] last year, and it is briefed here along with an application to determine the sensivitiy kernel of sound pressure variability due to changes in the index of refraction of the medium. Consider the following one-way parabolic wave equation,

$$\frac{\partial}{\partial x}u(x,y,z) = ik_{\mathrm{ref}}\left\{-1+\sqrt{k_{\mathrm{ref}}^{-2}\nabla_{\perp}^{2}+n^{2}(x,y,z)}\right\}u(x,y,z), \tag{2}$$

where u is the demodulated sound pressure with the baseline phase removed according to the reference wavenumber k_{ref}, i.e., $u = p\exp(-ik_{\mathrm{ref}}x)$, and n is the index of refraction with respect to k_{ref}. Note that the exact parabolic-equation (PE) operator consisting of the square-root Helmholtz operator is

$$\mathcal{L} = -1+\sqrt{k_{\mathrm{ref}}^{-2}\nabla_{\perp}^{2}+n^{2}(x,y,z)}. \tag{3}$$

Now, let

$$n^{2}(x,y,z) = \gamma_{0}(x,y,z)+\varepsilon\gamma_{1}(x,y,z), \tag{4}$$

where γ_0 is the square of the index of refraction of the background state, and γ_1 is a perturbation scaled by an arbitrary small parameter ε. The background PE operator is

$$\mathcal{L}_{0} = -1+\sqrt{k_{\mathrm{ref}}^{-2}\nabla_{\perp}^{2}+\gamma_{0}(x,y,z)}, \tag{5}$$

and there are various approximations made for $\mathcal{L}$ with respect to the perturbation of the index of refraction γ_1. Hursky et al. [9] and Smith [10] have showed the tangent linear operators for the standard narrow-angle and wide-angle PE's. In this project, the higher-order operator splitting algorithm (1) is used to derived a higher-order tangent linear operator that unifying previous formula:

$$\mathcal{L}_3 = \mathcal{L}_0 + \frac{1}{2}\left[(1-\mathcal{L}_0)\left(-1+\sqrt{1+\varepsilon\gamma_1}\right) + \left(-1+\sqrt{1+\varepsilon\gamma_1}\right)(1-\mathcal{L}_0)\right]. \tag{6}$$

The corresponding higher-order tangent linear PE solution with $\mathcal{L}_3$ is:

$$u(x+\Delta x, y, z) \cong e^{ik_{\mathrm{ref}}\Delta x \mathcal{L}_0}\left[1 + \frac{ik_{\mathrm{ref}}}{2}\Delta x(1-\mathcal{L}_0)\varepsilon\gamma_1\right]u(x,y,z), \tag{7}$$

where $\mathcal{L}_0$ and γ_1 are assumed to be commutative.

Lastly, in this year the square-root Helmholtz operator splitting method has been applied to PE models to handle 3D acoustic forward scattering from non-planar sea surface, and a numerical implementation employing the Step-step Padé PE method will be shown in the next section.

WORK COMPLETED

The tasks completed in the year are described below.

1. 3-D sound pressure sensitivity kernel

From the higher-order tangent linear PE solution (7), we can deduce the following local tangent kernel to determine the gradient of the sound pressure with respect to n^2 at a given position $\vec{x}'$.

$$\frac{\partial(p|_{\vec{x}'})}{\partial(n^2|_{\vec{x}'})} = \frac{ik_{\mathrm{ref}}}{2}\,(1-\mathcal{L}_0)\,p(\vec{x}') \tag{8}$$

By incorporating the Green's function between the perturbation position $\vec{x}'$ and the receiver position $\vec{x}$, one can obtain the sensitivity kernel of the sound pressure at $\vec{x}$ due to the medium perturbation at $\vec{x}'$:

$$\frac{\partial(p|_{\vec{x}})}{\partial(n^2|_{\vec{x}'})} = G(\vec{x};\vec{x}')\frac{\partial(p|_{\vec{x}'})}{\partial(n^2|_{\vec{x}'})}. \tag{9}$$

Sensitivity analysis is implemented for an idealized Gaussian canyon model as an example.

The example showing the sound propagation effects caused by submarine canyons is done with the 3-D Split-step Fourier PE model [5]. A strong focusing effect can be seen when sound propagates along the canyon axis (see Figure 2). In addition to the propagation study, the sensitivity kernels of sound pressure variations at three receiver locations due to medium perturbations in the canyon are shown in Figure 3. The dependency of the sensitivity kernel on receiver locations is significant. These sensitivity maps essentially tell how much the receiving sound pressure can change due to the medium perturbation at a given position.

2. 3-D sound propagation under non-planar surface waves

In this year, a numerical model employing the ADI Padé PE method has been developed to simulate underwater sound propagation under a non-plananr surface boundary. An illustration of the ADI method separating two-dimensional derivatives to one-dimeniosnal derivative is shown in Figure 4(a). In order to better capature the curvature of the surface boundary, a boundary-fitted grid is developed and shown in Figure 4(b), and a variable grid discrization scheme is implemented.

A semi-circular waveguide with pressure-release boundaries is employed to benchmark the numerical model. A normal mode solution consisting of Fourier-Bessel bases is first determined:

$$P(r,\phi,x) = i2\pi \sum_m \sum_n F_m(\phi_0) R_{mn}(r_0) F_m(\phi)\, R_{mn}(r) \frac{e^{i\sqrt{k^2-\kappa_{mn}^2}\,x}}{\sqrt{k^2-\kappa_{mn}^2}}, \tag{10}$$

where the Fourier bases $F_m(\phi) = \sqrt{\frac{2}{\pi}} \sin m\phi$, the Bessel bases $R_{mn}(r) = \sqrt{2}\,\frac{J_m(\kappa_{mn} r)}{a J_{m+1}(\kappa_{mn} a)}$, and a is the radius of the waveguide. The propagation angle (inclination angle from x) can be determined by $\theta_{mn} = \cos^{-1}\left[\sqrt{k^2-\kappa_{mn}^2}/k\right]$, where $\kappa_{mn} = j_{m,n}/a$, and $j_{m,n}$ indicates zeros of the Bessel functions. We can in fact use the Fourier Bessel modes to test the higher order PE angle limit. The computation results are shown in Figure 5, and a brief discussoin is provided in the next section.

3. Joint ocean and seabed effects

In this year, a integrated numerical model has been constructed with the sub-bottom layering stucture of the New Jersey shelf reported by Ballard et al. [11]. Nonlinear internal waves will be later added into this model to study the joint ocean and seabed effects on 3D underwater sound propoagation.

RESULTS

The major results of this project are summaried here, along with a publication list provided later. First, A higher-order tangent linear PE solution of 3D sound propagation has been derived, and it unifies other tangent linear PE solutions by employing a higher-order splitting algorithm for the square-root Helmholtz operator. This higher-order tangent linear solution also provides a sensitivity kernel of sound pressure variations due to changes in the index of refraction of the waveguide medium. Numerical examples of 3D sound propagation in an idealized Gaussian canyon are presented to show the performance of the solution. The examples clearly show the focusing effect for sound propagation along the canyon axis and the spatial distribution of the sound pressure sensitivity.

Secondly, a 3D sound propagation model with non-planar surface has been developed. The applications of this model will include studies of 3D propagating sound scattered from rough sea surface. A benchmark problem of semi-circular waveguides is employed to validate the model, and the benchmarking result is excellent. Numbers from error analyses are provided in the caption of Figure 5. Lastly, a numerical model has been constructed to include the sub-bottom layering structure of the New Jersey shelf. The purpose of this model is to investigate the 3D sound propagation effects caused jointly by marine geological features and oceanographic dynamics in shallow water. This integrated model will also be used later to study the sound pressure sensitivity due to time-varying water column dynamics.

IMPACT/APPLICATIONS

The potential relevance of this work to the Navy is on increasing the capability of sonar systems in shallow water areas. The contributions of the effort on studying 3-D sound propagation effects will also be on assessing the environment-induced acoustic impacts.

RELATED PROJECTS

Experimental data used in this project were collected from the ONR SW06 and QPE projects. Strong collaborations have been initiated and continued between the PI's of an ONR MURI project on integrated ocean dynamics and acoustics.

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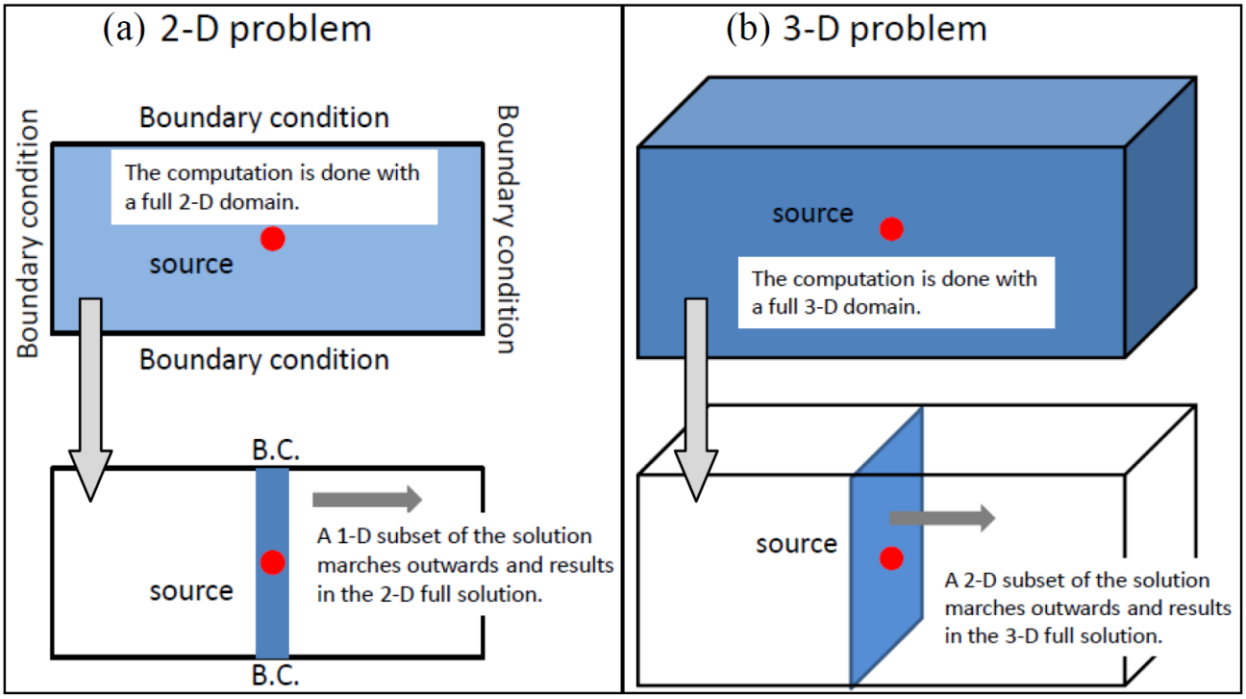

Figure 1. Dimension reduction of the Helmholtz equation by the parabolic-equation approximation. [The parabolic-equation (PE) approximation method has long been recognized as one of the most efficient and effective numerical methods to predict sound propagation in complex environments. This method approximates the Helmholtz wave equation of elliptic type to a one-way wave equation of parabolic type. The conversion allows efficient marching solution algorithms for solving the boundary value problem posed by the Helmholtz equation.]

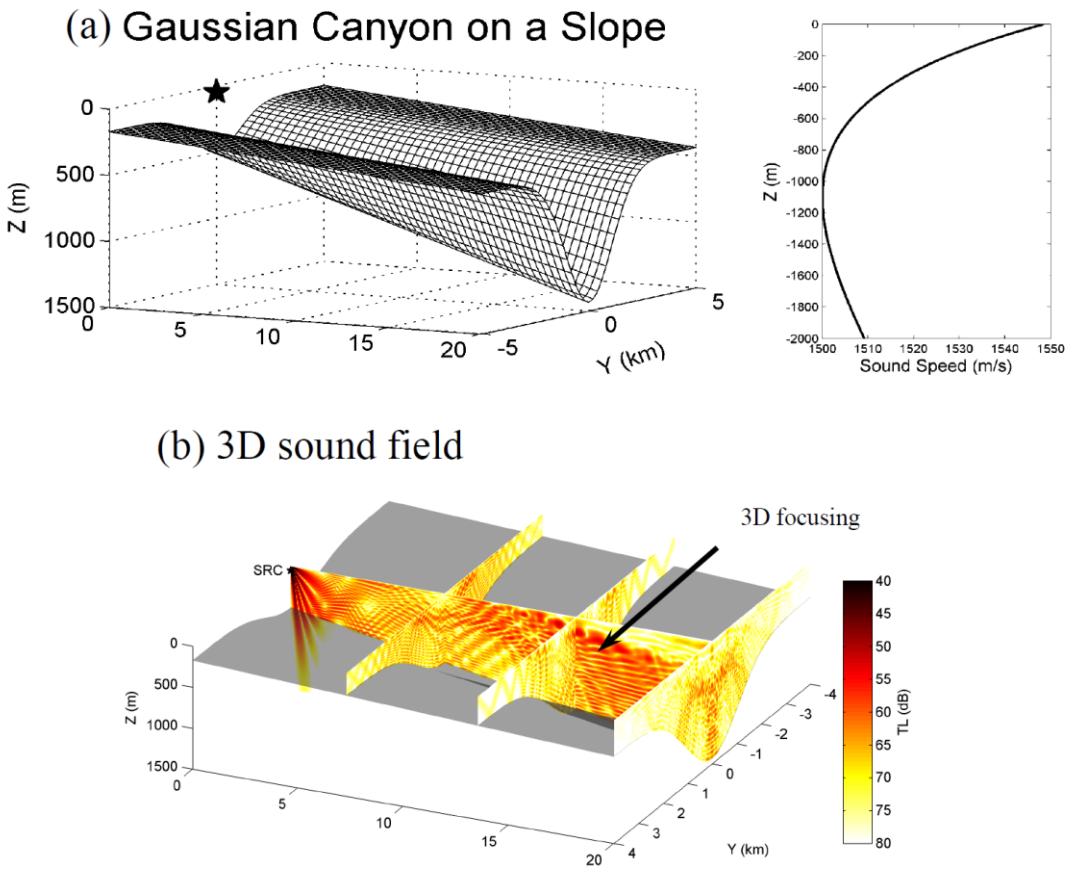

Figure 2. Idealized Gaussian canyon model.
[(a) Geometry of the canyon model. (b) The 3D sound field excited by a point source placed at the canyon axis, and strong 3D focusing effect is observed.]

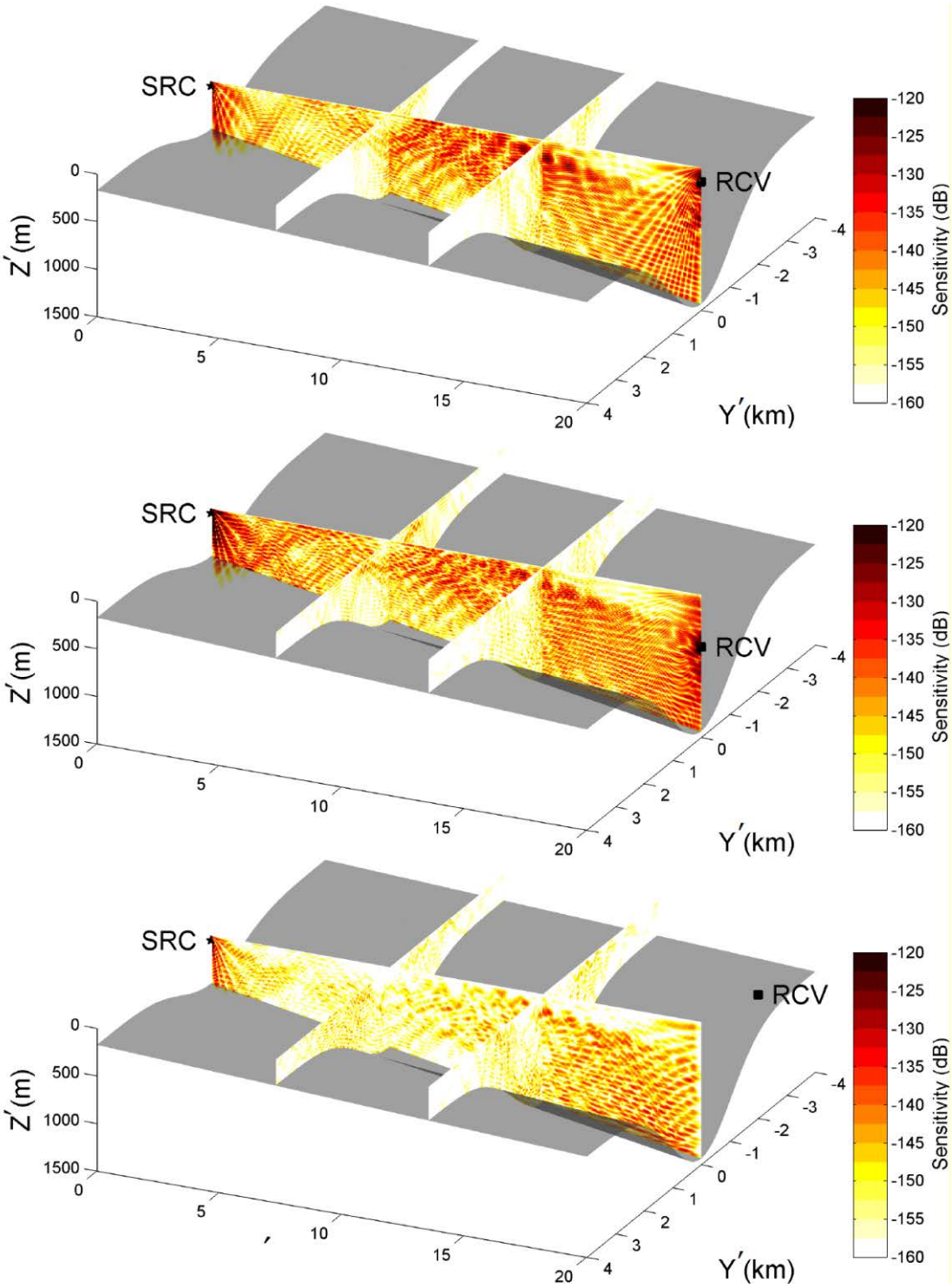

Figure 3. The sensitivity kernels of sound pressure variations at three receiver locations due to medium perturbations in the idealized Gaussian canyon model.
[One can see intensified sensitivity caused by the 3D focusing of sound. The dependency of the sound pressure sensitivity on receiver locations is significant. These sensitivity kernel maps essentially present how much the receiving sound pressure changes due to the medium perturbation at a given position.]

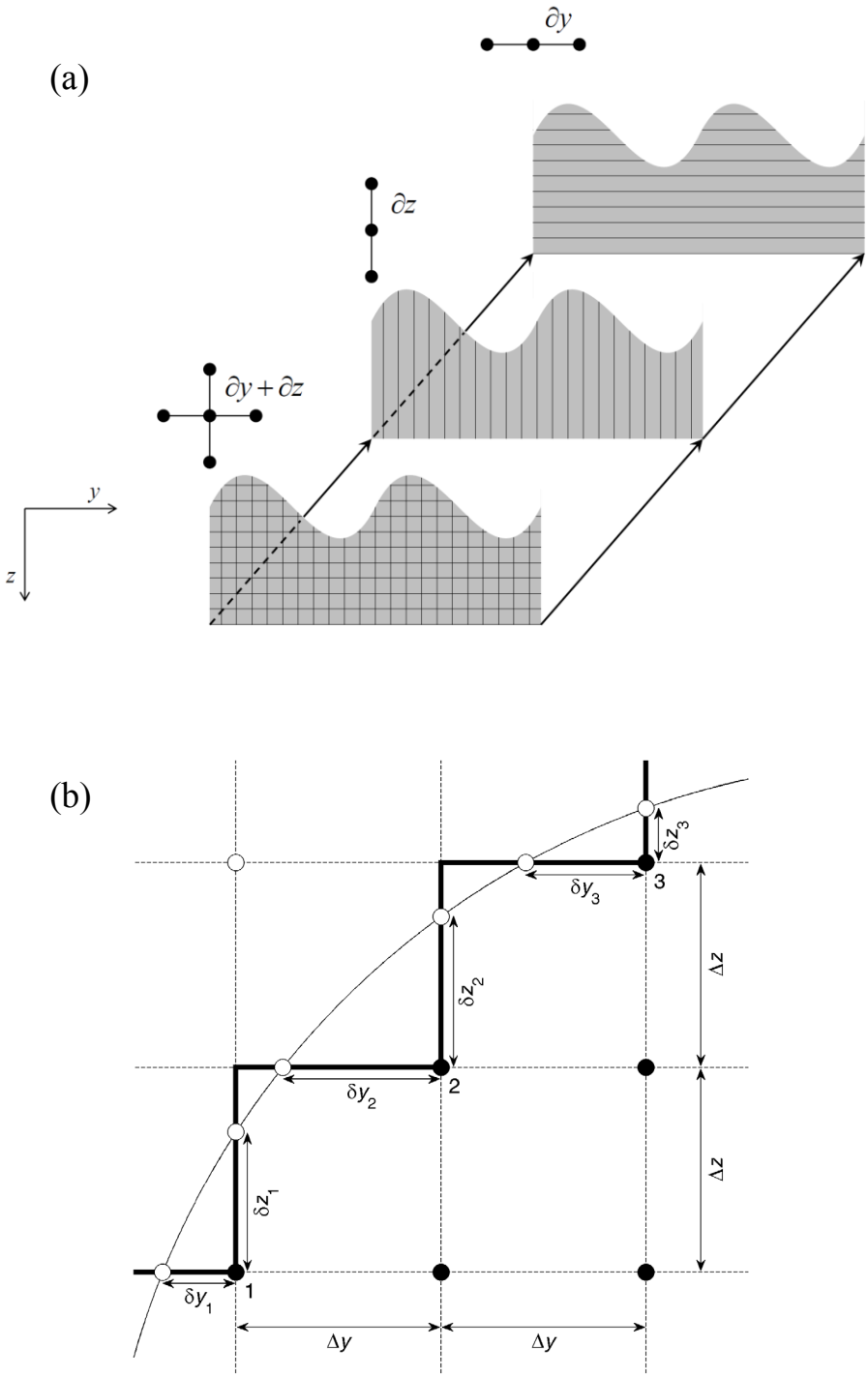

Figure 4. (a) The alternating direction implicit (ADI) method with a nonpanar surface, and (b) the boundary fitted grid for handling surface wave boundary conditions.

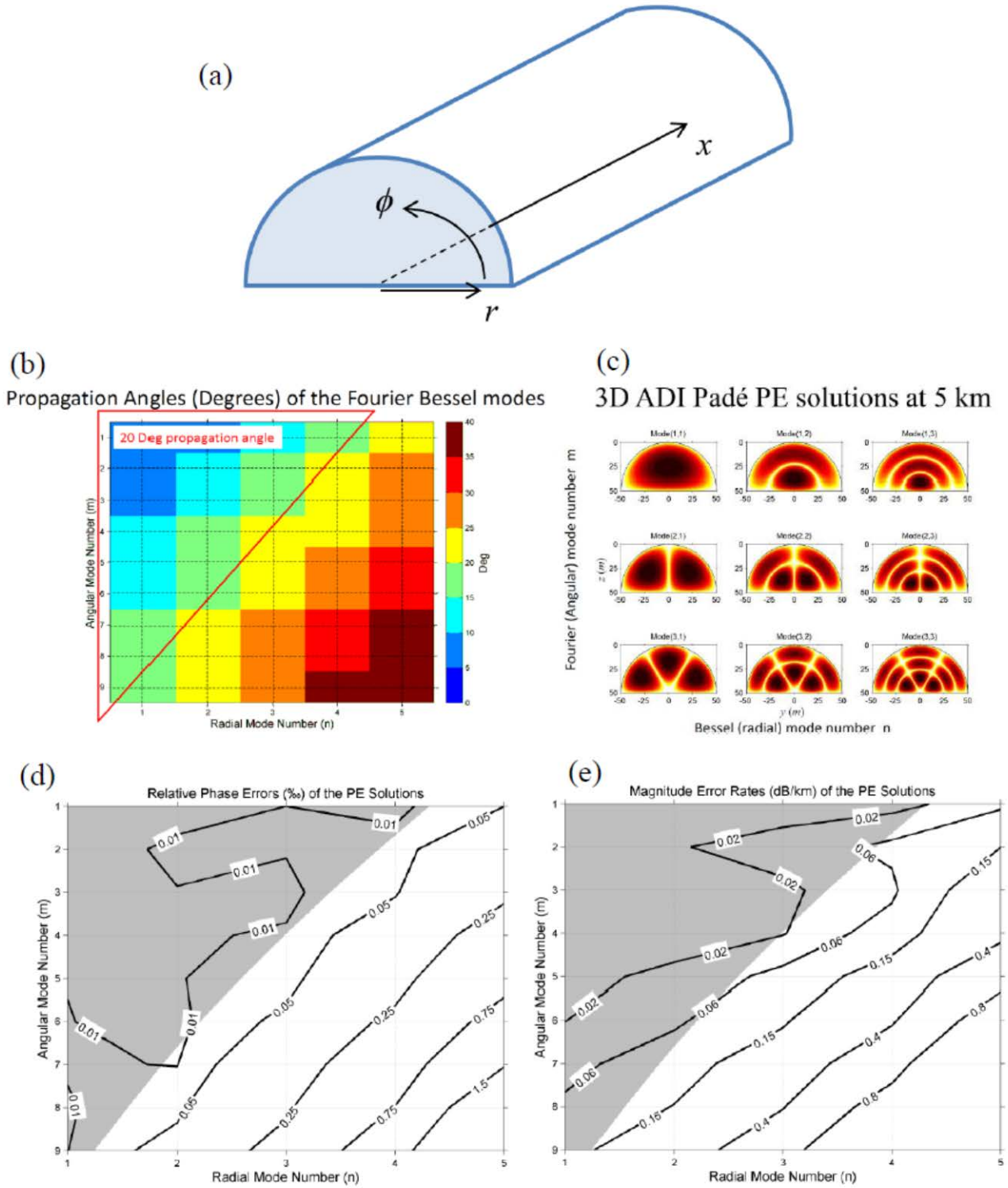

Figure 5. The semi-circular waveguide benchmark problem for the ADI Padé PE model with non-planar surface boundaries.
[(a) the gemetry of the benchmark problem, (b) the propagation angles of the Fourier-Bessel modes, (c) 3D ADI Padé PE solutions at 5 km, (d) relative phase errors (as small as 0.01 ‰), and (e) magnitude error rates (as small as 0.02 dB/km)]