Bayesian Inversion of Seabed Scattering Data
(Special Research Award in Ocean Acoustics)

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LONG-TERM GOALS

The long-term goals of this work are to improve ocean acoustic reverberation modeling and sonar performance predictions in shallow waters by developing inversion procedures to estimate seabed scattering and geoacoustic properties with uncertainties, as well as investigating the importance of various scattering processes. Important issues include: investigating the angular and frequency dependence of scattering (defining the scattering kernel), determining the dependence of scattering on physical properties of the seabed, and establishing the relative importance between scattering due to rough boundaries at the seafloor and sub-bottom interfaces or at volume heterogeneities. These issues are all key to the ability to invert scattering and/or reflection data for seabed geoacoustic and scattering parameters, and ultimately to the practical inversion of active source reverberation data for rapid environmental assessment applications.

OBJECTIVES

The specific objectives of this project are to carry out joint Bayesian inversion of scattering and reflection data to estimate the *in-situ* seabed scattering and geoacoustic parameters. This involves the development and implementation of efficient forward modeling and inversion algorithms, and the inversion of several sets of simulated data (to ensure the validity of the inversion algorithm) as well as the inversion of measured data.

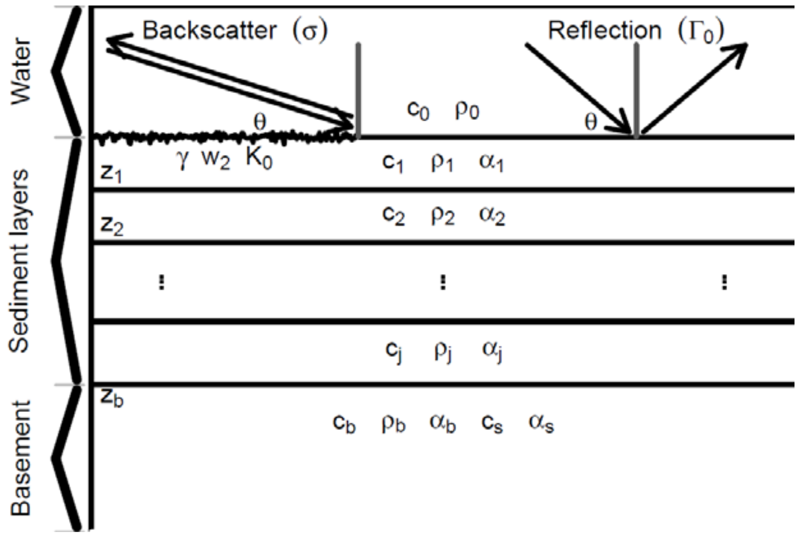

Figure 1: Schematic diagram of the environmental parameterizations for the monostatic-scattering kernel and reflection-coefficient forward and inverse models.

APPROACH

The data used in this work were collected by Charles Holland, who measured direct-path scattering[1] and reflection[2] data over a wide frequency band at a number of shallow-water test beds (Malta Plateau and near Elba Island in the Mediterranean Sea, and New Jersey and Scotian Shelves in the North Atlantic). These measurements probe the seabed on an intermediate spatial scale (patch-size radius ~500 m for both reflection and scattering), which generally allows a laterally-invariant parameterization of the seabed and reduces effects of spatial and temporal variability in the water column and other experimental uncertainties. While such data have been interpreted based on forward modeling studies and supporting geoacoustic measurements,[2] rigorous scattering inversion algorithms have not been developed previously, nor have quantitative inversion studies been carried out.

Two forward models are used in this research to predict mono-static scattering data and spherical-wave reflection data, both of which are applied to a seabed model consisting of a layered half-space. Figure 1 shows the assumed layered model structure. The top (zeroth) layer is seawater which is assumed to have a known sound-speed profile (measured during an experiment). Below this is a series of j flat sediment layers, terminated by a homogeneous semi-infinite basement ($j + 1$ layers with j interfaces in all). Layer properties include interface depth z (the lower boundary of a sediment layer), sound velocity c, density ρ, and attenuation α. In addition, the basement is assumed to be elastic with a shear-wave velocity c_s and attenuation α_s. The only difference between the seabed model for scattering and reflection calculations is that the first (water-sediment) interface is assumed to be rough for scattering and planar for reflection.

Details of the derivation of the scattering forward model can be found in Ref. 3. In this research the von Karman formulation[4] of the 2D spatial interface roughness power spectrum (W) is used: $W_I(\mathbf{K}) = w_2(|\mathbf{K}|^2 + K_0^2)^{-\gamma/2}$, where γ, w_2, and K_0 are known as the interface spectral exponent, spectral strength, and spectral cutoff, respectively, and $\mathbf{K}$ is the transverse component of the incident wave vector with magnitude $|\mathbf{K}| = k_0\cos(\theta)$, with k_0 the wavenumber in the zeroth layer (water).

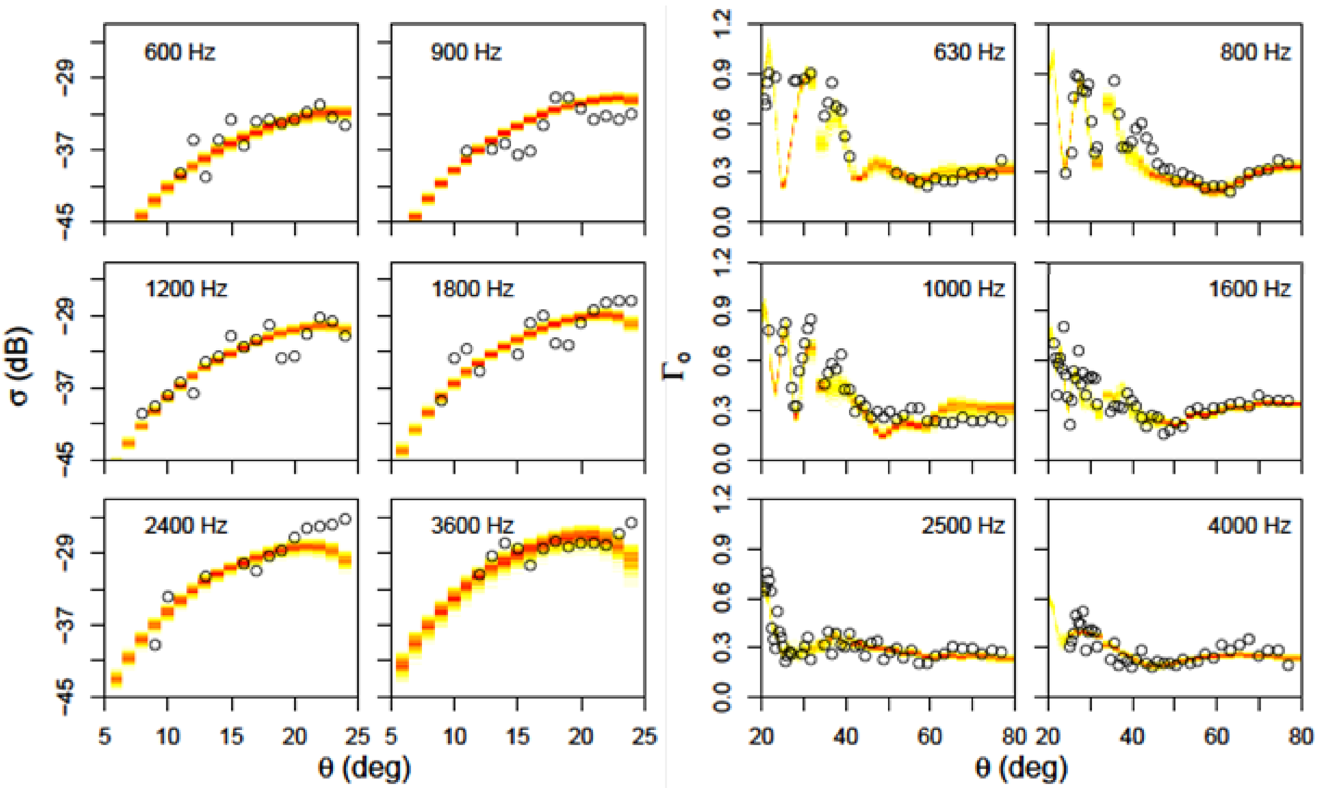

Figure 2: Measured data (o) and predicted data (color scales indicate marginal probability densities) at indicated frequencies. Left two columns: scattering data; right two columns: reflection-coefficient data.

Reflection-coefficient data are modeled here using a spherical-wave reflection model instead of the more-common and computationally-faster plane-wave reflection model, as the source-receiver distances in the experimental procedure are insufficient to ignore spherical-wave effects. Spherical-wave reflection coefficients Γ_0 for an arbitrary *N*-layer half-space are computed as in Brekhovskikh and Godin[5] as a superposition of plane waves (the Sommerfeld integral).

A Bayesian framework is applied for inversions in this work as it provides rigorous estimation of parameter uncertainties as well as parameter values, thereby quantifying the information content of the data to resolve the model parameters. Bayesian inversion is based on formulating the posterior probability density (PPD) of the model parameters of interest, which combines both data and prior information.[6,7] The multi-dimensional PPD is interpreted in terms of parameter estimates (e.g., the most-probable model), parameter uncertainties (variances, marginal densities, credibility intervals), and parameter inter-relationships (correlations, joint marginal densities), which provide a complete solution to the inverse problem. Since scattering and reflection inversion are strongly non-linear problems, these PPD properties are computed numerically using global optimization and Markov-chain Monte Carlo (MCMC) sampling algorithms.

One of the goals of this work is to carry out inversions trans-dimensionally, a relatively new approach in ocean acoustic inversion.[6] Trans-dimensional (trans-D) inversion allows the natural parsimony of the Bayesian approach to determine the complexity (e.g., number of seabed layers) of the estimated models. It also allows the parameter uncertainties to be marginalized over the number of layers, hence accounting for the uncertainty of the parameterization within parameter uncertainty estimates. Trans-D inversion has been shown to model complex layering structures without over-parameterizing simple structures.[8] However, the approach is computationally intensive, and of particular interest here is implementing population techniques[9] for increasing the efficiency.

The inversions of the two types of data (reflection and scattering) are conducted both individually and jointly. In addition, simulation studies have been conducted. The ability to resolve scattering parameters (spectral strength, spectral exponent, and spectral cutoff) and geoacoustic parameters (sound speed, density, attenuation, and layering structure) via inversion is investigated in terms of posterior parameter uncertainty distributions, which quantify the effective data information content. In particular, scattering/geoacoustic parameter resolution are investigated for inversion of scattering data alone versus joint inversion of scattering and reflection data (joint inversion is can overcome inter-parameter correlations which limit parameter resolution in scattering inversion[10]).

WORK COMPLETED

Items 1-5 were completed and included in previous reports. Items 6 has been updated to reflect that the corresponding paper[16] has now been accepted (in press) and item 7 represent new work conducted in 2014.

1) A fixed-dimensional inversion of the measured scattering data was conducted; the results have been presented at the 2011 meeting of Acoustic Society of America.[11] The inversion assumed only one sediment layer above a fluid basement and that the spectral cutoff was fixed at zero. Geoacoustic parameter estimates were found to be unresolved (their PPDs were approximately uniform). An informative prior distribution constraining the relationship between sediment sound speed and density was developed based on a large collection of measurements reported by Hamilton.[12] Geoacoustic parameter estimates were found to be adequately resolved (their PPDs had a single clear peak) when the joint prior was incorporated in the inversion. The scattering parameters were well resolved in both cases, with estimated values consistent with expectations for the site and reasonably small uncertainties.

2) A joint trans-D scattering/reflection inversion algorithm was developed and applied to measured data. During the inversion it was assumed that the basement was inelastic and that data residuals (errors) were uncorrelated. Both of these common assumptions were found to be inadequate to fully account for the measured data.

3) A new mathematical formulation of the prior distribution for the location of interfaces between sediment layers was developed. This formulation allowed for the explicit evaluation of the prior distribution (which is not possible in other trans-D inversions[6,13]), which in turn allowed an elastic basement to the incorporated into the inversion. A joint trans-D inversion was carried out which allowed errors to be correlated at all frequencies and modeled by first-order autoregressive [AR(1)] process with unknown coefficients. The results were presented at the 2012 European Conference on Underwater Acoustics (Edinburgh UK).

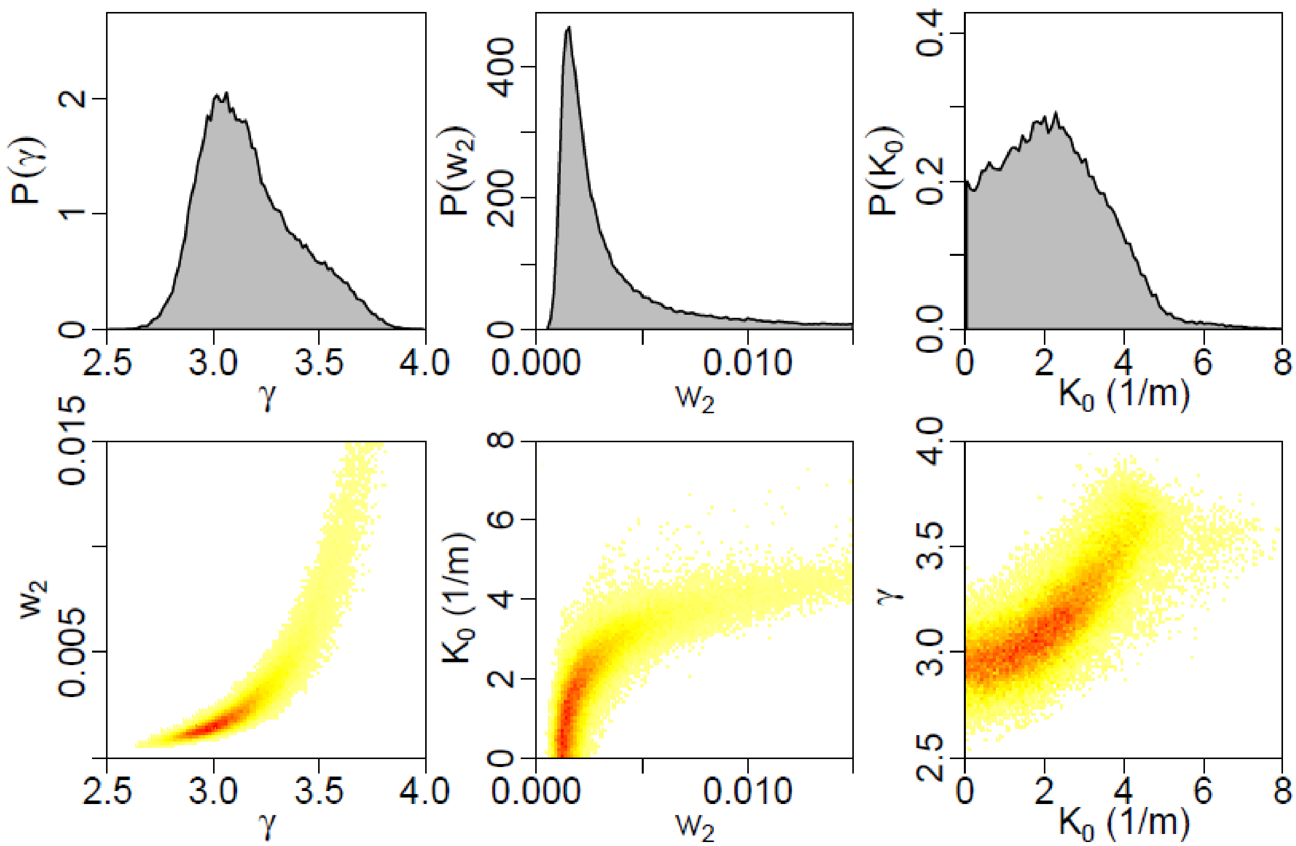

Figure 3: The 1D (top-row) and 2D (bottom row) marginal posterior probability distributions of the scattering parameters (scaled so each distribution has the same area).

4) A simulation study on the impact of data-residual models was conducted. The study considered inversion of simulated data with significant residual correlation only at low frequencies. Inversions were conducted assuming (incorrectly) no residual correlation, correlation at all frequencies, and using a trans-D auto-regressive model. These results have been published in a paper[14] in JASA and presented at the 2012 Canadian Acoustical Association meeting in Banff AB.

5) A joint inversion of scattering and reflection-coefficient data collected at the Malta Plateau was conducted. The results have been published in JASA[15] and were presented at the joint meeting of the International Congress on Acoustics, Acoustical Society of America, and Canadian Acoustical Association in Montreal, 2013. Some highlights of this work are shown in the results section.

6) A trans-D polynomial spline based parameterization for seabed geoacoustic structure has been developed and applied to bottom-loss data inversion. The parameterization is shown to be a superior model for geoacoustic profiles dominated by gradients (rather than distinct layers) such as muds. This work has been accepted and is now in-press in JASA (3 in list of Refereed Journal Articles at end of this report).

7) An inversion of scattering data using a clone GABIM[3] (developed as part of this work) as the

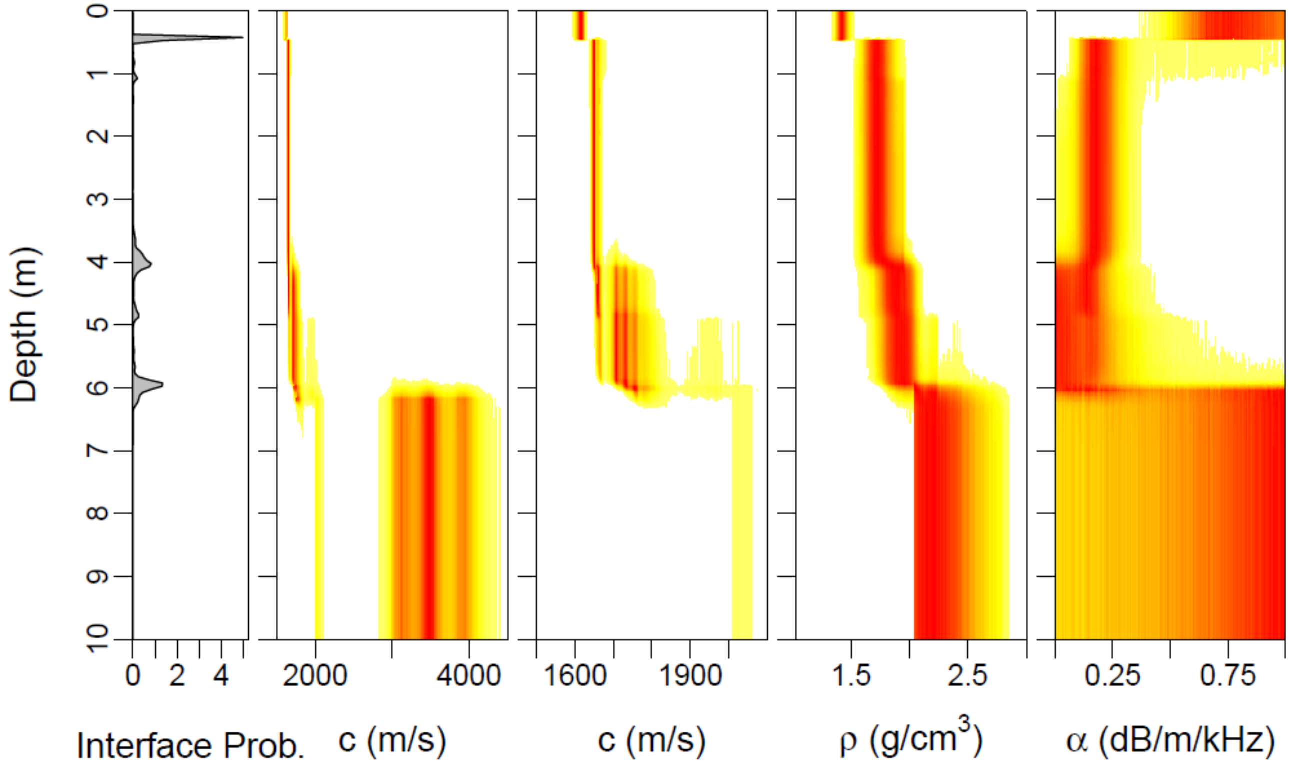

Figure 4: Marginal posterior profiles for geoacoustic properties from joint scattering reflection inversion. Plot boundaries correspond to prior bounds.

forward model has been conducted. The inversion allows for both volume and interface scattering and introduces a method based on deviance information criterion to determine the dominant scattering mechanism is in development. This work has been compiled in two submitted JASA papers, the first has been accepted and is in-press and the second is in review (4 and 6, respectively, in list of Refereed Journal Articles at end of this report).

RESULTS

This section presents the joint trans-D scattering and reflection inversion results from measured data. The scattering data are at frequencies of 600, 900, 1200, 1800, 2400, and 3600 Hz and an angular range of 5−24°. The reflection data have a similar frequency range (630, 800, 1000, 1600, 2500, and 4000 Hz) and an angular range of 20-85°. Figure 2 shows the scattering data (left) and reflection data (right) with the marginal predicted data from the inversion. There is generally good agreement between the measured and predicted data. However, there is residual error correlation present for many of the frequencies; this is accounted for in the inversion by a trans-D autoregressive error model.

The inversion results in terms of marginal probability densities for the scattering parameters are shown in Fig. 3. These distributions indicate that γ, w_2, and K_0 are well resolved with reasonable estimated values. Additionally the shapes of the distributions for γ and w_2 appear to be approximately Gaussian and log-Gaussian, respectively (which represent their theoretical distributions if all other parameters are known), and they are in good agreement with the values presented in Ref. 4. In addition, the parameter values indicate an RMS roughness of ~0.08 m and a correlation length of ~0.4 m, both of which are consistent with the expected marine environment.

The marginal posterior profiles of the geoacoustic parameters are displayed in Fig. 4. Overall, 3-5 sediment layers are resolved/supported by the data. The seabed sound-speed profile appears better constrained within its prior bounds than the density and attenuation profiles. The sound speed and density of the uppermost layer is consistent with sand (known to represent the surficial sediments). The high sound speed and density below ~6-m depth is consistent with a strong reflector at this depth on a high-resolution seismic survey of the experimental site. Further, these results are consistent with a limestone basement, which is known to exist in the region. It is the existence of this limestone layer which necessitated the elastic model (including shear properties) for the basement in the inversion algorithm.

IMPACT/APPLICATIONS

The performance of naval sonar systems in shallow waters is strongly influenced by acoustic interaction with the bottom, and therefore knowledge of seabed scattering and geoacoustic parameters and their uncertainties is required to model, predict, and optimize sonar performance. Bayesian inversion methods offer a powerful framework for parameter extraction and uncertainty estimation, thereby quantifying the geoacoustic information content of the data. The proposed inversion methodology has been applied previously to reflection data, and the current project combines this with monostatic scattering inversion to constrain seabed scattering parameters and investigate scattering mechanisms. Inversion results for both measured and simulated data indicate reasonably good ability to resolve seafloor scattering parameters from acoustic measurements.

RELATED PROJECTS

1) *Quantifying Geoacoustic Uncertainties and Seabed Variability for Propagation Uncertainty (Award Number*: *N000140910394, J. Dettmer, 2008-2011).* This project involves Bayesian inversion of reflection data to investigate the uncertainty and variability of seabed geoacoustic parameters, and the effects on acoustic propagation. Inversion methodologies similar to that in the present project are developed and applied.

2) *Bayesian Ambient Noise Inversion for Geoacoustic Uncertainty Estimation (Award Number: N000141110214, J. Quijano, 2011-2012).* This project involved trans-D Bayesian inversion of oceanic ambient noise for seabed geoacoustic parameters. The ambient noise is processed to produce bottom-loss as a function of angle and frequency, which is similar to the reflection data inverted in this project. Similar trans-D inversion procedures are also applied.

3) *Automated geoacoustic inversion and uncertainty: Meso-scale seabed variability in shallow water environments (Award Number: N000140910394, J. Dettmer, 2011-2014).* The project advances and carries out geoacoustic inversions in 2D and 3D shallow-water environments. The resulting geoacoustic models will represent benchmarks for meso-scale variability and uncertainty estimation, and also allow the study of compressional- and shear-wave dispersion and attenuation-frequency dependence. Bayesian hierarchical models and trans-D inversions allow increasingly automated data analysis in challenging shallow-water environments.

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PUBLICATIONS

Refereed Journal Articles

6. C. W. Holland, J. Dettmer, G. Steininger, and S. E. Dosso. Acoustic measurements of sediments with pebbles and cobbles J. Acoust. Soc. Am., 2014 [submitted, refereed].
5. S. E. Dosso, J. Dettmer, G. A. M. W. Steininger, and C. W. Holland. Efficient trans-dimensional Bayesian inversion for geoacoustic profile estimation Inverse Problems, 2014 [in press, refereed].
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3. G. A. M. W. Steininger, S. E. Dosso, C. W. Holland, and J. Dettmer. A trans-dimensional polynomial-spline parameterization for gradient-based geoacoustic inversion. J. Acoust. Soc. Am., 2014 [in press, refereed].

2. G. A. M. W. Steininger, C. W. Holland, S. E. Dosso, and J. Dettmer. Seabed roughness parameters from joint backscatter and reflection inversion at the Malta Plateau. J. Acoust. Soc. Am., 134:1833–1842, 2013a [published, refereed].
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4. G. Steininger, S. E. Dosso, C. W. Holland, and J. Dettmer. Seabed roughness parameters for the Malta Plateau from joint backscatter and reflection inversion. Proceedings of Meetings on Acoustics 19, 070003 (2013) [published].
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