Effect of Environmental Uncertainty on Target State Estimates

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Outline

- Definition of Uncertainty
- Bayesian Target State Estimation
- Example of the Effect of Environmental Uncertainty on Target State Estimates

Definition of Uncertainty

- Let

  P = the estimated value of some quantity

  A = the actual value (unknown to us)

  $E = P - A$.
- The *uncertainty* in P is represented by the probability distribution of E.
- Note: This distribution need not be normal, have mean 0, or be unimodal.

Bayesian Target State Estimation: Discrete Bayesian Tracking

Bayesian Tracking Framework

Prior Distribution on Target State

Define target state space, e.g., (position, velocity, type)

Specify prior probability distribution on the target state

Account for Target Motion

Specify distribution on target motion and state transitions.

Likelihood Functions

Convert observations (contacts) into likelihood functions

Posterior Distribution

Combine likelihood functions with prior to compute the posterior distribution on target state.

Note: The Posterior Distribution is the Bayesian "Estimate" of Target State

Single Target Recursion

Let

s = target state; S = target state space

t_0 = the start time; $t_1 < t_2 < \ldots$ be the observation times

$Y_k = y_k$ be the observation at time t_k

Recursion

Initial Distribution: $p(t_0, s_0) = q_0(s_0)$ for $s_0 \in S$

For $k \geq 1$ **and** $s_k \in S$

Motion Update: $p^-(t_k, s_k) = \int_S q_k(s_k | s_{k-1})\, p(t_{k-1}, s_{k-1})\, ds_{k-1}$

Calc Likelihood: $L(y_k | s_k) = \mathbf{Pr}\{Y_k = y_k | X(t_k) = s_k\}$

Compute Posterior: $p(t_k, s_k) = \frac{1}{C} L(y_k | s_k)\, p^-(t_k, s_k)$

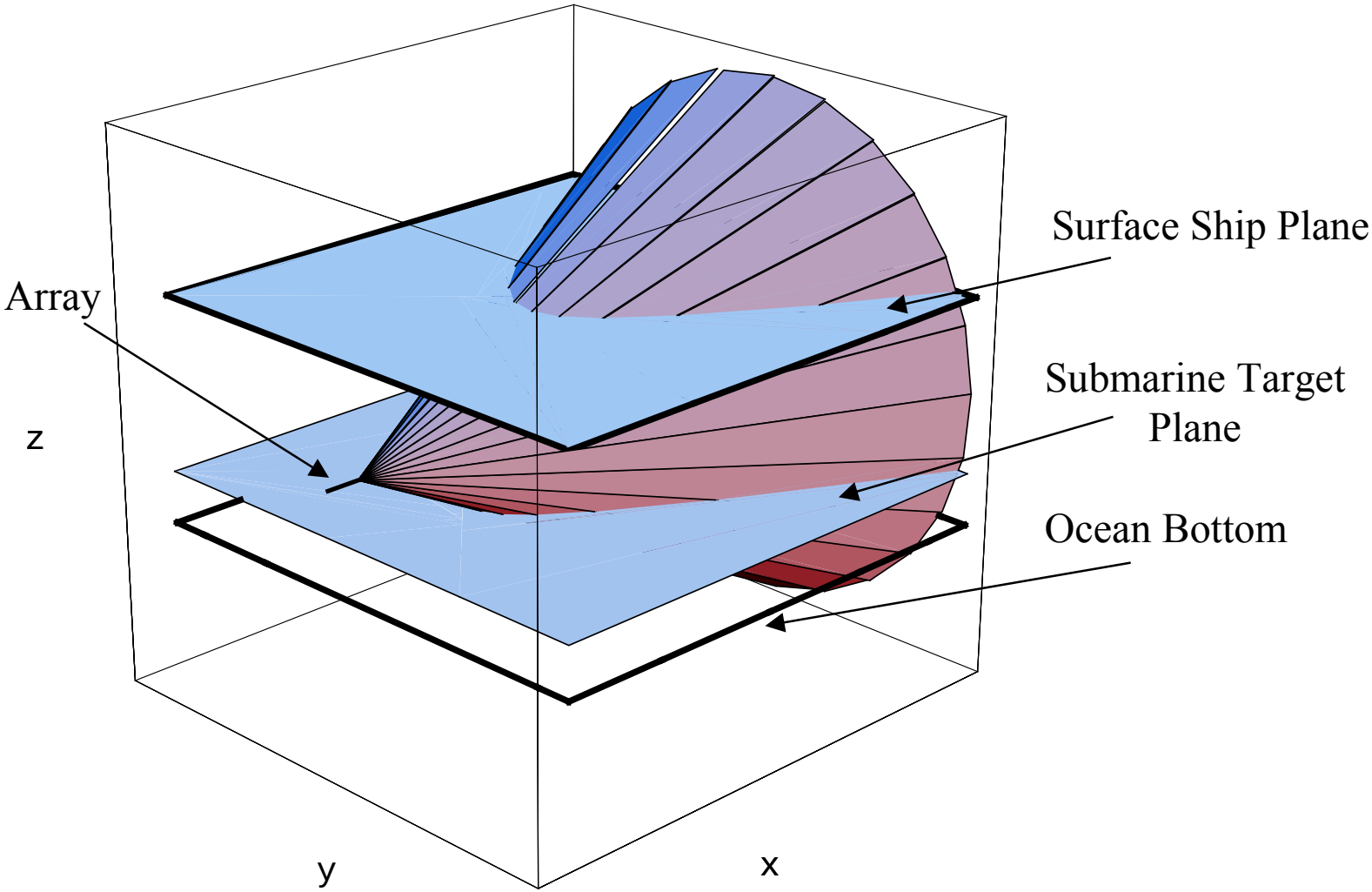
Likelihood Functions

- Likelihood functions replace and extend the notion of contact in Bayesian Tracking
- Let Y be an observation, e.g.,
  - detection of a target by a sensor
  - a line of bearing measurement from a towed array
  - measured frequency of an acoustic signal
- The likelihood function converts an observation to a function on the target state space S:

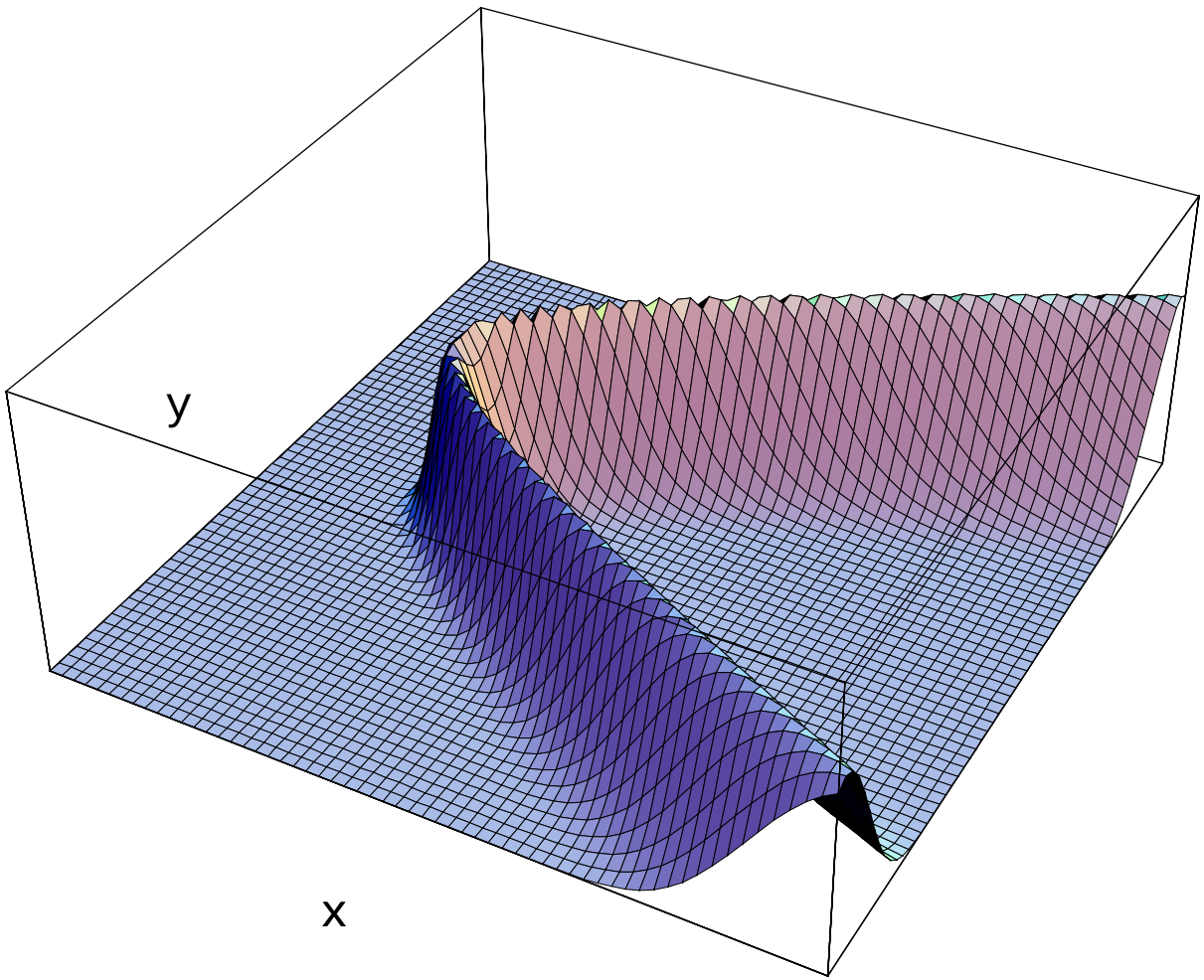
$$L(y|s) = \mathbf{Pr}\{Y = y | X(t) = s\} \text{ for } s \in S$$

- The observation (data) $Y = y$ is known. The target state s is not. Thus $L(y \mid \bullet)$ is a function on S.

Example of Likelihood Function: Conical Bearing



Direct Path Likelihood Function (Plane of Submarine Target)



Combining Two Likelihood Functions

- Combine a detection and line of bearing observation
- Line of bearing likelihood function

  Let θ be the observed bearing and $b(s)$ be the bearing of a target at state s to the sensor. Then

$$L_b\left(\theta \mid s\right) = \frac{1}{\sigma_b\sqrt{2\pi}} \exp\left(\frac{-(b(s)-\theta)^2}{2\sigma_b^2}\right) \text{ for } s \in S$$

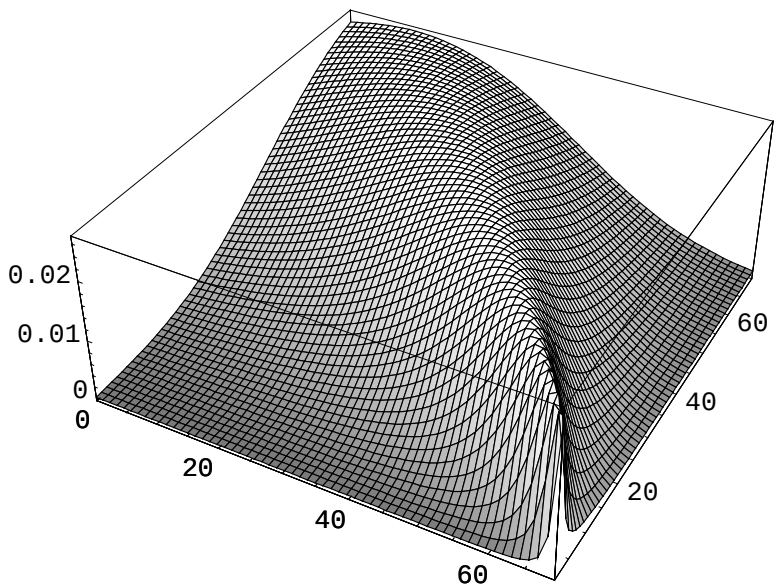
- Detection Likelihood Function

  $r(s) =$ range of state s from sensor

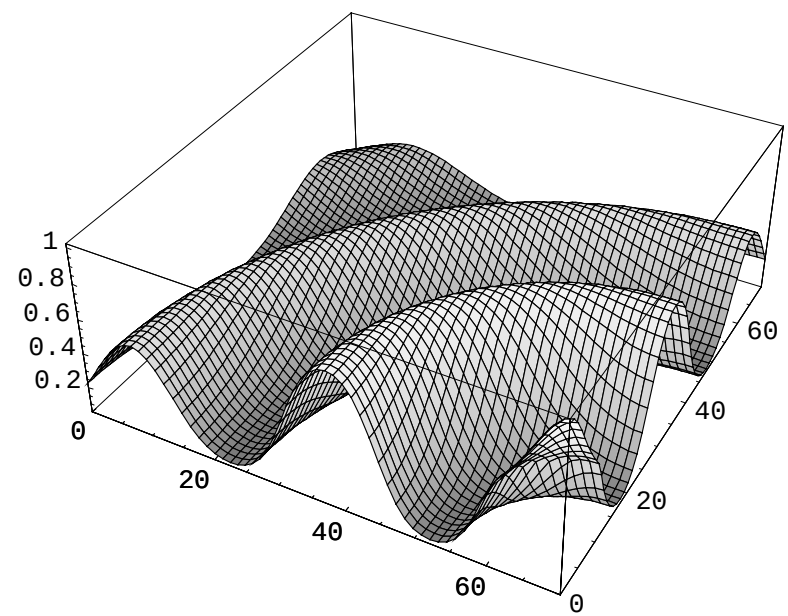
  $\mathrm{P_d}(r) =$ probability of detecting a target at range r from sensor

  $\mathrm{L_d}(\text{detection} \mid s) = P_d\left(r(s)\right)$ for $s \in S$

Bearing and Detection Likelihood Functions

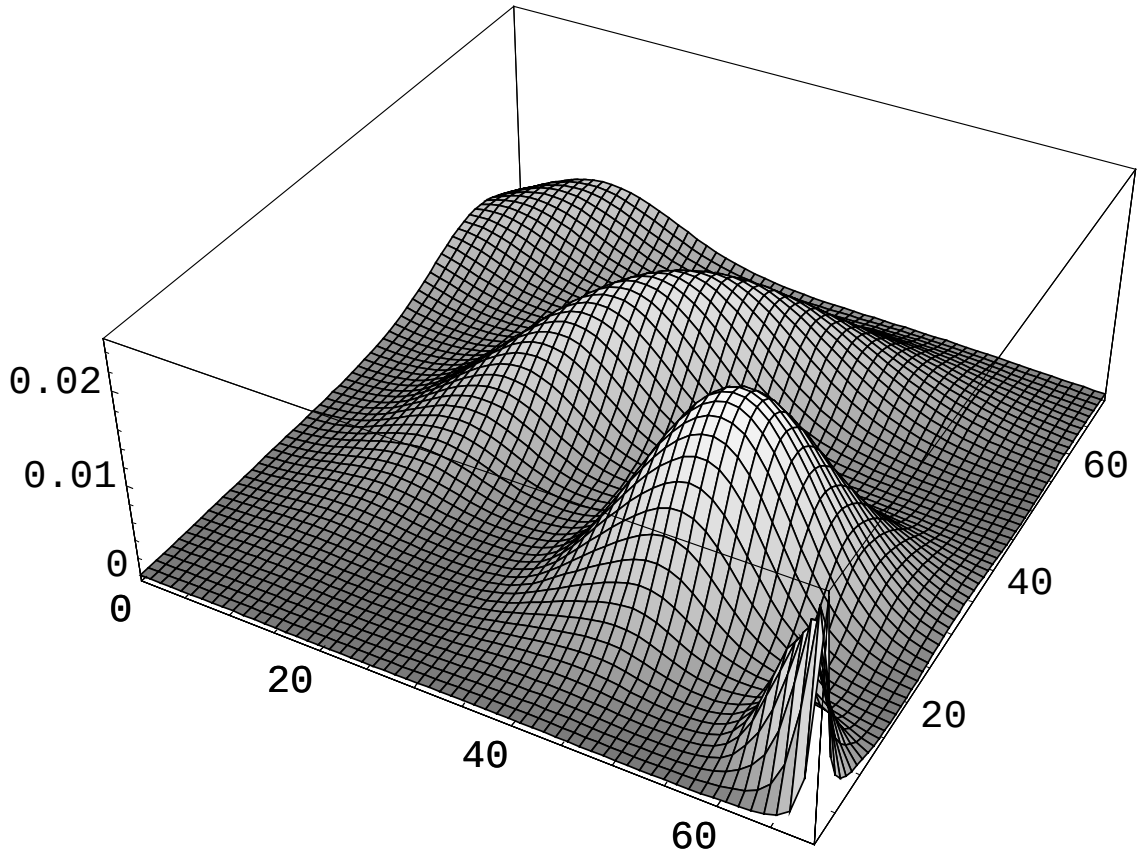


Bearing Likelihood



Detection Likelihood

COMBINED LIKELIHOOD FUNCTION
Bearing and Detection



Example: Effect of Environmental Uncertainty on Target State Estimation

- Problem:
  - Compute state estimate and uncertainties (due to environmental uncertainties) for towed array detection.
- Approach
  - Use standard discrete Bayesian tracking formulation with an additional uncertainty for estimate of propagation loss.
  - Combine bearing and detection estimate to produce probability distribution on target state
  - Show the effect of environmental uncertainty in target state estimate

Description of Example

- Detection of a Submarine by a Towed Array
  - Source:
    - Depth 32m; frequency 400 Hz; Source level SL = 125 dB
  - Sensor:
    - Location 31.9^o N 71.25 ^{o} W
    - Length 61m, Depth 15m
    - Directivity index DI = 15 dB at 400 Hz
    - Beam width (3 dB down) 3.5^o at 400 Hz
    - Detection threshold DT = -5 dB
  - Environment
    - Ambient noise, NL = 65 dB at 400 Hz (sea state 3)

Detection Assumptions

- Mean Signal Excess, $SE(s)$, at the array given target at s is

$$SE(s) = SL - TL(s) - LE - DT$$

where

$$TL(s) = \text{ transmission loss from } s \text{ to the array}$$

$$LE = (NL - AG) \oplus (SN - AG)$$

$$\approx NL - AG = NL - DI$$

- Actual Signal Excess, $\tilde{S}_E(s)$, equals mean signal excess perturbed by a random term so that

$$\tilde{S}_E(s) = SE(s) + \xi$$

- Detection Occurs when $\tilde{S}_E(s) > 0$

- Figure of Merit: $FOM = SL - LE - DT = 125 + (65 - 15) + 5 = 80 \text{ dB}$

Environmental Uncertainties

- The random term, ξ, consists of

$$\xi = \xi_A + \varepsilon(r)$$

where

ξ_A represents the time varying fluctuations in the ambient noise

$\varepsilon(r)$ is the error in the transmission loss estimate at range r

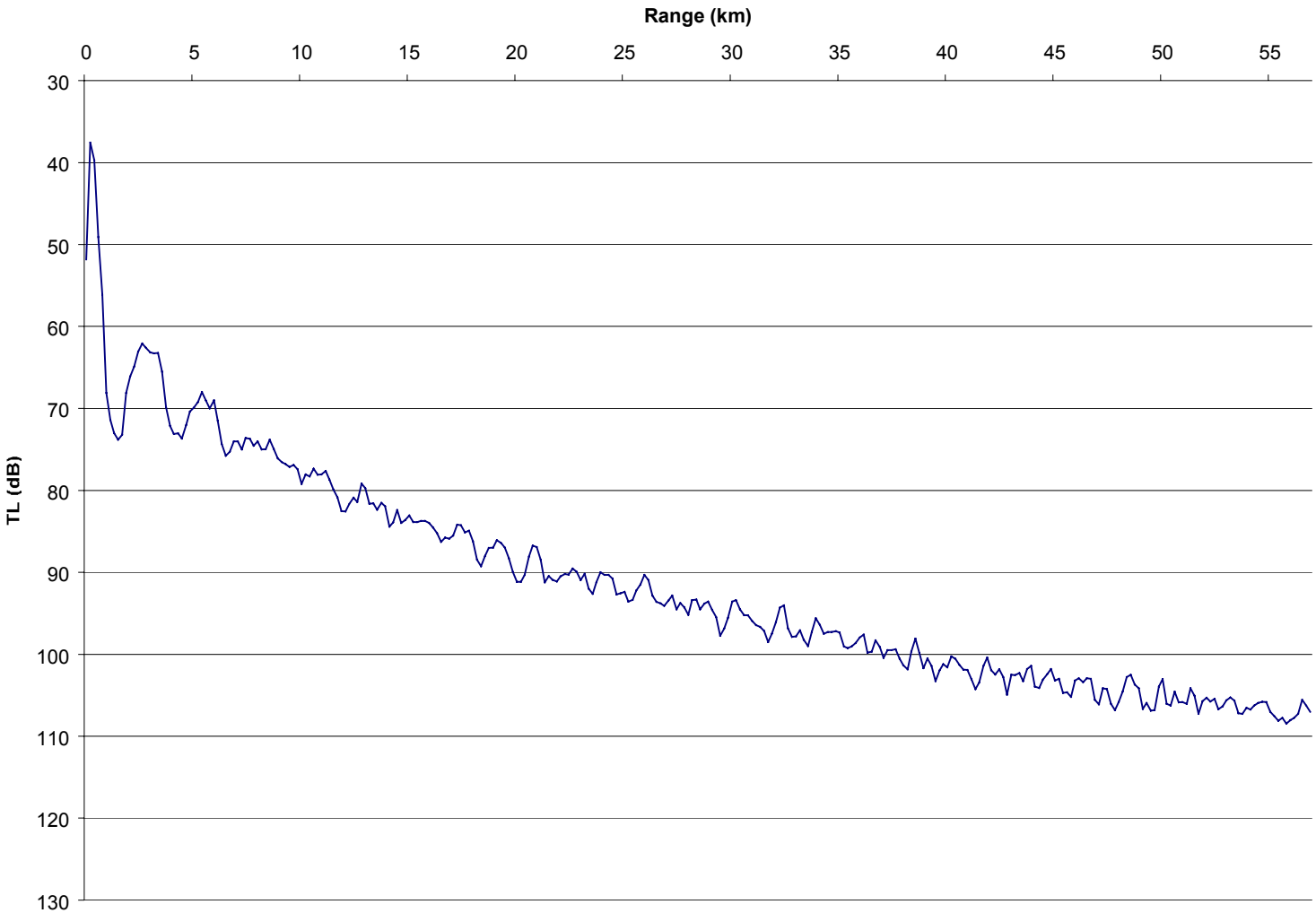
We assume

$$\xi_A \sim \mathrm{N}(0, 4\,\mathrm{dB})$$

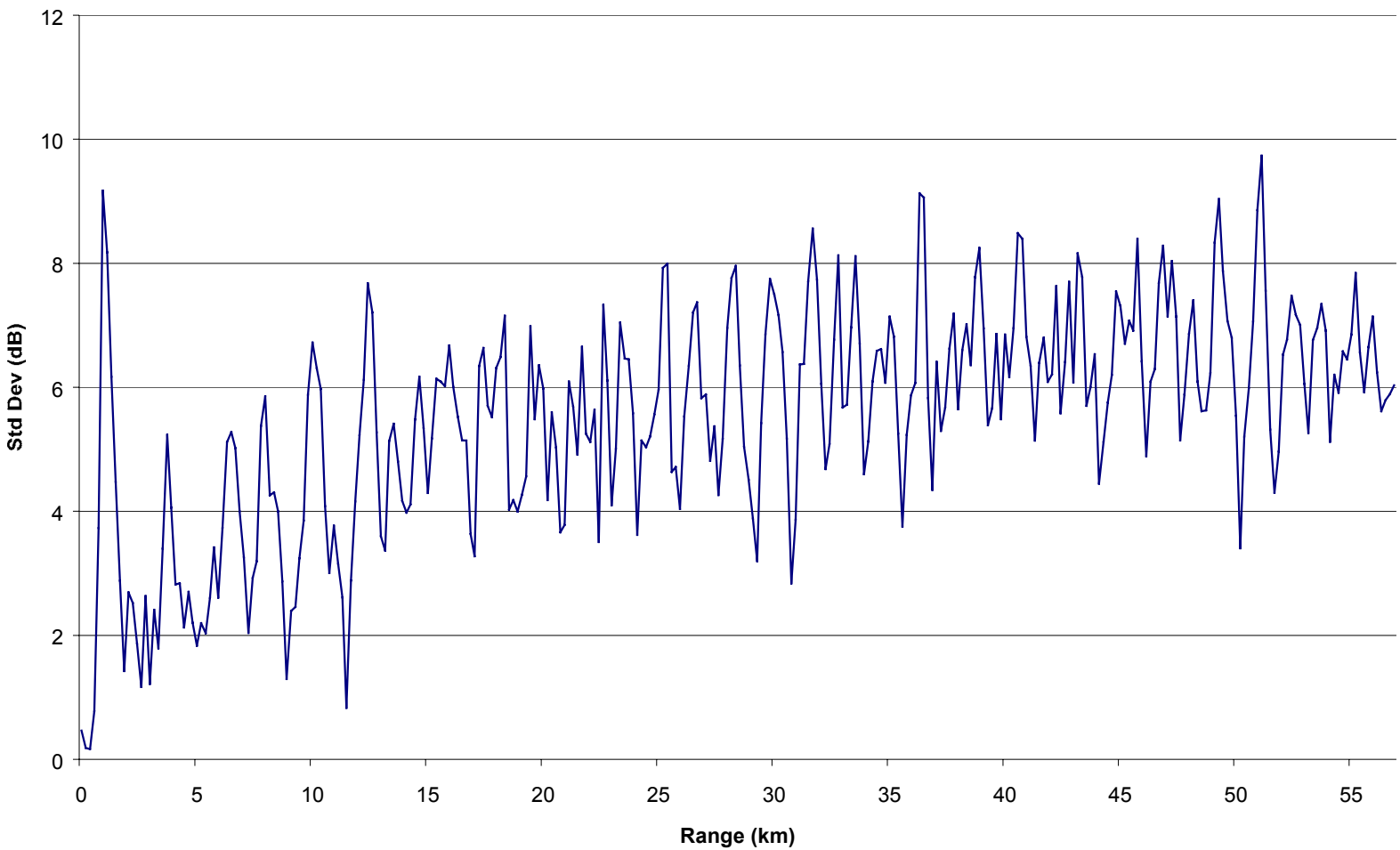
$$\varepsilon(r) \sim N\left(0, \sigma_T(r)\right)$$

Estimated Transmission Loss at 400 Hz

Northeast Line



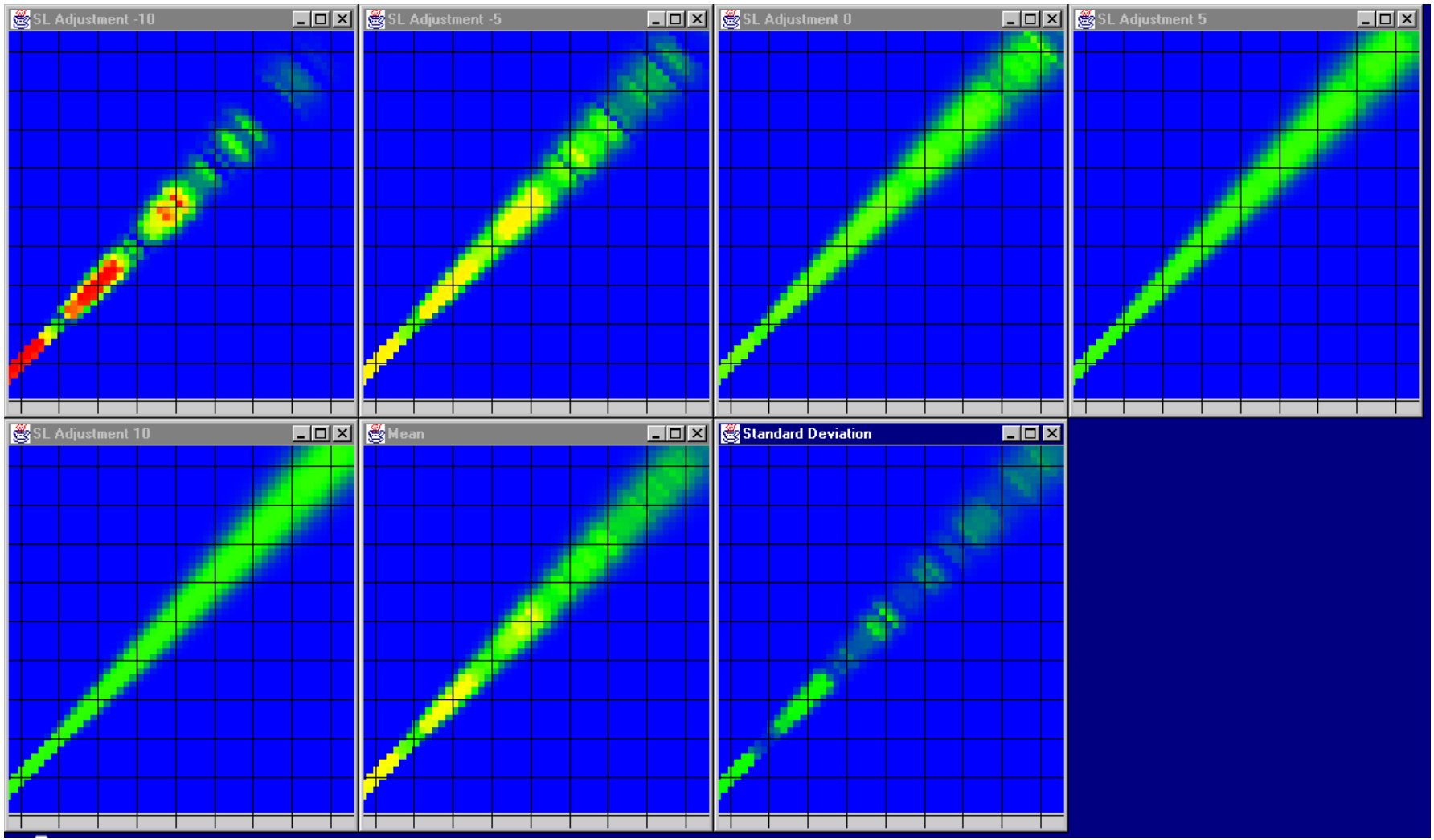
Standard Deviation of Error in Transmission Loss Estimate $\sigma_T(r)$



Single Line of Bearing Detection

Grid Spacing = 1 km

ε = -10 dB ε = -5 dB ε = 0 dB ε = 5 dB

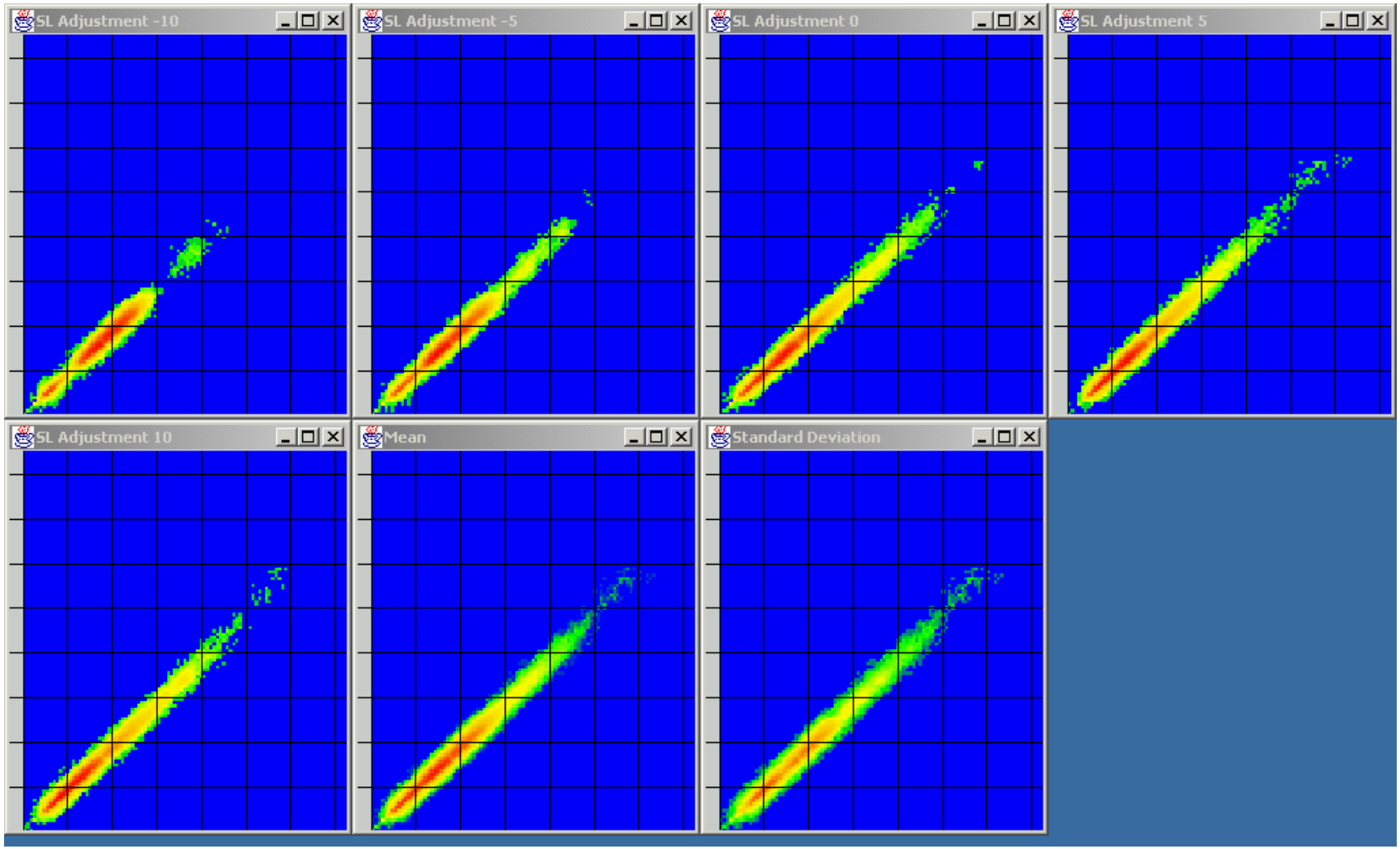


ε = 10 dB Mean Std Deviation

Fifteen Minutes of Tracking

Grid Spacing = 1 km

ε = -10 dB ε = -5 dB ε = 0 dB ε= 5 dB



ε = 10 dB Mean Std Deviation

Likelihood Function: Conical Bearing

Let $a = (a_1, a_2, a_3)$ be the location of the array and φ be the orientation of the array. Specifically, the array is centered at a and oriented in the direction of the unit vector $\varphi = (\varphi_1, \varphi_2, \varphi_3)$ whose tail is located at a. Let $\alpha(x, a, \varphi)$ be the conical angle from the point x to the array. Then

$$\alpha(x, a, \varphi) = \arccos\left(\frac{\varphi \cdot (x - a)}{\|x - a\|}\right)$$

Suppose that θ is the bearing observation and that that measurement error in the bearing is Gaussian with mean 0 and standard deviation σ_b. We assume straight-line propagation of the sound so that the (direct path) likelihood function for this conical bearing observation is

$$L_C\left(\theta \,|\, x\right) = \frac{1}{\sigma_b \sqrt{2\pi}} \exp\left(\frac{-(\alpha(x, a, \varphi) - \theta)^2}{2\sigma_b^2}\right)$$