Sonar Array Performance Assessment with Uncertain Signal and Noise Models

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"...as we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns -- the ones we don't know we don't know." Donald Rumsfield (February, 2002).

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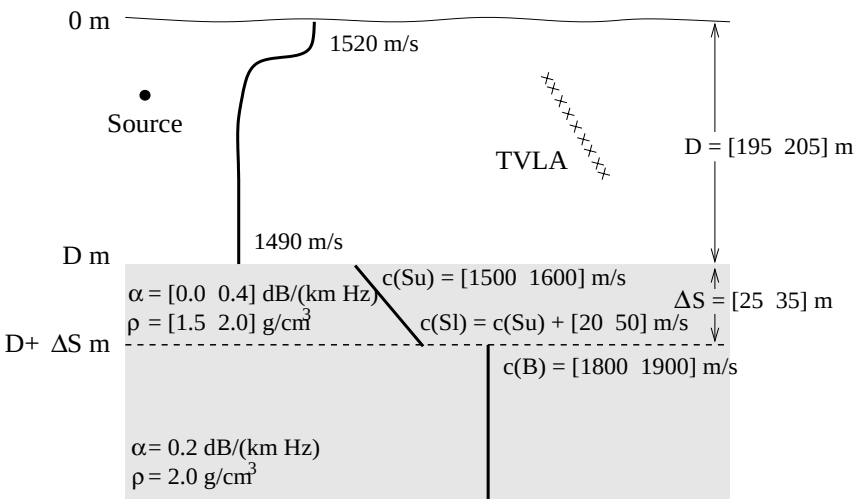
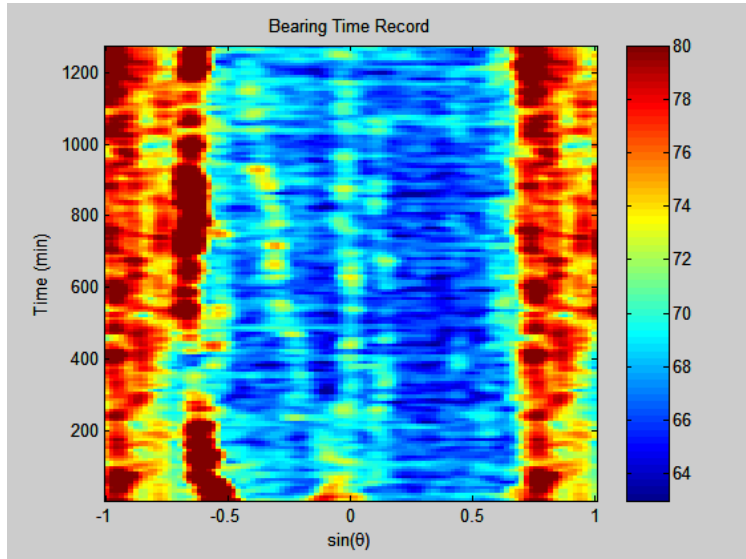
Sonar Detection Performance in an Uncertain Environment

OBJECTIVE: To achieve *in situ* assessment of sonar detection performance when the ocean environment and the noise field directionality are uncertain.

Modeling Environmental Uncertainty vs. In Situ Performance Assessment





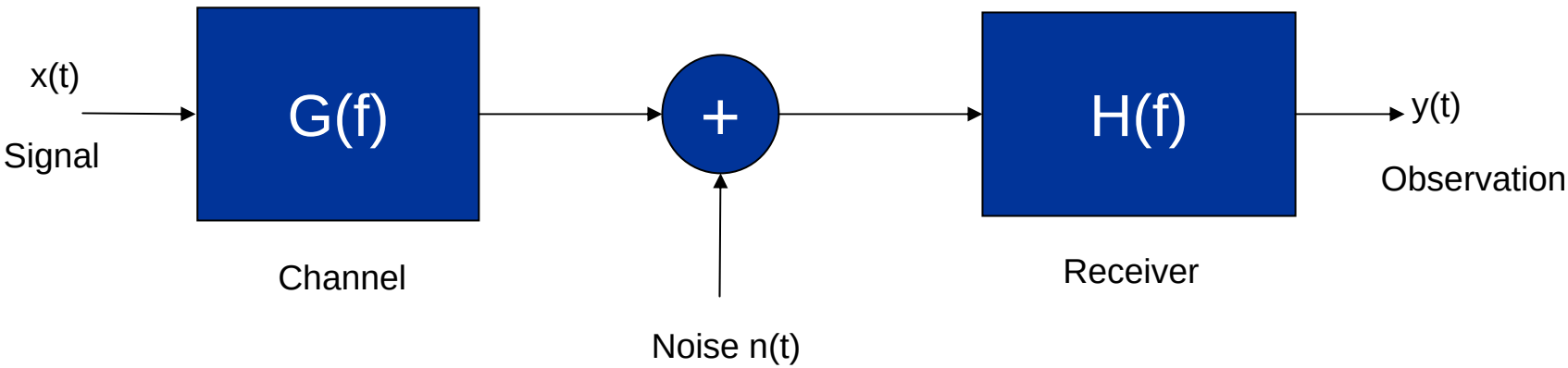
e.g. What's the uncertainty in TL vs. Range?

e.g. What's my probability of detection for a given PFA as a function of hypothesized SNR?

- To achieve agreement between predicted and actual detection performance, signal and noise model uncertainty must be captured in the detection statistic.

A Simple Example: The "Uncertain" Matched Filter

- When the signal, channel response, and statistics of the Gaussian noise are known exactly, the optimum detector is the matched-filter (MF) whose performance depends only on SNR.

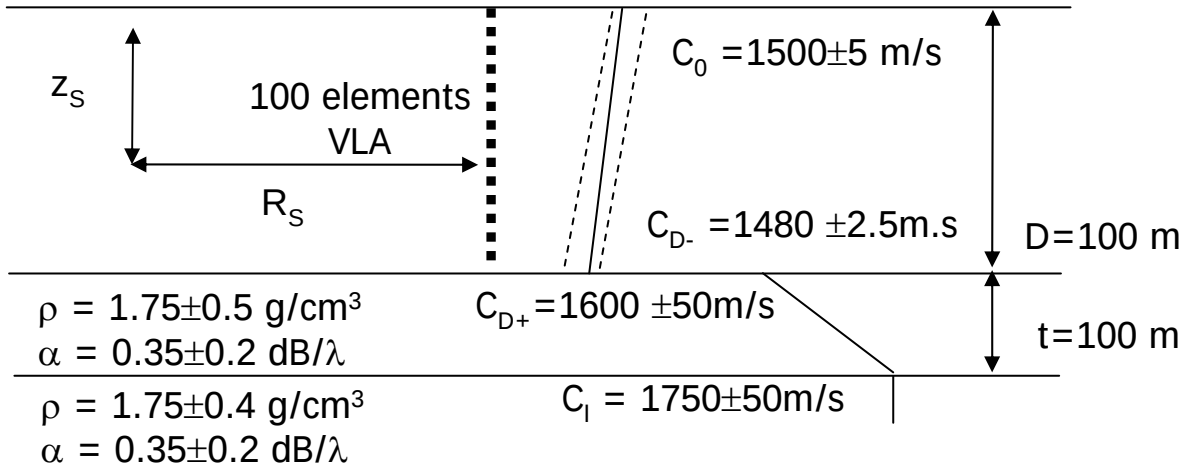


- When the receiver is *mismatched* to the channel response, theoretical predictions assuming the channel is known exactly often significantly over-predict performance.
- To bridge the gap between theoretical predictions and actual performance, channel uncertainty must be built into the assumed receiver design. It doesn't work simply to treat SNR as a random variable in the formulas for matched-filter performance.
- The Bayesian approach: Treat G(f) as a random process with prior statistics and compute the performance of the likelihood ratio test (LRT).
- The adaptive CFAR approach: Treat G(f) as being a function of unknown parameters and compute the performance of the generalized likelihood ratio test (GLRT).

Sonar Performance Assessment in an Uncertain Ocean Environment

- Detection performance depends on the prediction algorithm and the uncertain environment in which it is evaluated
- What are the random variables and what do we average?
- Examples of prediction scenarios:
    - Predictor assumes a known environment and is evaluated in a known environment; i.e. no environmental uncertainty (Matched ocean detector performance)
    - Average the performances of the predictors that are optimum for each possible environment over the environmental uncertainty (Average performance of matched ocean detectors)
    - Using the average ocean as *the* ocean, average its performance over the environmental uncertainties (Average performance of mismatched detectors)
    - Optimally incorporate the environmental uncertainty in the prediction algorithm and obtain its performance in that uncertain environment (Optimum performance prediction)
    - For a particular sonar system, could average its performance over environmental uncertainty as the prediction for that particular system in that uncertain environment.

NRL Benchmark Ocean Environment



f=450Hz

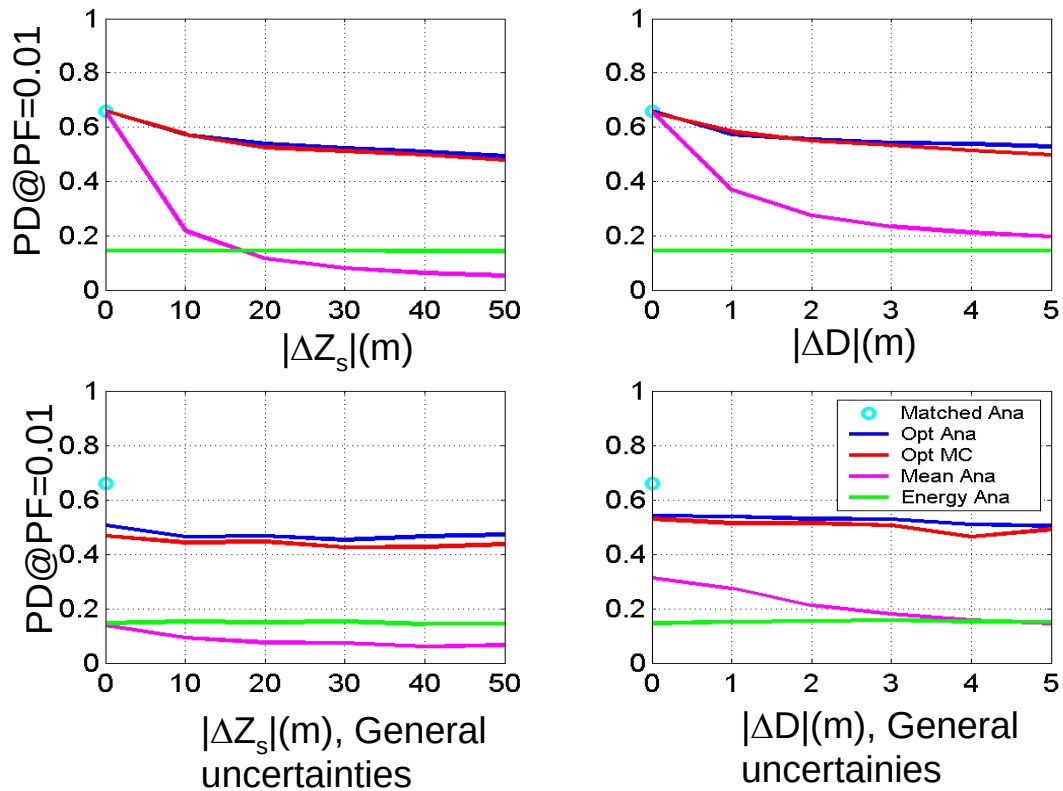
1:1:100m

full spanning

vertical array

Abbreviation	variable	mean value	Range of uncertainty
D	Ocean depth	100m	$\|\Delta D\|$=5m
C_0	Ocean surface sound speed	1500m/s	$\|\Delta C_0\|$=2.5m/s
C_{D-}	Ocean bottom sound speed	1480m/s	$\|\Delta C_{D-}\|$=2.5m/s
C_{D+}	Upper sediment sound speed	1600m/s	$\|\Delta C_{D+}\|$=50m/s
C_l	Lower sediment and sub-bottom sound speed	1750m/s	$\|\Delta C_l\|$=100m/s
t	Sediment thickness	100m	0m
ρ	Sediment and sub-bottom density	1.75g/cm^3	$\|\Delta\rho\|$=0.25g/cm^3
α	Sediment and sub-bottom attenuation	0.35dB	$\|\Delta\alpha\|$=0.25dB
Z_s	Source depth	50m	0m
R_s	Source range	6km	0m

Comparison of performance (ROC) of several possible sonar detection prediction methods as a function of environmental uncertainties



Matched-Ocean predictor

$$\lambda'(r) = \left| r^H s \right|^2$$

$$P_D = P_F^{1/(SNR+1)}$$

Mean-Ocean Predictor

$$\lambda'(r) = \left| r^H s_m \right|^2$$

Mean ocean

$$P_D = \sum_{i=1}^{M} P_F^{1/(\rho(s_i, s_m) SNR_i + 1)} / M$$

Correlation Coefficient

Energy Predictor

$$\lambda'(r) = r^H r$$

Optimal Analytical Performance prediction

$$P_D = 1-(1-P_F)^{(R-1)/R}(1-(1-(1-P_F)^{1/R})^{1/(SNR+1)})$$

$$SNR = E_{\Psi, S_s}(SNR(\Psi, S_s))$$

Optimal Bayesian detector: Monte Carlo method

$$\lambda(r) = \sum_{i=1}^{L} \lambda_i, \quad \lambda_i = \frac{1}{1+SNR_i} \exp\left(\frac{SNR_i \left| r^H s_i \right|^2}{1+SNR_i}\right)$$

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From Environmental Uncertainty to Wavefront Uncertainty

$$H_1 \quad r = \sqrt{SNR(\psi, S_s)}\, a s(\psi, S_s) + n_1, \quad n_1 = \sum_{k=1}^{K} \sqrt{INR_k(\psi, S_k)}\, b f_k(\psi, S_k) + n_0$$

$$H_0 \quad r = n_1, \qquad a \sim N(0,1), b \sim N(0,1), n_0 \sim N(0, I_N)$$

- **Environmental uncertainty** characterized by probability density functions of environmental parameters and ocean acoustic model (or by realizations)

$$\psi \sim p(\psi), S_s \sim p(S_s), S_k \sim p(S_k)$$

- **Uncertain signal wavefront** $s(\psi, S_s) = \mathbf{H}(\psi, S_s) / \|\mathbf{H}(\psi, S_s)\|$

  where $\mathbf{H}(\psi, S_s)$ is the ocean transfer function sampled at an N sensor array*

- **Signal matrix**: matrix of signal wavefronts due to environmental uncertainty

- **Total signal-to-noise ratio** at the receivers

$$SNR(\psi, S_s) = \frac{\sigma_a^2}{\sigma_n^2} \mathbf{H}(\psi, S_s)^H \mathbf{H}(\psi, S_s)$$

*Note: ocean transfer function $\mathbf{H}(\psi, S_s)$ typically computed using KRAKEN code(1991, M. B. Porter).

From Wavefront Uncertainty to Bayesian Sonar Detection Performance Prediction

- **Model-based Monte Carlo methods for ROC**: Can obtain optimal performance prediction incorporating environmental uncertainty but no insight into how to interpret the ROC in terms of fundamental parameters representing the uncertainty in the environmental parameters. Can be computationally intensive.
- **Model-based analytical approaches for ROC**: Have derived simple analytical expressions for the ROC for optimal performance prediction incorporating environmental uncertainty. The ROC is expressed in terms of the SNR and the rank of the signal matrix, R, where R captures the environmental uncertainty

Model-based Bayesian Approaches

Signal & Noise Models that incorporate probabilistic models of environmental uncertainty and acoustic propagation model

Likelihood ratio

$$P_F = \int_\beta^\infty p(\lambda \mid H_0)\, d\lambda$$

$$P_D = \int_\beta^\infty p(\lambda \mid H_1)\, d\lambda$$

A posteriori prob

PCL = Probability of correct localization

Analytical (and MC) methods: ROC and PCL expressions

Analytical Bayesian Sonar Detection Performance Prediction (ROC)

Environment		Opt Detector statistic	ROC performance
Known	Multiple Interferers	$\lambda'(r) = \left\| r^H M_0^{-1} s \right\|^2$ $M_0 = \sum_{k=1}^{K} INR_k f_k f_k^H + I_N$	$P_D = P_F^{1/(SNR\, s^H M_0^{-1} s + 1)}$
	Single Interferer	$\lambda'(r) = \left\| r^H (I - \frac{INR}{1+INR} f f^H) s \right\|^2$	$P_D = P_F^{1/(SNR\,(1 - \frac{INR}{1+INR} \left\| s^H f \right\|^2) + 1)}$
	Diffuse noise	$\lambda'(r) = \left\| r^H s \right\|^2$	$P_D = P_F^{1/(SNR + 1)}$
Uncertain Environmental uncertainty captured by R: rank of the signal covariance matrix	Multiple Interferers	$\lambda(r) = \sum_{i=1}^{L} \lambda_i$ $\lambda_i = \frac{1}{1 + SNR_i G_i} \exp\left(\frac{SNR_i \left\| r^H M_0^{-1} s_i \right\|^2}{1 + SNR_i G_i}\right)$ $SNR_i = SNR((\Psi, S_s)_i), \quad G_i = s_i^H M_0^{-1} s_i$ $M_0 = \sum_{m=1}^{M} \sum_{k=1}^{K} INR_k \quad ((\Psi, S_k)_m)$ $f_k((\Psi, S_k)_m) f_k((\Psi, S_k)_m)^H + I_N$	$P_D = 1 - (1 - P_F)^{(R-1)/R}$ $(1 - (1 - (1 - P_F)^{1/R})^{1/(SNR\,G+1)})$ $SNR = E_{\Psi, S_s}(SNR(\Psi, S_s))$ $G = E_{\Psi, S_s}(s(\Psi, S_s)^H M_0^{-1} s(\Psi, S_s))$
	Diffuse noise	$\lambda(r) = \sum_{i=1}^{L} \lambda_i$ $\lambda_i = \frac{1}{1 + SNR_i} \exp\left(\frac{SNR_i \left\| r^H s_i \right\|^2}{1 + SNR_i}\right)$	$P_D = 1 - (1 - P_F)^{(R-1)/R}$ $(1 - (1 - (1 - P_F)^{1/R})^{1/(SNR + 1)})$

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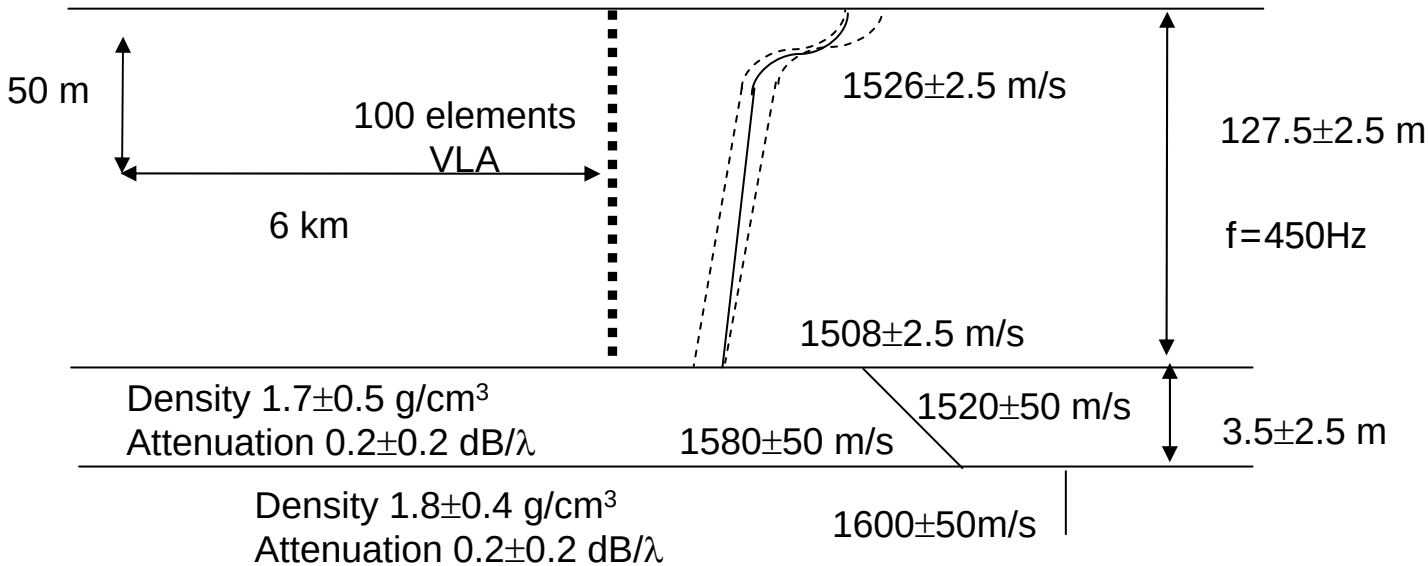
Example of capturing environmental uncertainty in signal matrix rank estimation



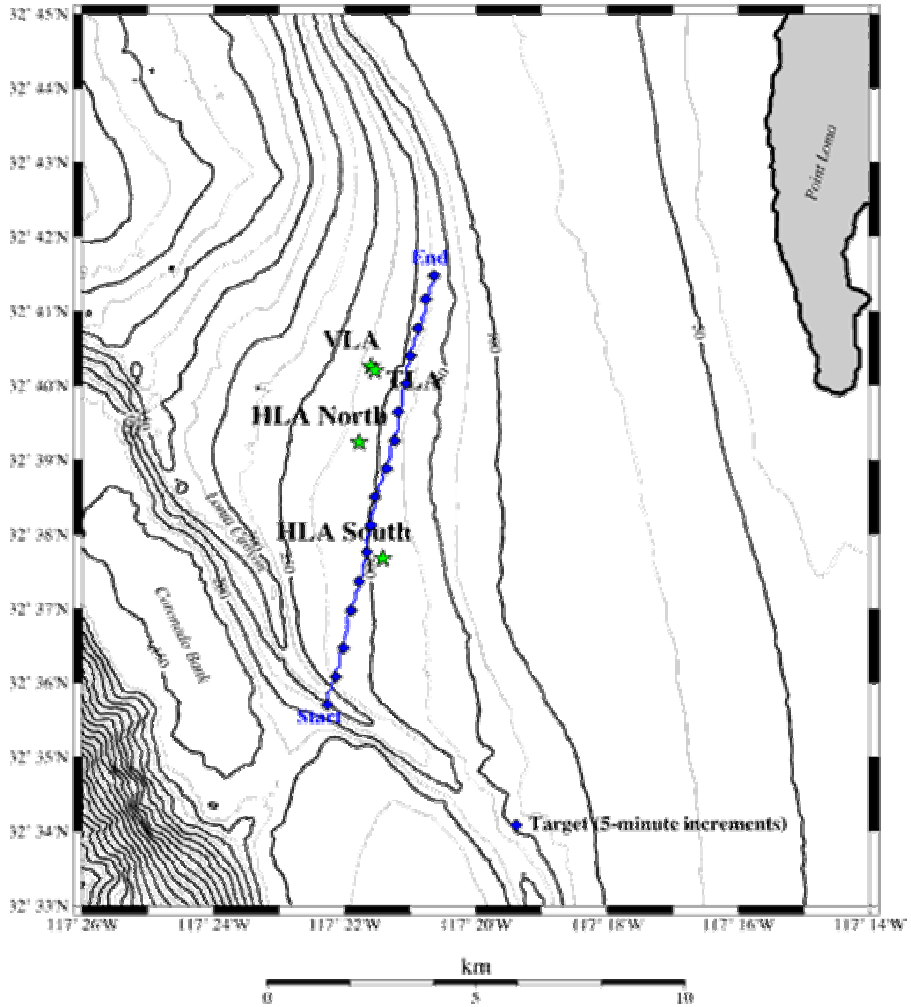
Rank of the signal covariance matrix	Uncertain water depth $\lvert\Delta D\rvert \le 2.5$ m	Uncertain Surface Speed $\lvert\Delta s\rvert \le 2.5$ m/s	Uncertain Bottom Speed $\lvert\Delta b\rvert \le 2.5$ m/s	Uncertain source depth $\lvert\Delta Z_s\rvert \le 5$ m	General uncertain
NRL benchmark ocean	8	3	3	3	13
NATO Ocean	6	3	3	3	9

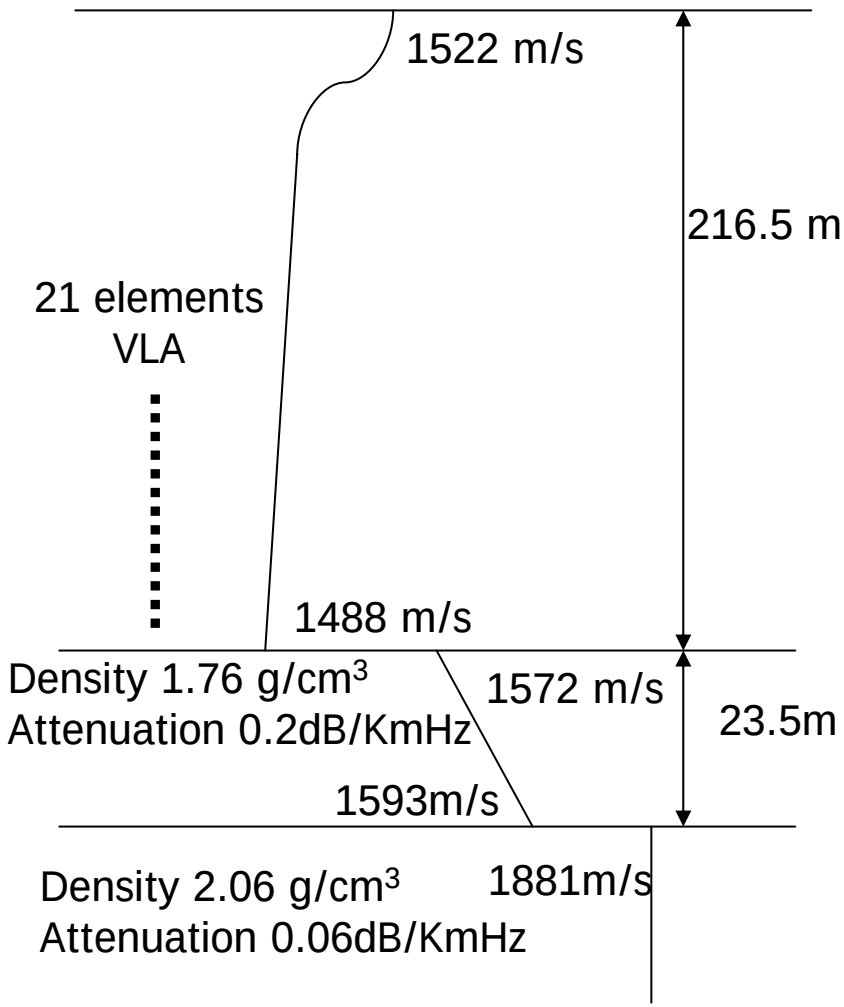
Note: NATO ocean parameters come from reference (1995, Gingras and Gerstoft).

SWellEx-96 Event S5

5/10/96 23:15-5/11/96 00:30 GMT

Environmental Model

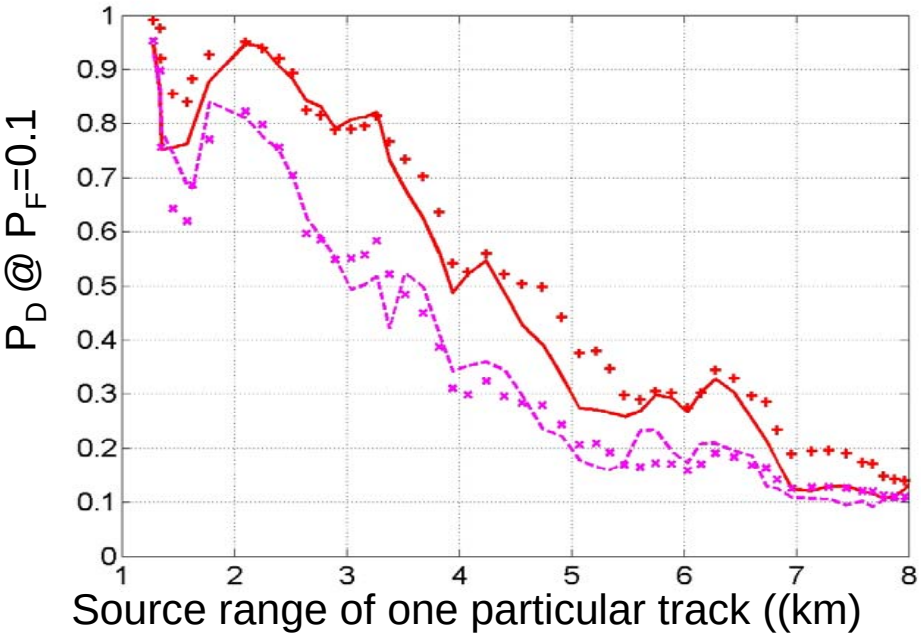




Courtesy of MPL: http://www.mpl.ucsd.edu/swellex96

Detection Performance Prediction Experimental Results

(SWellEx-96 Event S5): Wave Front Uncertainty Due to Source Motion



Matched-ocean detector $\lambda' = \left| r^H s \right|^2$

+ Monte Carlo method
- Analytical Prediction

$$P_D = Q\left(\sqrt{SNR}, -\sqrt{2\log(P_F)}\right)$$

Bayesian detector $\lambda(r) = \sum_{i=1}^{L} \lambda_i / L$

$$\lambda_i = e^{-SNR/2} I_0\left(\frac{\mu}{\sigma_n^2}\left|r^H s_i\right|^2\right)$$

+ Monte Carlo method
- Analytical Prediction

$$P_D = 1-(1-P_F)^{(R-1)/R}\left(1-Q\left(\sqrt{SNR}, -\sqrt{2\log(1-(1-P_F)^{1/R})}\right)\right)$$

- Matched-ocean scenario: known source range for each receiving spatial data vector
- Source motion scenario: source range uncertainty of ± 450m for each data frame.

Frame: 21 channels x 900snapshots (5mins), updated per 100snapshots

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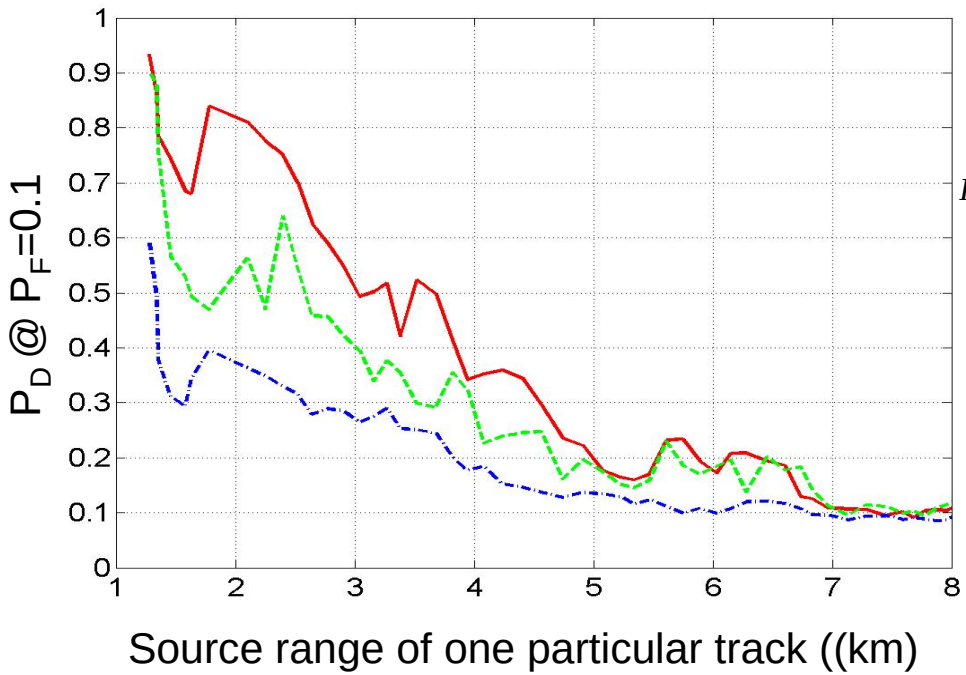
Detection Performance Prediction Experimental Results

(SWellEx-96 Event S5): Impact of prediction method



-- Bayesian detector, analytical prediction

$$P_D = 1-(1-P_F)^{(R-1)/R}\left(1-Q\left(\sqrt{SNR},-\sqrt{2\log(1-(1-P_F)^{1/R})}\right)\right)$$

-- Mean-ocean detector, Monte Carlo method

-- Energy detector, Monte Carlo method

Adaptive CFAR Detection in an Uncertain Ocean

- In an uncertain waveguide, the *M*-sensor passive detection problem is given by:

$$\mathrm{H}_o:\ x_n = \eta_n \quad \text{versus} \quad \mathrm{H}_1:\ x_n = s_n U(\theta_s)a + \eta_n$$

where $[U(\theta_s)]_{ml} = \phi_l(z_m)e^{-jk_l md\sin\gamma\sin\theta_s}, [a(r_s,z_s)]_l = \phi_l(z_s)e^{-jk_l r_s}$, s_n are unknown amplitudes, θ_s, r_s, z_s are source bearing, range, depth, and γ is array tilt.

- We use $E(Uaa^H U^H) = H\Lambda H^H$ to define a reduced-dimensional signal subspace whose rank *p* (or *r* in Loren's slides) increases with environmental uncertainty.

- Adaptive detection assumes a set of i.i.d "training vectors" are available to estimate the unknown Gaussian noise covariance, $R_\eta = E(\eta_n \eta_n^H)$

- For adaptive detection in an uncertain ocean, the CFAR generalized likelihood ratio test (GLRT) is given by:

$$\lambda(x_n) = \frac{x_n^H \hat{R}_\eta^{-1} H(H^H \hat{R}_\eta^{-1} H)^{-1} H^H \hat{R}_\eta^{-1} x_n}{K + x_n^H \hat{R}_\eta^{-1} x_n}$$

where $\hat{R}_\eta = \frac{1}{K}\sum_1^K x_n x_n^H$ estimated from K "signal-free" training data snapshots.

Analytic Probability of Detection vs. Output SNR

- If the signal wavefront and noise covariance are known exactly, the likelihood ratio test (LRT) detection statistic is non-central Chi-square distributed with PD:

$$PD(\tau) = \int_{\tau}^{\infty} \chi^2_{[2,2\cdot SNR]}(x)\mathrm{dx}$$

- Setting the level τ to achieve a specified PFA, this equation can be numerically solved to find the SNR associated with a specified PD.

- For an uncertain signal wavefront and noise covariance matrix, the statistics of the optimal adaptive CFAR GLRT have been derived by (Kraut et.al. 2001) with:

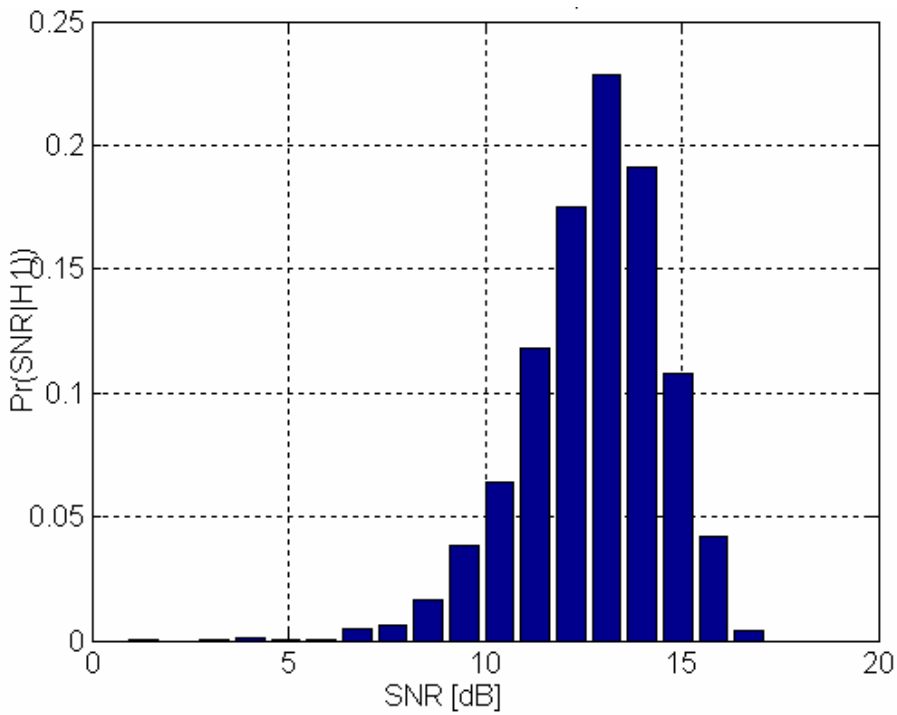
$$PD(\tau) = \int_{\tau}^{\infty} \int_{-\infty}^{\infty} F_{[2\cdot p,2\cdot(K-M+1),2\cdot SNR\cdot b]}(x)\mathrm{p}(b)\mathrm{d}b\mathrm{d}x \text{ with } \mathrm{p}(b) \sim \beta_{[K-M+p+1,M-p]}$$

- Now PD also depends on the ocean uncertainty through the signal subspace dimension, p, the number of training snapshots available to estimate the noise field, K, and the number of sensors in the array, M.

Estimating Output Signal-to-Noise Ratio

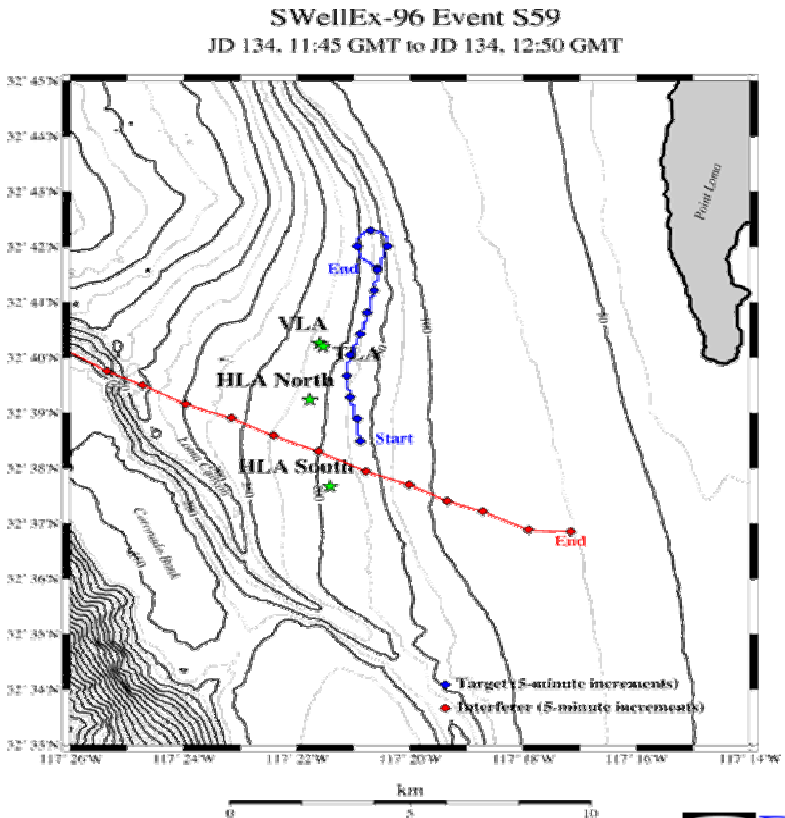
- Detection performance depends on output SNR defined by $SNR = |s|^2 a^H U^H R_\eta^{-1} U a$
- Prediction of output SNR at *a particular time and place* would require modeling of the signal wavefront, noise covariance, and source level which are all uncertain.
- In practice, it would therefore make sense to model output SNR as a random variable with *a priori* density determined by environmental and source level uncertainty.

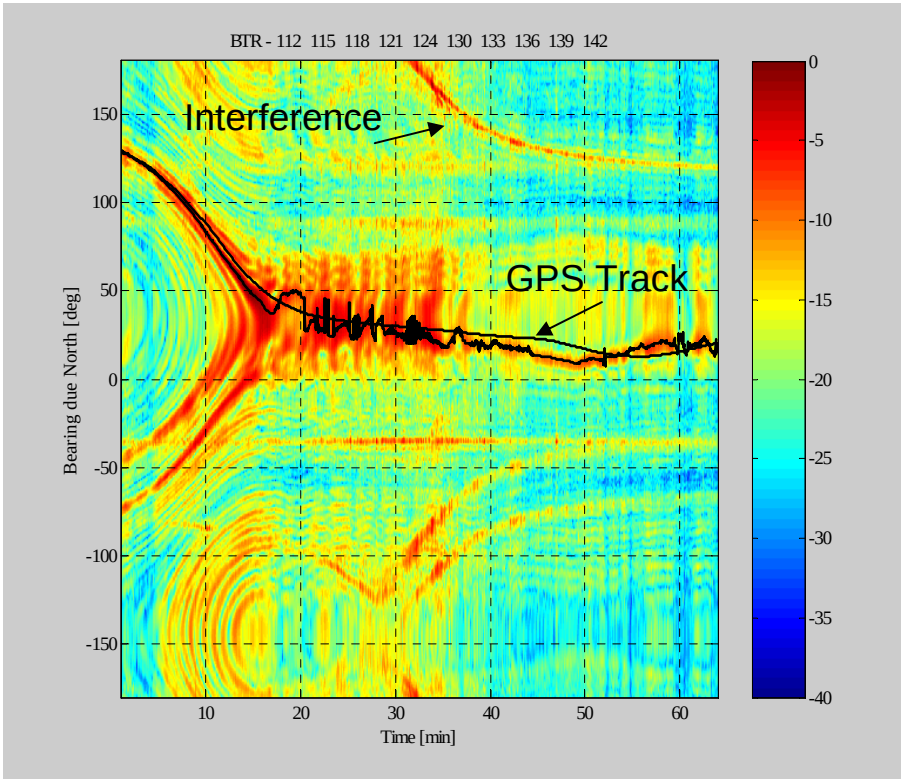
- Alternatively, to compare our performance predictions for *a particular track* of real data, we used a maximum likelihood estimator (MLE)output SNR which is robust with respect to wavefront uncertainty (large *p*) and noise covariance (large *K*).
- Histogram (right) of simulated SNR MLE for signal rank *p=8* and noise training data *K=300* versus true SNR of 13 dB.



Horizontal Array Data from SWELLEX-96 Event S59

- The SWELLEX-96 dataset (courtesy MPL/SIO) was used to validate analytic PDF's and detection performance prediction. Previously analyzed S5 event with source only.
- Narrowband data from the *M=27* sensor, 240-meter long, non-uniformly spaced horizontal line array AODS North was compared with detection performance predictions for the S59 event. Scenario (left) 10 frequency BTR (right)





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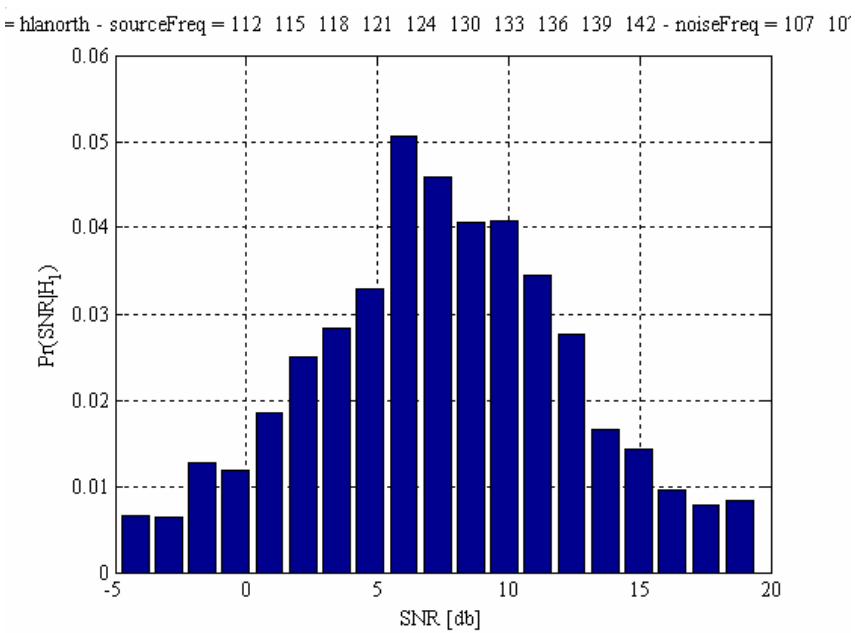
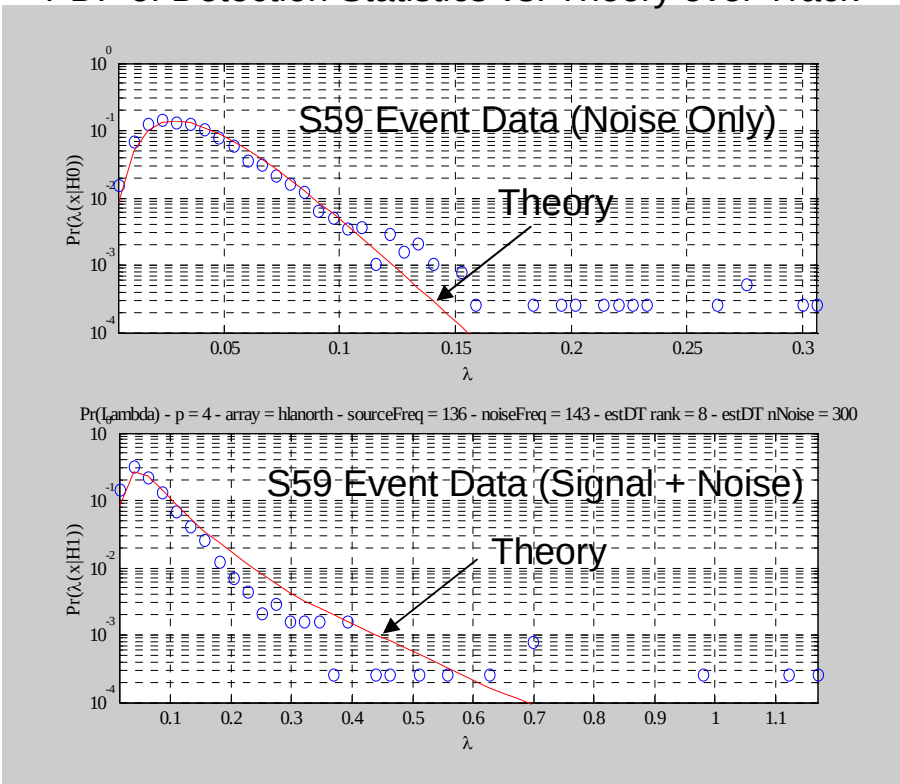
Detection Statistics Over Entire S59 Event Track

- Comparison of analytic PDF under H0 (noise only) and H1 (signal+noise) versus SWELLEX-96 data at 136 Hz for *p=4* and *K=128* (right figures) using estimated distribution of SNR's (left figure) over entire 65 minute track.
- Note good agreement between observed and predicted PDF's for this level of signal wavefront uncertainty.

Estimated PDF of SNR over S59 track over 10 frequencies between 112 and142 Hz.

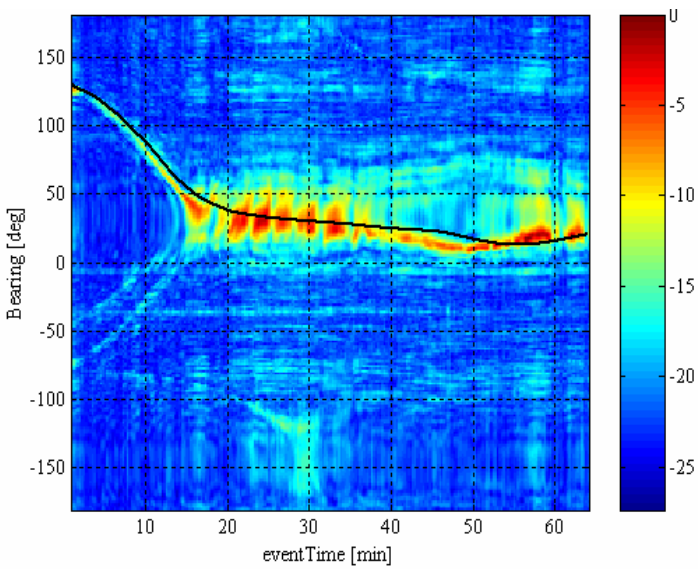


PDF of Detection Statistics vs. Theory over Track



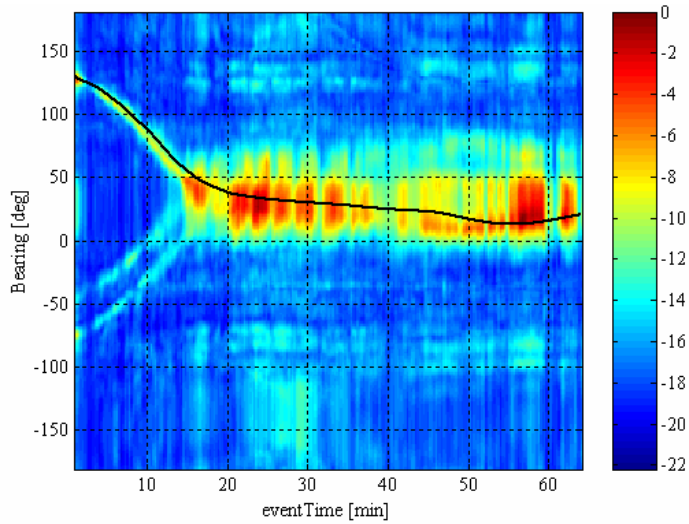
Adaptive CFAR GLRT vs. Bearing for S59 Event

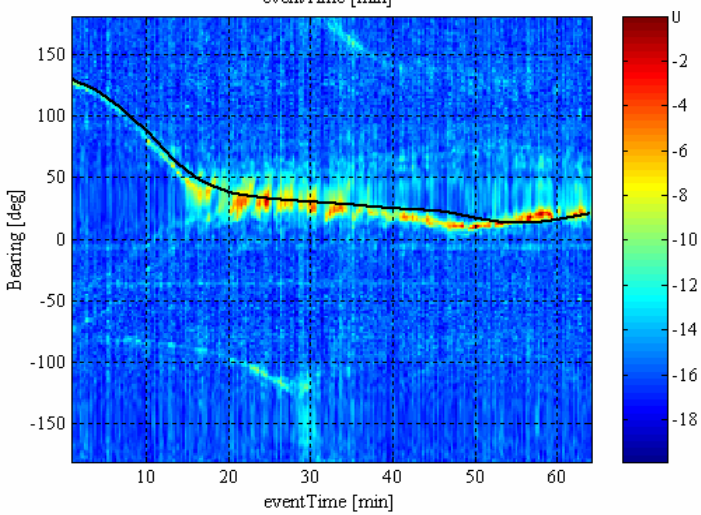
- Difficult to predict match between theory and data from visual inspection, although for *p* = 1, under-resolution and noisy background for K=32 suggests sub-optimum performance.



K=128, p = 1

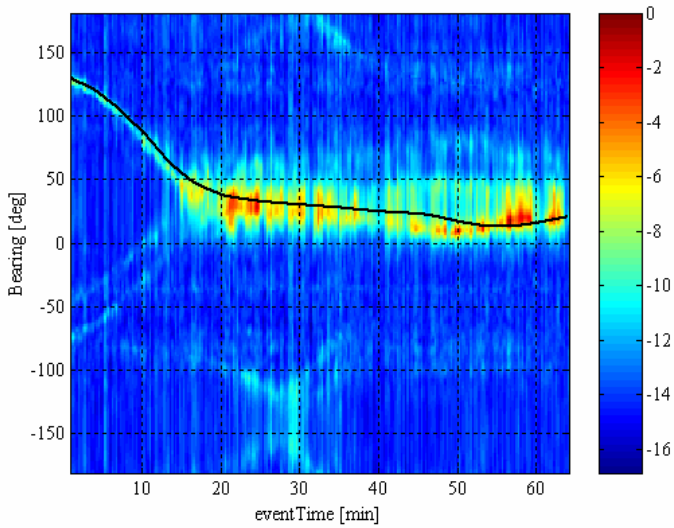
K=128, p = 4





K=32, p = 1

K=32, p = 4



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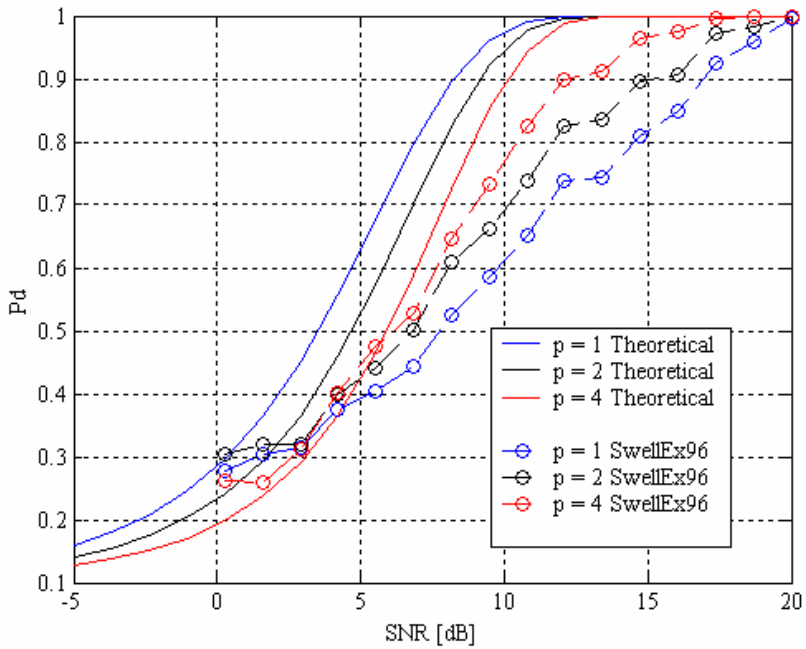
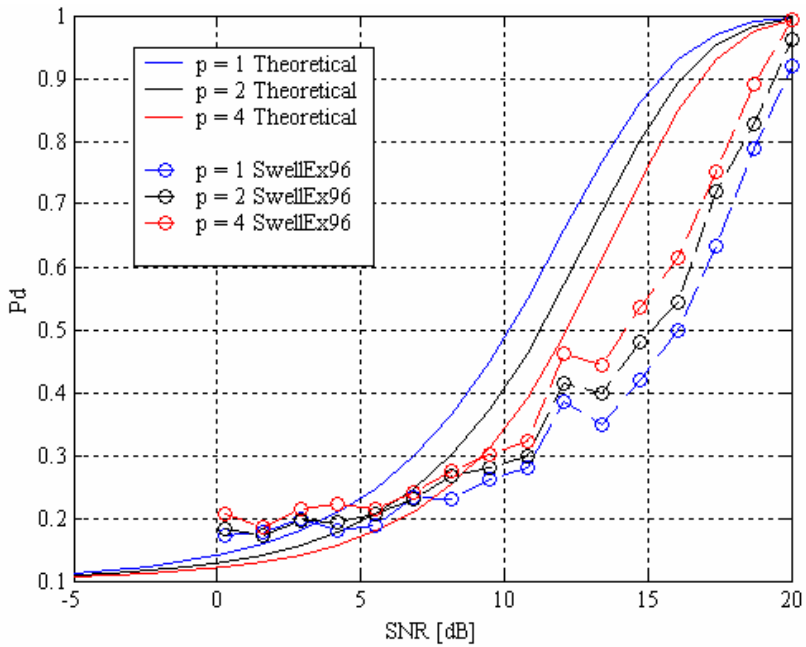
Event S59 Detection with Signal Wavefront Uncertainty

- Probability of detection versus SNR for the adaptive GLRT detector for real S59 M=27 HLA North event data versus analytic prediction with increasing signal wavefront uncertainty (i.e. subspace rank p). Snapshot support *K=128* (left) and *K=32* (right).
- Good agreement achieved between theory and data for uncertain wavefront (*p=4)* model (red curves). Note mismatch for *p=1* prediction versus real data when plane-wave beamforming assumed which, in part, represents prediction error of current practice.

PD vs. SNR for K=128 over 10 tones



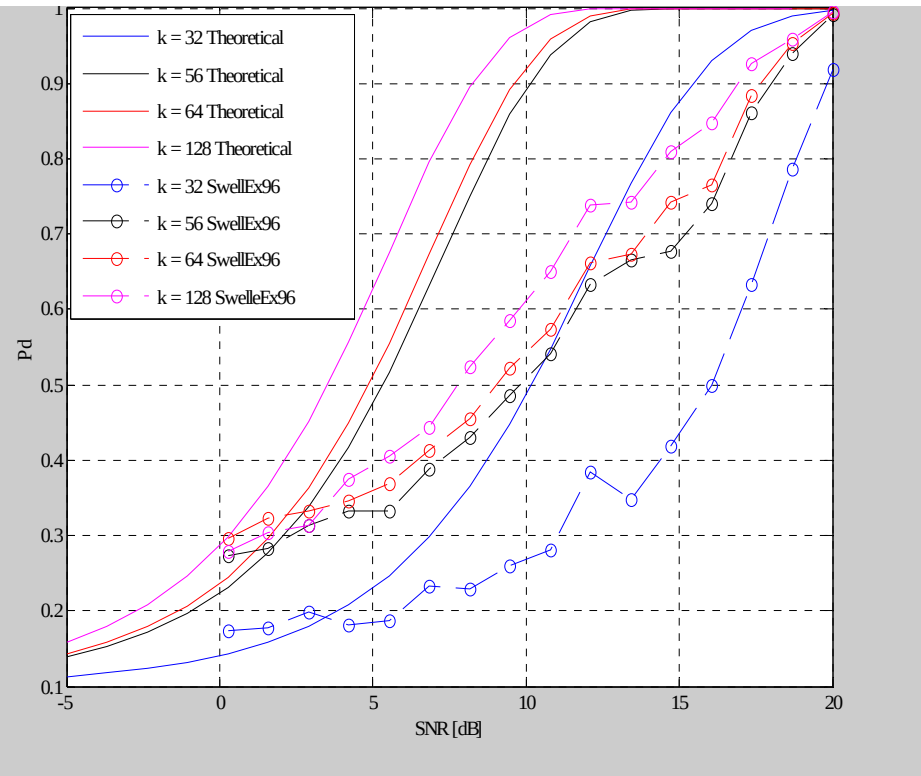
PD vs. SNR for K=32 over 10 tones



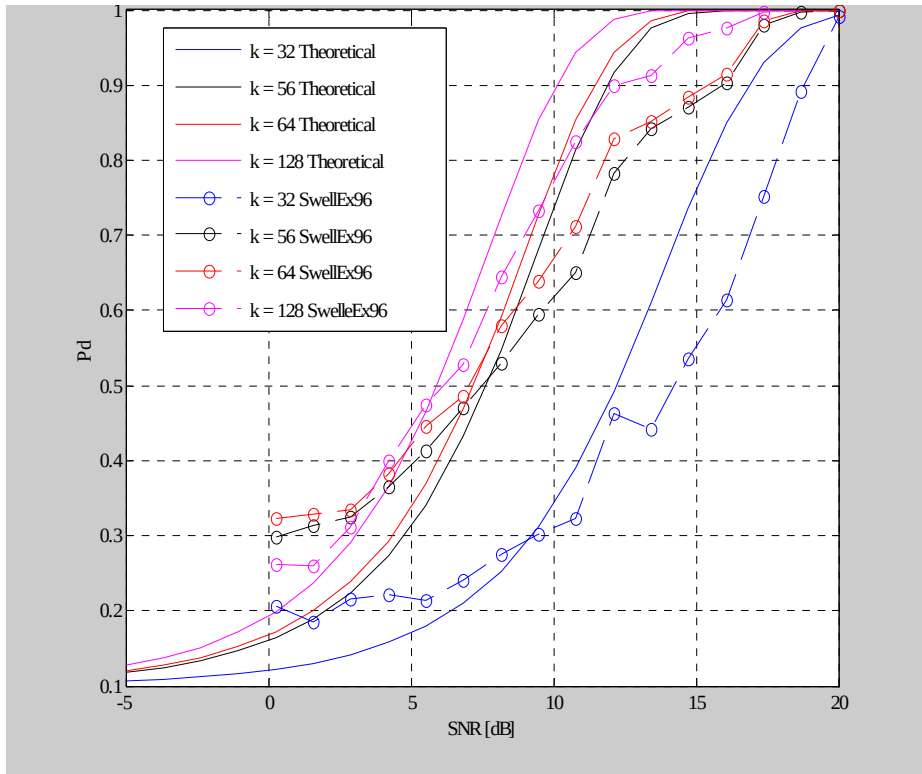
Event S59 Detection with Limited Noise Training Data

- GLRT PD versus SNR for real S59 AODS North data versus theory with limited snapshots K. Signal wavefront uncertainty: *p=1* (left) and *p=4* (right).
- Good agreement achieved between theory and data for uncertain wavefront (*p=4)* model (right) with mismatch evident when *p* = 1 plane-wave modeling assumed (left). Note signal contamination in noise training data results in PD reduction at high SNR's.

PD vs. SNR (p=1)



PD vs. SNR (p = 4)



Summary and Conclusions

- Bridging the gap between predicted and actual sonar performance requires capturing signal, noise and environmental model uncertainty in the detection statistic.
- Fast, analytical methods for characterizing the performance of both optimum Bayesian and adaptive CFAR detectors have been derived.
- Signal covariance matrix rank is an efficient measure of ocean environmental uncertainty for both Bayesian and adaptive CFAR detection performance prediction.
- Analytical methods derived that approximate the ROC for optimum Bayesian detection for an uncertain ocean environment model in terms of the signal covariance matrix rank, SNR, and a signal-to-noise interference coefficient.
- Good agreement between theoretical and real data performance for single-source SWELLEX-96 S5 event achieved for both vertical and horizontal arrays when uncertainty in the signal wavefront is accounted for.
- Recent results for the multiple source SWELLEX-96 S59 event using the AODS North horizontal array yield similar agreement between theory and real data detection performance when wavefront uncertainty and limited snapshot support are accounted for.
- At higher SNR's, signal contamination of the noise training data can cause the measured PD to be notably less than analytical prediction when the training data limited.
- Further work is required to extend theoretical adaptive CFAR detection results to the case where signal is present in the training data and highly dynamic scenarios where $K < M$.
- Further work is required to determine the priorities of specific ocean environmental parameters on detection performance prediction, extending and using the analytical Bayesian performance prediction methods derived.