Statistical Properties of the Acoustic Field in Inhomogeneous Oceanic Environments

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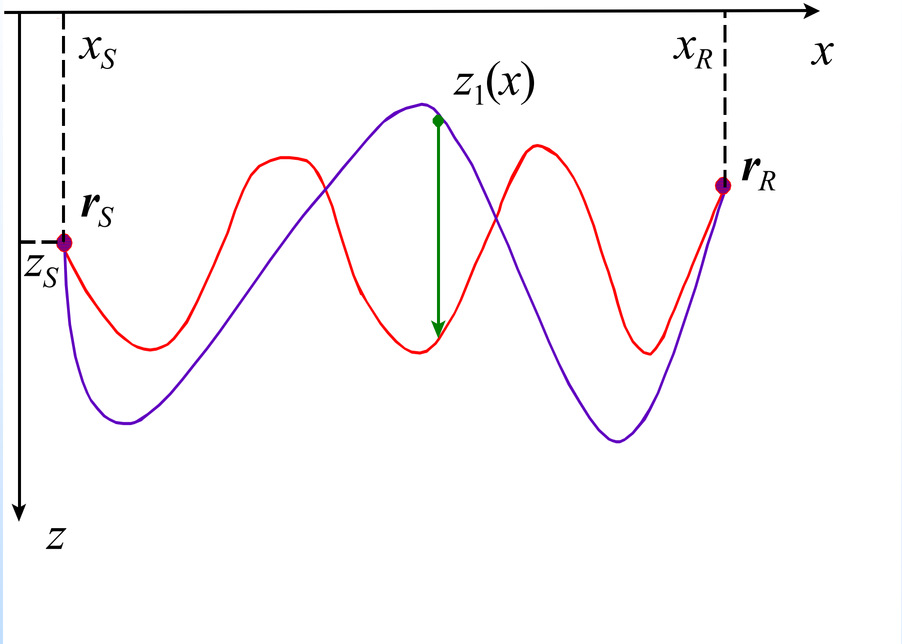
Ray perturbation theory



Differential ray equation:

$$\frac{d^2 z}{d x^2} = \left[1 + \left(\frac{dz}{dx}\right)^2\right]\left(\frac{c_x}{c}\frac{dz}{dx} - \frac{c_z}{c}\right), c = c(x, 0, z(x))$$

Ansatz:

$$z(x) = z_0(x) + \varepsilon z_1(x) + \varepsilon^2 z_2(x) + \ldots,$$
$$z_j(x_{S,R}) = 0,\ j = 1, 2, \ldots$$

Equations for ray trajectory variations:

$$\frac{d^2 z_1}{d x^2} = A_1 \frac{dz_1}{dx} + A_2 z_1 - \frac{a}{\cos^3 \chi}, \quad \frac{d^2 \varphi}{d x^2} = A_1 \frac{d\varphi}{dx} + A_2 \varphi, \quad \varphi \equiv \left(\frac{\partial z_0}{\partial \chi_0}\right)_x, \quad a = \left(\cos\chi \frac{\partial}{\partial z} - \sin\chi \frac{\partial}{\partial x}\right)\frac{c_1}{c}$$

$$z_1(x) = \int_{x_S}^{x_R} G(x, x') \frac{a\, dx'}{\cos^3 \chi}, G(x, x') = \frac{p(x_<) q(x_>)}{w(x')}, x_< = \min(x, x'), x_> = \max(x, x')$$

Ray perturbation theory (continued)

Travel time variation:

$$T = T_0 + \varepsilon\,\delta T_1 + \varepsilon^2\left(\delta T_{21} + \delta T_{22} + \delta T_{23}\right) + O\left(\varepsilon^3\right)$$

$$T_\alpha = \int_{x_S}^{x_R} \frac{B_\alpha dx}{c\cos\chi},\quad B_0 = 1,\quad B_1 = -\frac{c_1}{c},\quad B_{21} = \frac{c_1^2 - cc_2}{c^2},$$

$$B_{22} = -az_1\cos\chi,\ B_{23} = -\frac{c^2}{2g^2}\left(\int_{x_S}^{x}\frac{\partial c_1}{\partial y}\frac{g\,dx'}{c^2\cos\chi}\right)^2,\ g = \int_{x_S}^{x}\frac{c\,dx'}{\cos\chi}$$

Bearing variation:

$$\psi(\boldsymbol{r}) = \frac{c}{g\cos\chi}\int_{x_S}^{x}\frac{\partial c_1}{\partial y}\frac{g\,dx'}{c^2\cos\chi}$$

Arrival angle variation:

$$\chi_1 = \cos^{-2}\chi \quad dz_1/dx$$

False Doppler:

$$\frac{f(\boldsymbol{r},t) - f_S}{f_S} = \int_{x_S}^{x}\frac{dx'}{\cos\chi}\frac{1}{c^2}\frac{\partial c}{\partial t}\bigg|_{z=z_0(x'),\,t=t_0(x')},\quad t_0(x') = t - \int_{x'}^{x}\frac{dx''}{c\cos\chi}$$

In-plane travel time bias: Internal gravity waves

$$\left\langle \delta T_1 \right\rangle = \int_{x_S}^{x_R} \frac{\left(E_1^2 - E_2\right)_{z=z_0(x)} \sigma_\varsigma^2\left(x, 0, z_0(x)\right) dx}{c^3\left(x, 0, z_0(x)\right) \cos\chi\left(x, 0, z_0(x)\right)} = \int_{x_S}^{x_R} \frac{\left(1 - E_1^{-2} E_2\right)_{z=z_0(x)} \sigma_c^2\left(x, 0, z_0(x)\right) dx}{c^3\left(x, 0, z_0(x)\right) \cos\chi\left(x, 0, z_0(x)\right)}$$

$$\left\langle \delta T_2 \right\rangle = \int_{x_S}^{x_R} \left. \frac{c\, A \varphi_1 \varphi_2\, dx}{w \cos^2 \chi} \right|_{y=0,\, z=z_0(x)}$$

Here:

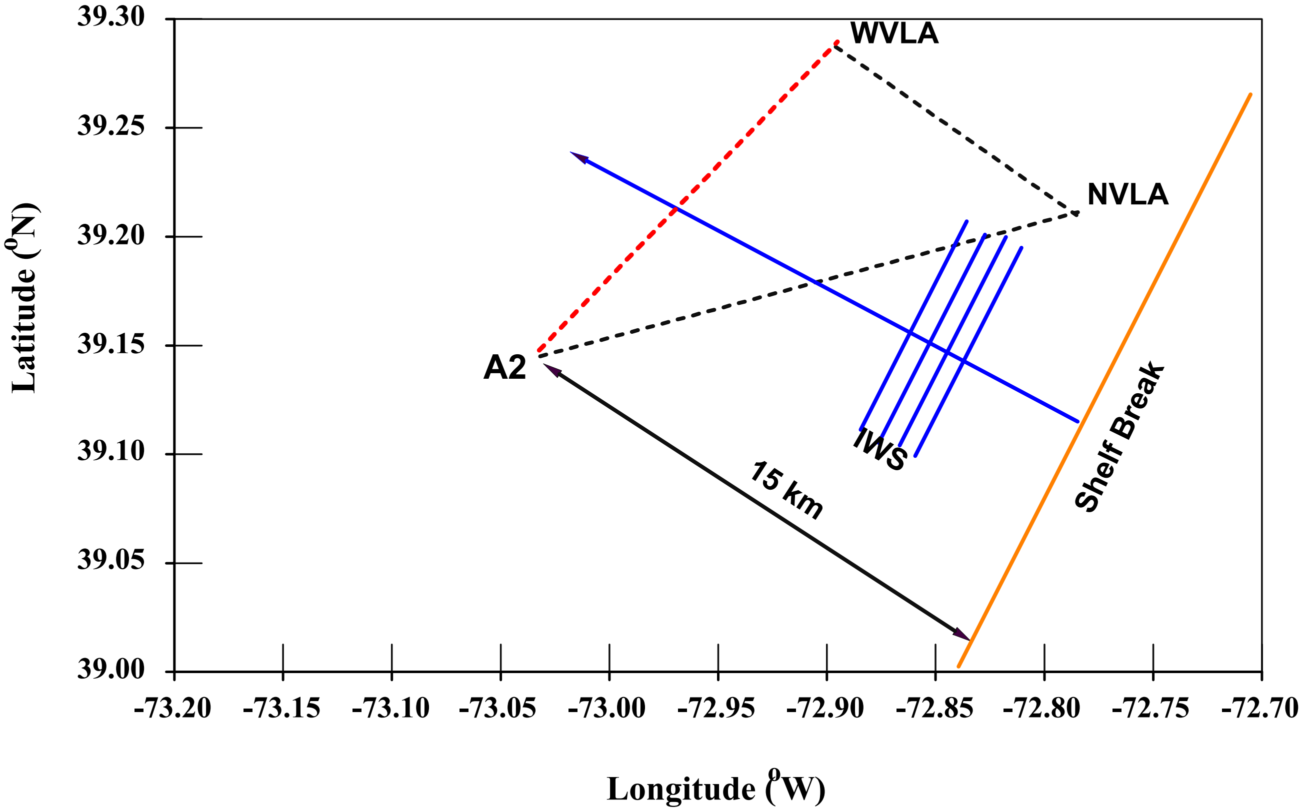
$$c_{pert} = c\left(z + \zeta(z)\right) - \gamma_a c(z) \zeta(z), \quad c_j = c E_j \zeta^j, \quad E_1 = -\gamma_a \frac{N^2(z)}{N^2(z_a)}, \quad \frac{E_2}{E_1^2} = \frac{1}{2c\gamma_a^2} \frac{\partial^2 c}{\partial z^2} \frac{N^4(z_a)}{N^4(z)}$$

$$A(\chi; \mathbf{r}) = \frac{1}{2\cos\chi} \int_{-\infty}^{+\infty} \left(\sin^2\chi \frac{\partial^2 W}{\partial x^2} - 2 \sin\chi \cos\chi \frac{\partial^2 W}{\partial x \partial z} + \cos^2\chi \frac{\partial^2 W}{\partial z^2} \right)_{y=0,\, z=x\tan\chi} dx$$

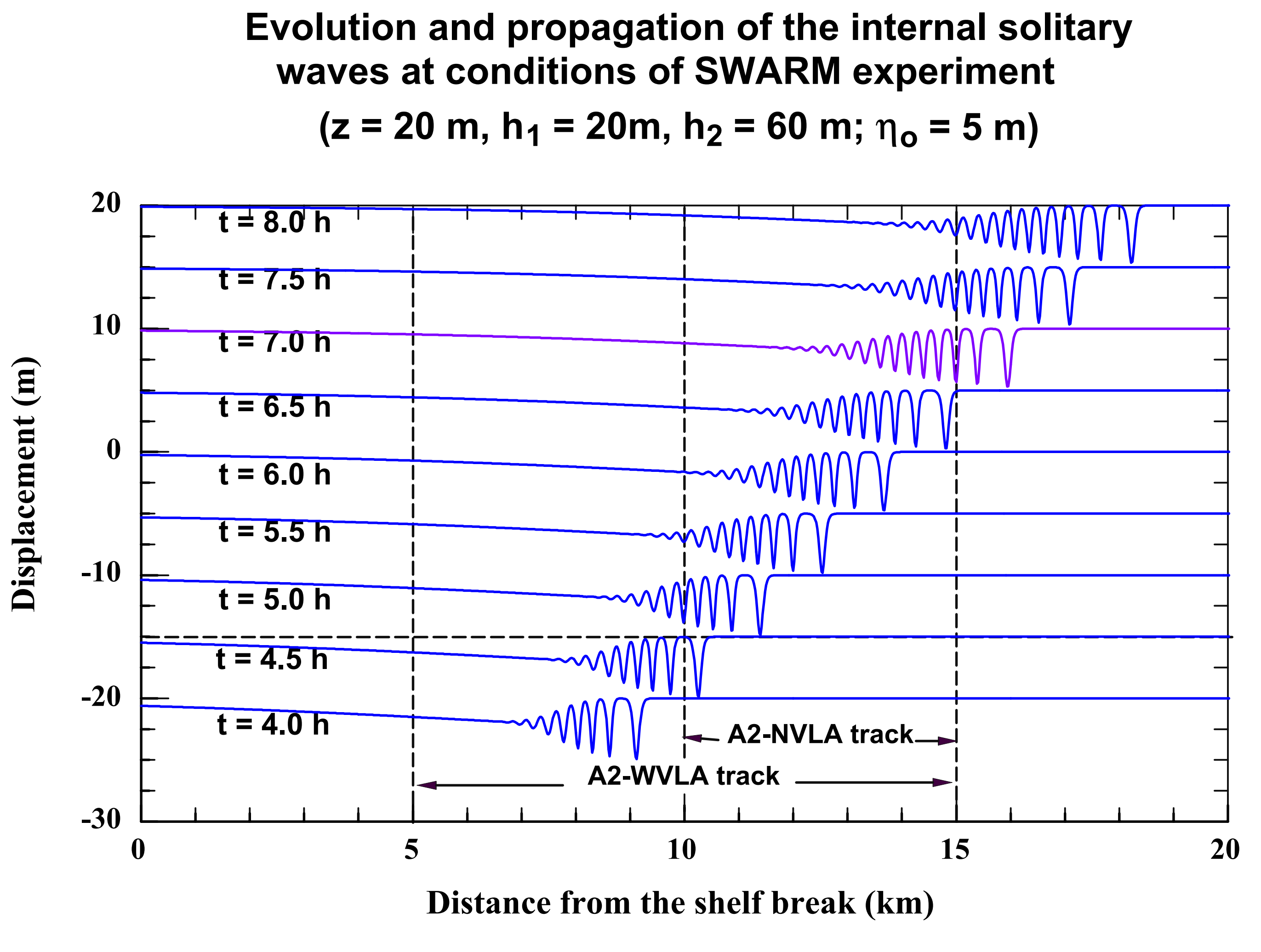
Horizontal refraction due to solitons of internal gravity waves

Geometry of the 1995 SWARM Experiment

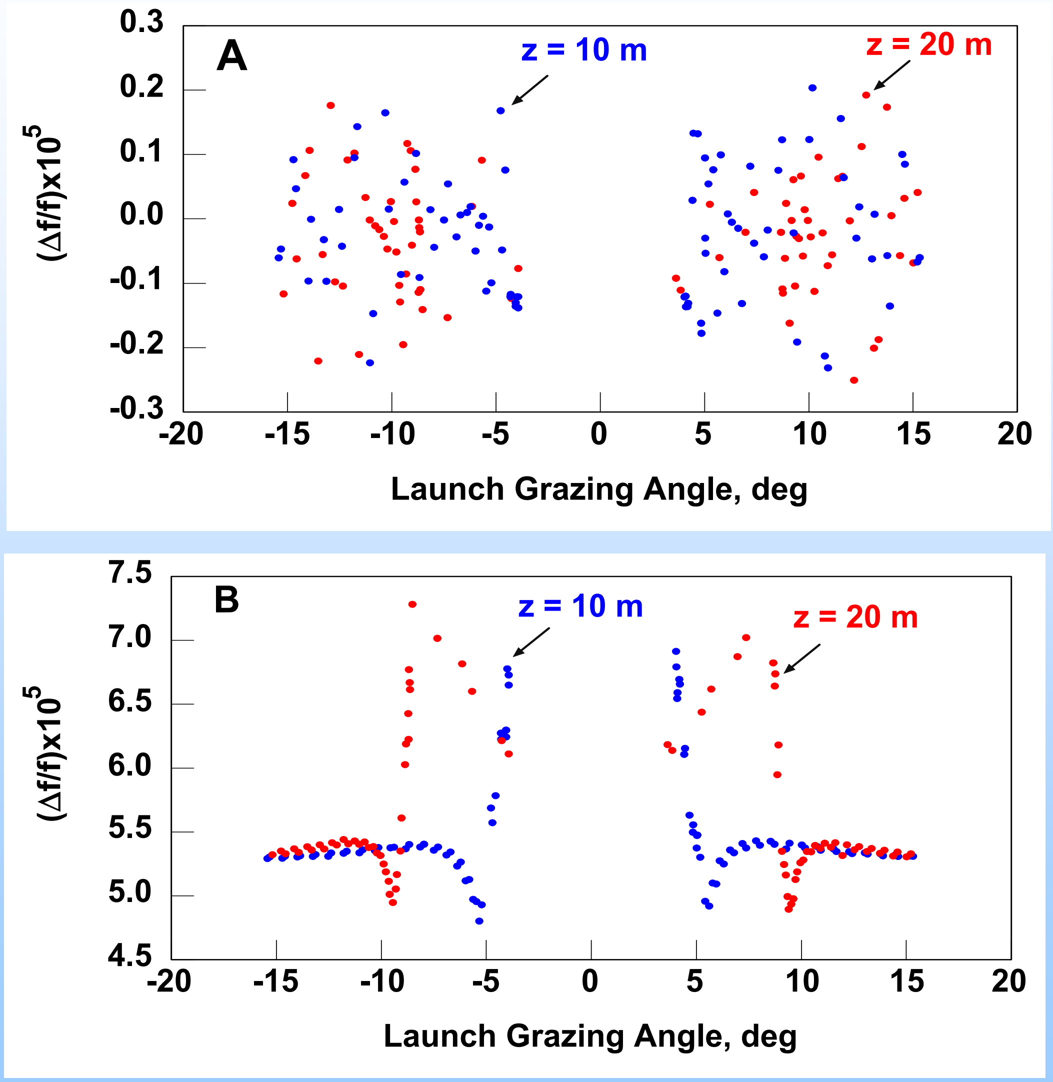
J. Apel et al., IEEE J. Ocean. Eng., 22(3), pp. 465- 499, (1997)
M. Badiey, Y. Mu, J. Lynch, J. Apel, and S. Wolf, IEEE J. Ocean. Eng., 27(1), pp. 117-129 (2002)



Horizontal refraction due to solitons of internal gravity waves



Acoustic frequency wander due to an IW soliton



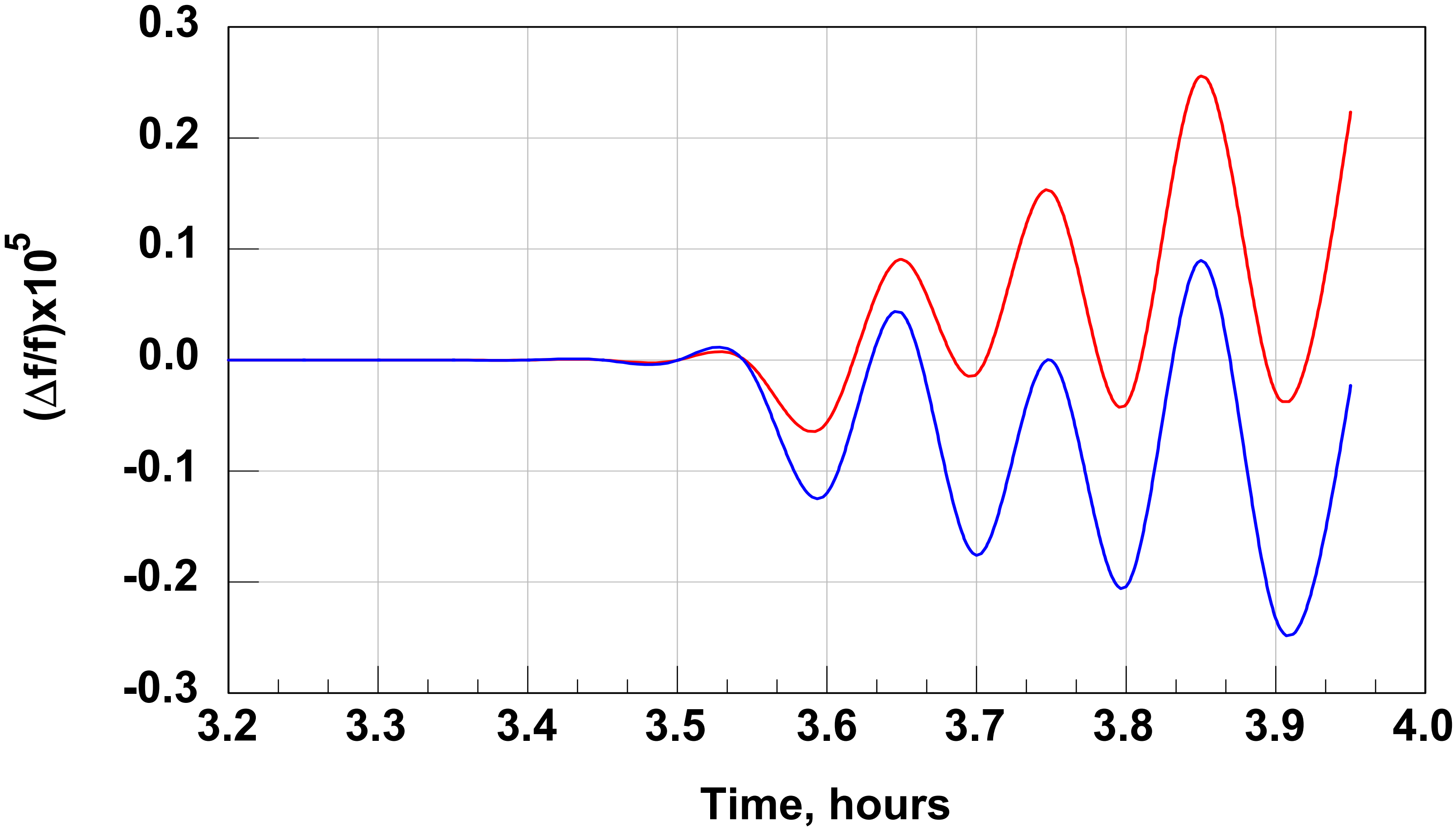
False Doppler shift on various eigenrays for two propagation paths of equal lengths:

$\psi = 45^{\mathrm{o}}$ (A) **and** $\psi = 90^{\mathrm{o}}$ (B);

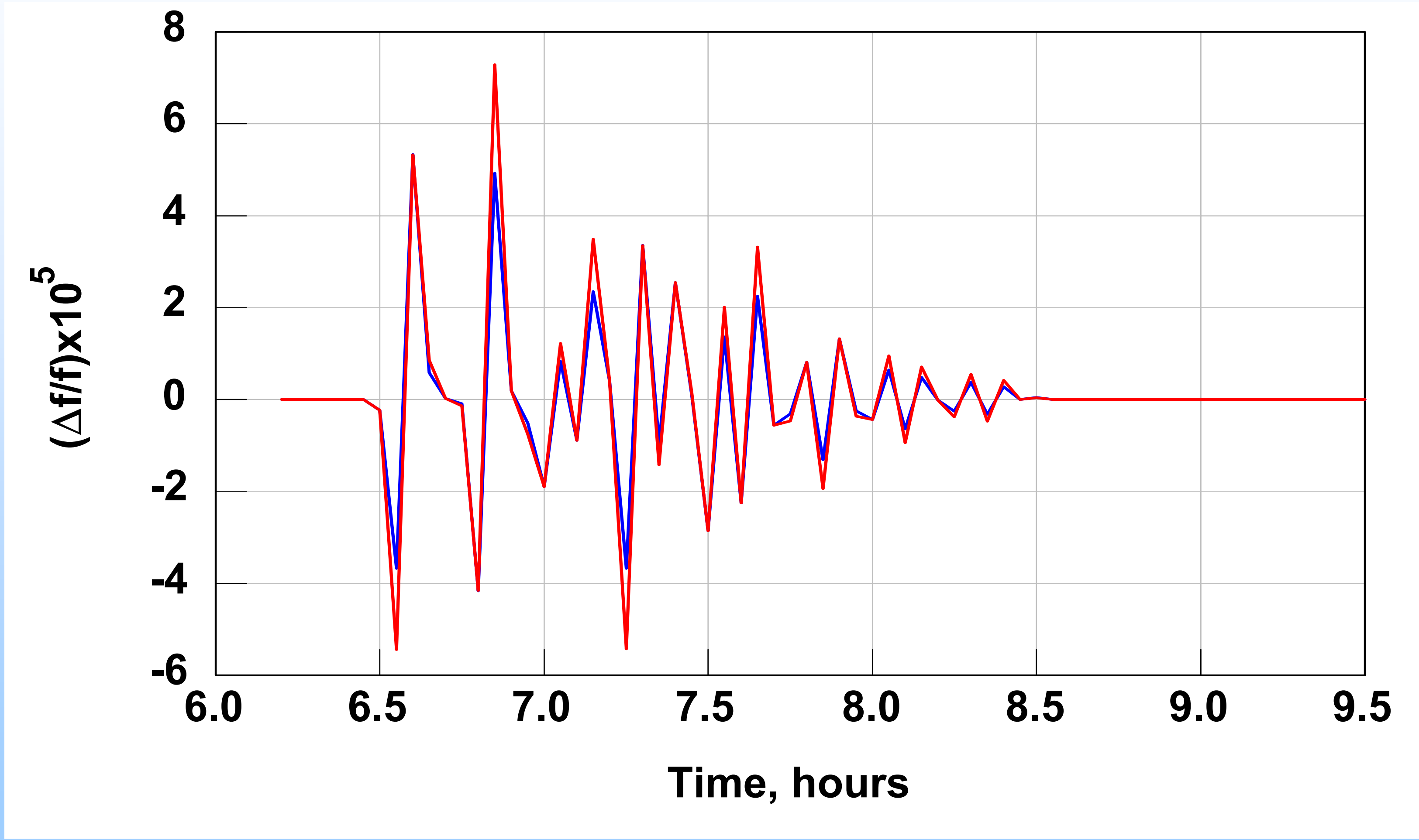
R = 13.5 km, t = 6.6 h

$$\Delta f / f = 10^{-5} \Leftrightarrow u_s = 1.5\ \mathrm{cm/sec}$$

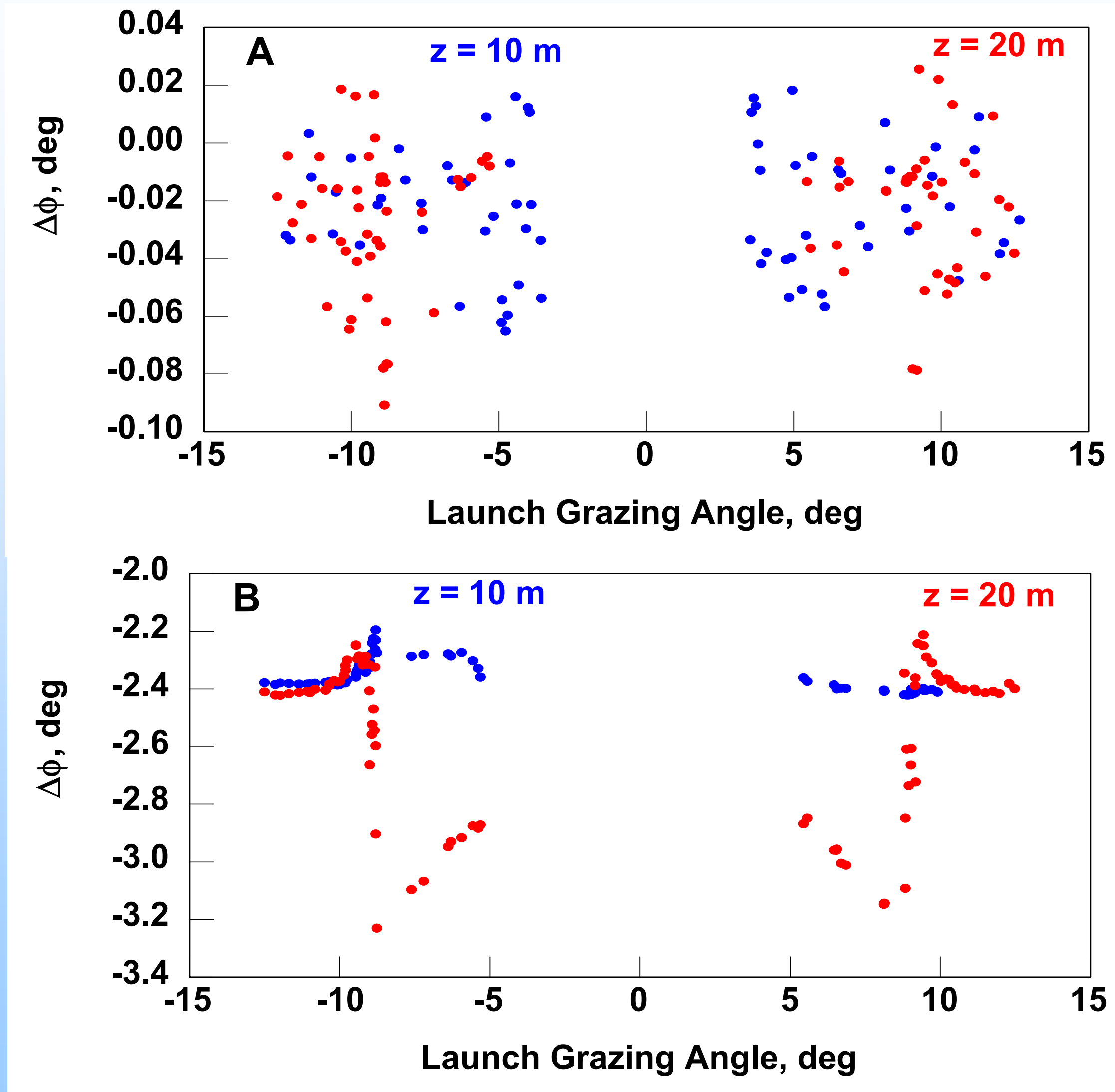
Envelope of frequency fluctuations
(r = 13.5 km, $z_s = z_r$ = 20 m, $\varphi = 0°$)



Envelope of frequency fluctuations
(r = 13.5 km, $z_s = z_r$ = 20 m, $\varphi = 90°$)



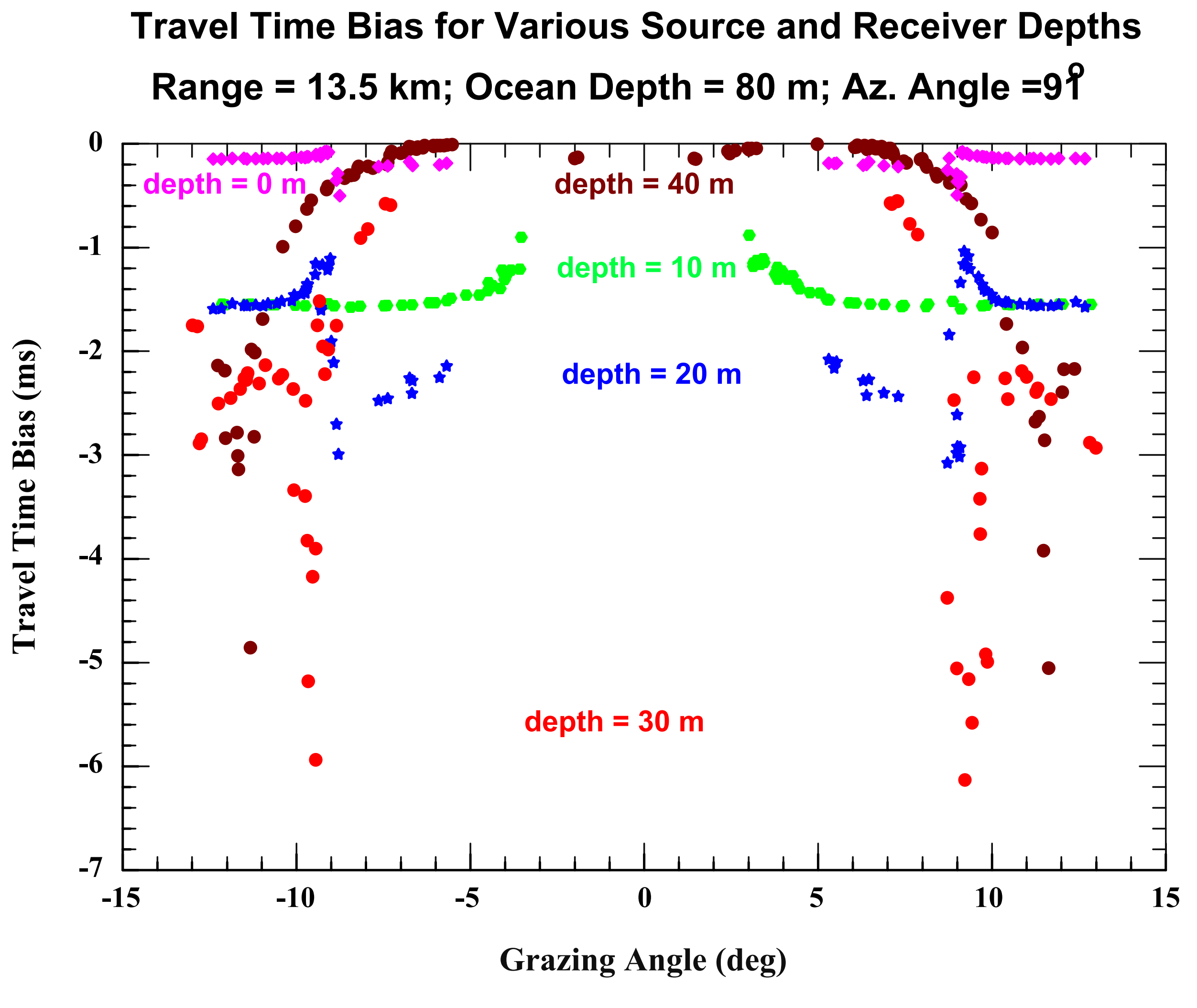
Horizontal refraction due to an IW soliton: azimuthal angles



Bearing perturbations for two propagation paths of equal lengths: $\varphi = 45^{\circ}$ (A) **and** $\varphi = 90^{\circ}$ (B);

$R = 13.5$ km, $t = 6.6$ h

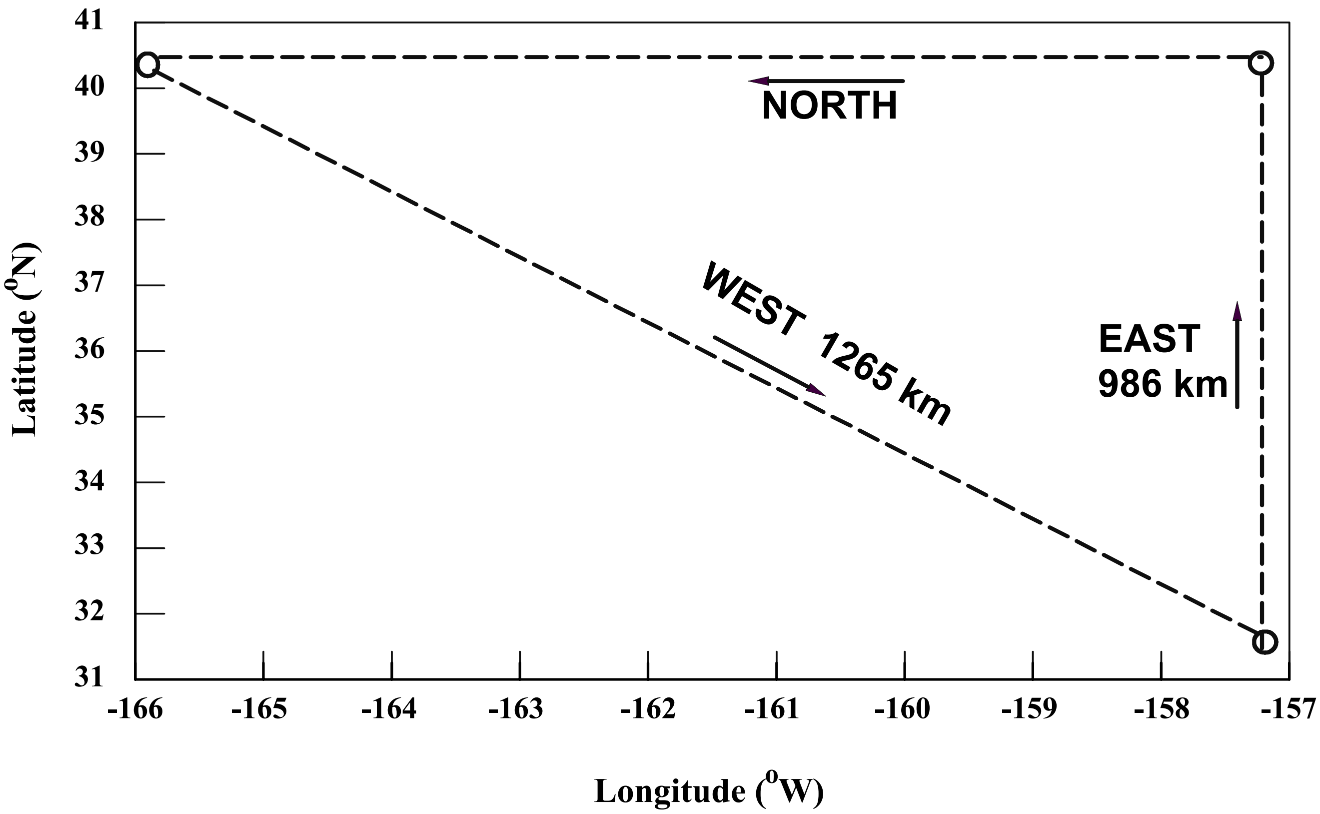
Horizontal refraction due to solitons of internal gravity waves: travel time bias



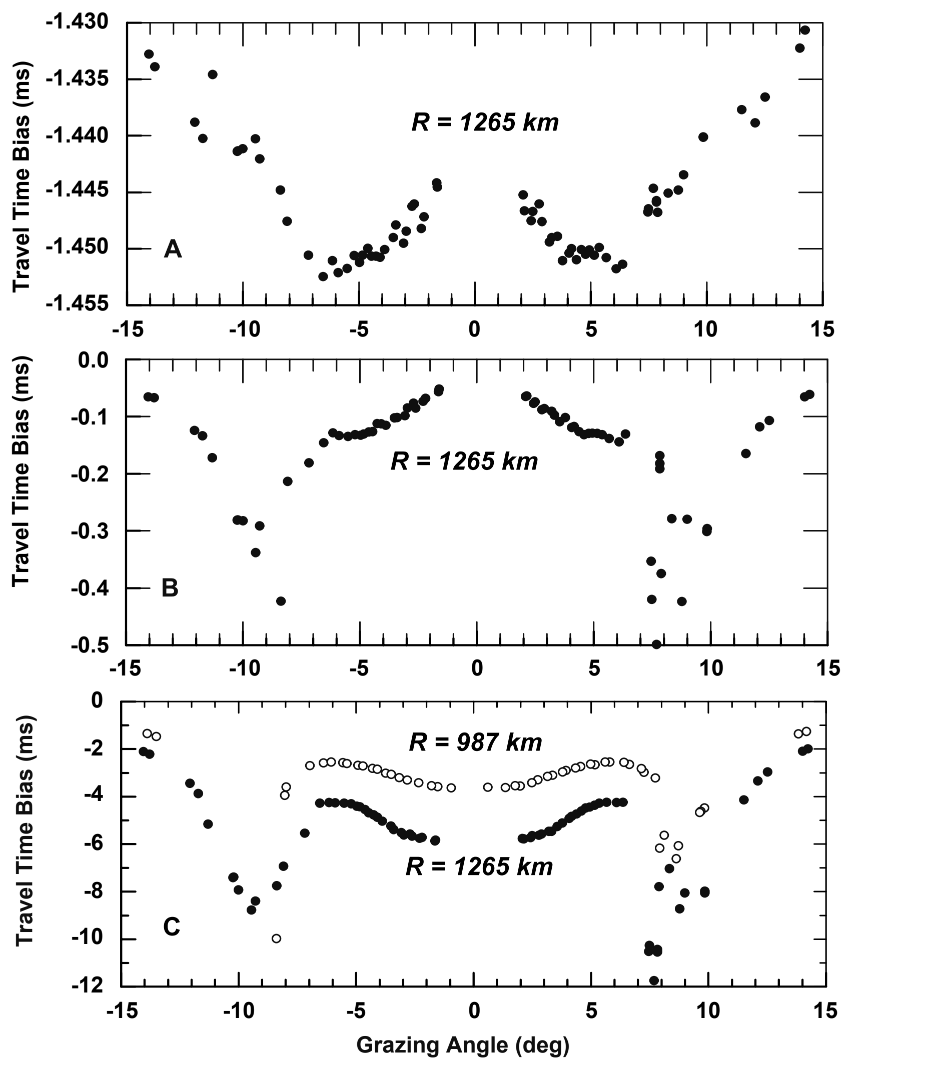
Horizontal refraction in deep water

Geometry of the 1987 Reciprocal Transmission Experiment (RTE)

(B. Dushaw, P. Worcester, B. Cornuelle, and B. Howe, JASA, 93(1), pp. 255-275 (1993))

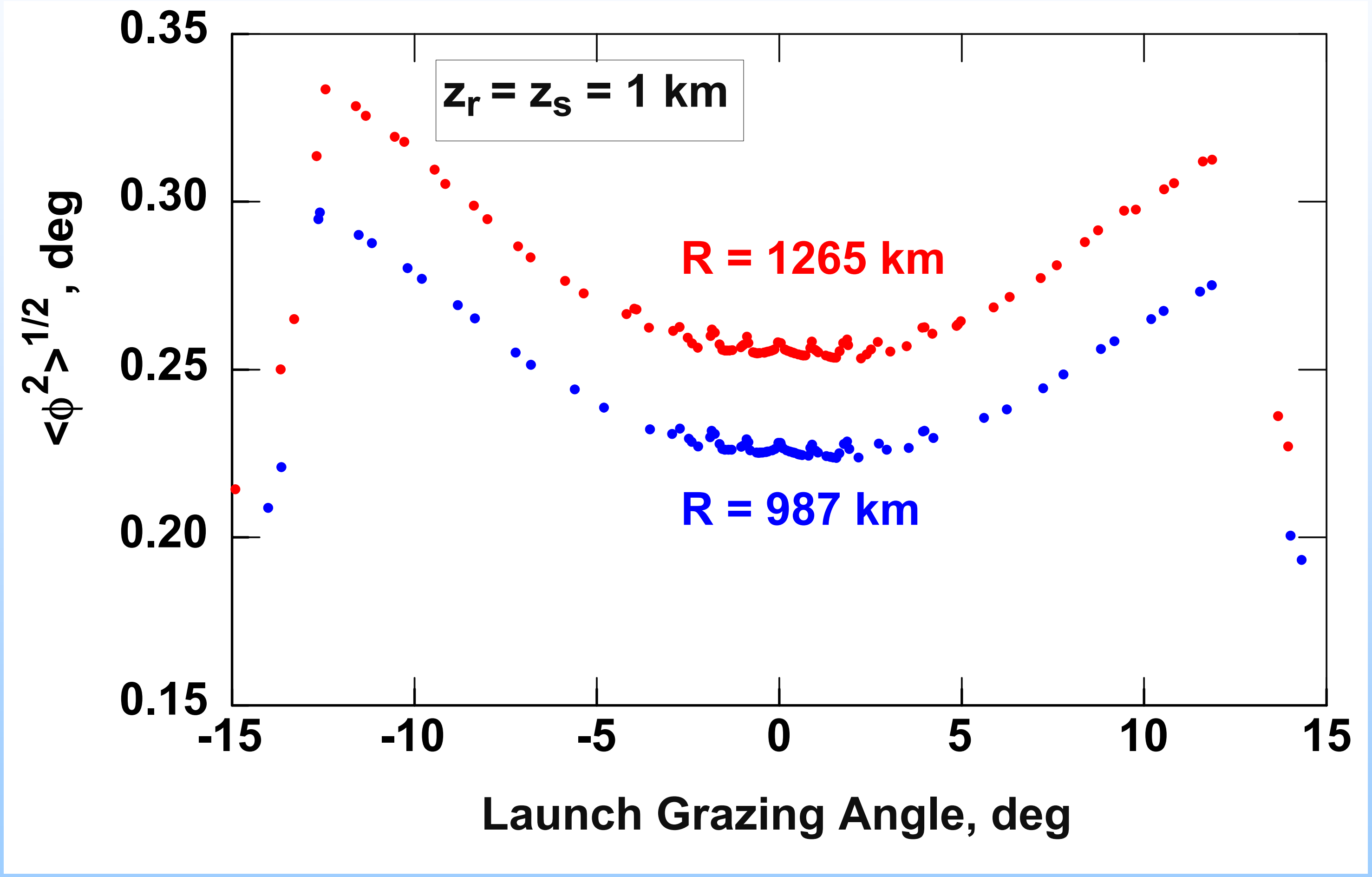


Horizontal refraction in deep water

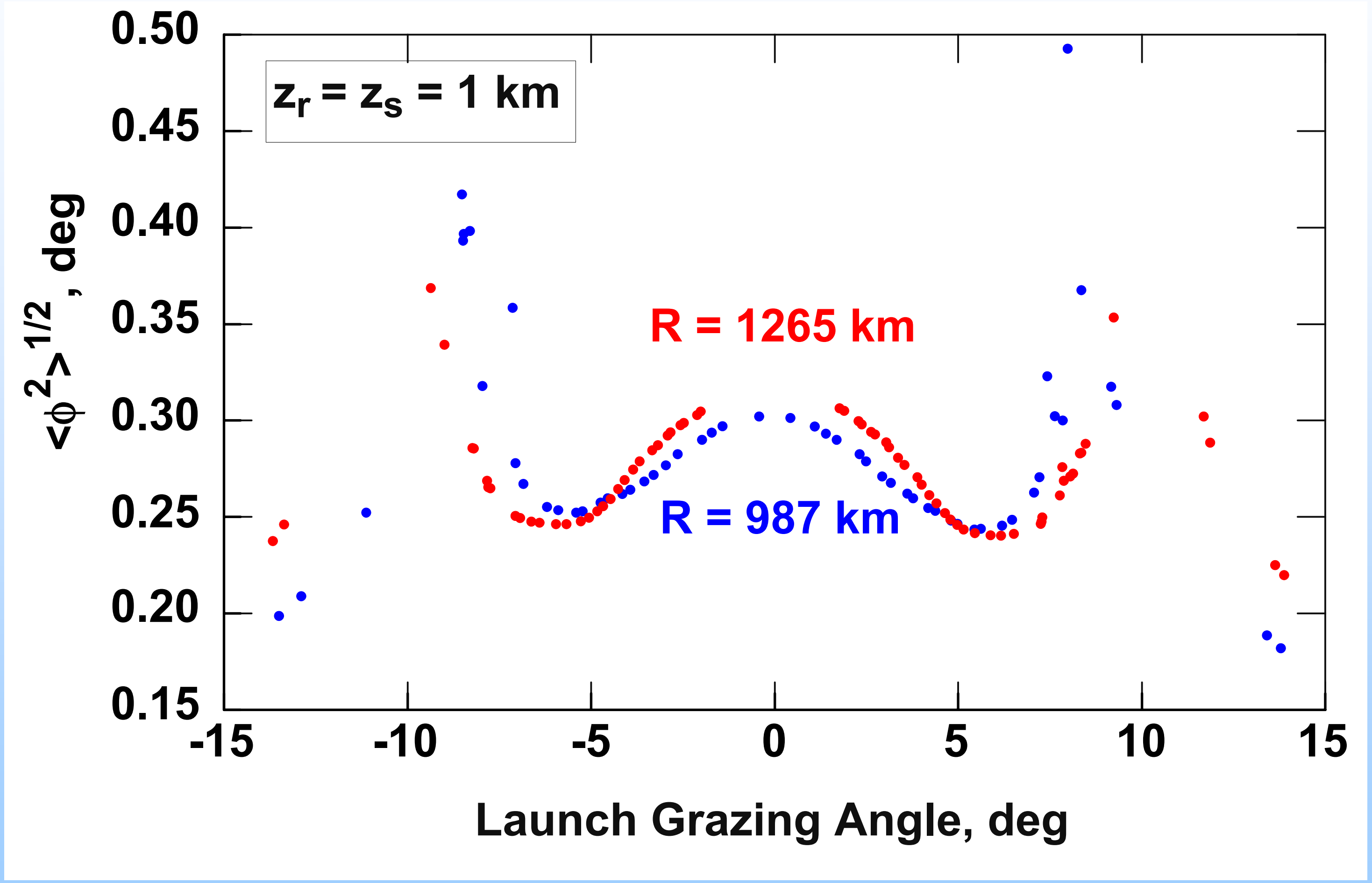


Ray-travel-time bias due to horizontal refraction as a function of launch angle of the ray. Travel times are calculated under conditions of RTE87 experiment for three types of hydrological processes: global variations in the sound speed (A), mesoscale inhomogeneities (B), and internal gravity waves (C).

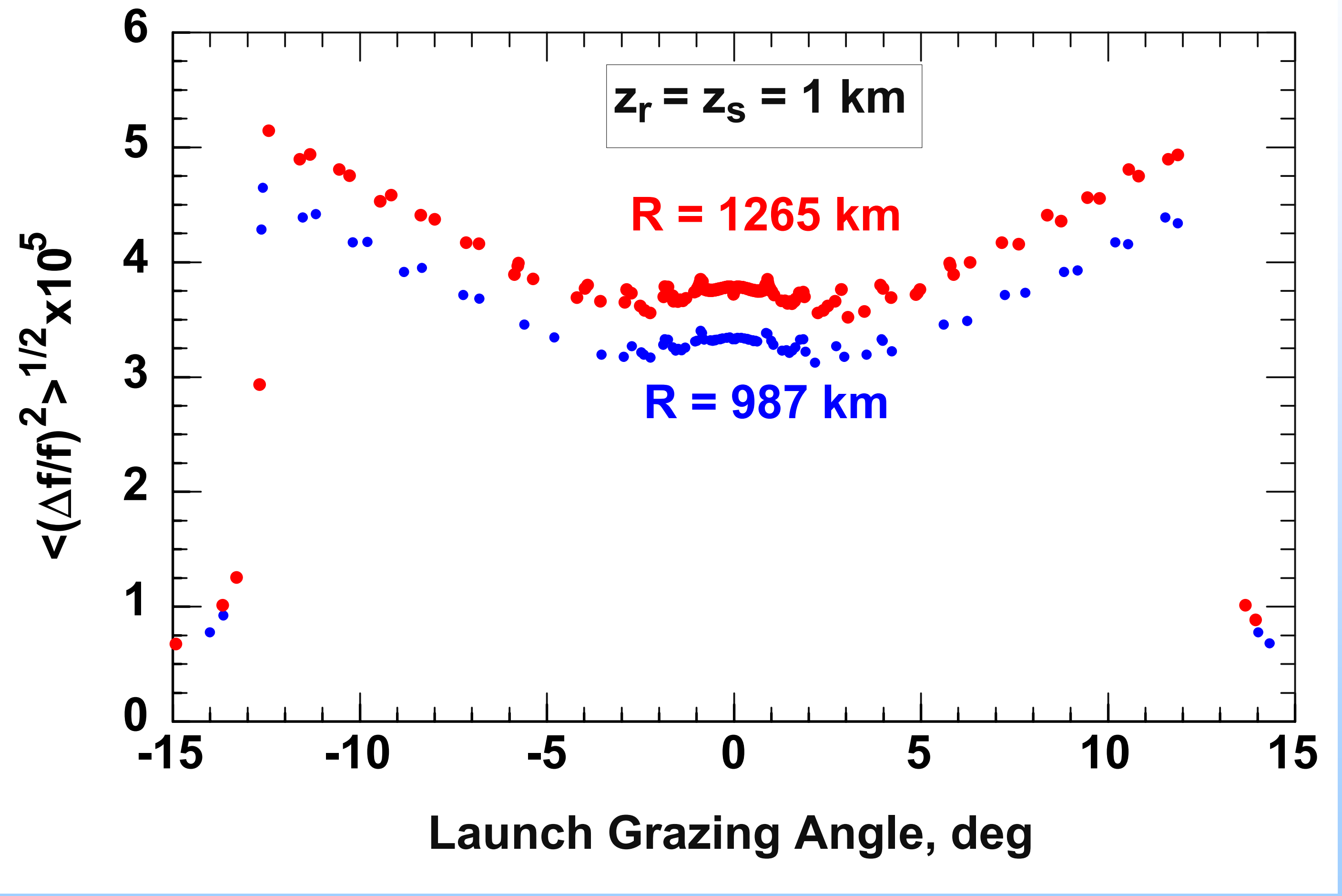
Fluctuations of the horizontal refraction angle for the canonical sound speed profile



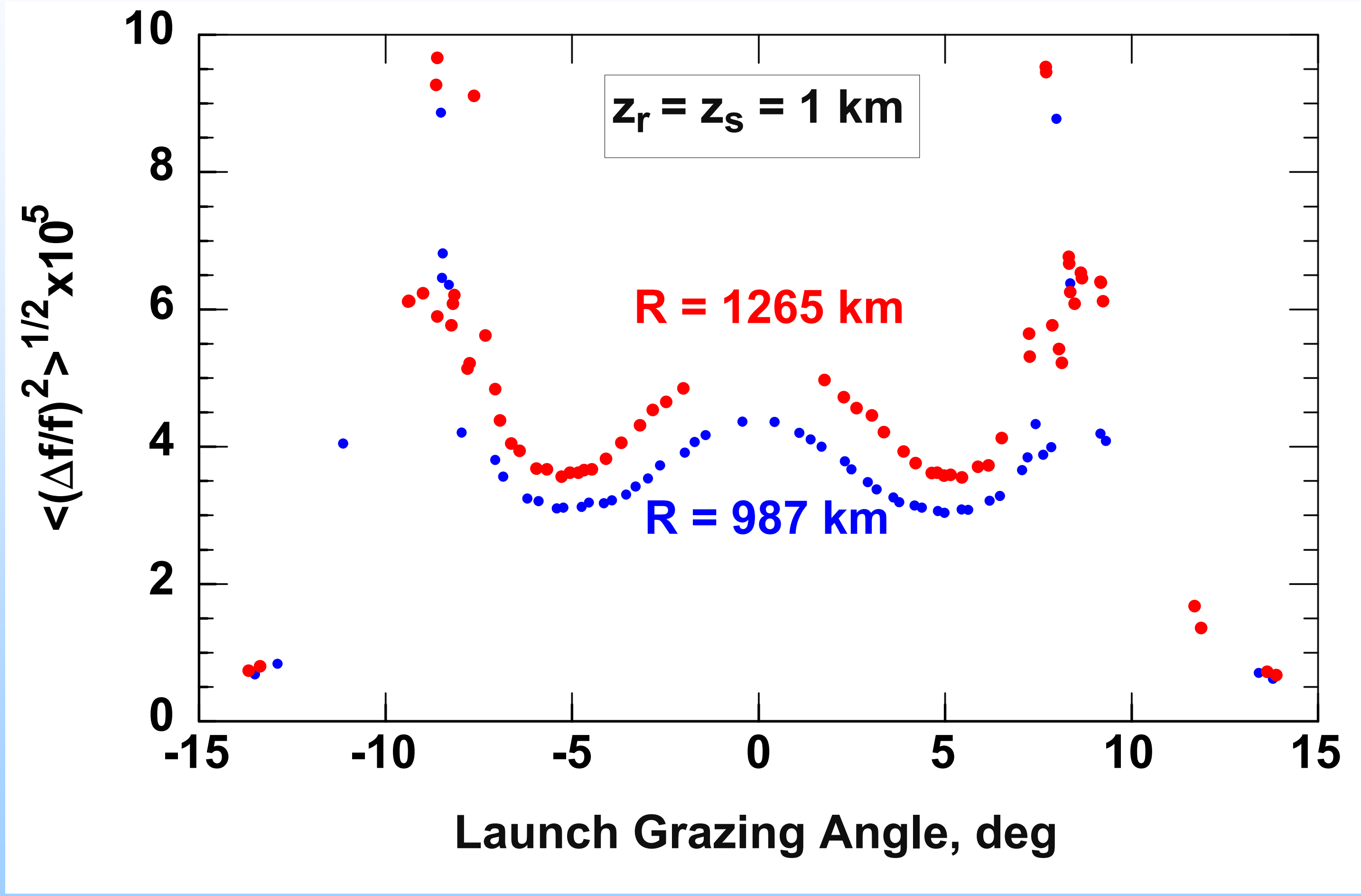
Fluctuations of the horizontal refraction angle under conditions of the RTE87 experiment



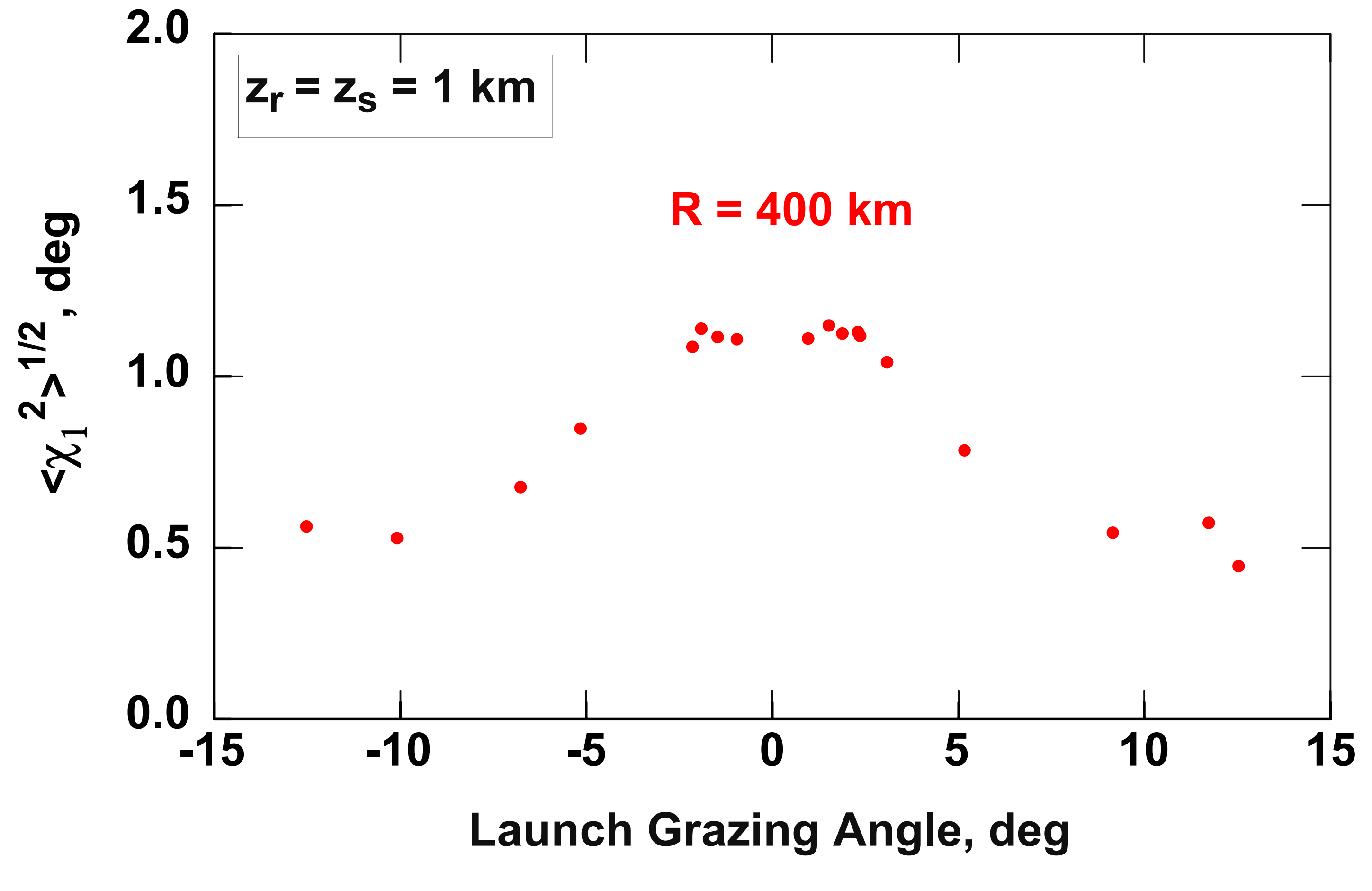
Acoustic frequency wander for the canonical sound speed profile



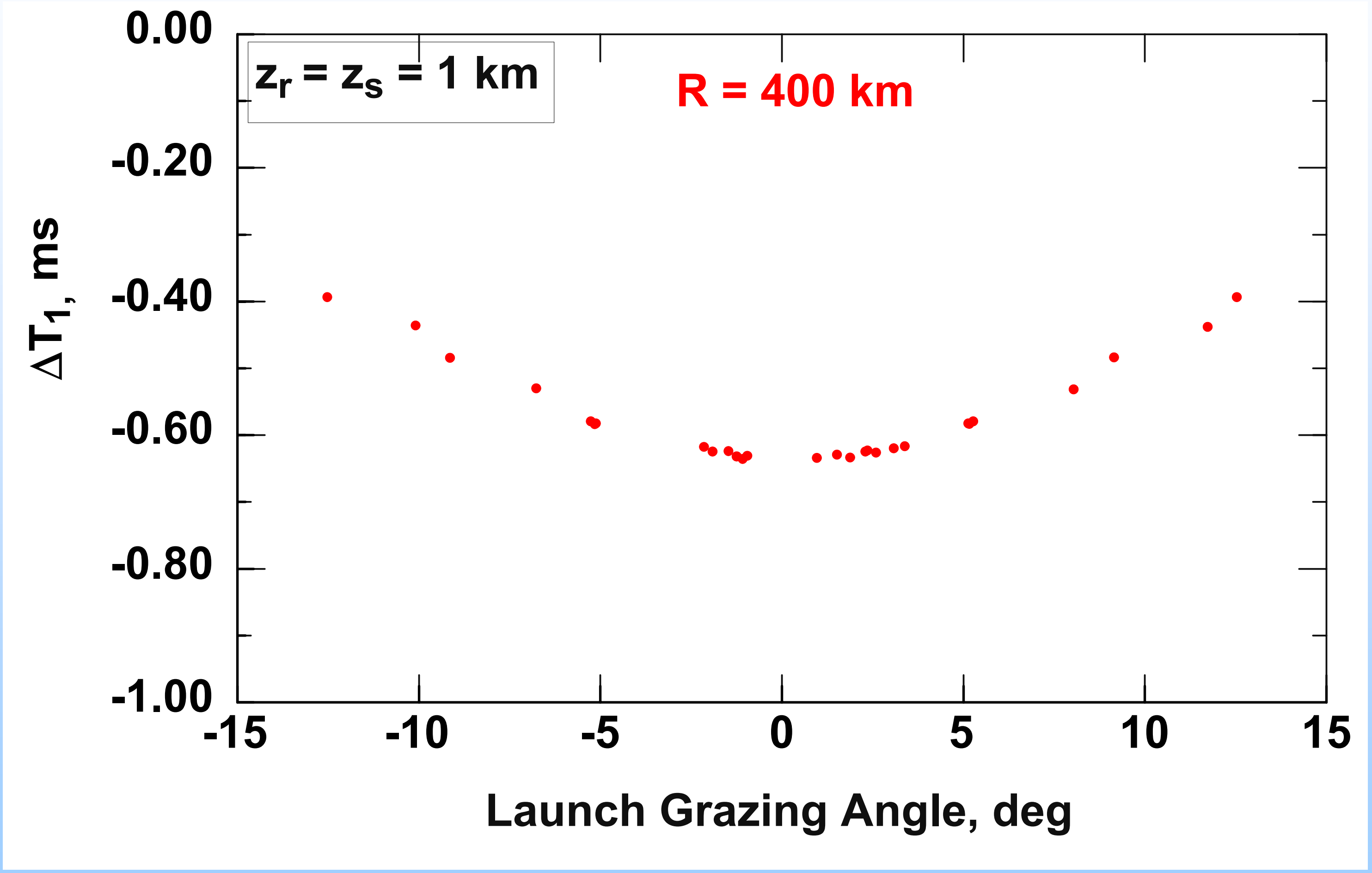
Acoustic frequency wander under conditions of the RTE87 experiment



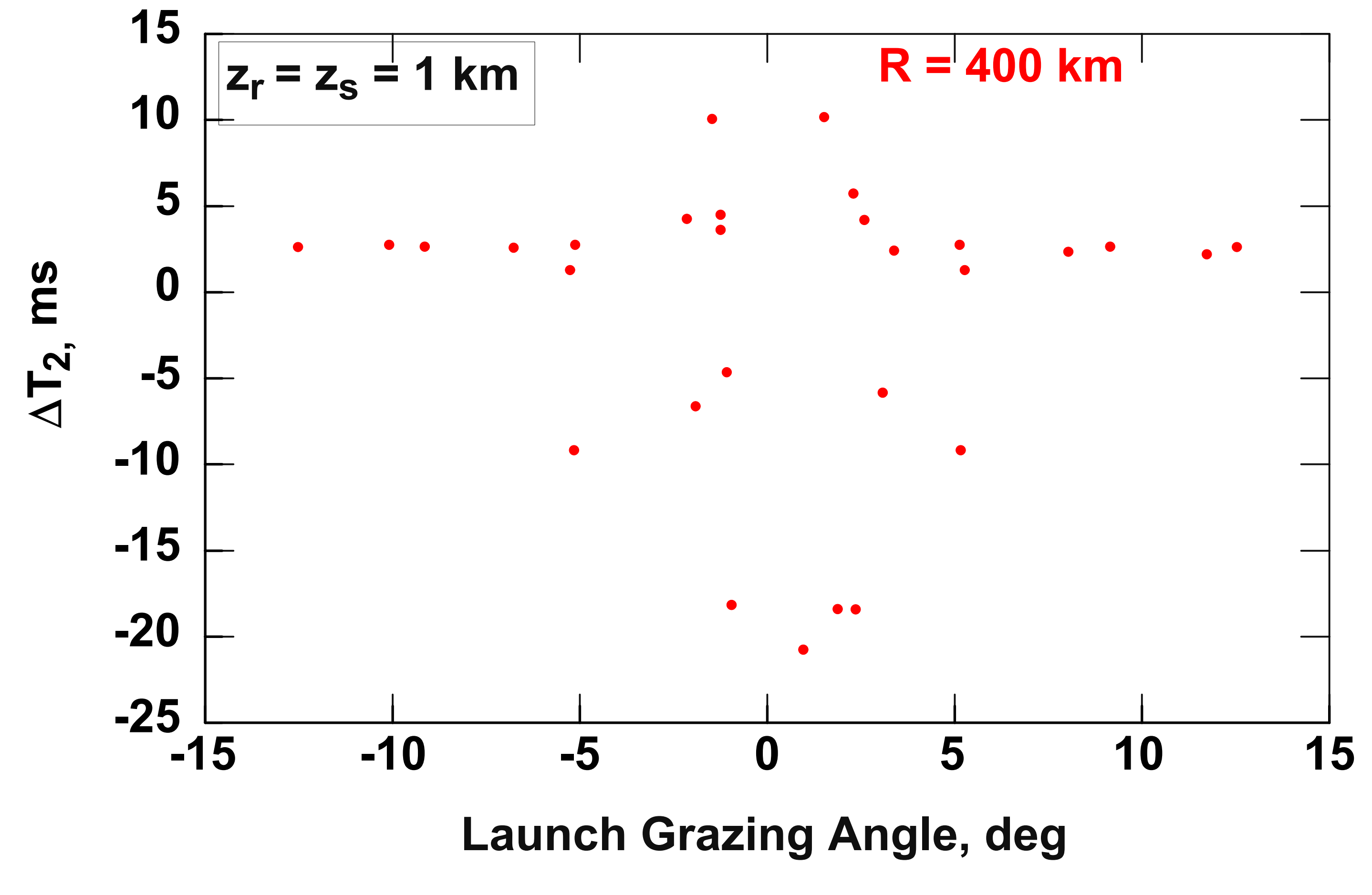
Ray grazing angle fluctuations for canonical sound speed profile



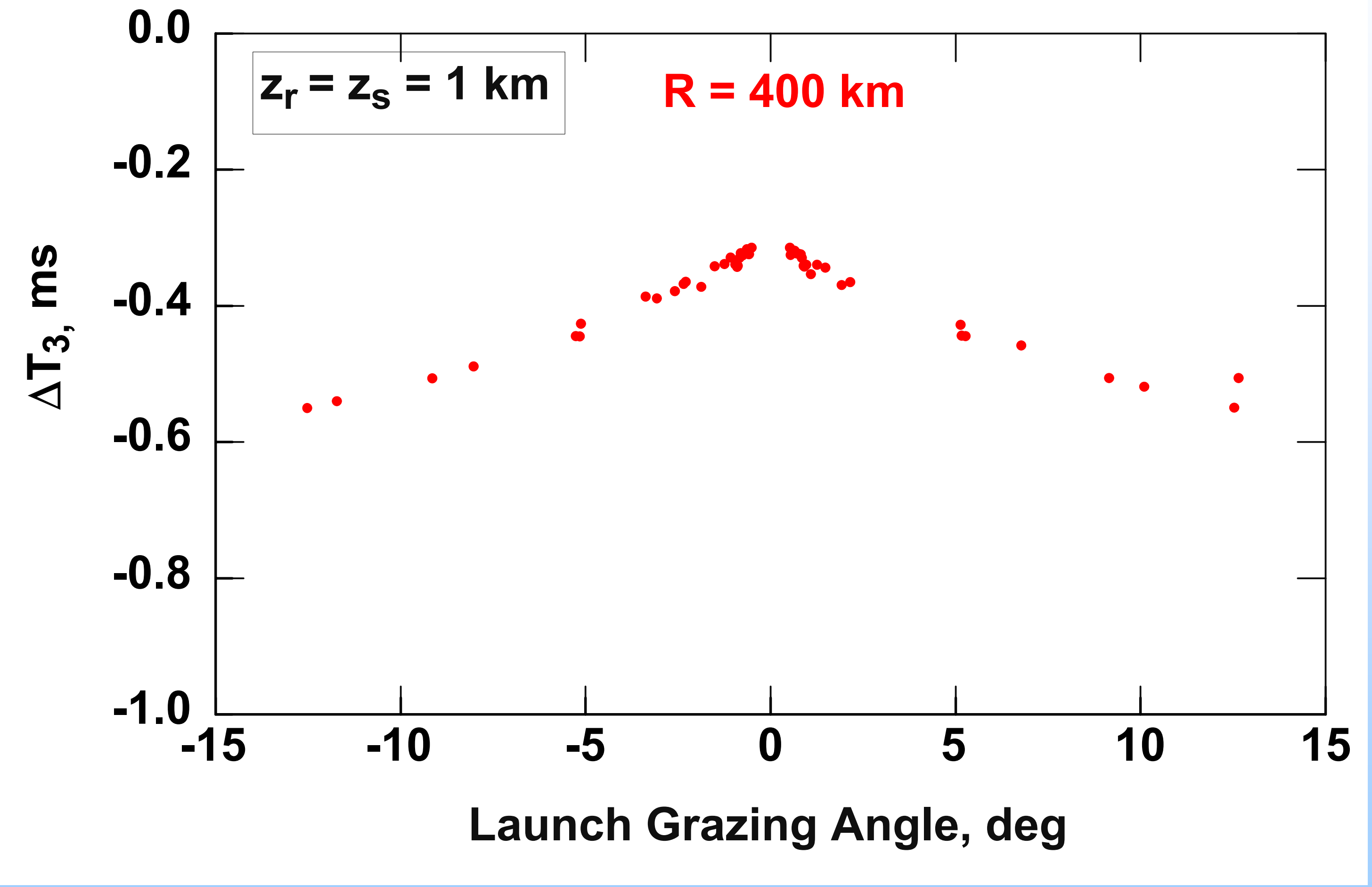
Travel time bias: sound speed nonlinearity



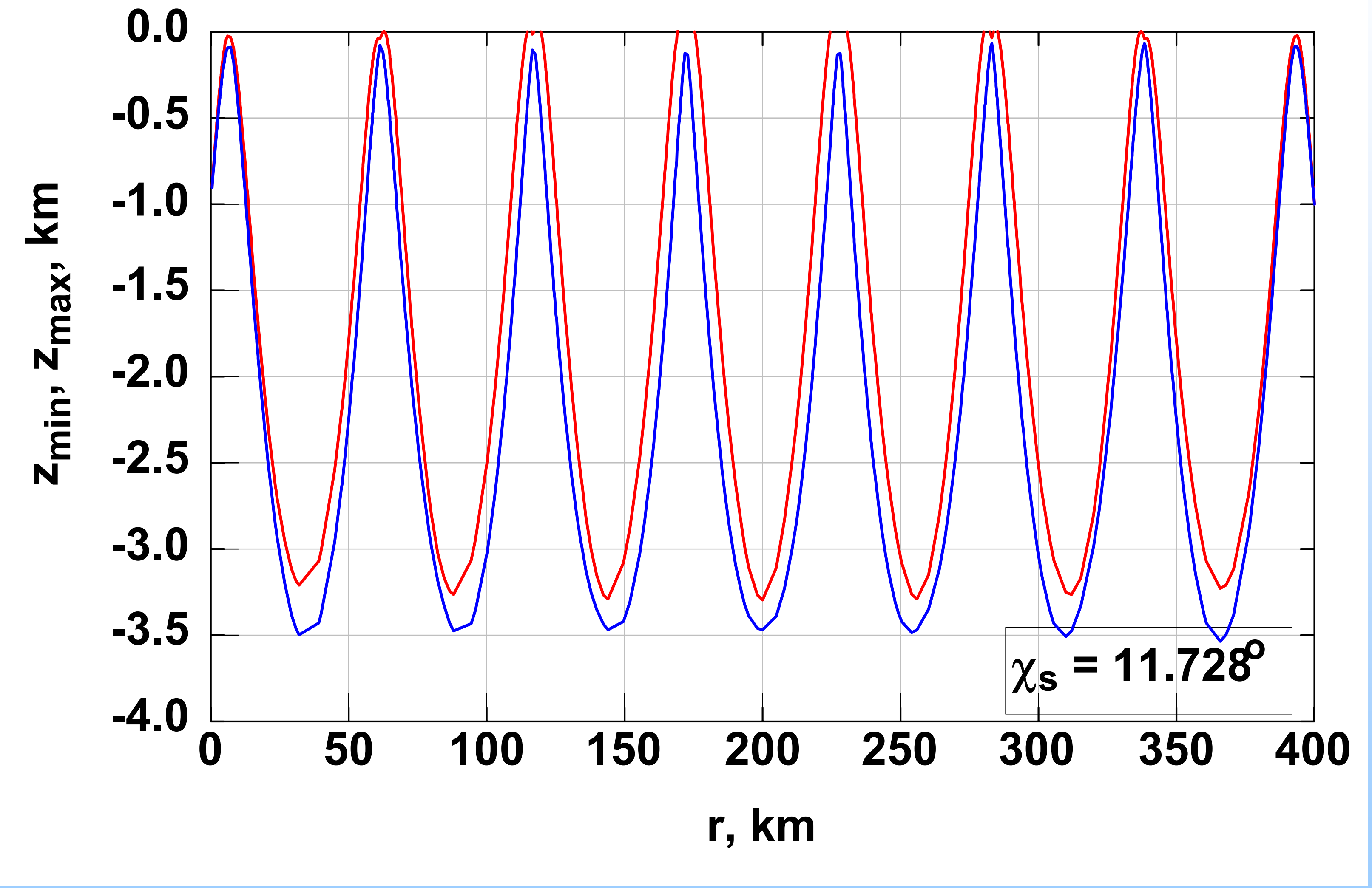
Travel time bias: ray path variation



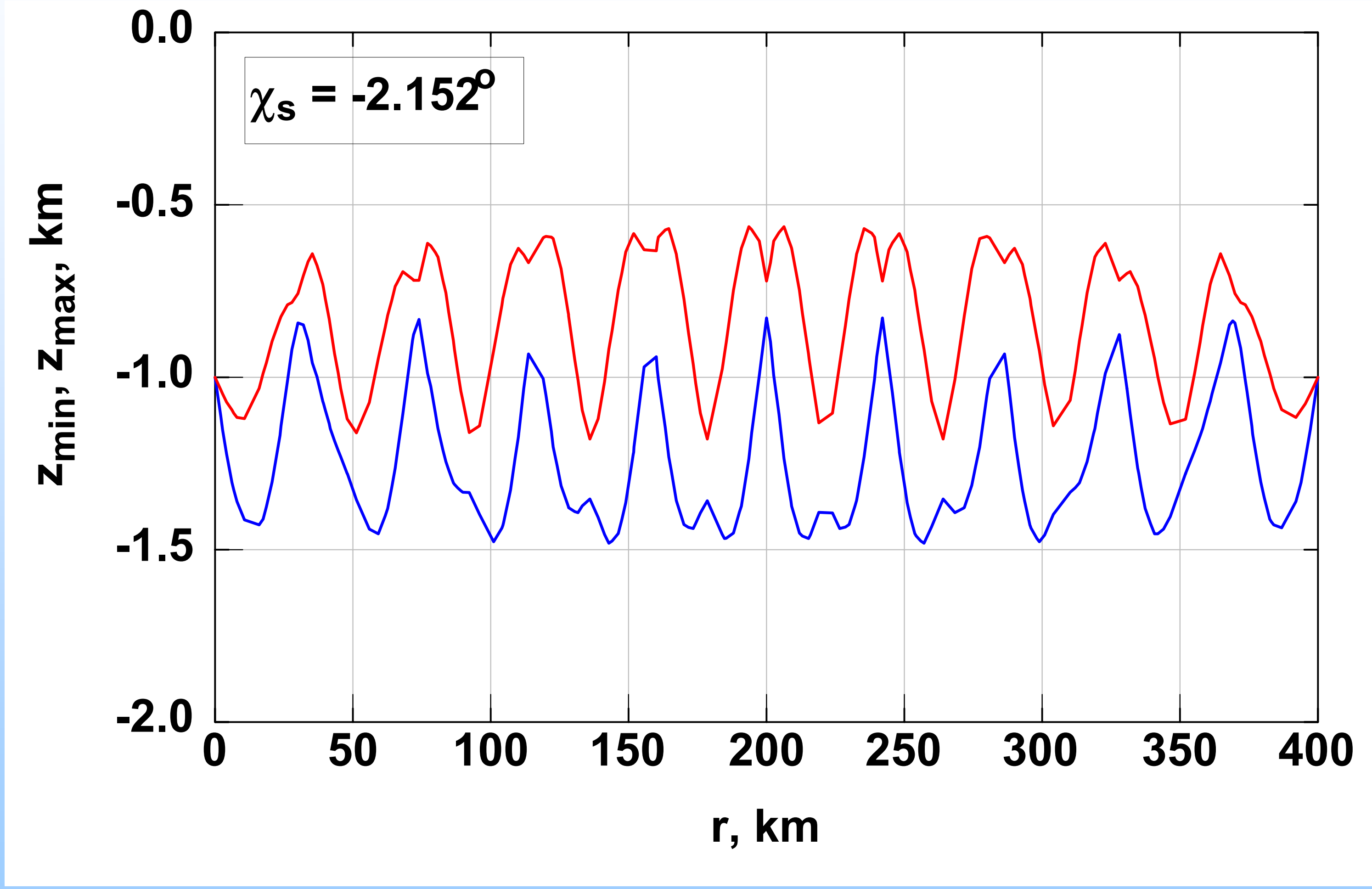
Travel time bias: horizontal refraction



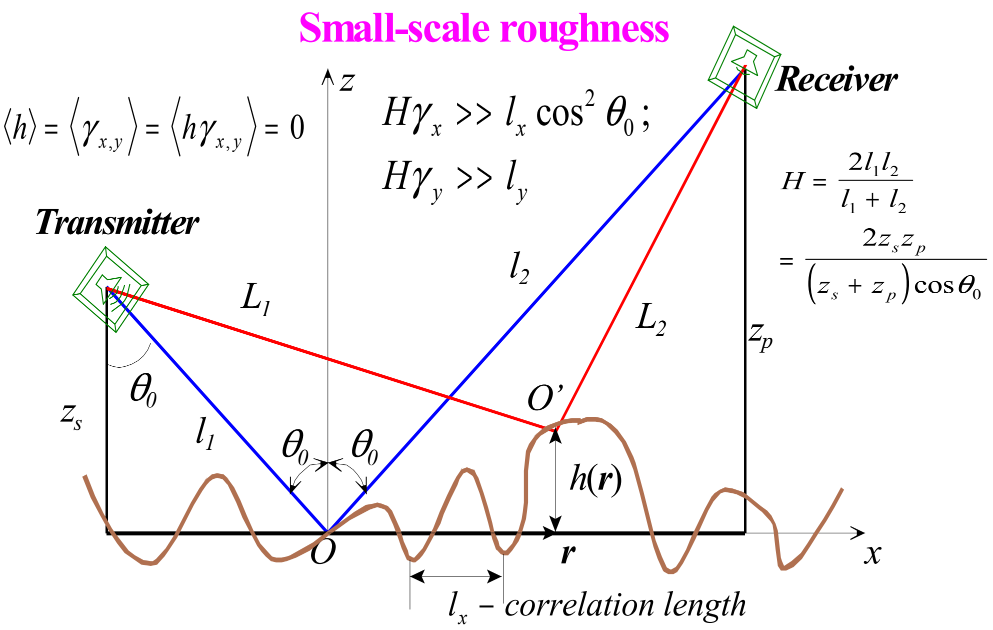
IW-induced ray variations: a steep ray



IW-induced ray variations: a near-axial ray



Travel-Time Statistics for Waves Scattered at a Rough Surface



$$c\langle T\rangle - cT_0 = \frac{\sin^2\theta_0}{H}\langle h^2\rangle + H\left[\langle\gamma_x^2\rangle + \langle\gamma_y^2\rangle\cos^2\theta_0\right]$$

PDF of random travel times

$$\tau = (T - T_0)/T_0, \quad b = \langle\gamma_y^2\rangle\cos^2\theta_0 / \langle\gamma_x^2\rangle$$

Large-scale roughness:

$$W(\tau) = \frac{1}{\sqrt{2\pi\langle\tau^2\rangle}}\exp\left(-\frac{(\tau - \langle\tau\rangle)^2}{2\langle\tau^2\rangle}\right)$$

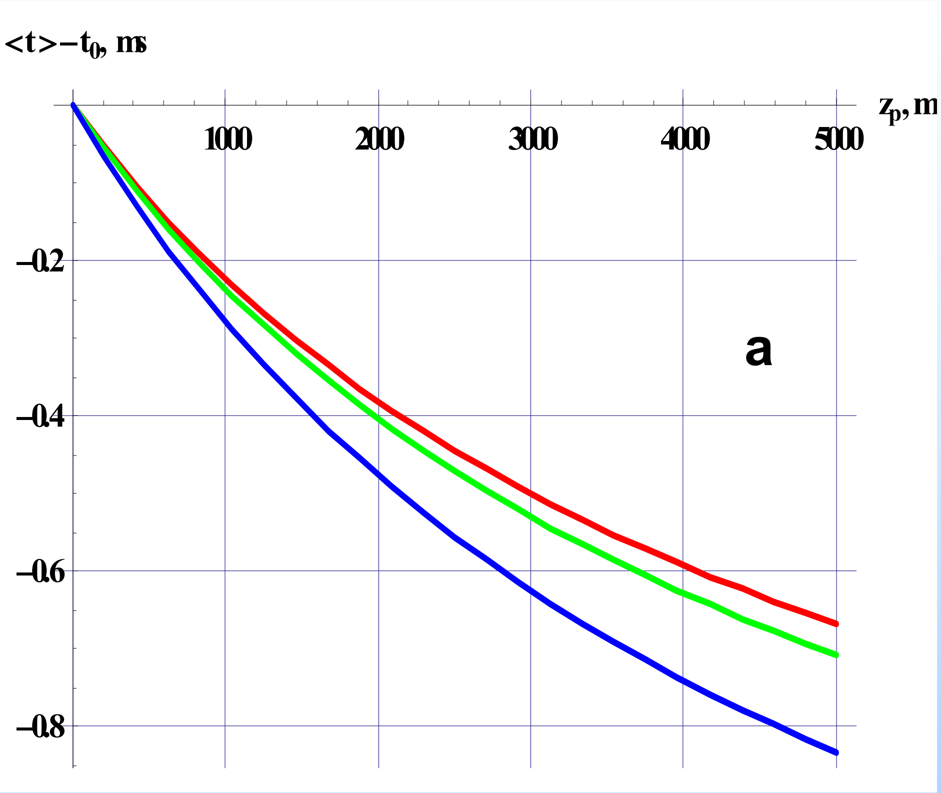
$$\langle\tau^2\rangle = 4\cos^2\theta\langle h^2\rangle/(l_s + l_p)^2$$

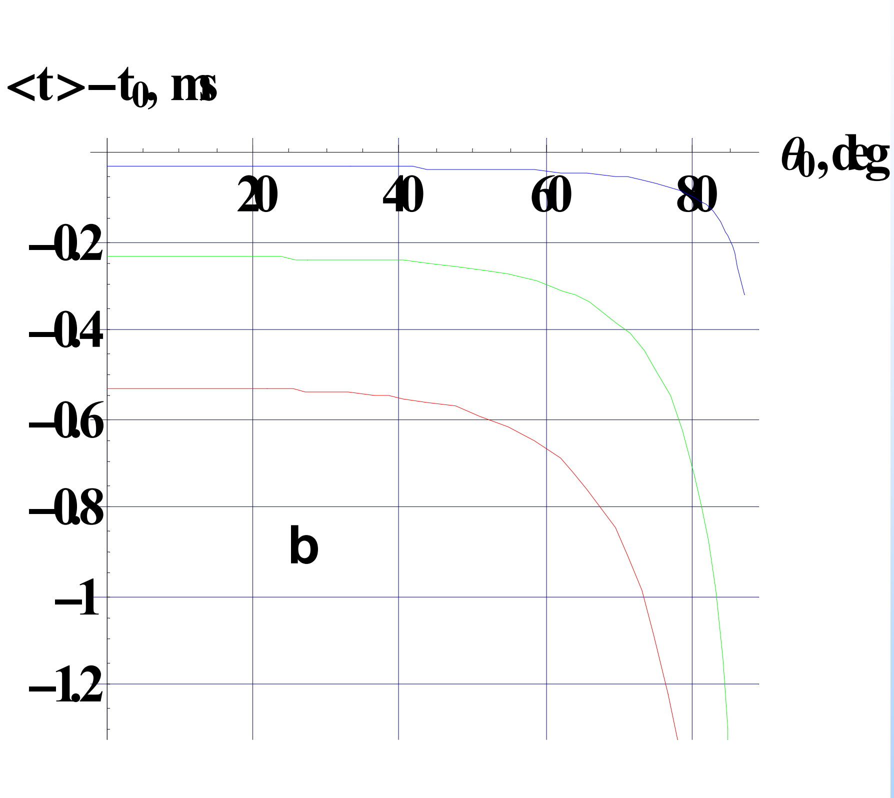
$$\langle\tau\rangle = \frac{2\langle h^2\rangle\sin^2\theta}{(l_s + l_p)^2} \mp \frac{2l_s l_p}{(l_s + l_p)^2}\left(\langle\gamma_x^2\rangle + \langle\gamma_y^2\rangle\cos^2\theta\right)$$

Small-scale roughness:

$$W(a) = \frac{1}{\sqrt{b}}\exp\left(-a\frac{1+b}{2b}\right)I_0\left(a\frac{1-b}{2b}\right), \quad a = \frac{\tau(l_s + l_p)^2}{4l_s l_p\langle\gamma_x^2\rangle}$$

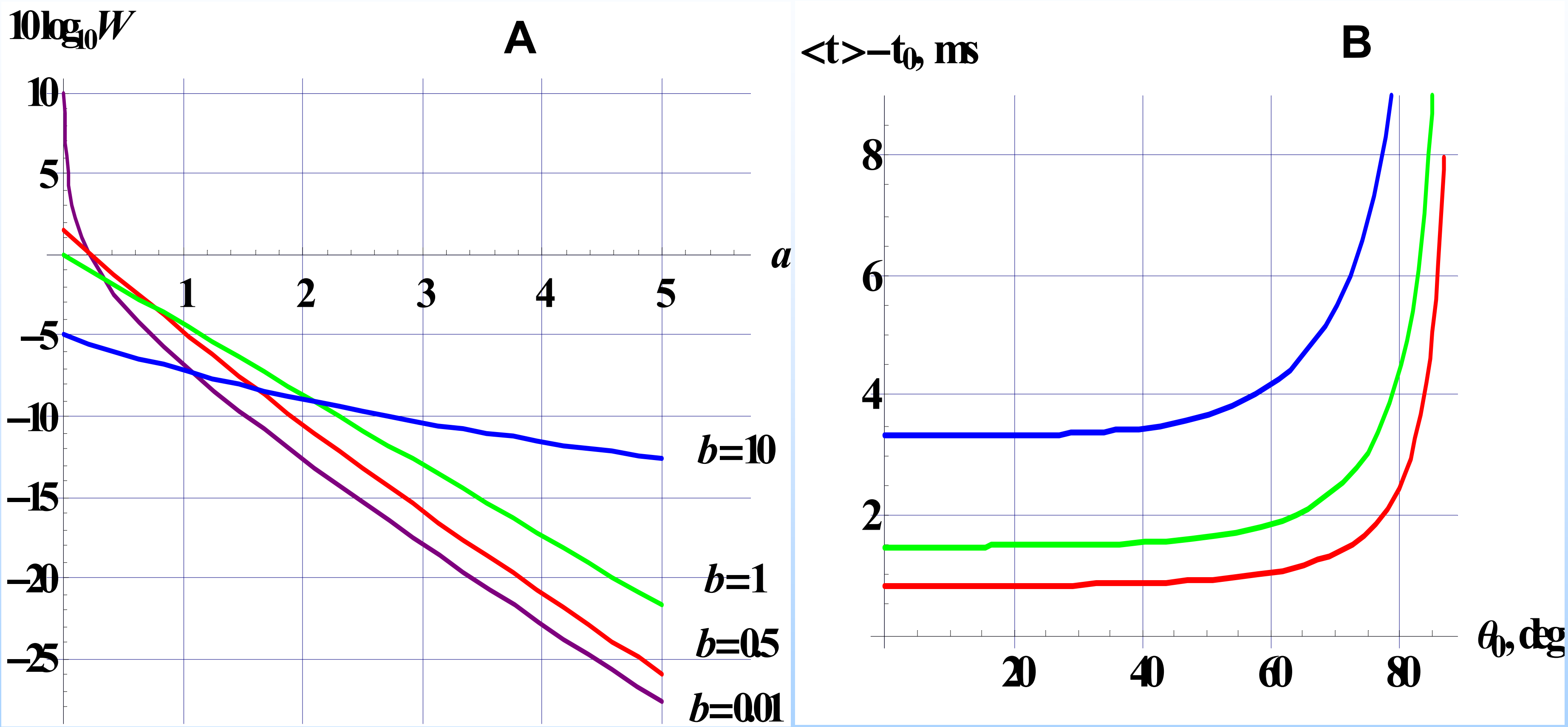
Travel-Time Statistics for Waves Scattered at a Large-Scale Rough Surface





Travel-time bias of reflected acoustic waves as a function of (a) the receiver depth (θ_0 = 0°, 45°, and 60°, L =200 m, z_s = 5000 m, $\langle h^2 \rangle$ = 4 m^2) and (b) the angle of incidence (L = 100 m, 150 m, and 400 m, z_p = z_s = 1000 m)

Travel-Time Statistics for Waves Scattered at a Small-Scale Rough Surface



Travel-time PDF (A) and bias of reflected acoustic wave as a function of the angle of incidence (B) (L = 20 m, 15 m, and 10 m, $z_p = z_s = 1000$ m , $\langle h^2 \rangle = 0.25\ \text{m}^2$)

Travel-time Fréchet kernel
(“Banana-doughnut theory”)

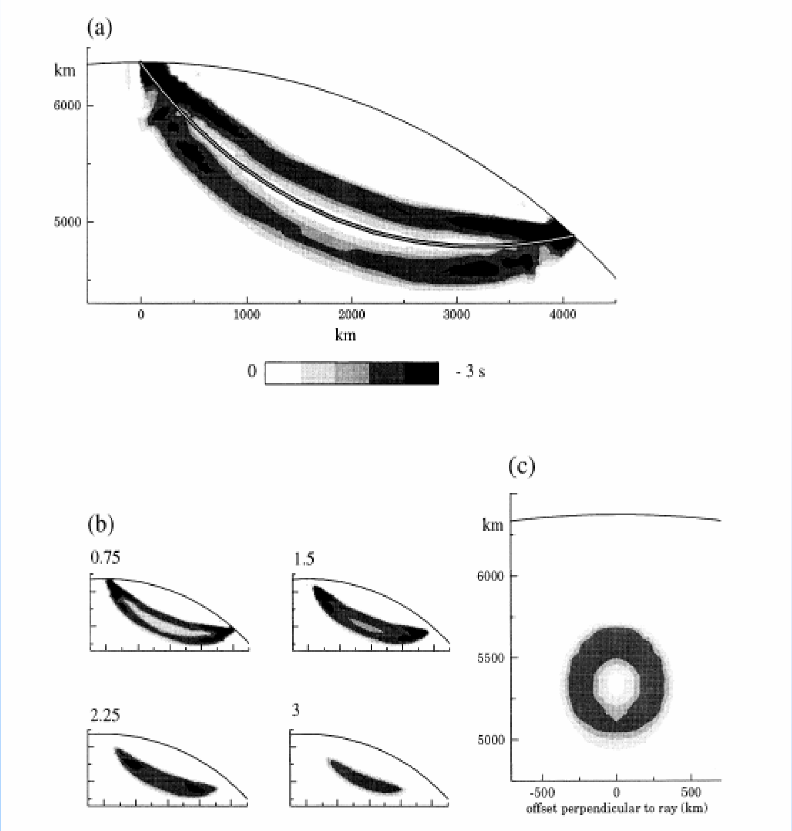
Definition:

$$\delta T(\mathbf{r}_R,\mathbf{r}_S)=\int K_\beta(\mathbf{r};\mathbf{r}_R,\mathbf{r}_S)\,\delta\beta(\mathbf{r})\,d\mathbf{r}$$

Computation:

$$K_\beta(\mathbf{r};\mathbf{r}_R,\mathbf{r}_S)=\int_{-\infty}^{+\infty}\frac{\partial p_0}{\partial t}\left(\frac{\partial p}{\partial \beta}\right)_{\beta_0}dt\Bigg/\int_{-\infty}^{+\infty}\frac{\partial^2 p_0}{\partial t^2}\,p_0(t)\,dt$$

$$=\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega}\omega\,\mathrm{Im}\left[\tilde{p}_0^{*}\left(\frac{\partial \tilde{p}}{\partial \beta}\right)_{\beta_0}\right]d\omega\Bigg/\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega}\omega\left|\tilde{p}_0(\omega)\right|^2 d\omega$$



Cross-sections through 3-D sensitivity kernel for 0.01 - 0.06 Hz *S* wave at epicentral distance 40º: ray-plane (a), off-plane (b), and perpendicular (c) cross-sections. (From *H. Marquering et al.*, Geophys. J. Int. **137**, 805-815 (1999)