Statistical Uncertainty in Bistatic Performance Prediction

Brian La Cour

Tim Hawkins

Robert Luter

Applied Research Laboratories

The University of Texas at Austin

Overview

We consider uncertainty in performance prediction through the various terms in the (bistatic) sonar equation:

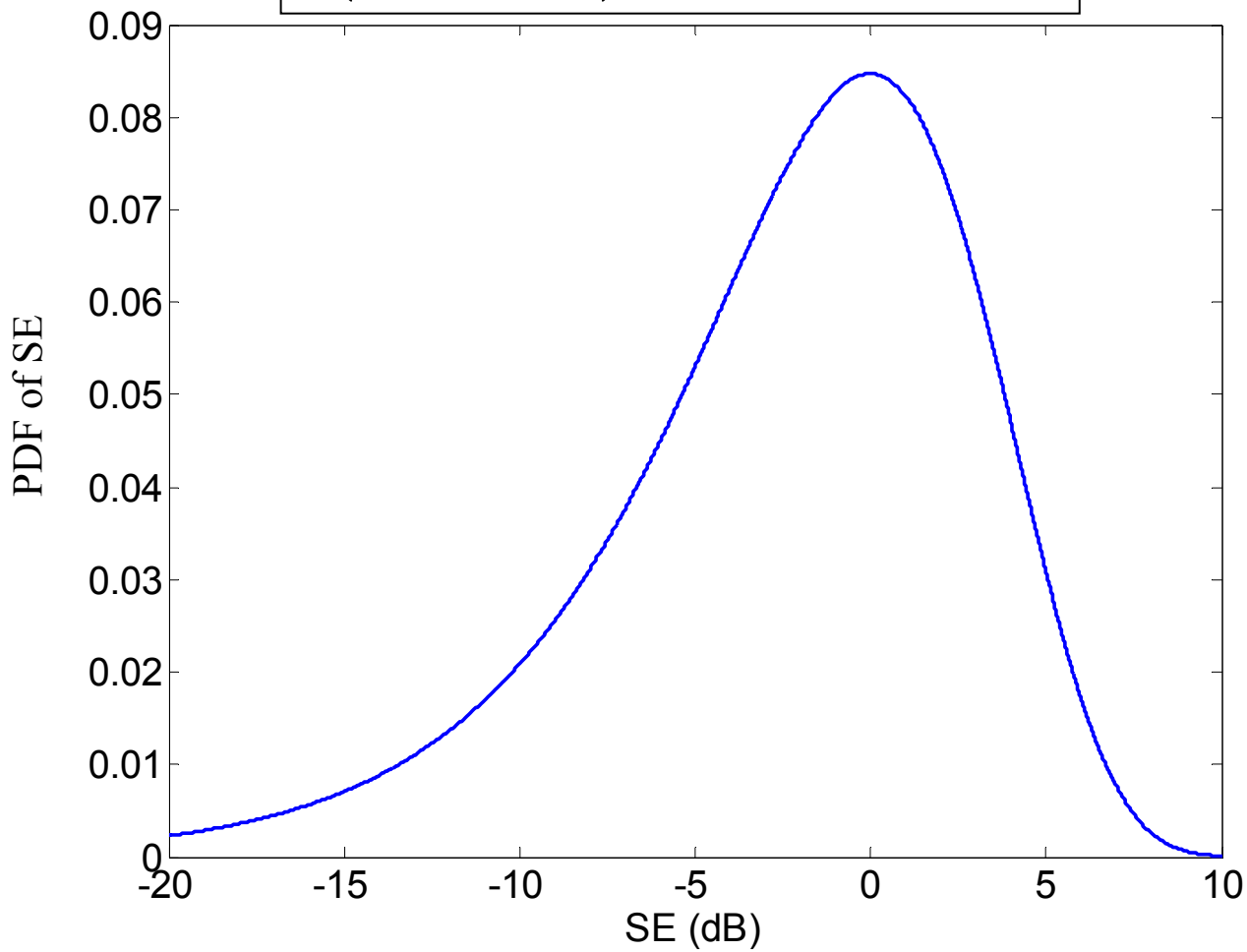
$$\mathbf{SE = SL - TL_1 - TL_2 - RL + TS},$$

which is a consequence of the normalized matched filter.

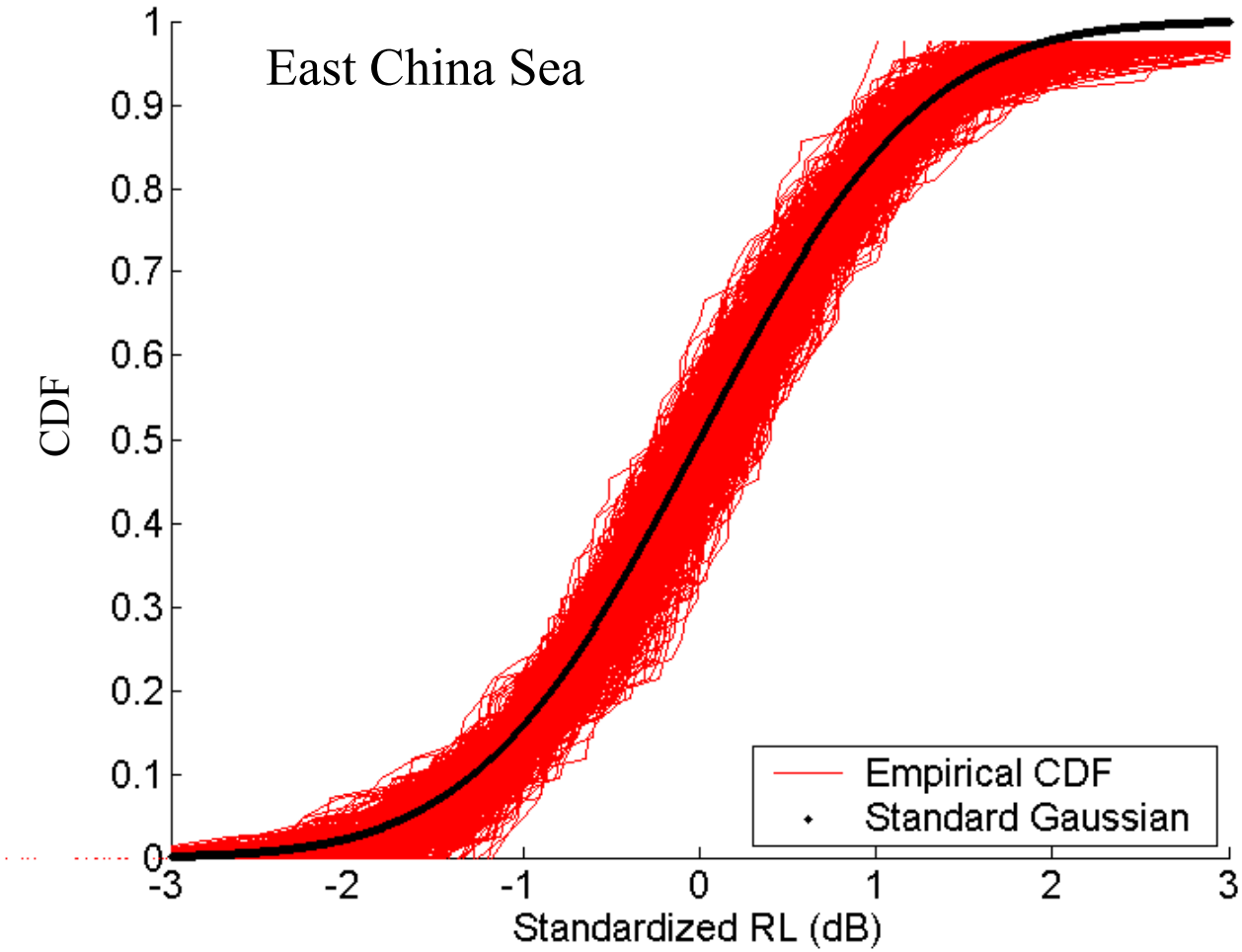
- Assume SL is fixed.
- TL and RL are modeled by CASS or PE and affected by internal waves and ill-known bottom properties.
- TS varies with source/receiver bearings and ping-to-ping.

"Off Target" SE Distribution

$$P\left(SE(s) \le y\right) = 1 - \exp(-10^{y/10})$$

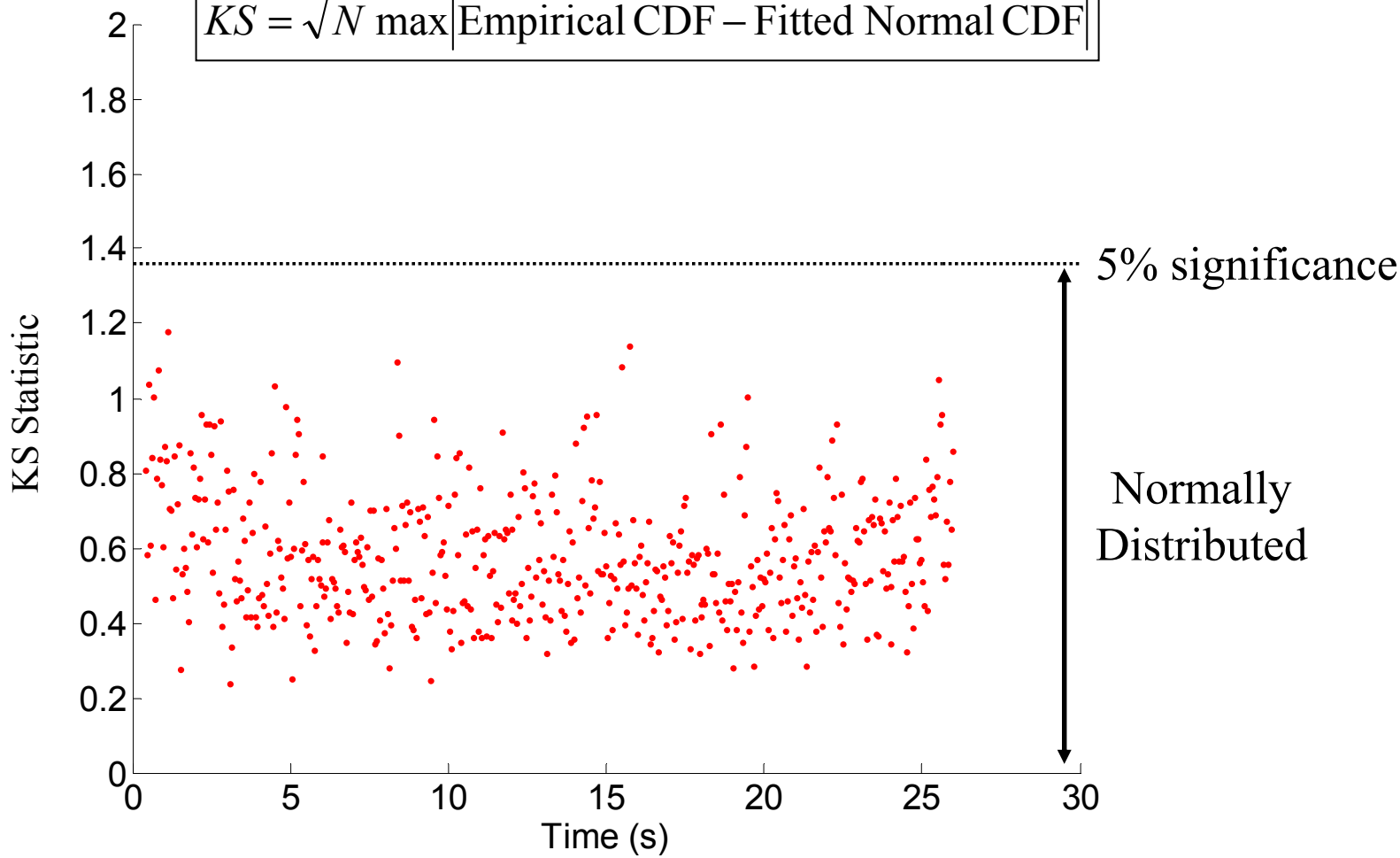


Effect of Internal Waves on RL

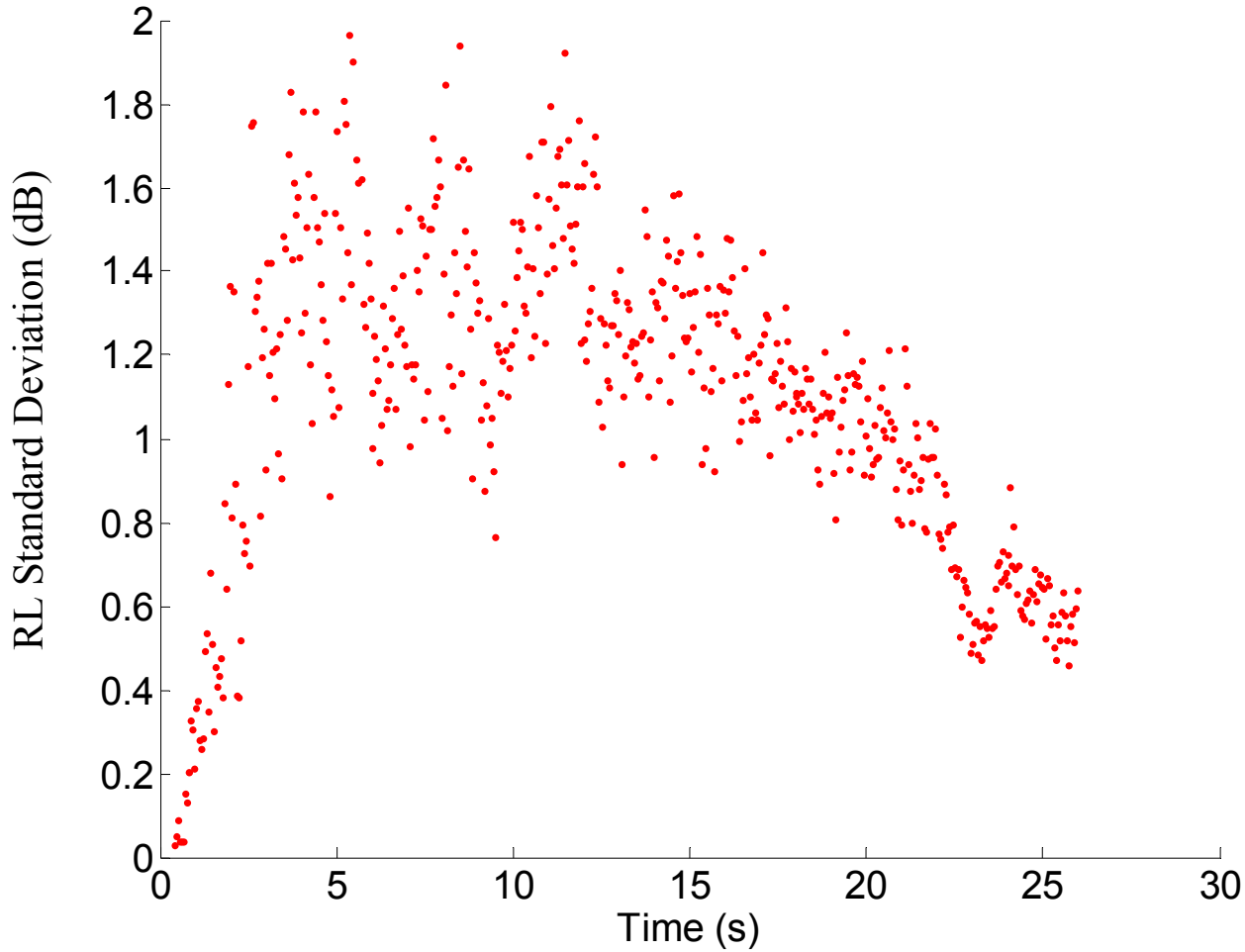


KS Test for Normality of RL

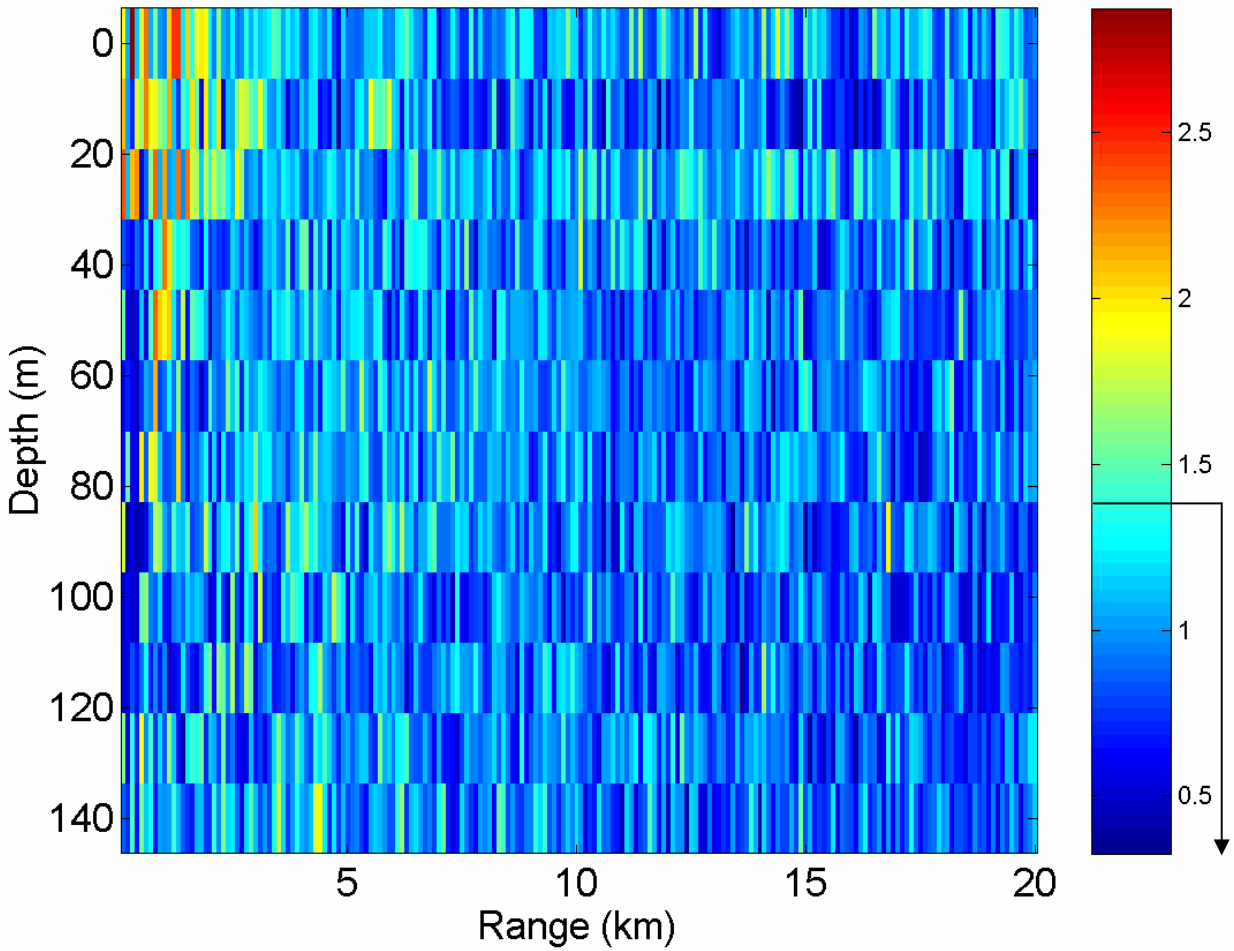
$$KS = \sqrt{N}\,\max\left|\text{Empirical CDF} - \text{Fitted Normal CDF}\right|$$



Time-Dependent Standard Deviation

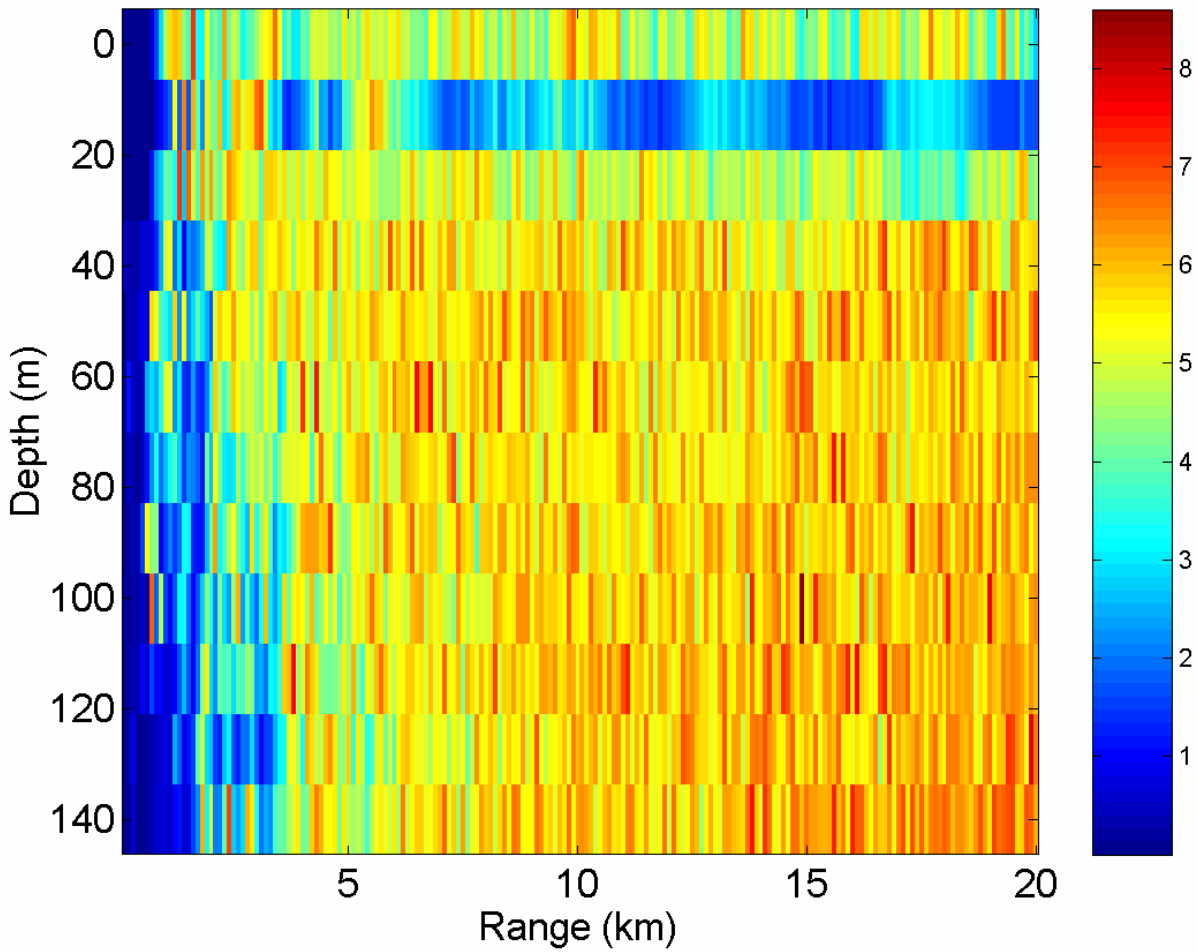


KS Test for Normality of TL



Normal at
5% significance

TL Standard Deviations

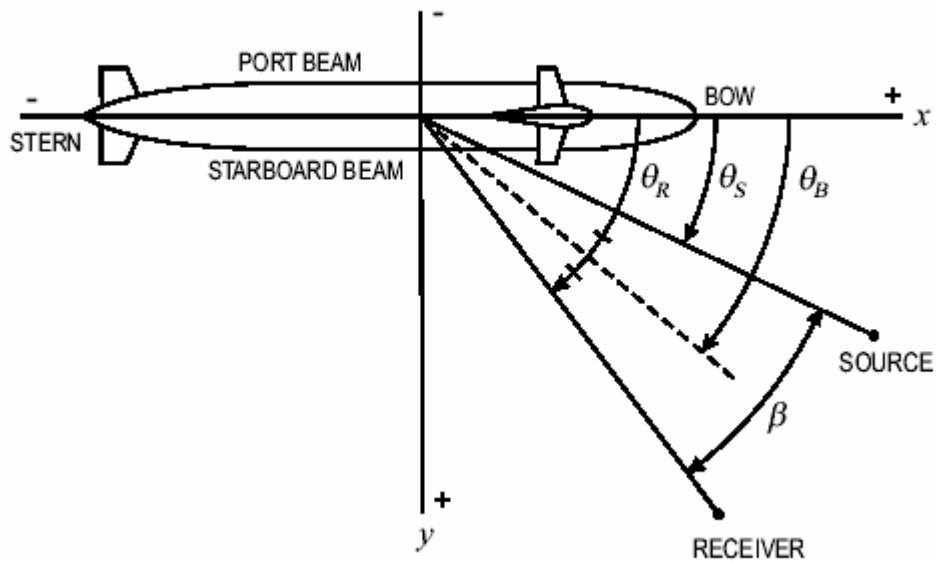


Summary of RL and TL Results

- Both RL and TL have normally distributed fluctuations due to internal waves.
- The standard deviation for RL is about 1 dB.
- The standard deviation for TL is about 6 dB.
- $TL_1 + TL_2 + RL$ has a normal distribution with a standard deviation of about 8.5 dB.

Modeling Bistatic Target Strength*

Target Geometry



*Based on David Drumheller's BASIS model (NRL/FR-MM/7140-02-10,019)

Outline of the Modeling Approach

- Model the target as a collection of **primitive shapes**.
- Assume only **specular reflections**.
- Compute bistatic TS using the **Bistatic Theorem**.
- Compute forward scattering using **Babinet's Principle**.
- Assume the final received signal has a **Rician distribution**.

Monostatic Target Strength

$$\mathrm{TS}(\theta) = 10\log_{10}\left|u(\theta) + n\right|^2$$

scattering function
$u(\theta) > 0$

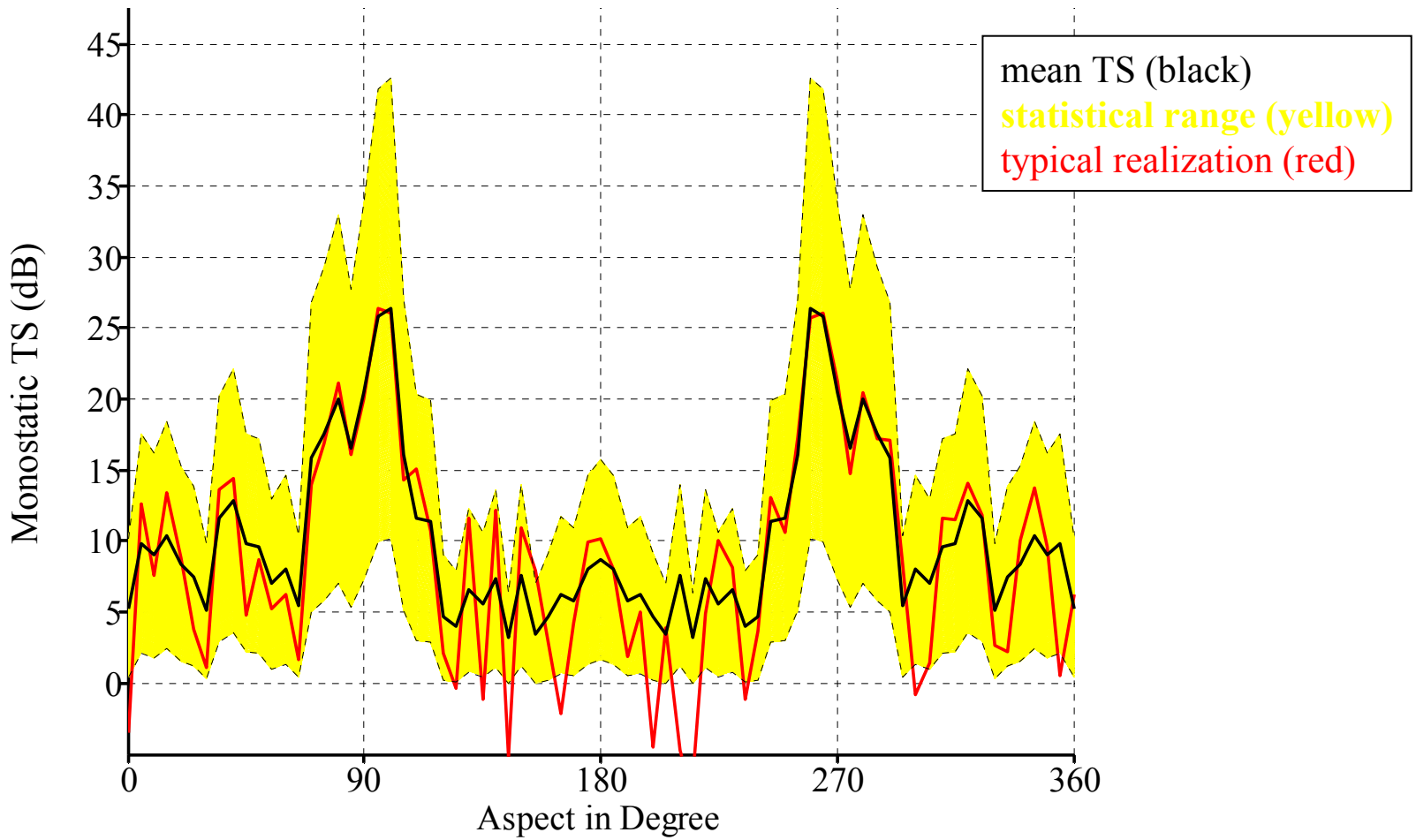
complex Gaussian noise
$E(n) = 0, \quad E(n\, n^*) = \sigma^2$

$$u(\theta) = \int_B \sum_n \left[g_n(\theta,k) + \tilde{g}_n(\theta,k)\right] S(k)\, dk$$

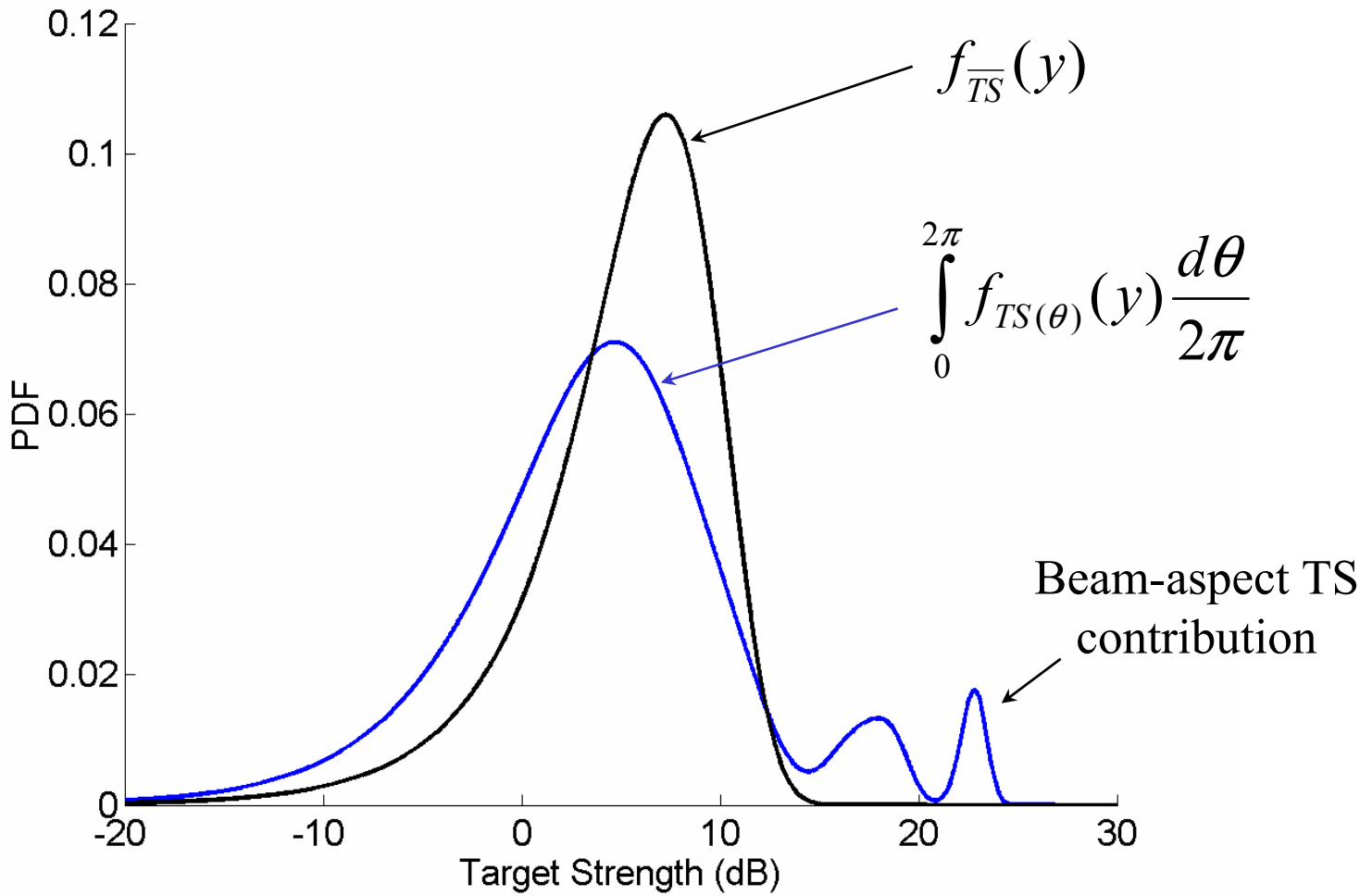
primitive
back scattering
function

primitive
forward scattering
function

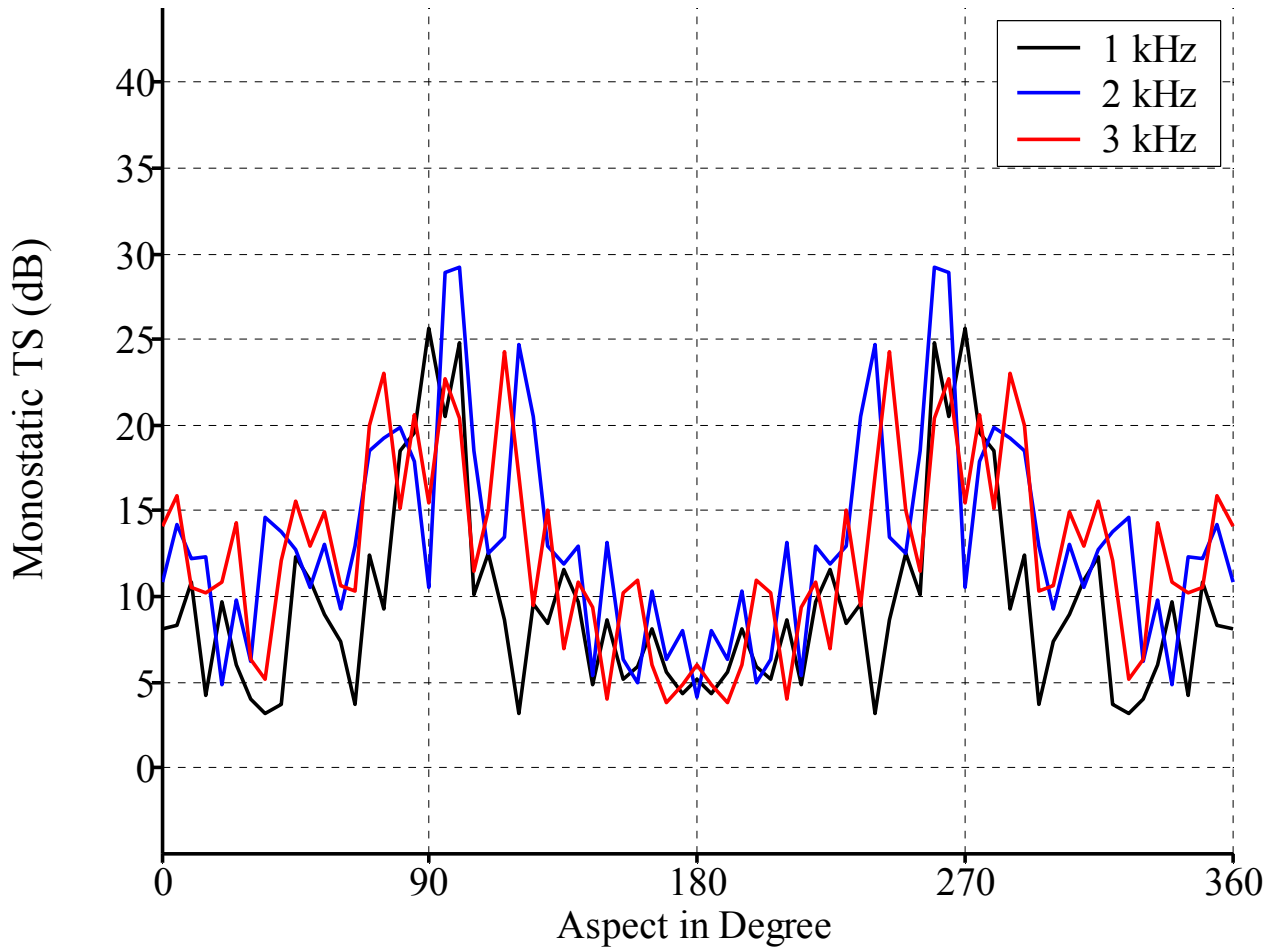
Monostatic Target Strength



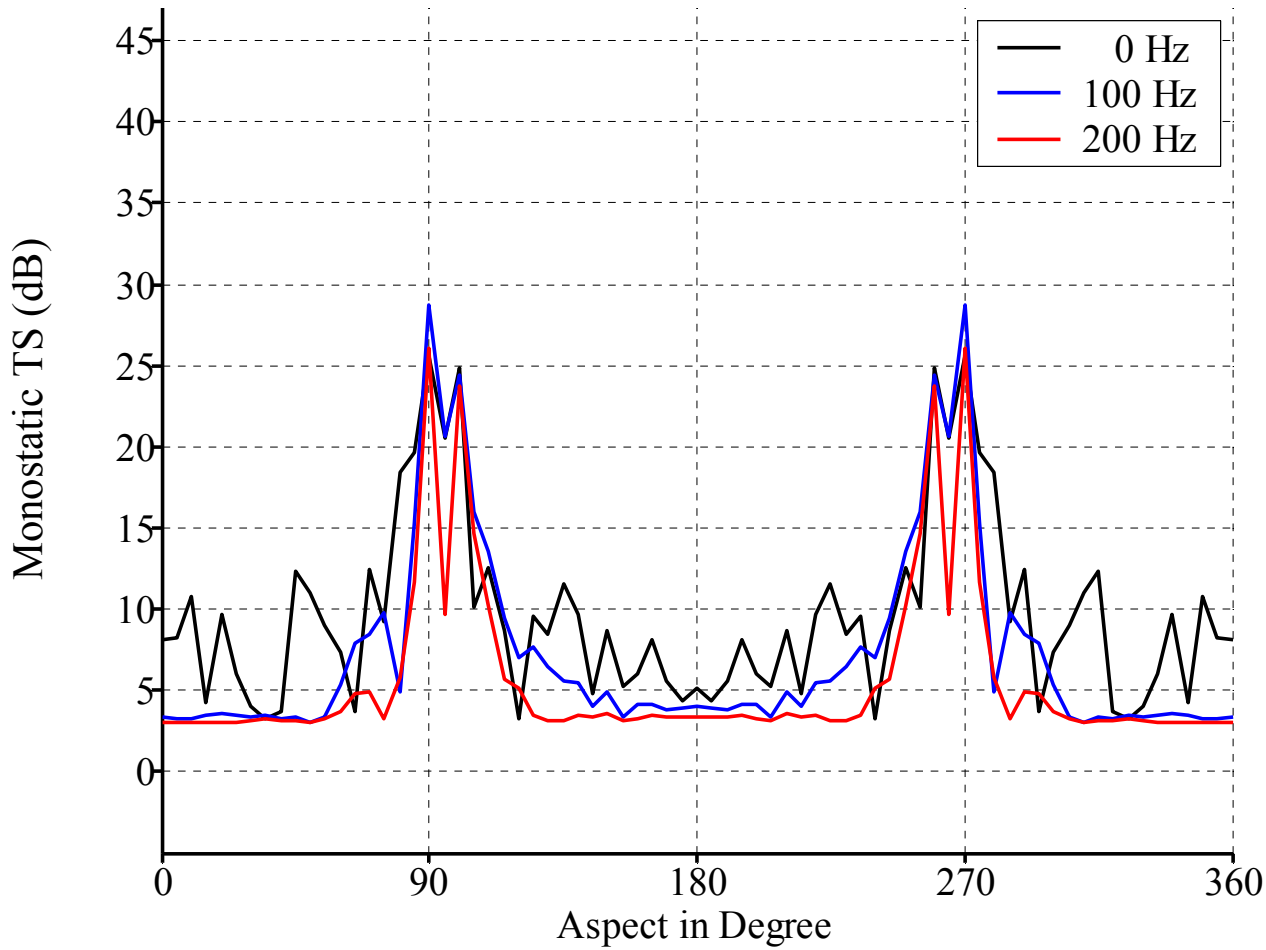
TS Distribution Averaged Over Aspect



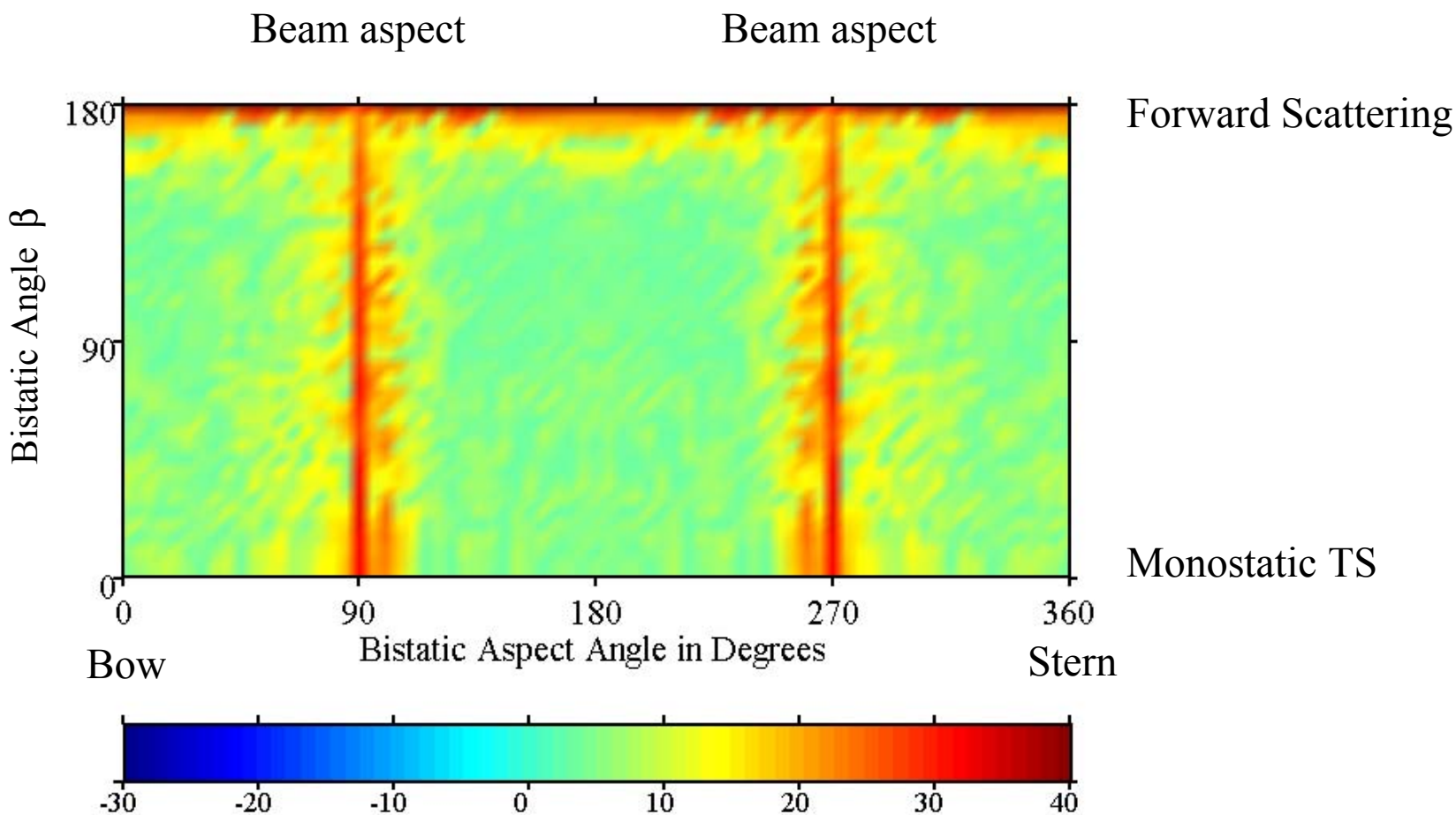
TS Frequency Dependence



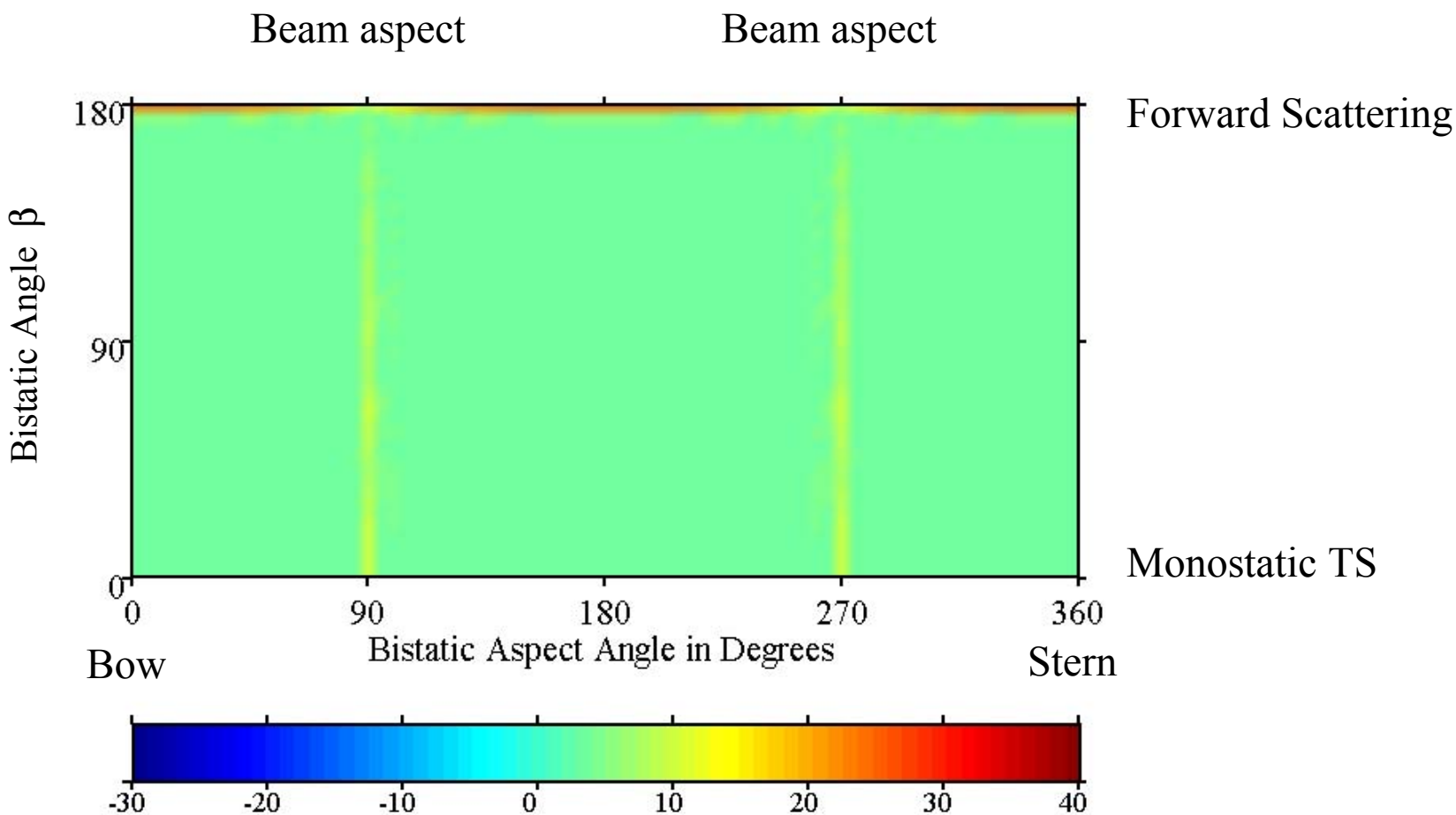
TS Bandwidth Dependence



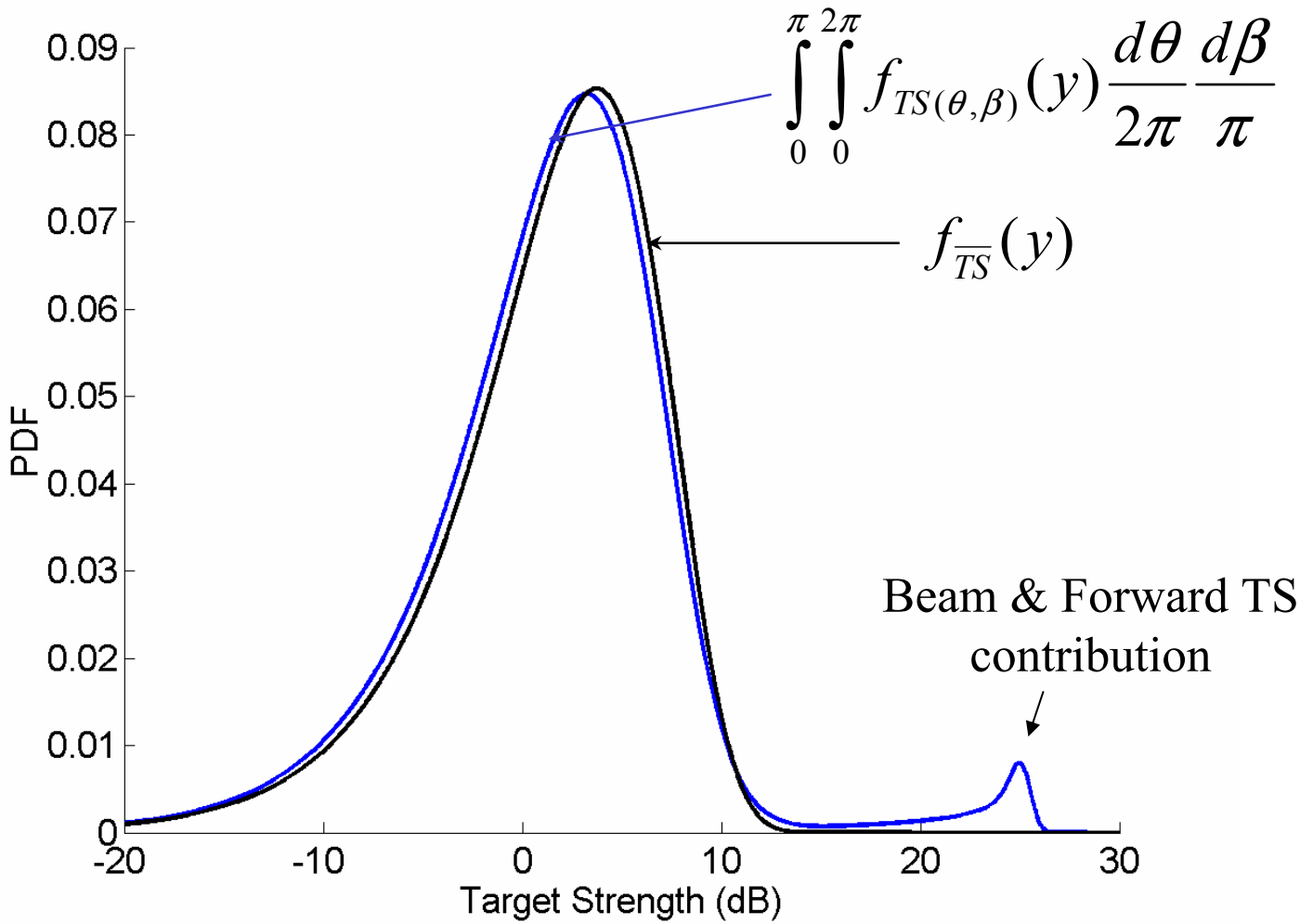
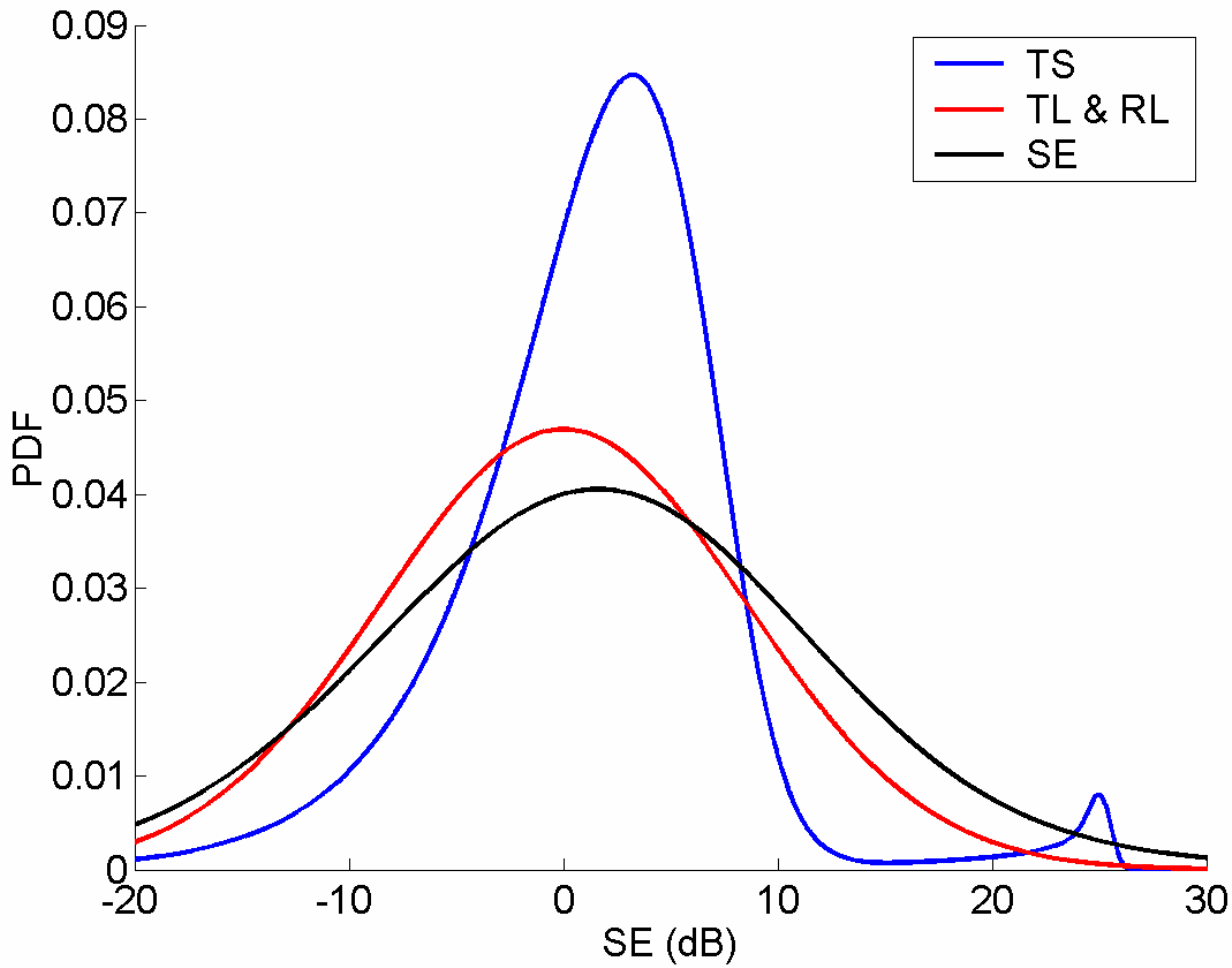
Bistatic Target Strength (Narrowband)



Bistatic Target Strength
(100 Hz BW)



Bistatic TS Averaged Over Aspect



Final "On Target" SE Distribution