A Coupled Modes Approach for Capturing Uncertainty in Towed Array Processing

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Goals

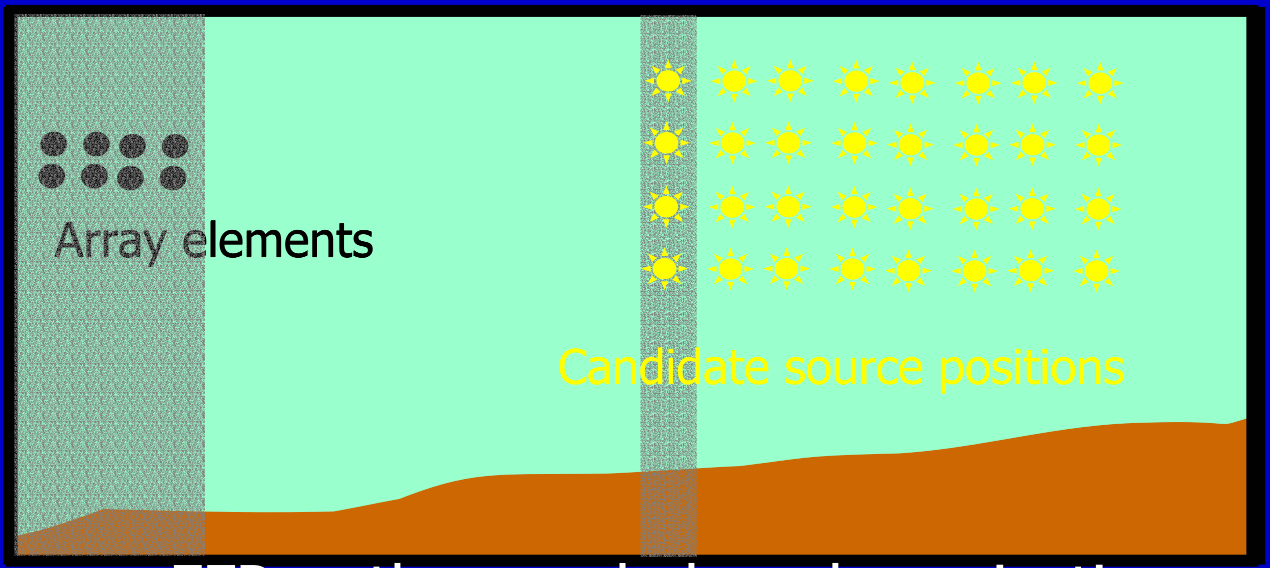
- Develop an accurate modeling capability for sonar system responses (in complicated environments with uncertain parameters)
- Include uncertain parameters:
  - *Bottom type*
  - *Sound speed profiles*
  - *Array element and source locations*
  - *Bathymetry*
  - *Coherence (for arrays with large no. elements)*
  - *Noise field*
- Develop method to efficiently simulate array response over 360 degrees (and allow Monte Carlo simulations over uncertain parameters)

Outline

- Describe coupled-normal mode approach using both:
  - Pressure projection method
  - Coupling matrix method
- Give examples for SWellEx-96 site

Coupled-normal modes

For modeling system response to unknown source locations, there are often more many candidate source positions than array elements so codes are run using reciprocity.



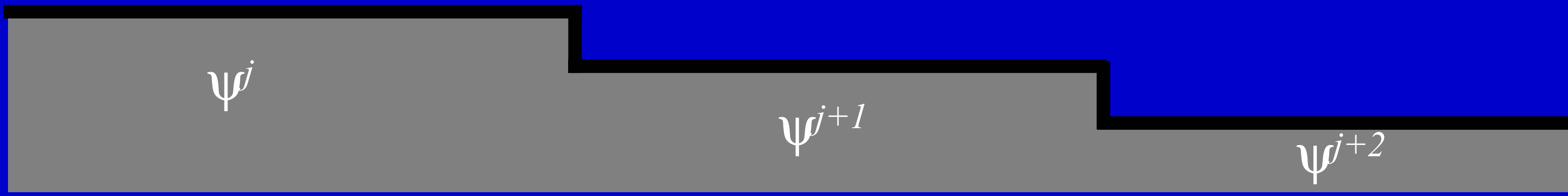
For PE, rays, FFP or the coupled mode projection method an *N* element array requires *N* forward calculations.
It is better if we can make a single run between the target range/depth cells to the array.

⇨ Mode methods have the possibility for a pre-calculation

Coupled-normal modes

- For range independent,

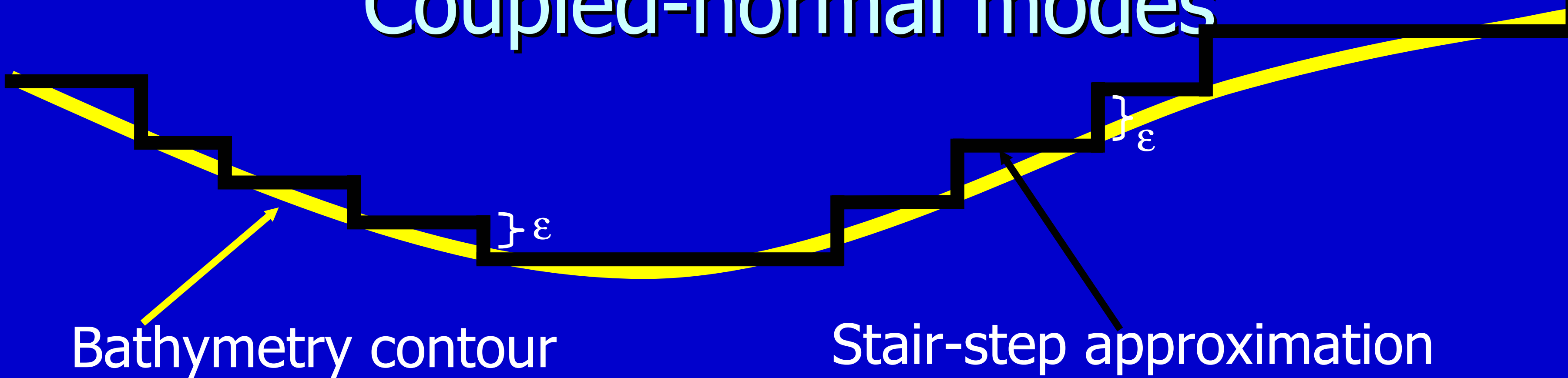
$$p(r,z) = \frac{i}{4\rho(z)} \sum_{m=1}^{\infty} \Psi_m(z_s)\Psi_m(z) H_0^{(1)}(k_{rm} r)$$



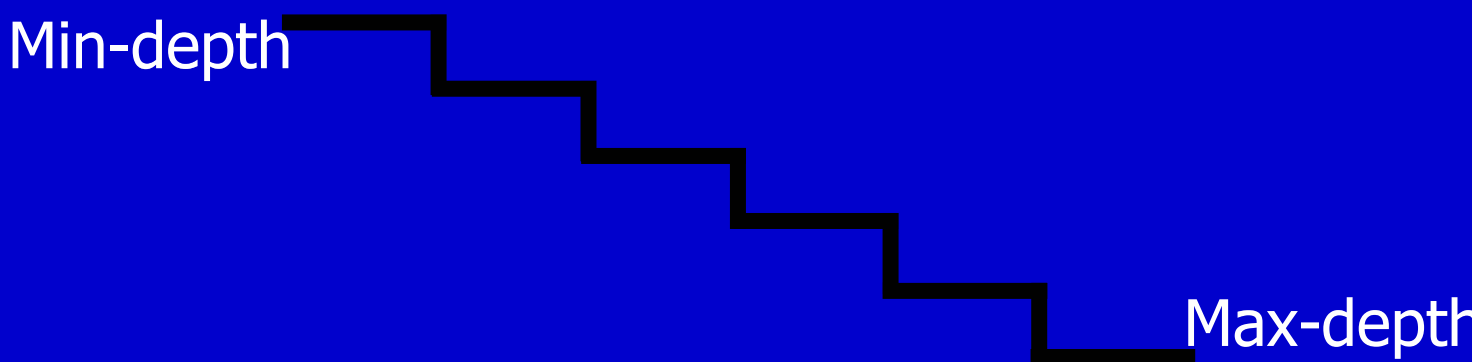
- For range dependent, mode functions are projected onto adjacent segment,

$$c_{lm} = \int \frac{\Psi_l^{j+1}(z)\Psi_m^{j}(z)}{\rho^{j+1}(z)} dz$$

Coupled-normal modes

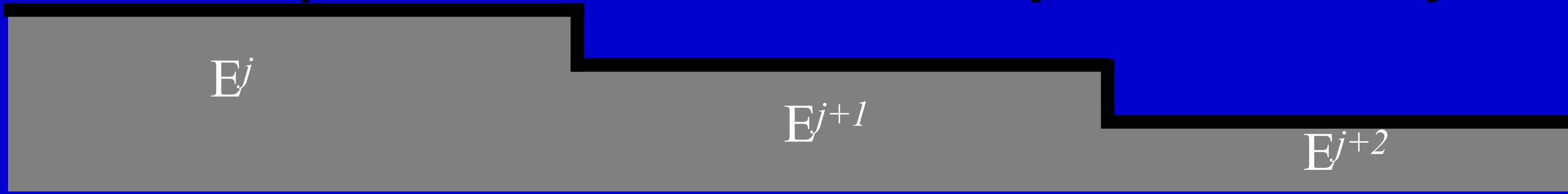


- New mode set is used when bathymetry changes by ε (depends on slope)



- All mode sets needed fall between the minimum and maximum depth- these modes are pre-computed and sorted later for bathymetry

Coupled-normal modes (2 methods)



Mode sets at each depth are represented as matrices,

$$\overline{E}^{j} = \begin{bmatrix} \Psi_1^j(z_1) & \Psi_2^j(z_1) & \cdots \\ \Psi_1^j(z_2) & \Psi_2^j(z_2) & \\ \vdots & & \ddots \end{bmatrix}$$

For a single source to set of receivers, the most efficient calculations are through projection (Method 1),

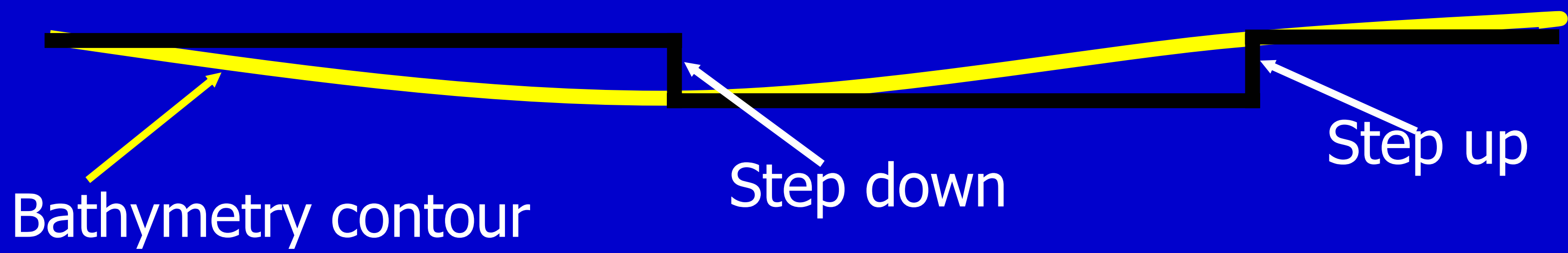
$$\overline{P}^{j} = \begin{bmatrix} p(z_1) & p(z_2) & \cdots \end{bmatrix}$$

$$\overline{P}^{j+1} = \overline{P}^{j}\,\overline{E}^{j+1}$$

Or through forming (and saving) coupling matrices (Method 2),

$$\hat{C} \approx E^{j+1} E^{j}$$

Forming Coupling Matrix



At any range, there is only the option to take a step up, or take a step down.

Therefore, we can pre-calculate and store up and down coupling matrices for each depth segment,

$$\overline{C}^{up} = \overline{E}^{T}\,\overline{E}^{up}, \qquad \text{and} \qquad \overline{C}^{down} = \overline{E}^{T}\,\overline{E}^{down}.$$

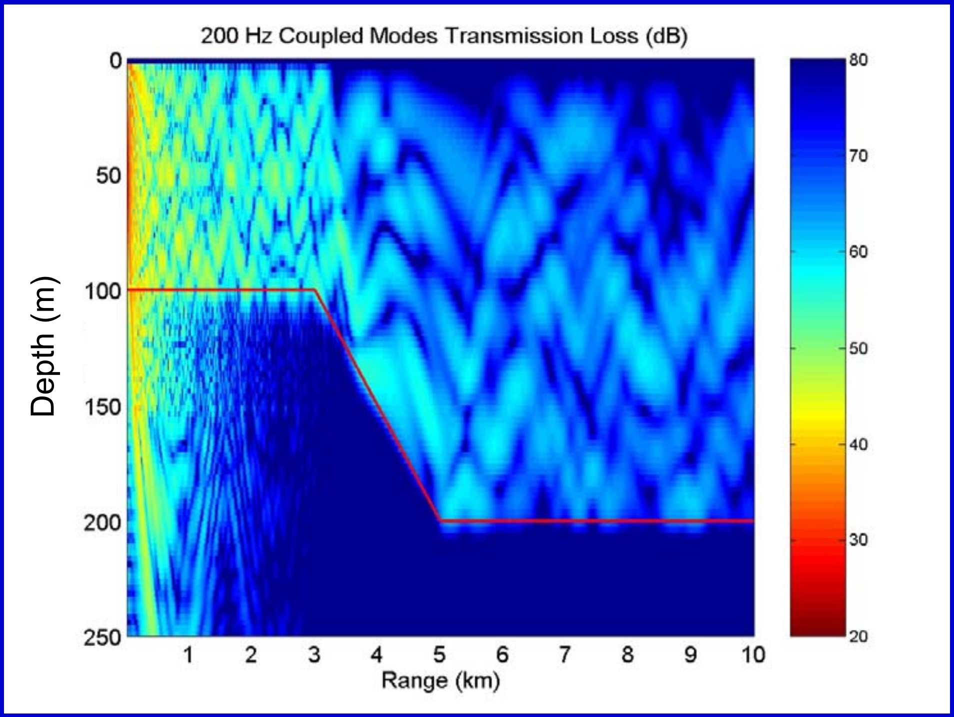
Pre-calculation is slightly slower (than projection method) but forward calculation efficiency is about the same (can be faster if limited to diagonal terms).

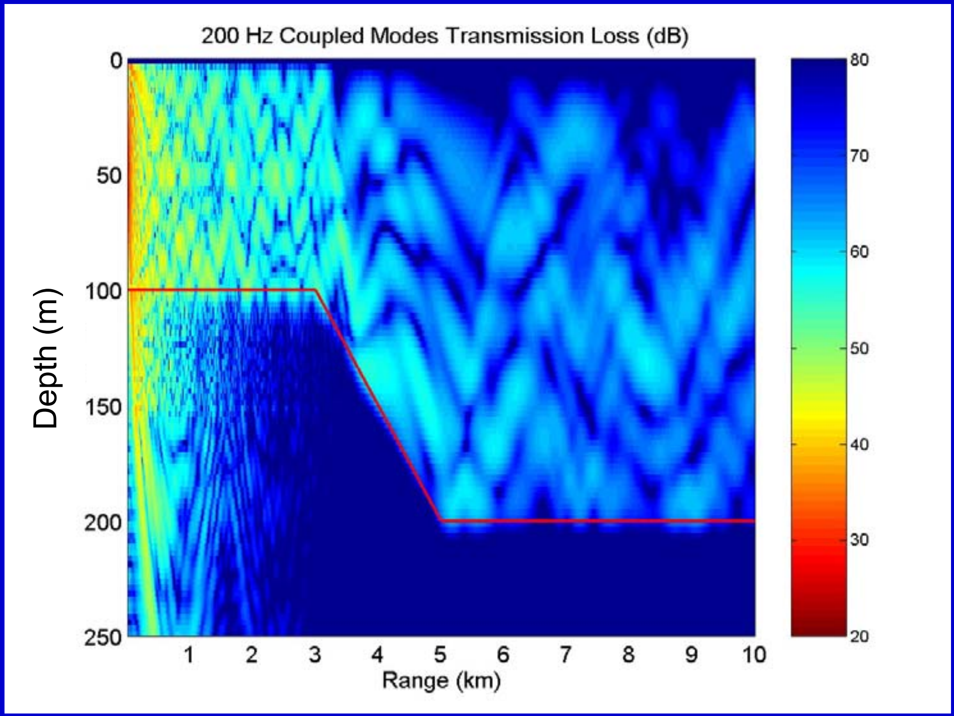
Projection vs Coupling Matrices (both take advantage of pre-calculations)

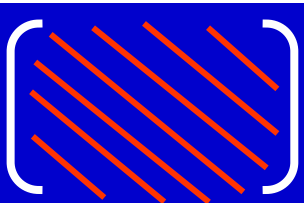
- Projection Method:
  - Fastest for a single source (or receiver)
  - Requires mode functions finely sampled in depth (can be a big storage requirement)
- Coupling matrix method:
  - Don’t need to store modes on fine grid (storage is reduced)
  - Always taking small depth change steps so can keep only near diagonal terms for reduced run-times and storage requirements
  - Can rapidly compute multiple source/receiver combinations
  - A little slower in the pre-calculation stage

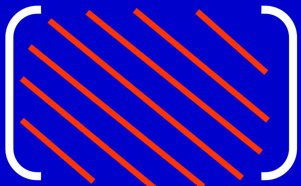
Diagonals of Coupling Matrix

To reduce storage and decrease run times, we can store only near diagonal elements of coupling matrices. Take for example a case of downslope propagation at 200 Hz:

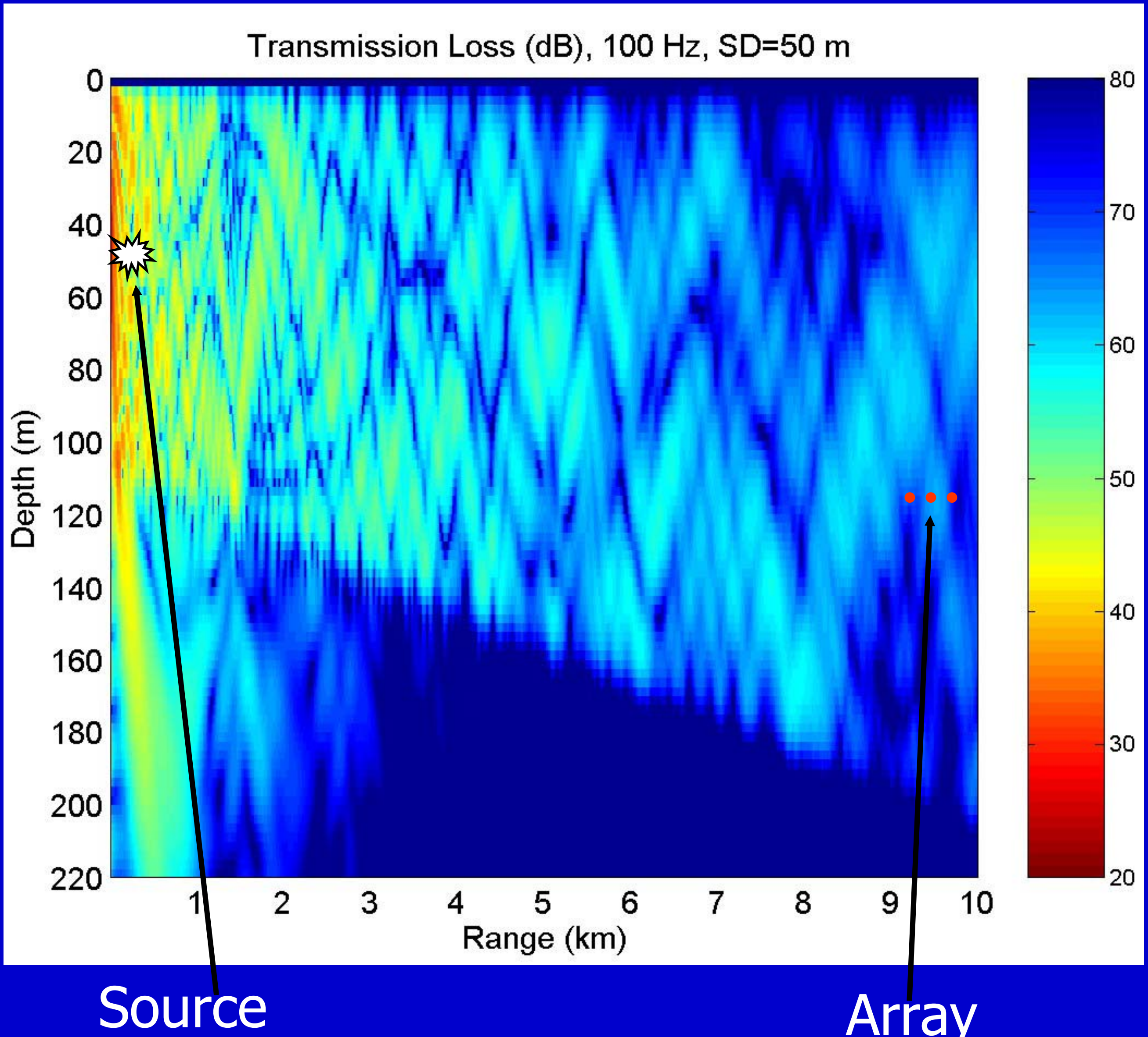


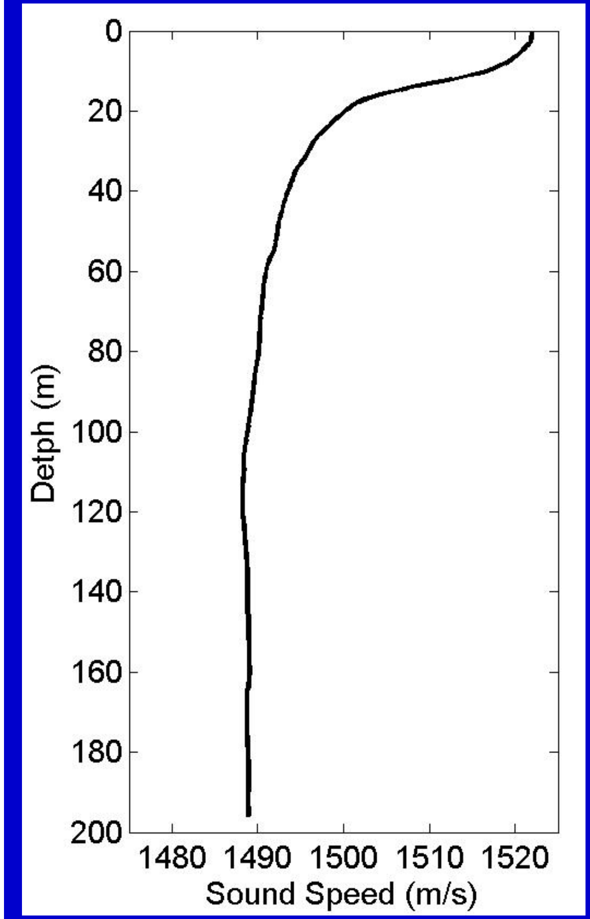


$\mathbf{C} =$ 

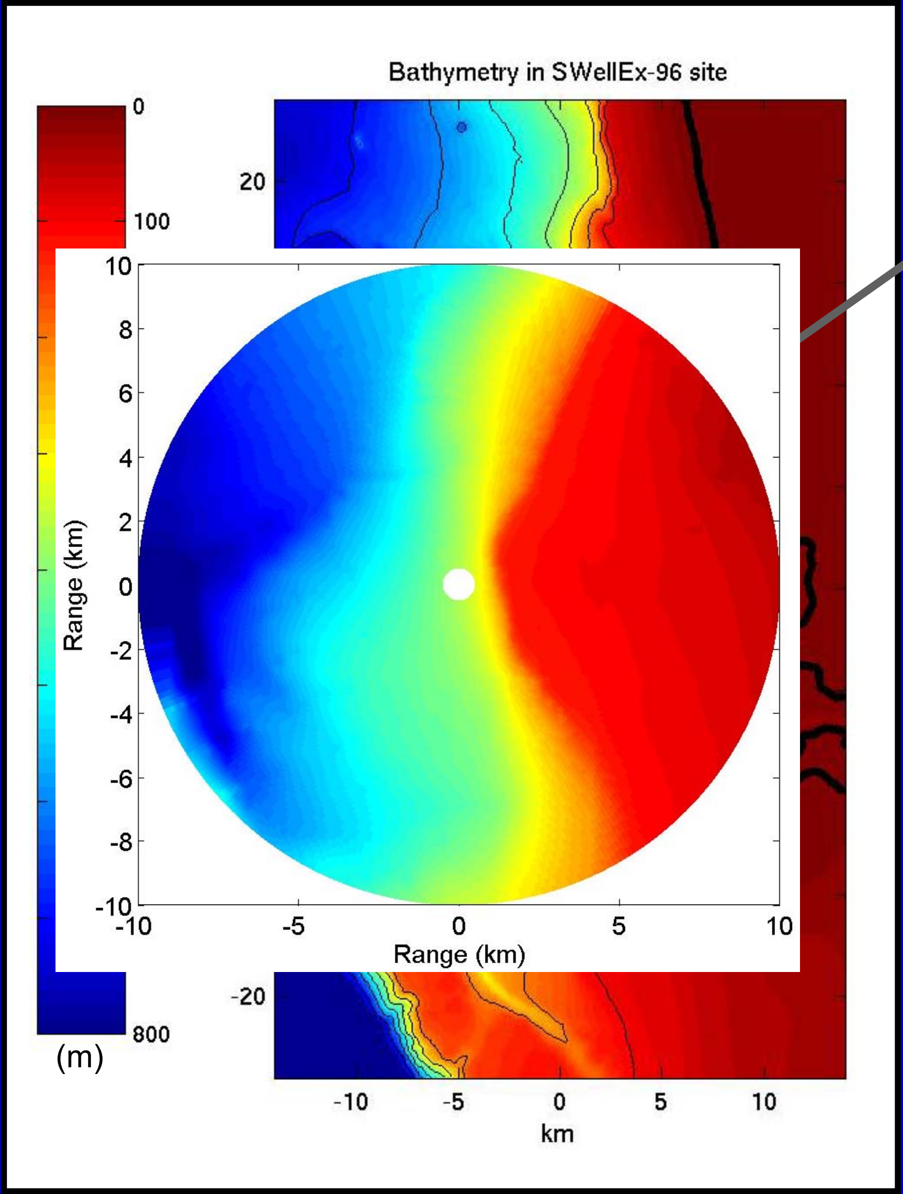
$\mathbf{C} =$ 

Example: SWellEx-96 Site





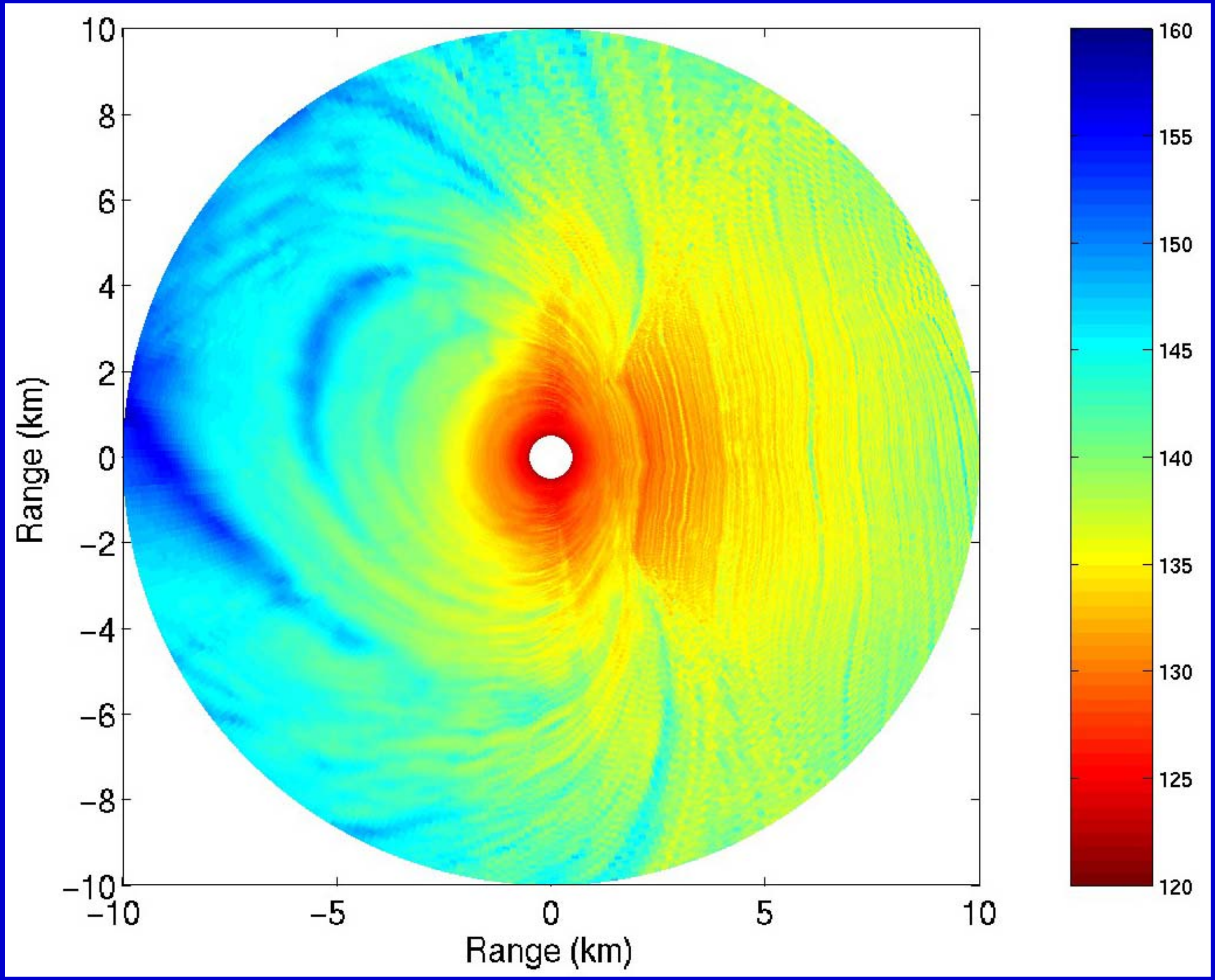
Example: Uncertainty in Minimum Detectable Level (Towed Array for SWellEx-96 Site)



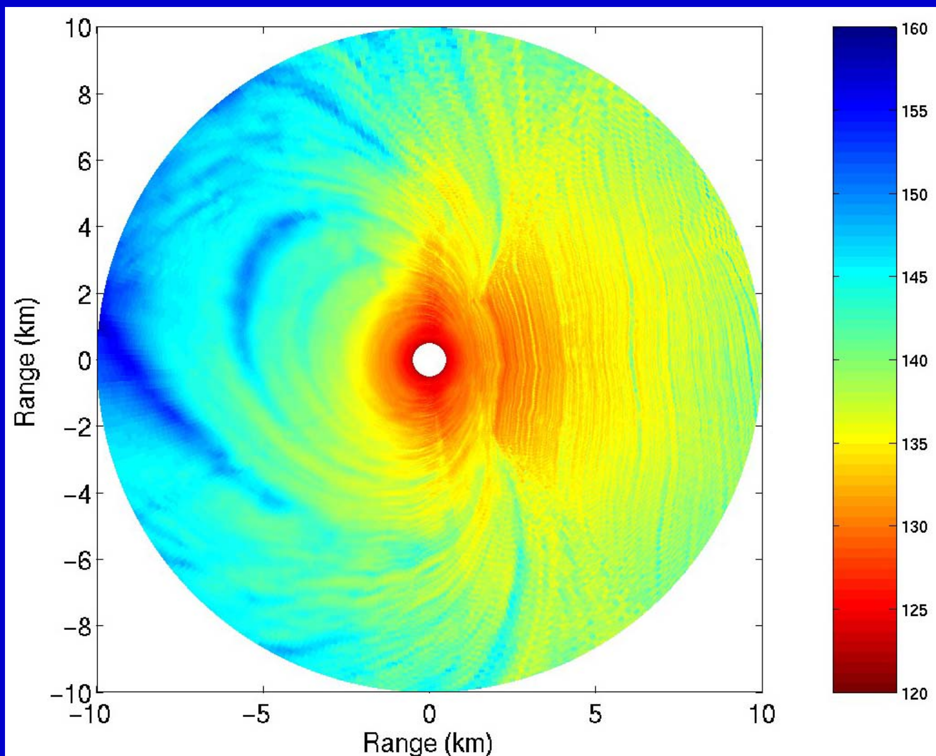
- A single line, horizontal array is placed here (51 elements, 100 Hz)
- Simulation includes uncertainty in:
  - *Target depth (0-100 m)*
  - *Bottom type (sand, silt)*
- Capability to rapidly make predictions for different array geometries (*tow depth, orientation*)

Minimum Detectable Level

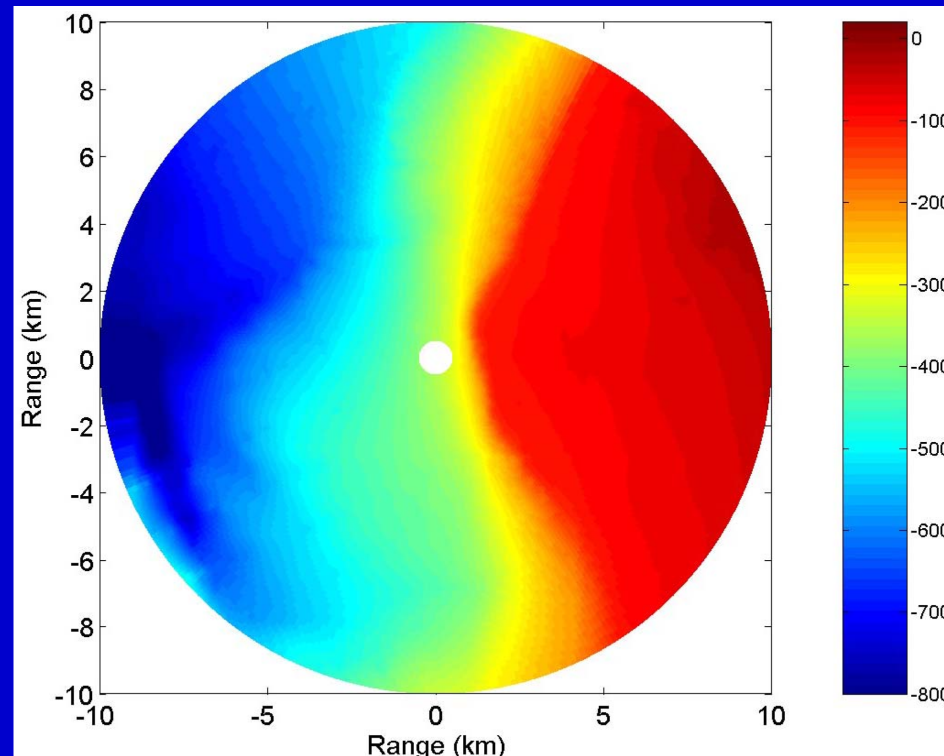
Complex pressure field is beamformed (Conv. plane-wave, NL=80 dB, DT=6 dB, array at 110 m depth). Below is mean MDL (over water column or top 100 m).



Effect of bathymetry on MDL

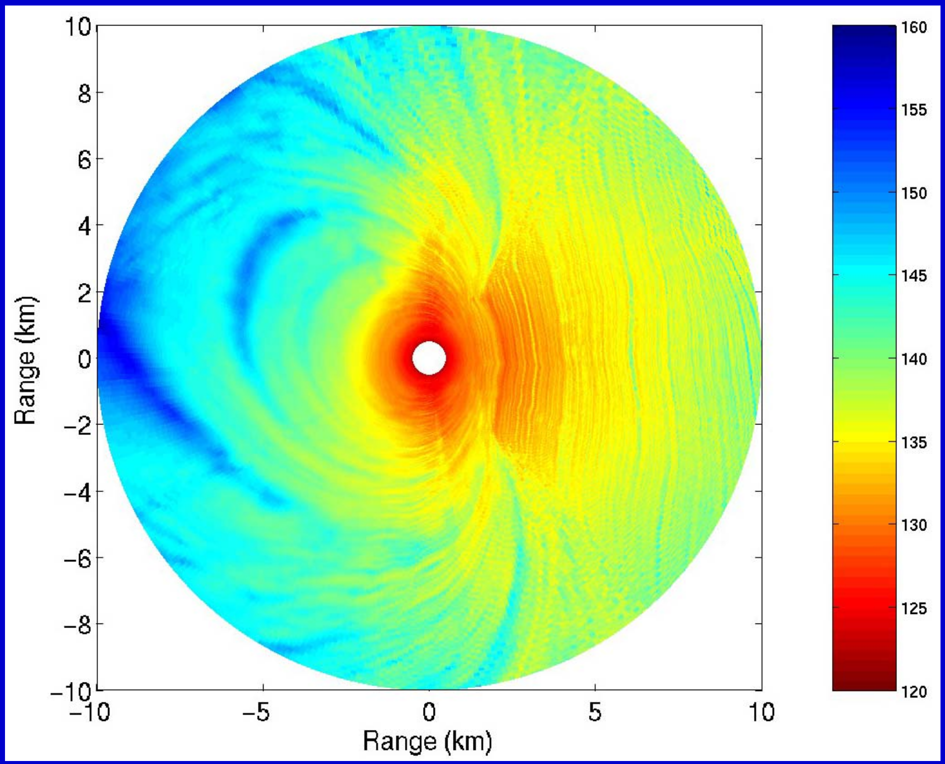


MDL for Array at 110 m depth

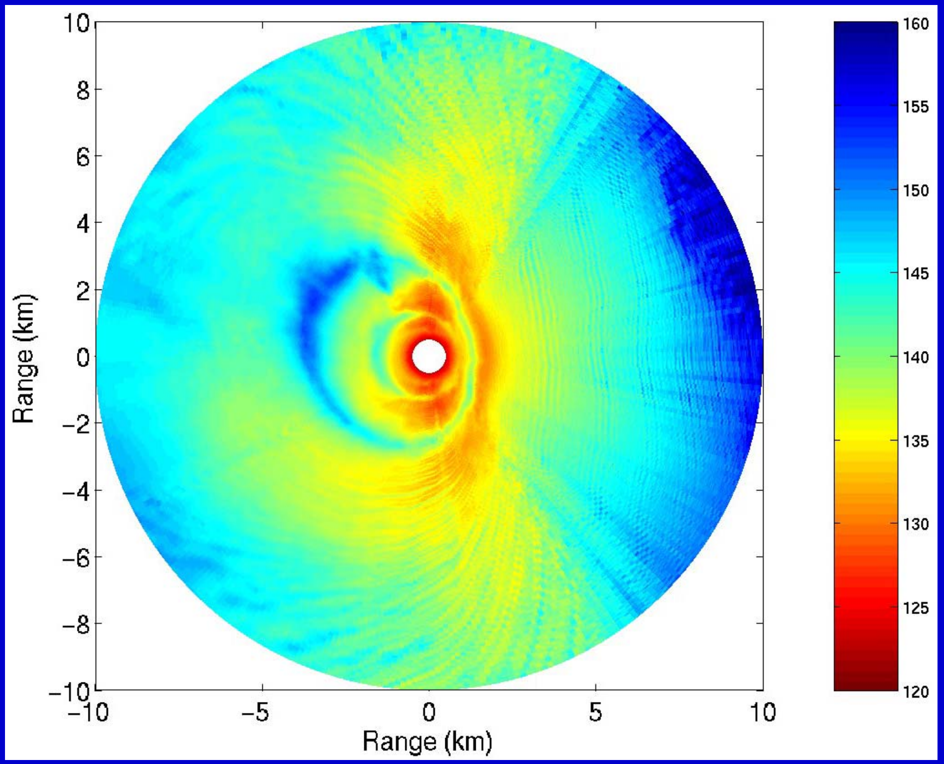


Bathymetry

Effect of tow depth on MDL

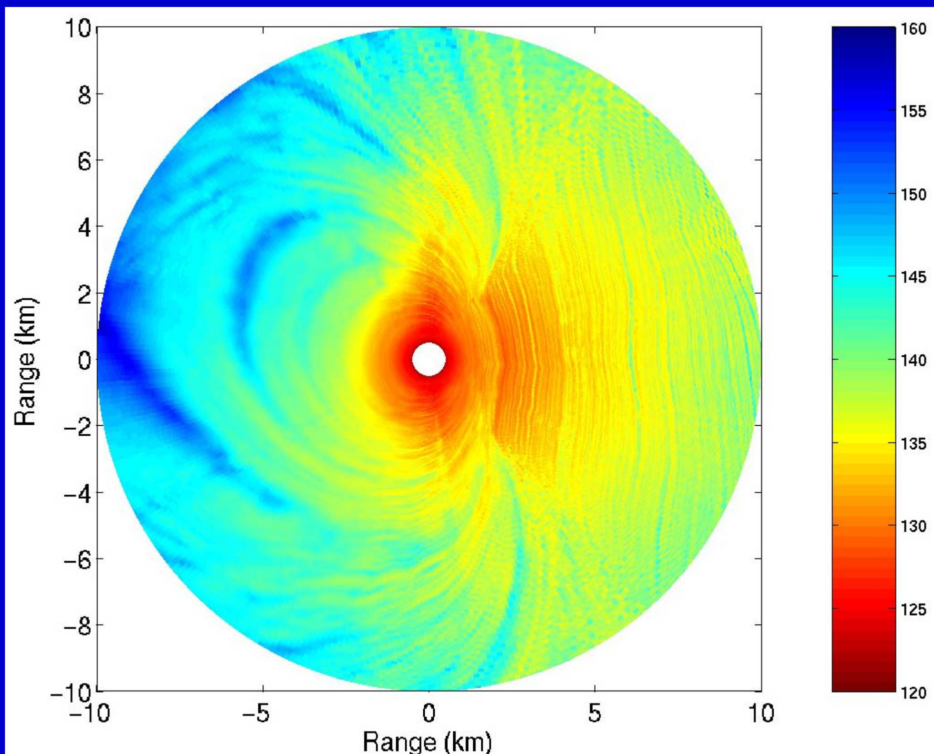


MDL for Array at 110 m depth

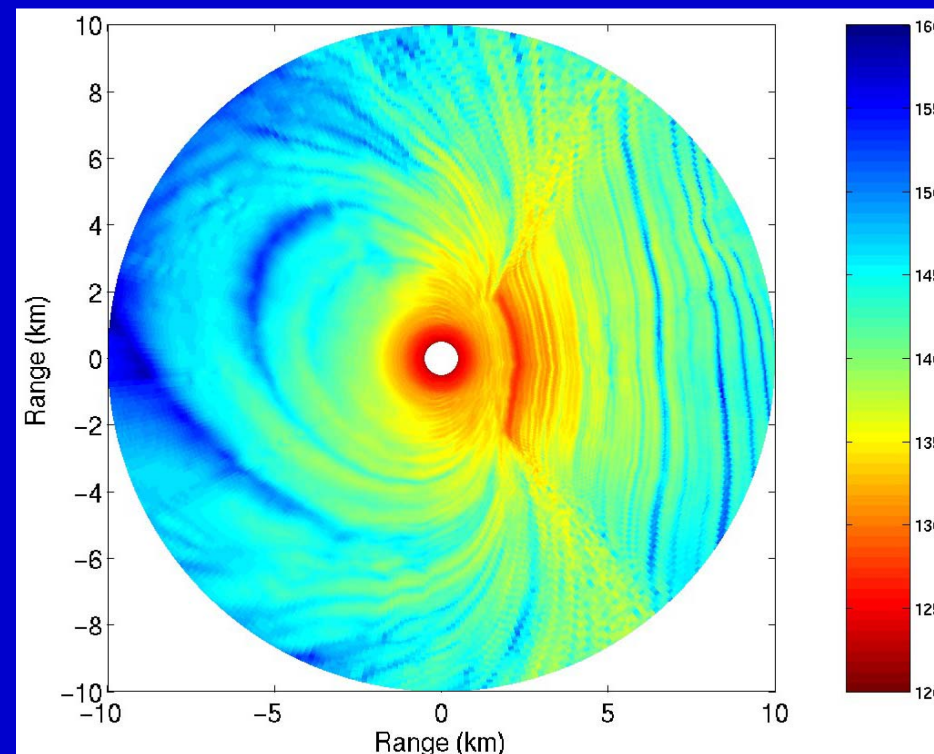


MDL for Array at 30 m depth

Effect of bottom type on MDL



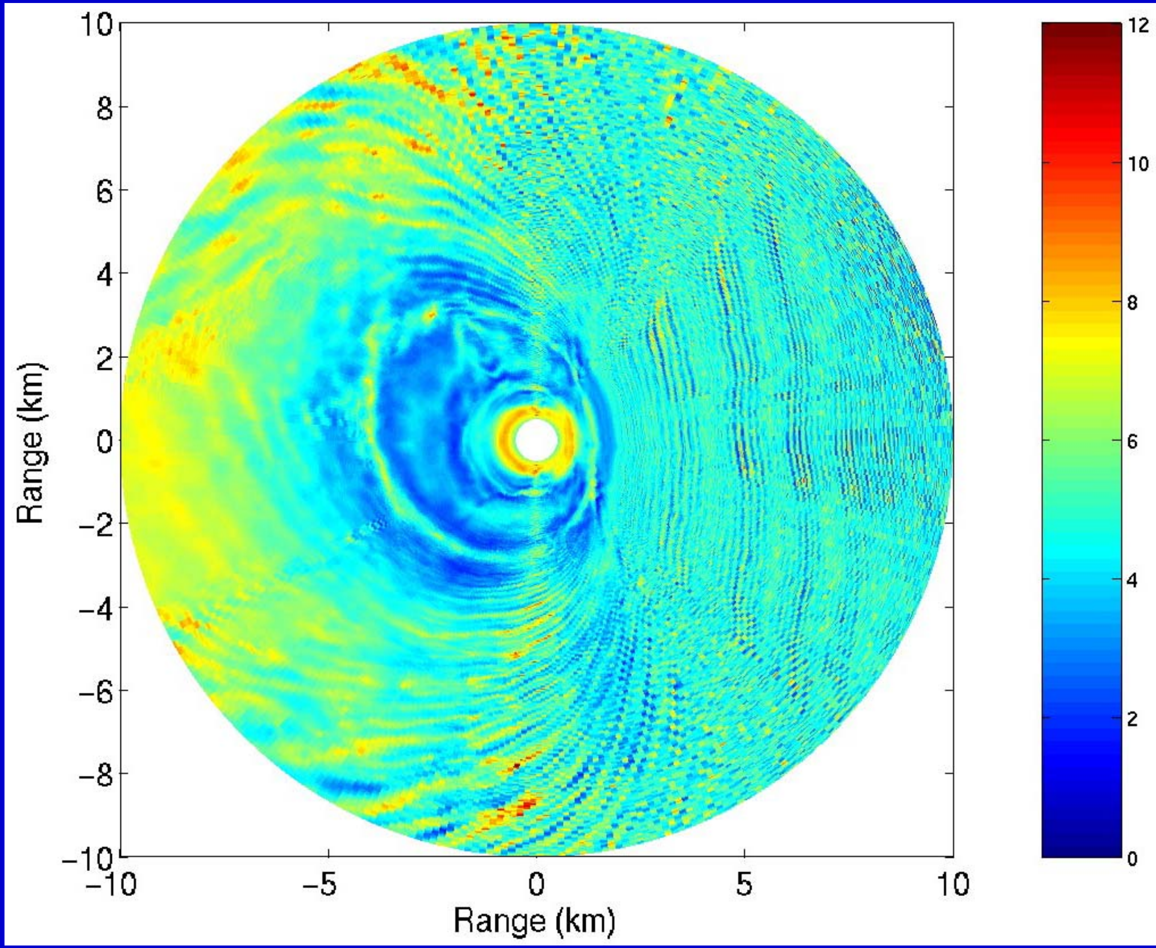
MDL for Array at 110 m depth, sand bottom properties



MDL for Array at 110 m depth, silt/clay bottom properties

Uncertainty in MDL

Complex pressure field is beam-formed (Conv. Plane-wave, NL=80 dB, DT=6 dB, array at 30 m depth). Below is standard deviation value (over water column or top 100 m) of min. source level.

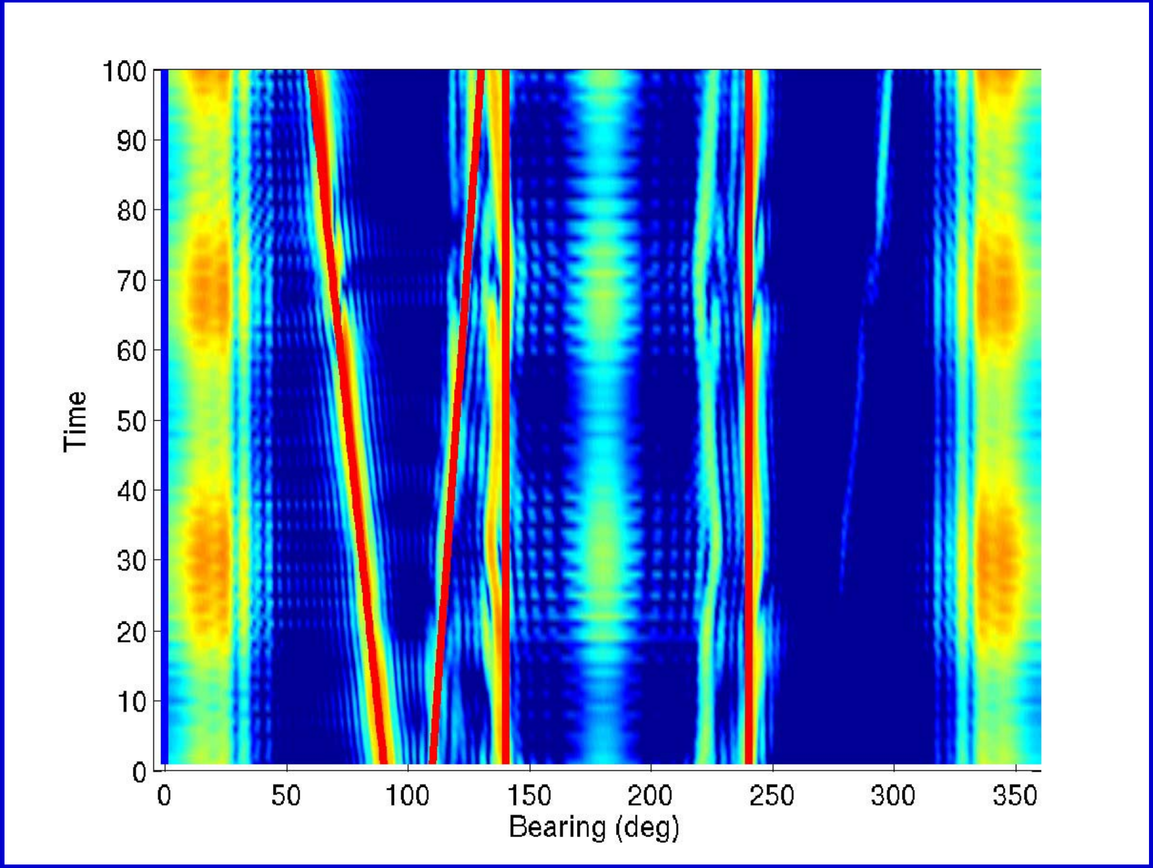


Summary

- We want to move beyond TL modeling to systems modeling but do it in a computationally efficient and accurate way.
- If calculations are fast we can model many uncertain parameters (e.g. Monte-Carlo)
- Array responses to self-noise and shipping in range-dependent areas can also be simulated
- The coupled-modes formulation provides a way to do a large amount of pre-calculation so many scenarios can be simulated very rapidly

Example: Simulated BTR

- Input environment, array geometry (e.g. towed array hydrophone positions) and specify ship tracks (SL, ranges, bearings, time)



Example: BTR from SWellEx-96