Statistical Properties of the Acoustic Field in Inhomogeneous Oceanic Environments

Oleg A. Godin
CIRES/University of Colorado and NOAA/OAR/Environmental Technology Laboratory

Alexander G. Voronovich
NOAA/OAR/ Environmental Technology Laboratory

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Motivation for a statistical description of the uncertainty associated with horizontal refraction and medium time-dependence:

— range-dependence and cross-range variation of environmental parameters affect the acoustic field in a different manner

— being relatively weak, effects of the cross-range variations are not necessarily negligible

— these effects accumulate with range rather rapidly (typically, as third power of range) and lead to ***biases*** in signal travel time and modal phases

— signal frequency wander due to medium non-stationarity introduces uncertainty in estimates of target velocity

— lack of the detailed knowledge of cross-range environmental inhomogeneities is likely to remain, for the foreseeable future, an obstacle for the accurate prediction of the underwater acoustic field

— environmental measurements cannot be repeated fast enough along the propagation path to make a deterministic prediction of the frequency wander possible

MODE PHASE BIAS: Deterministic theory

$$\theta_m = O\left(\varepsilon^3\right) + \int_{x_1}^{x_2} q_m(x,0)\,dx - \int_{x_1}^{x_2} \frac{(x-x_1)^{-2}\,dx}{2\,q_m(x,0)} \left(\int_{x_1}^{x} \frac{\partial q_m}{\partial y}(a,0)(a-x_1)\,da \right)^2$$

RAY TRAVEL TIME BIAS: Deterministic theory

$$\delta T_{HR} = O\left(\varepsilon^3\right) - \frac{1}{2c_0} \int_{x_1}^{x_2} \left(\int_{x_1}^{x} \frac{\partial n}{\partial y} \frac{(a-x_1)}{\cos\chi}\,da \right)^2 \frac{(x-x_1)^{-2}\,dx}{n\cos\chi}$$

RAY TRAVEL TIME BIAS: Random inhomogeneities

$$n(\boldsymbol{r}) = n_0(\boldsymbol{r})[1+\varepsilon G(\boldsymbol{r})], \quad \varepsilon^2 n_0^2 \langle G(\boldsymbol{r}_1) G(\boldsymbol{r}_2)\rangle = \sigma_n^2 W(\boldsymbol{r}_1 - \boldsymbol{r}_2 ; (\boldsymbol{r}_1 + \boldsymbol{r}_2)/2)$$

$$\langle \delta T_{HR} \rangle = \int_{x_1}^{x_2}\int_{x_1}^{x} \left.\frac{\sigma_n^2 B}{\cos\chi}\right|_{y=0,\, z=z_0(a)} \frac{(a-x_1)^2\, da}{c_0} \frac{dx}{\alpha(x-x_1)^2} + O(\varepsilon^4)$$

where

$$\alpha(x) = n(x, 0, z_0(x)) \cos\chi(x, 0, z_0(x))$$

$$B = \frac{1}{\cos\chi}\int_0^{+\infty} \frac{\partial^2 W}{\partial y^2}(x, 0,\, x\tan\chi ; \boldsymbol{r})dx = \int_0^{+\infty} \frac{dW_I}{da}(a;\boldsymbol{r})\frac{da}{a}$$

Special case:

$$\delta T_{HR} = \frac{D}{6\alpha c_0}(x_2 - x_1)^2, \quad D = l^{-1}(\alpha)\oint \sigma_n^2 B\, ds$$

MODE PHASE BIAS: Random inhomogeneities

$$q_m(x,y) = q_{m0}\left[1 + \varepsilon g(x,y)\right]$$

$$\varepsilon^2 q_{m0}^2 \left\langle g(x_1, y_1)\, g(x_2, y_2) \right\rangle = \sigma_q^2\, w\left(x_1 - x_2, y_1 - y_2; \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\left\langle \theta_m \right\rangle = q_{m0}\left(x_2 - x_1\right) + \left\langle \theta_{HR} \right\rangle + O\left(\varepsilon^4\right), \quad \left\langle \theta_{HR} \right\rangle = \int_{x_1}^{x_2} \sigma_q^2(x,0)\, b(x) \frac{\left(x - x_1\right)dx}{3q_{m0}}$$

where

$$b = \int_0^{+\infty} \frac{\partial^2 w}{\partial Y^2}\left(a, Y = 0; x, y\right) da = \int_0^{+\infty} \frac{dw_I}{da}\left(a; x, y\right) \frac{da}{a}$$

Statistically homogeneous medium:

$$\left\langle \theta_{HR} \right\rangle = b\sigma_q^2 \left(x_2 - x_1\right)^2 / 6q_{m0}$$

Conditions of applicability S large-scale inhomogeneities

Perturbation theory: $\eta_b r^2 / L^3 << 1$

Uncoupled azimuth approximation: $\eta_b^2 k_0 r^3 / L^2 << 1$

Conditions of applicability S small-scale inhomogeneities

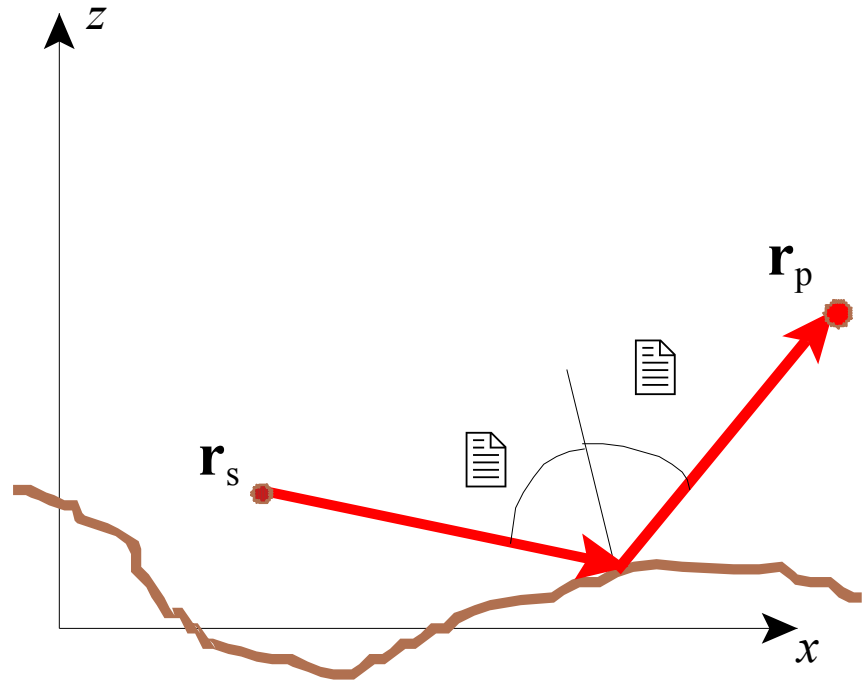
$$\left\langle y^2(x) \right\rangle = \frac{-2b\sigma_q^2}{3q_{m0}^2} \frac{(x - x_1)^2 (x_2 - x)^2}{x_2 - x_1} + O\left(\varepsilon^4\right)$$

Perturbation theory: $\eta_b (r/L)^{3/2} \ll 1$

Uncoupled azimuth approximation: $\eta_b^2 k_0 r^2 / L \ll 1$

Reflection at a rough surface

$$z = \varepsilon h(\boldsymbol{R}), \quad \boldsymbol{R} = (x, y)$$



Travel time perturbation:

$$cT = T_0 + \varepsilon T_1 + \varepsilon^2 T_2 + \ldots$$

First-order corrections:

$$cT_1 = -2\cos\theta_0\, h(\boldsymbol{R}_0), \quad \sigma_{cT} = 2\cos\theta_0 \sigma_h \left[1 + O(\varepsilon)\right]$$

Reflection at a rough surface (continued)

Second-order corrections:

$$cT_2 = -2\sin\theta_0 \frac{z_p - z_s}{z_p z_s} h h_x + \frac{K h^2}{2}\sin^2\theta_0 - \frac{2}{K}\left(h_x^2 + h_y^2\cos^2\theta_0\right)$$

where

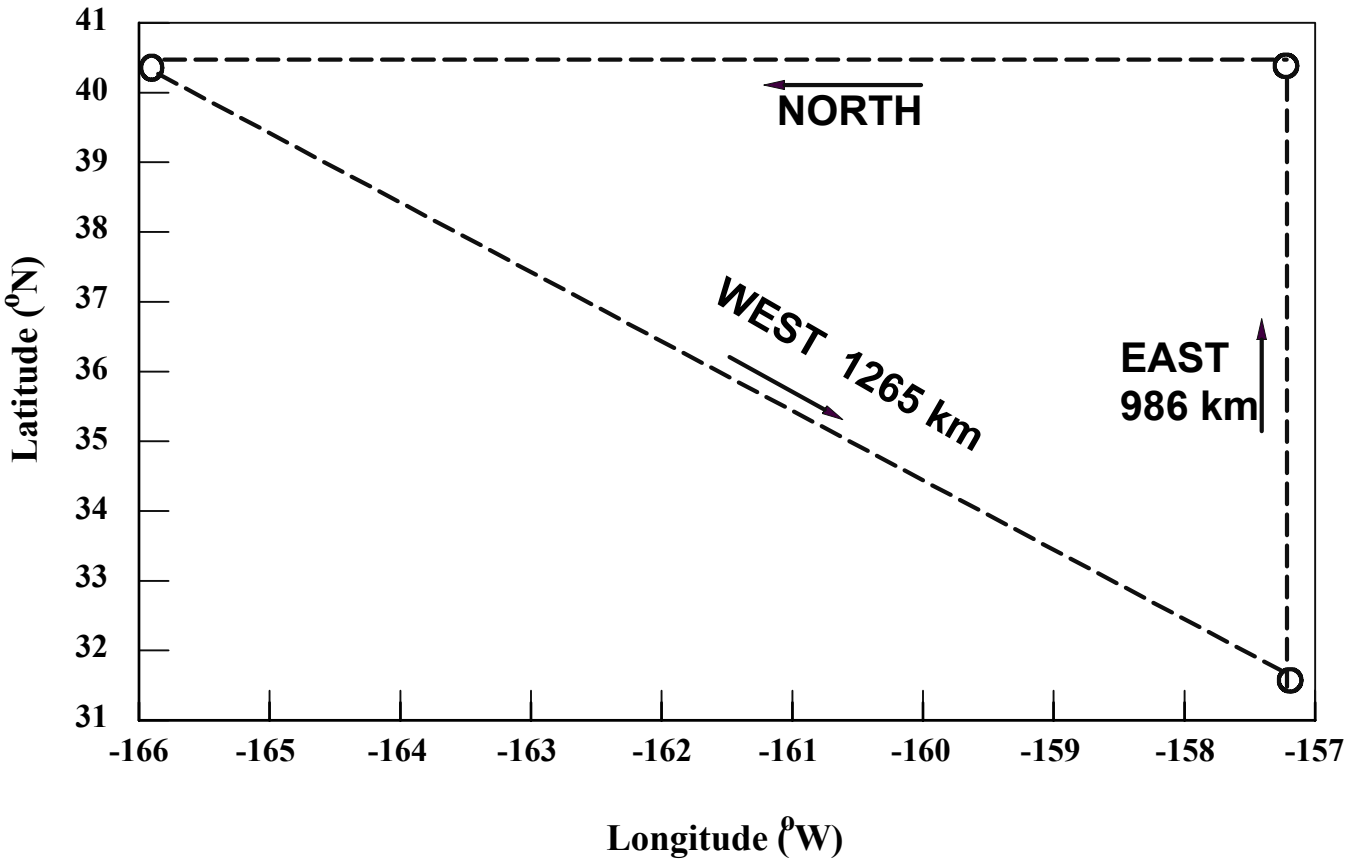
$$K = \frac{1}{\left|\boldsymbol{r}_p - \boldsymbol{r}_0\right|} + \frac{1}{\left|\boldsymbol{r}_s - \boldsymbol{r}_0\right|} = \cos\theta_0 \frac{z_p + z_s}{z_p z_s}$$

Travel time bias:

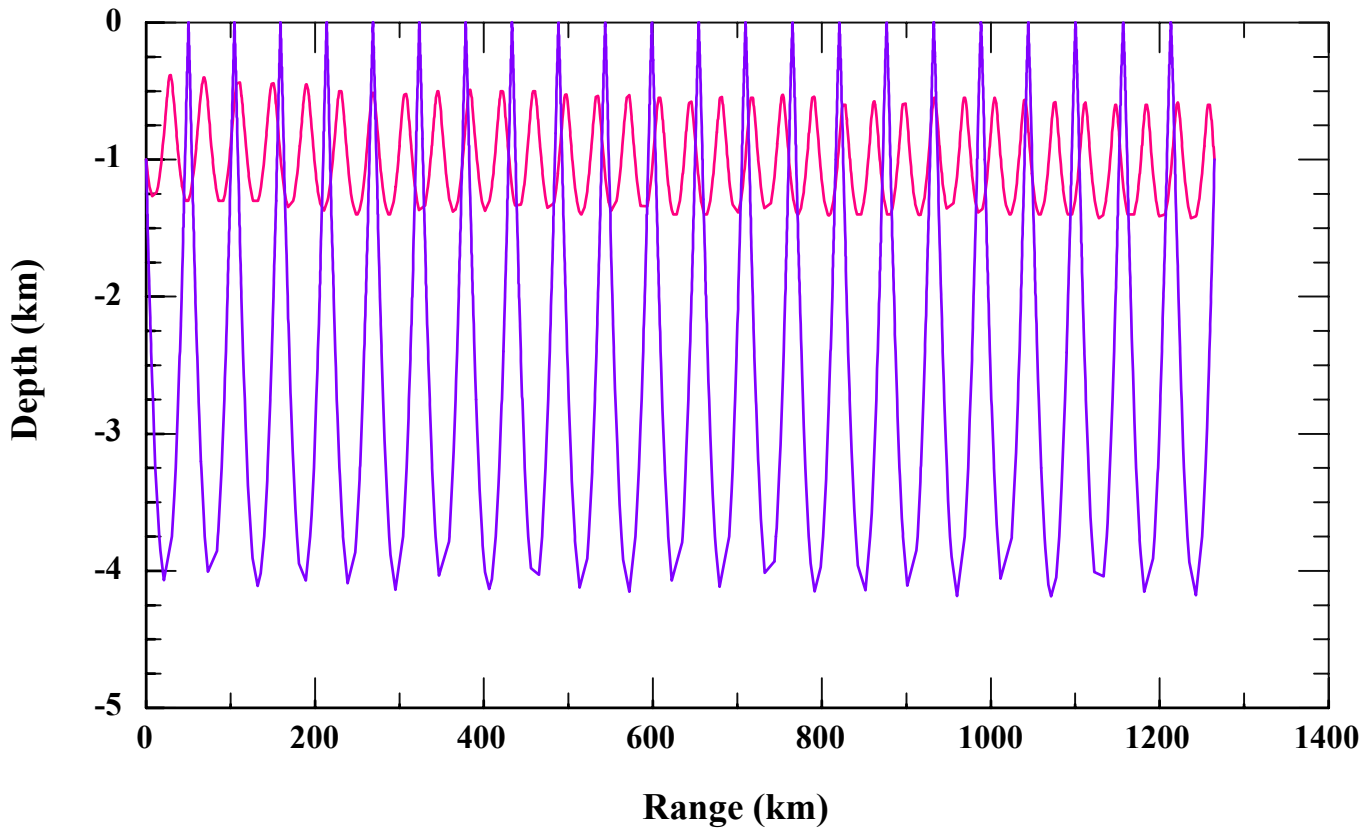
$$\langle cT\rangle\left[1 + O(\varepsilon)\right] = \frac{K\sigma_h^2}{2}\sin^2\theta_0 - \frac{2\sigma_h^2}{KL^2}\left(1 + \cos^2\theta_0\right), \quad L^2 \equiv \frac{\langle h^2\rangle}{\langle h_x^2\rangle}$$

Geometry of the 1987 Reciprocal Transmission Experiment (RTE)

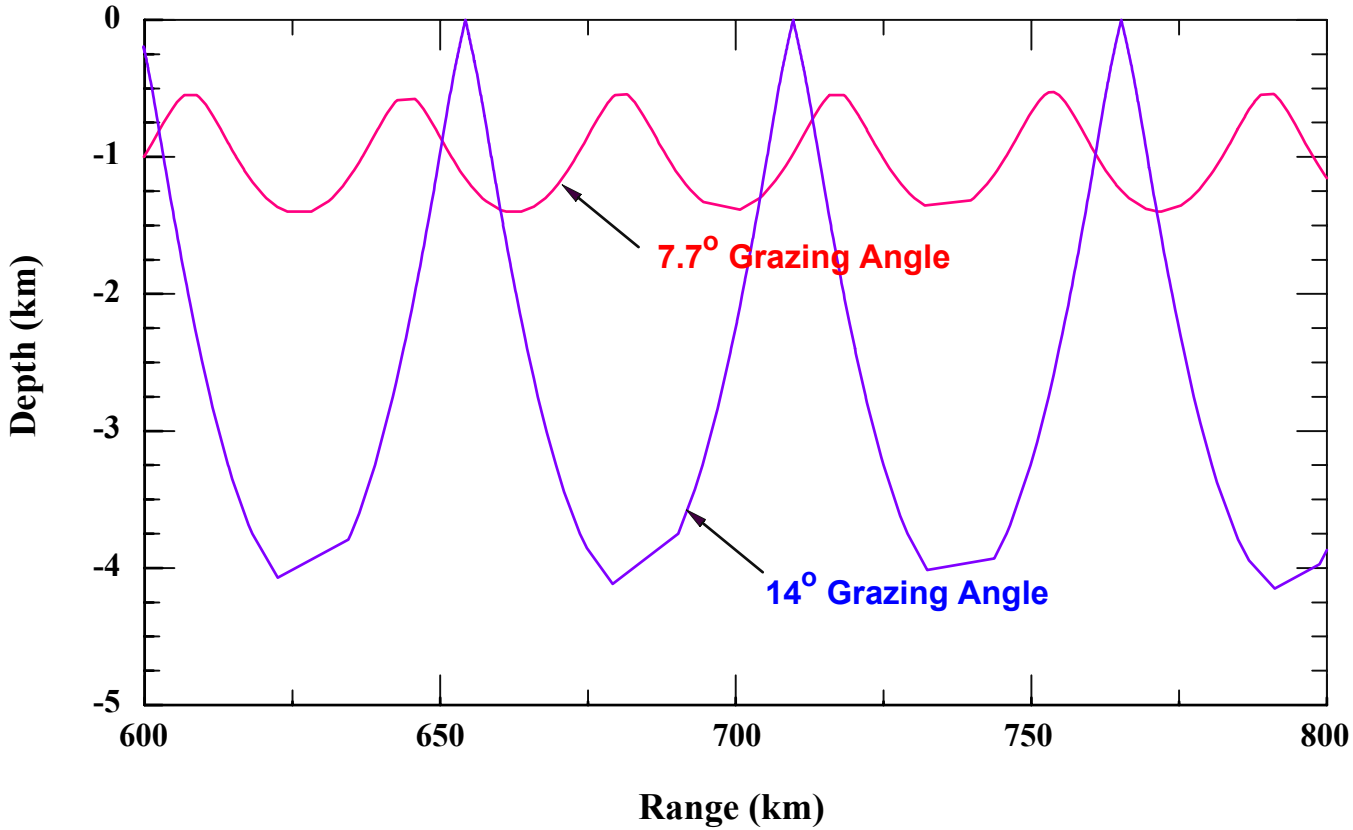
(B. Dushaw, P. Worcester, B. Cornuelle, and B. Howe, JASA, 93(1), pp. 255-275 (1993))

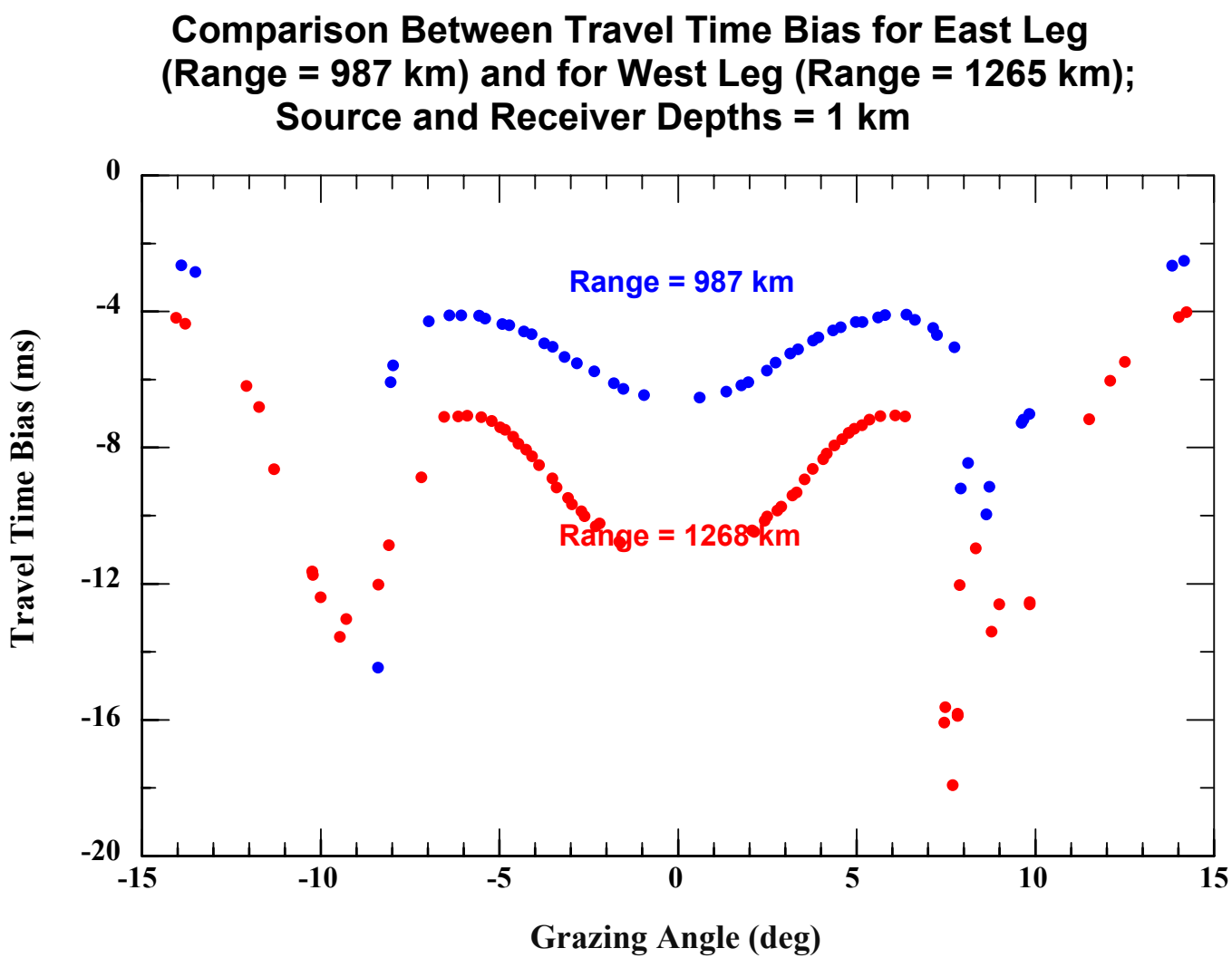


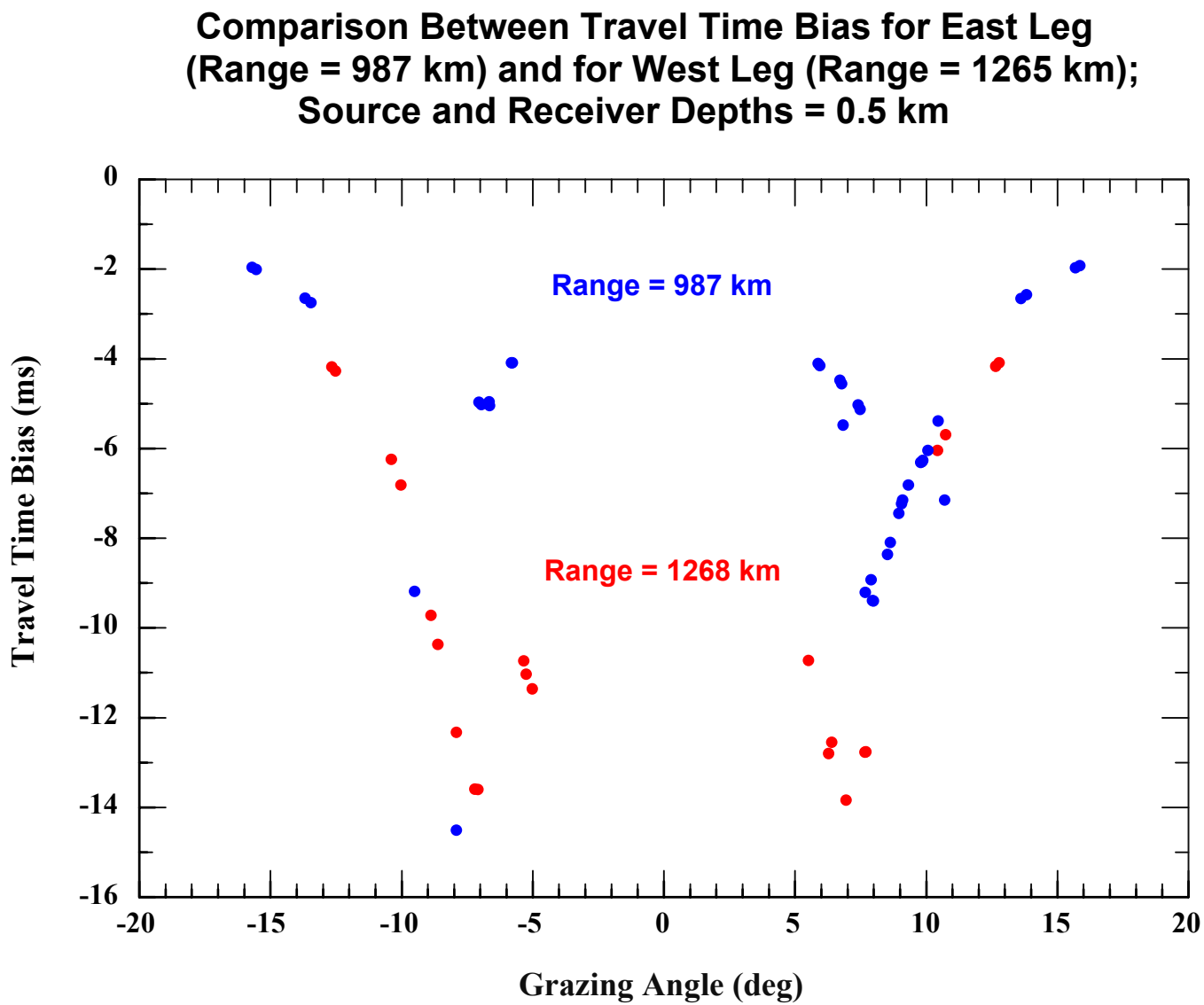
Example of Rays for RTE'87 Conditions (Source-Receiver Range = 1265 km; Source and Receiver Depths = 1km; Ocean Depth = 5.25 km)

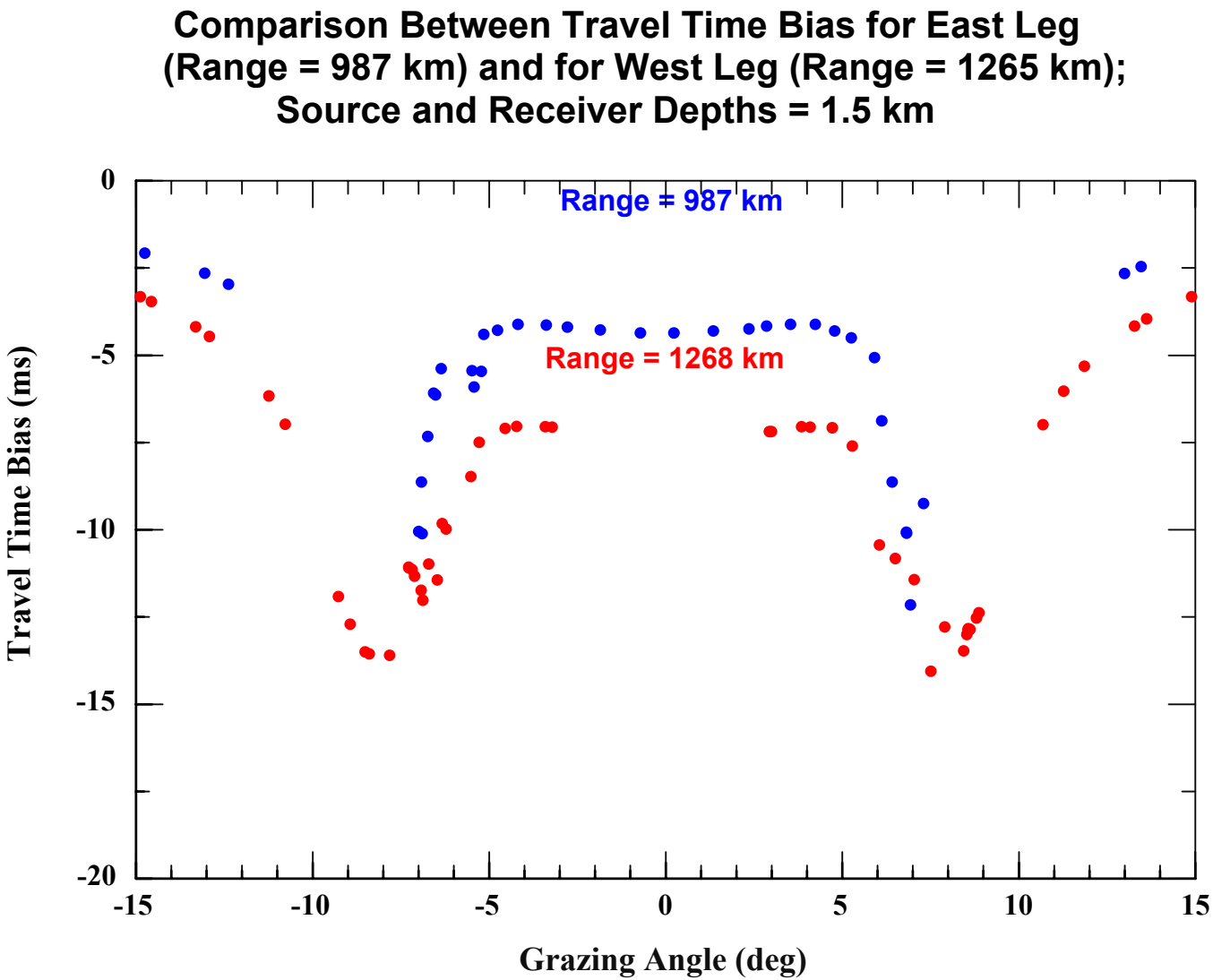


Enlarged Fragment of the Above Plot (Source-Receiver Range = 1265 km; Source and Receiver Depths = 1km; Ocean Depth = 5.25 km)



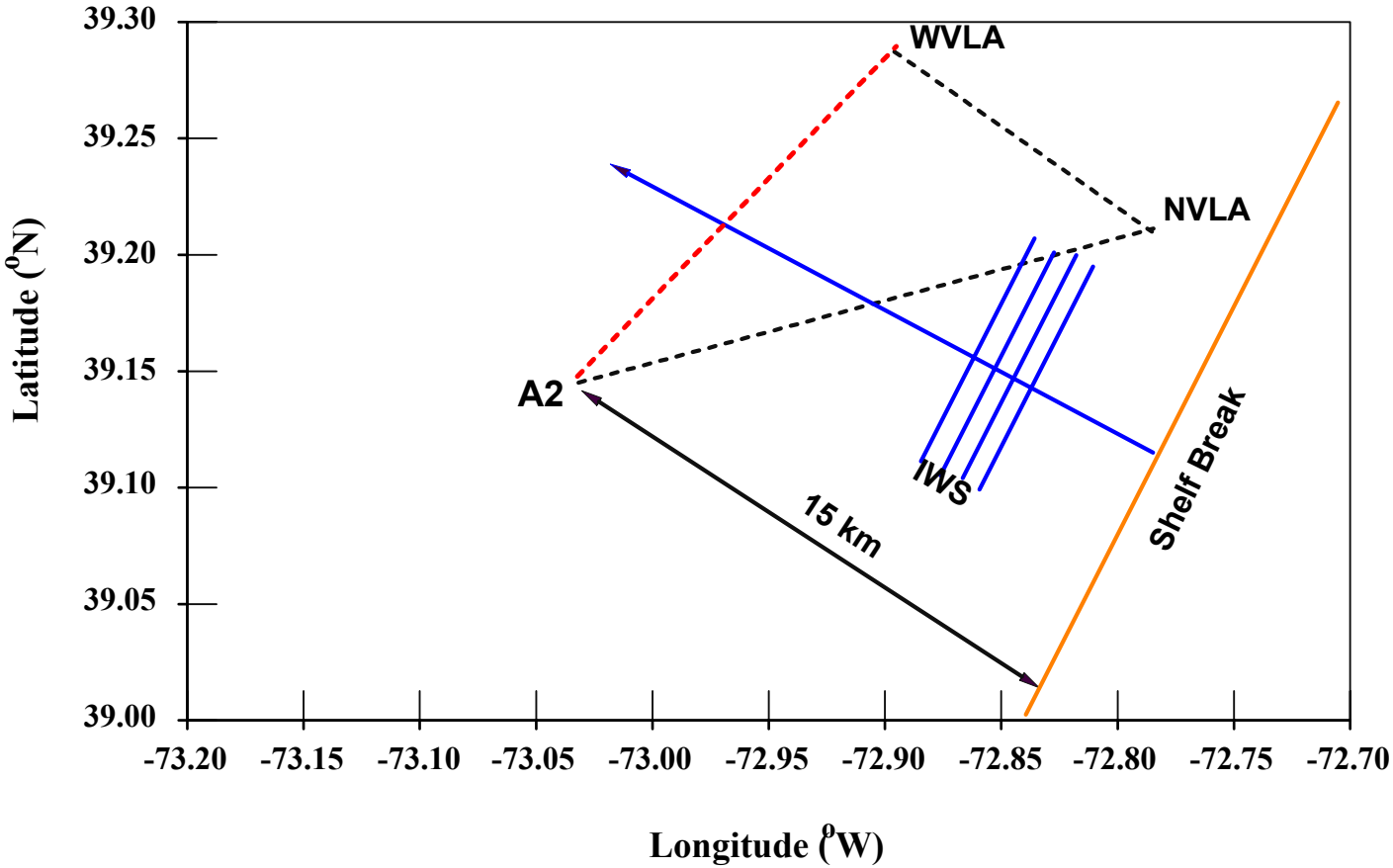

Comparison Between Travel Time Bias for East Leg
(Range = 987 km) and for West Leg (Range = 1265 km);
Source and Receiver Depths = 1 km
Range = 987 km
Range = 1268 km
Travel Time Bias (ms)
Grazing Angle (deg)
0
-4
-8
-12
-16
-20
-15
-10
-5
0
5
10
15


Comparison Between Travel Time Bias for East Leg
(Range = 987 km) and for West Leg (Range = 1265 km);
Source and Receiver Depths = 0.5 km
Range = 987 km
Range = 1268 km
Travel Time Bias (ms)
Grazing Angle (deg)
0
-2
-4
-6
-8
-10
-12
-14
-16
-20
-15
-10
-5
0
5
10
15
20


Comparison Between Travel Time Bias for East Leg
(Range = 987 km) and for West Leg (Range = 1265 km);
Source and Receiver Depths = 1.5 km
Range = 987 km
Range = 1268 km
Travel Time Bias (ms)
Grazing Angle (deg)
0
-5
-10
-15
-20
-15
-10
-5
0
5
10
15

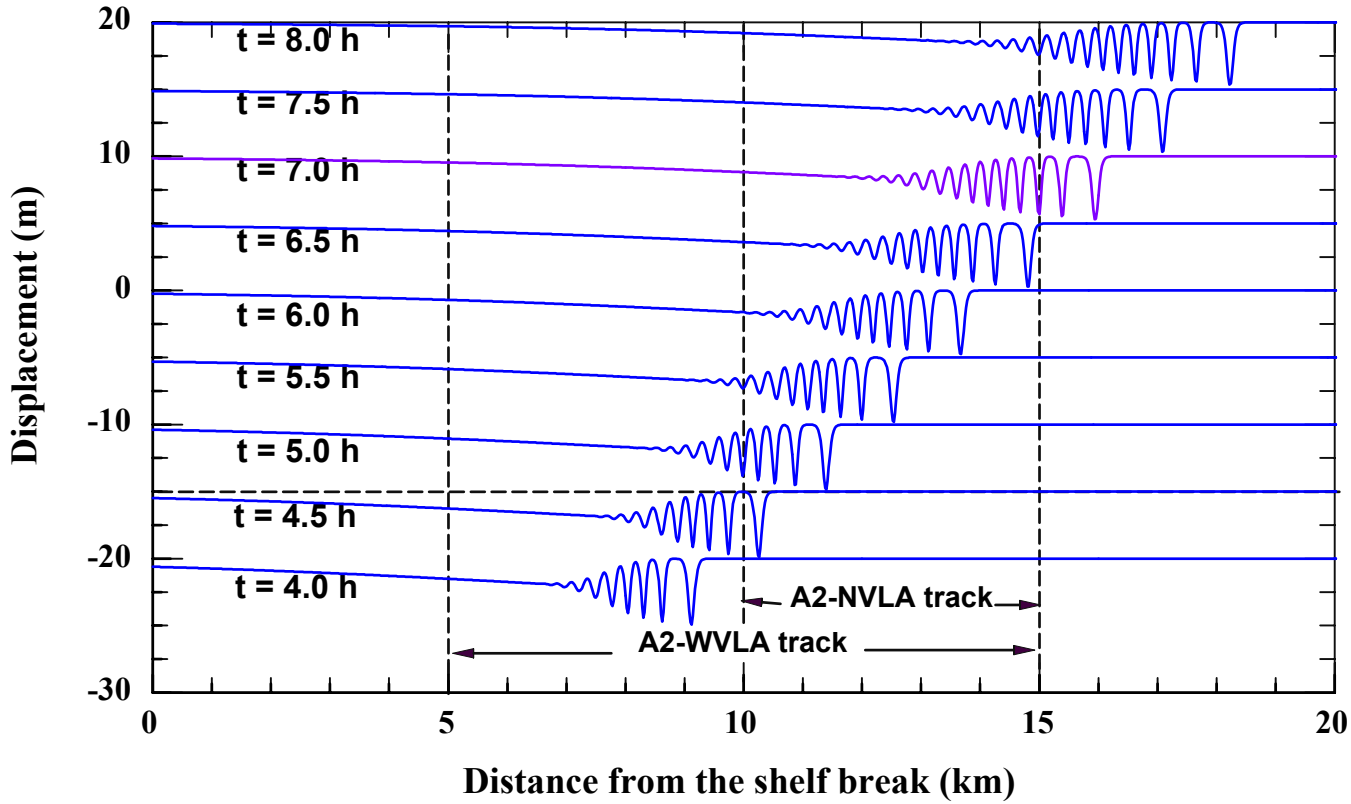
Geometry of the 1995 SWARM Experiment

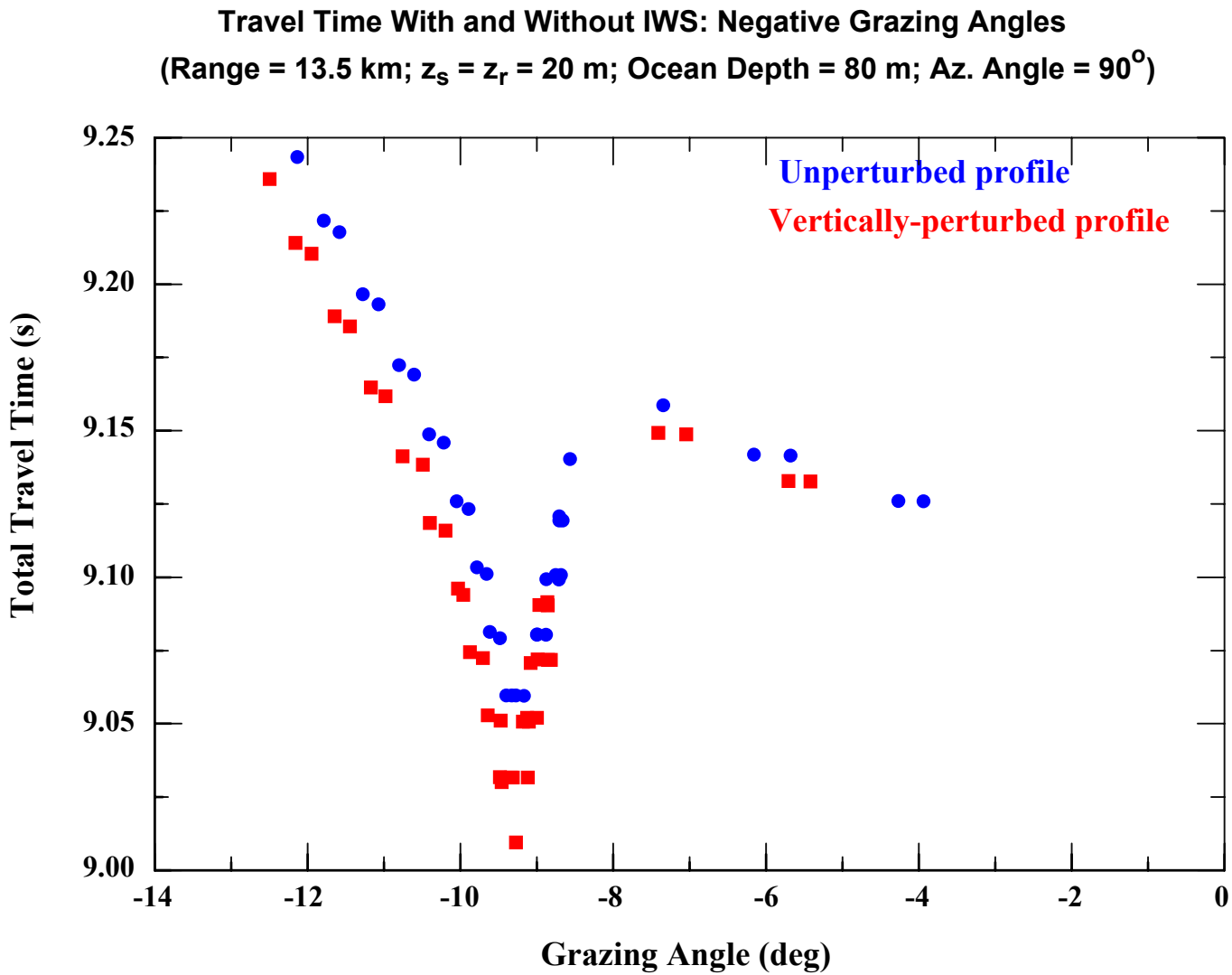
J. Apel et al., IEEE J. Ocean. Eng., 22(3), pp. 465- 499, (1997)
M. Badiey, Y. Mu, J. Lynch, J. Apel, and S. Wolf, IEEE J. Ocean. Eng., 27(1), pp. 117-129 (2002)

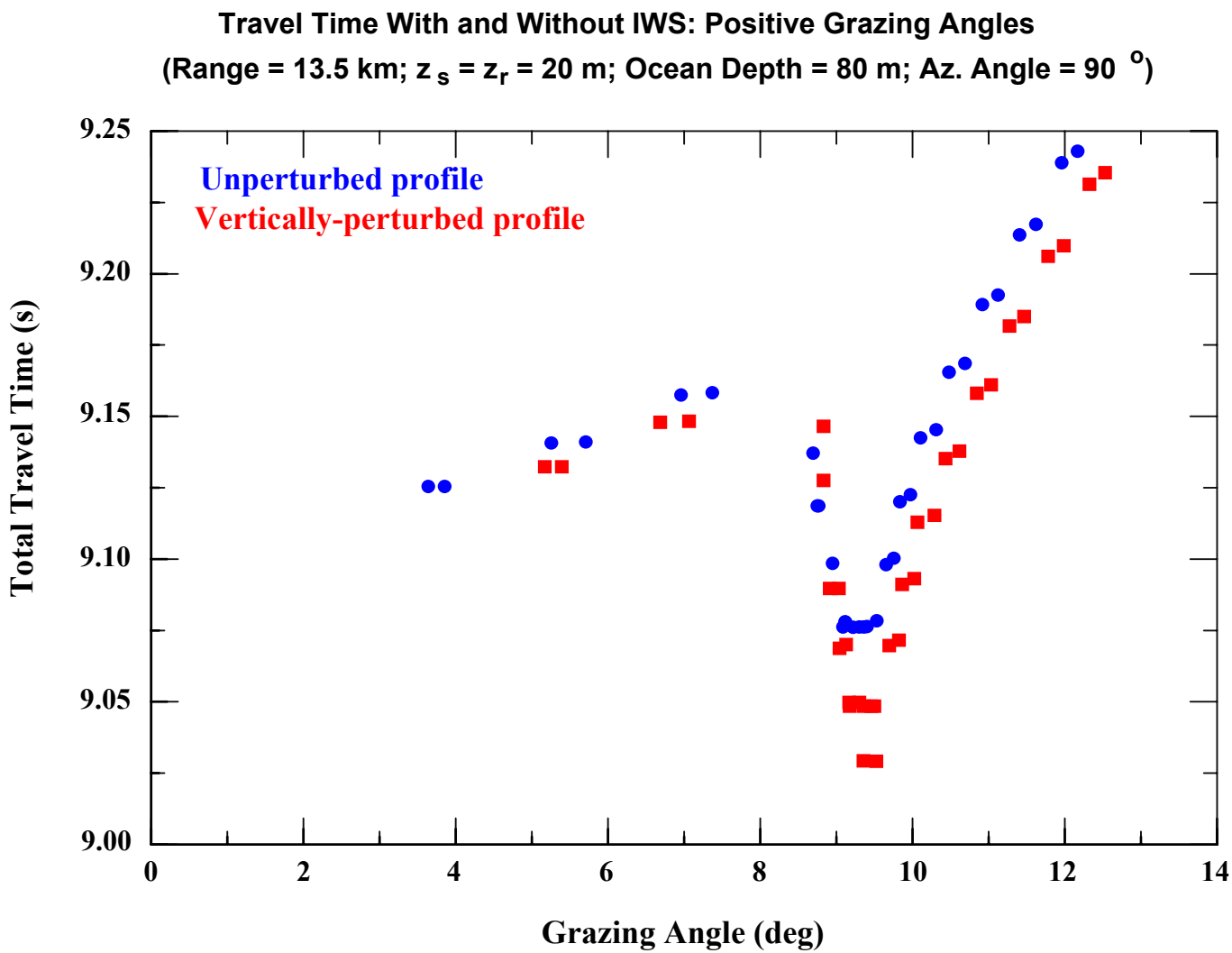


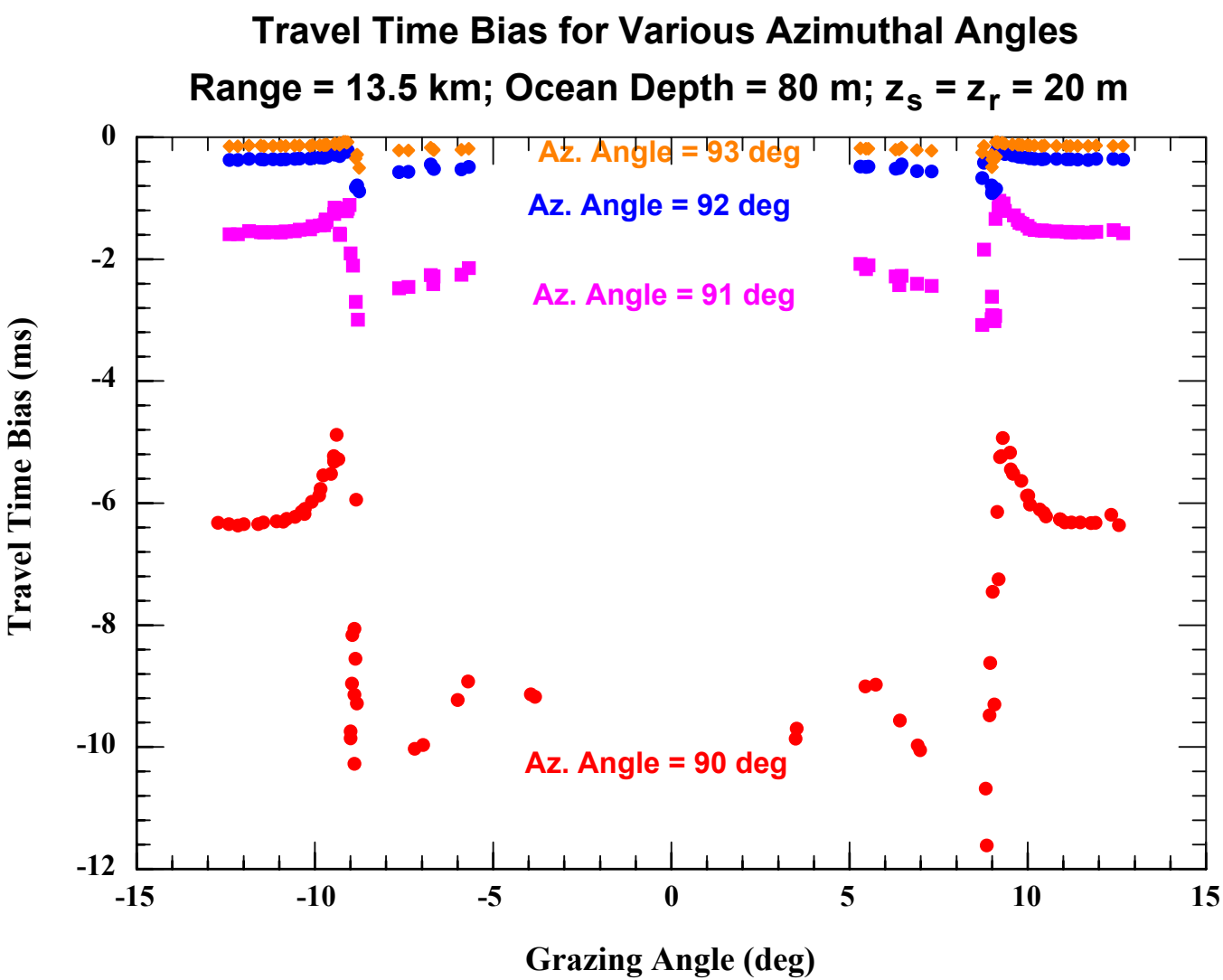
Evolution and propagation of the internal solitary waves at conditions of SWARM experiment

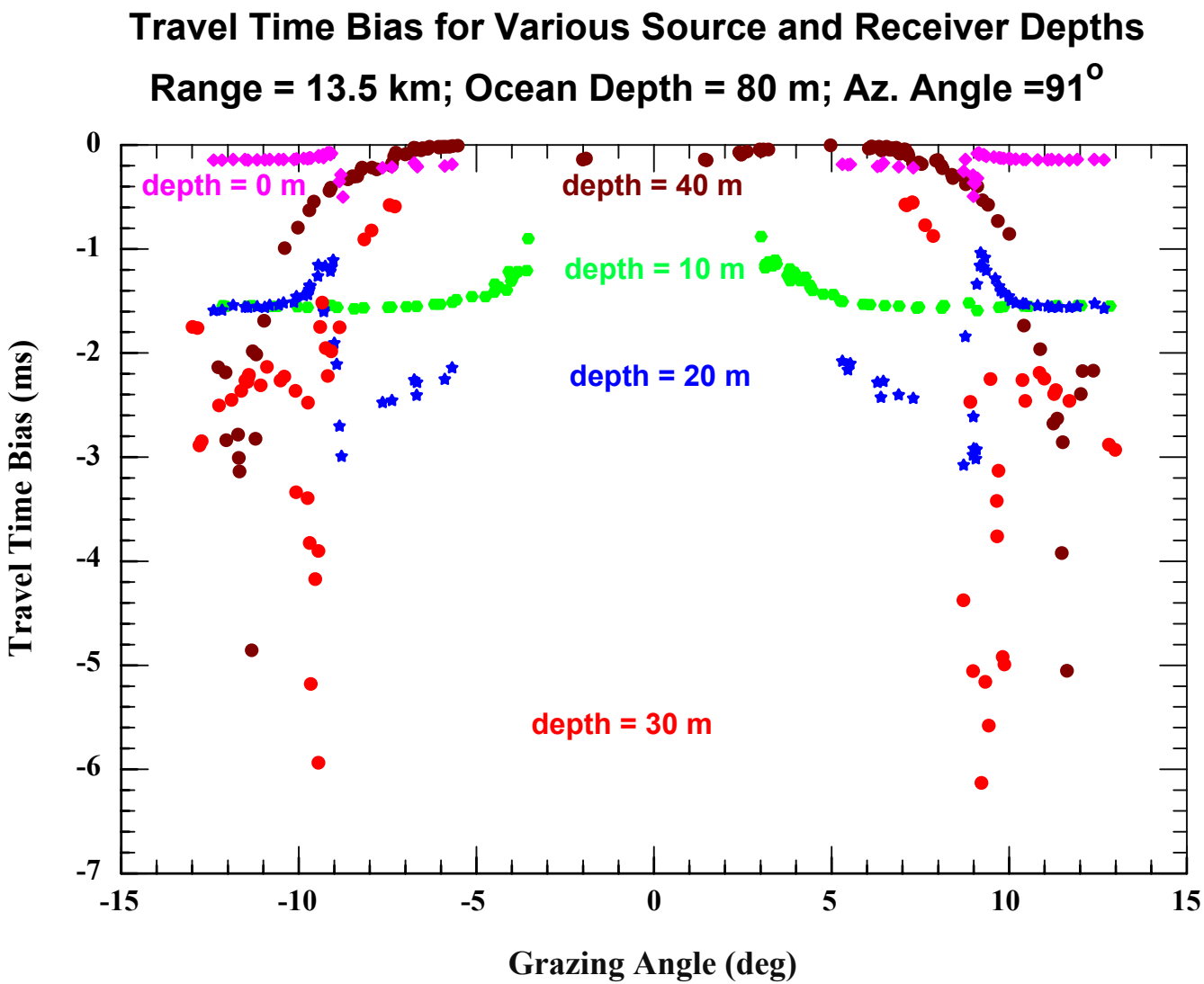
(z = 20 m, h_1 = 20m, h_2 = 60 m; η_o = 5 m)

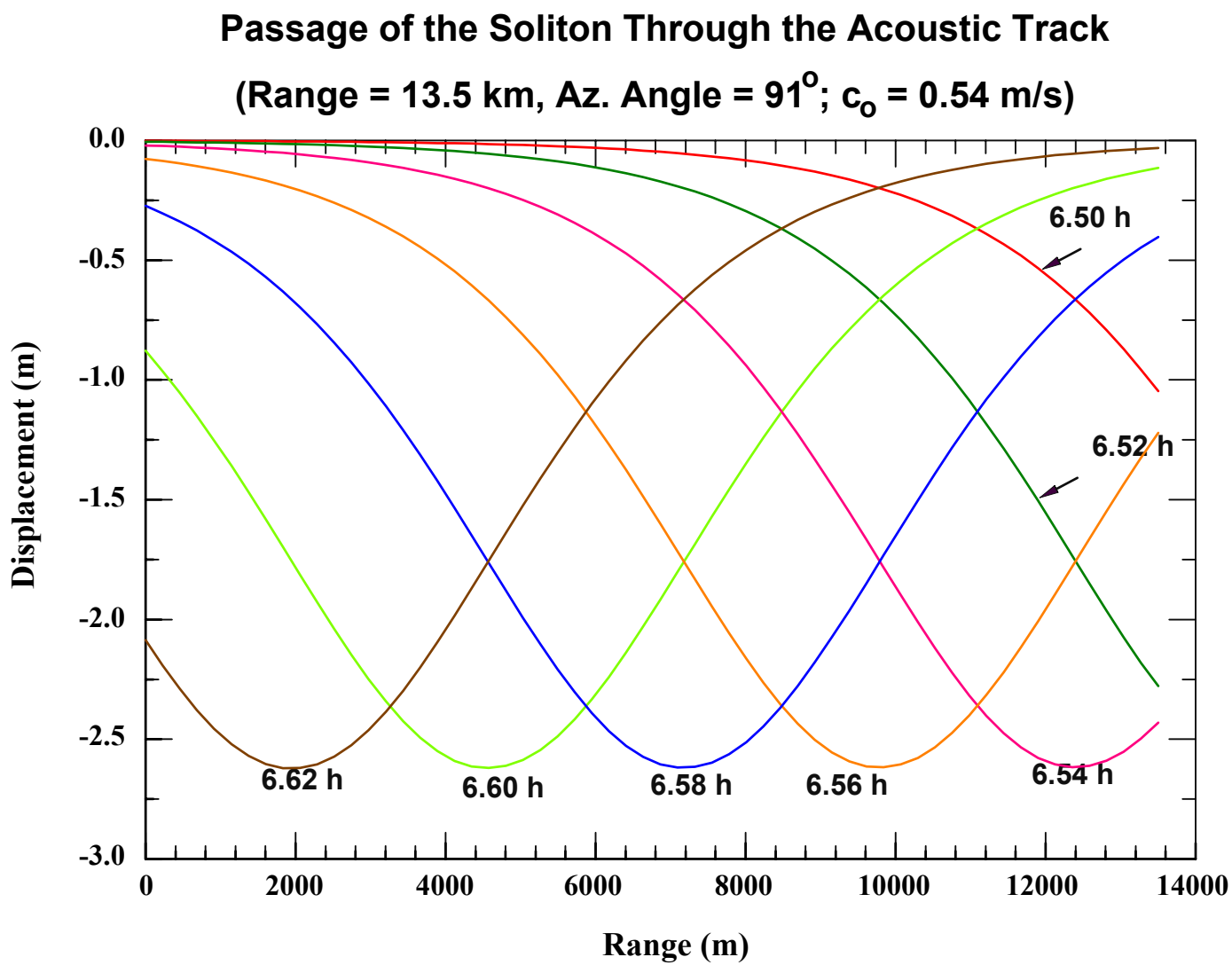


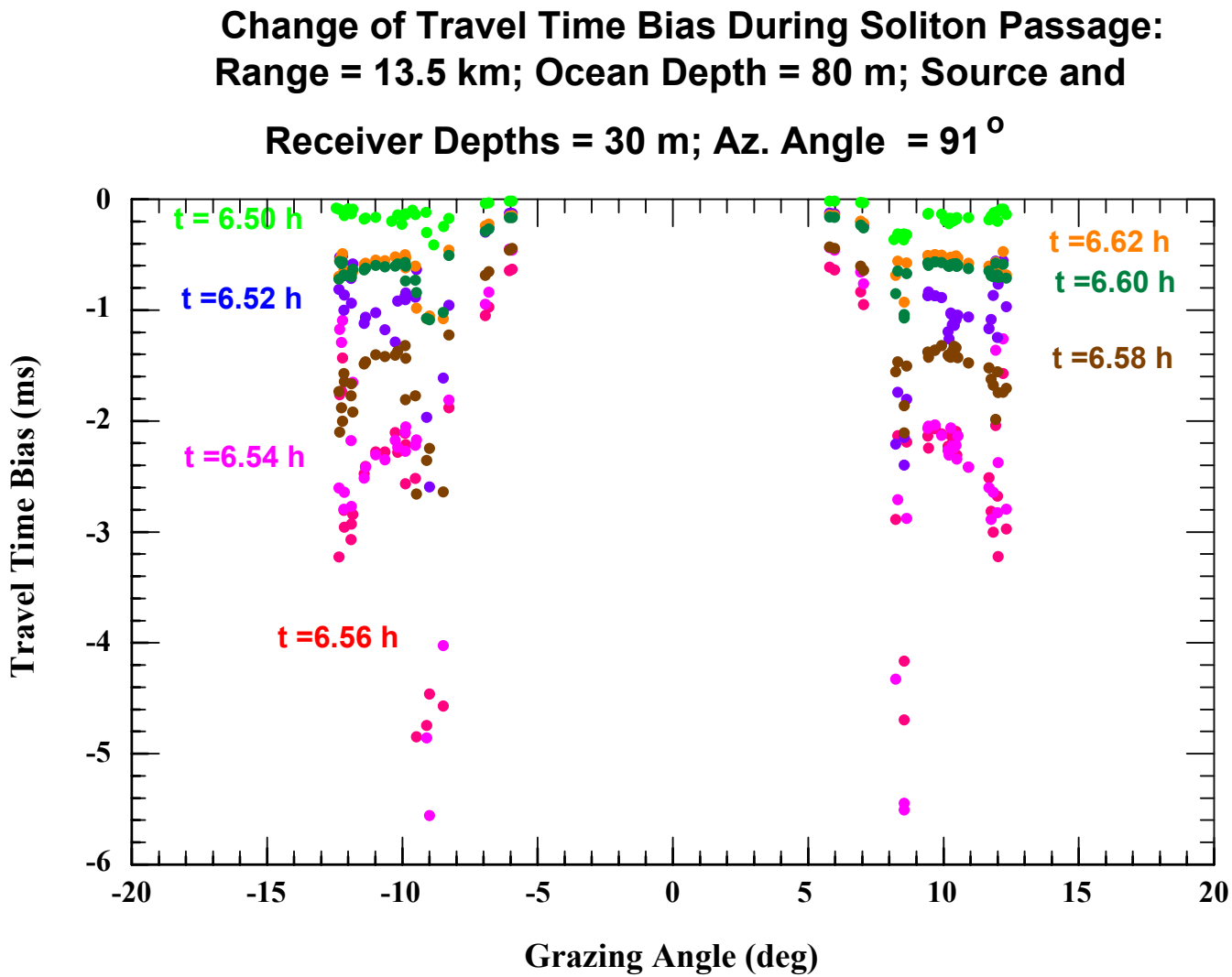

Travel Time With and Without IWS: Negative Grazing Angles
(Range = 13.5 km; $z_s = z_r$ = 20 m; Ocean Depth = 80 m; Az. Angle = 90°)
Unperturbed profile
Vertically-perturbed profile
Total Travel Time (s)
9.25
9.20
9.15
9.10
9.05
9.00
-14
-12
-10
-8
-6
-4
-2
0
Grazing Angle (deg)


Travel Time With and Without IWS: Positive Grazing Angles
(Range = 13.5 km; z s = zr = 20 m; Ocean Depth = 80 m; Az. Angle = 90 o)
Unperturbed profile
Vertically-perturbed profile
Total Travel Time (s)
9.25
9.20
9.15
9.10
9.05
9.00
0
2
4
6
8
10
12
14
Grazing Angle (deg)

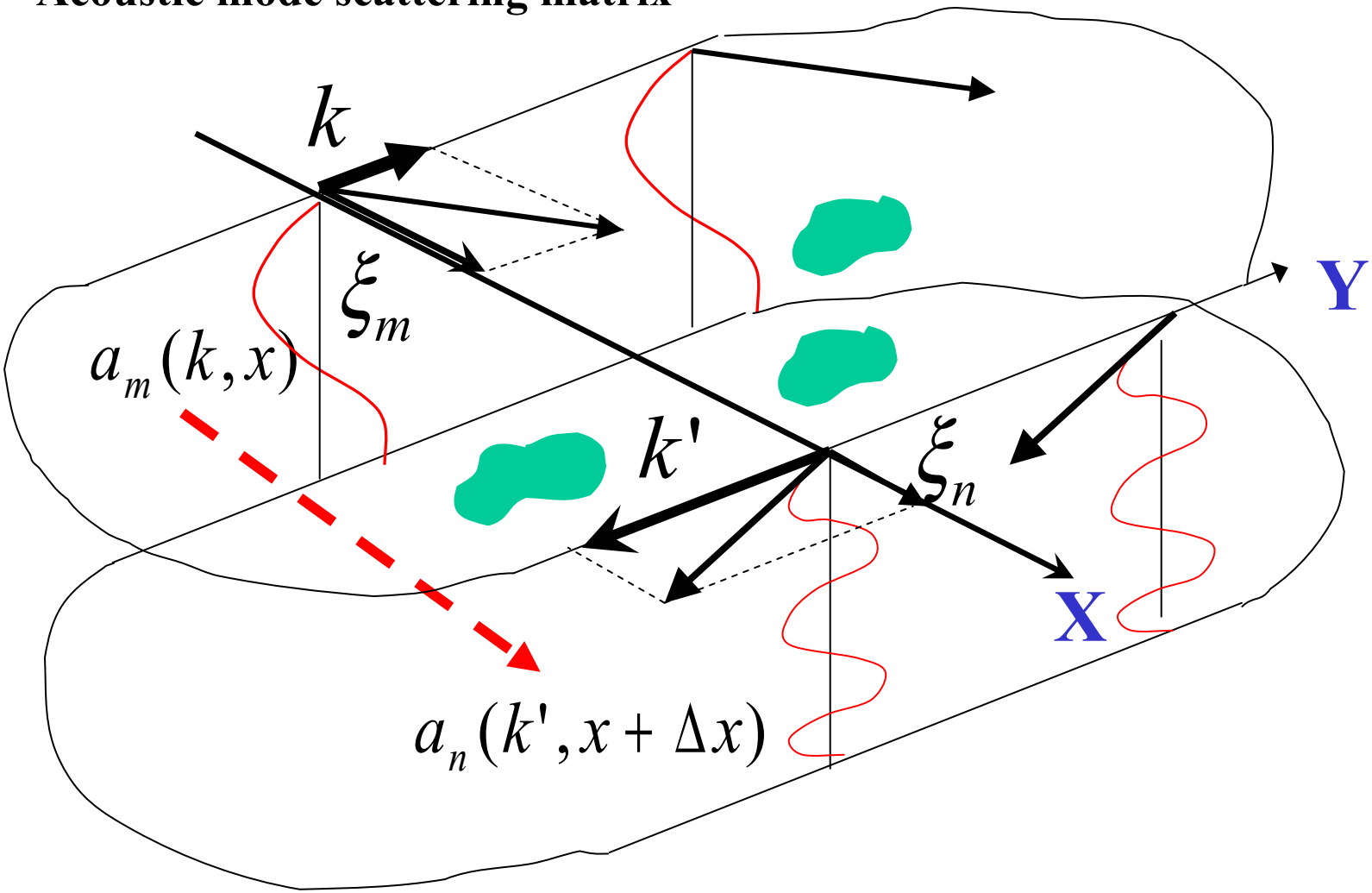

Travel Time Bias for Various Azimuthal Angles
Range = 13.5 km; Ocean Depth = 80 m; $z_s = z_r = 20$ m
Az. Angle = 93 deg
Az. Angle = 92 deg
Az. Angle = 91 deg
Az. Angle = 90 deg
Travel Time Bias (ms)
Grazing Angle (deg)
0
-2
-4
-6
-8
-10
-12
-15
-10
-5
0
5
10
15


Travel Time Bias for Various Source and Receiver Depths
Range = 13.5 km; Ocean Depth = 80 m; Az. Angle =91°
depth = 0 m
depth = 40 m
depth = 10 m
depth = 20 m
depth = 30 m
Travel Time Bias (ms)
Grazing Angle (deg)
0
-1
-2
-3
-4
-5
-6
-7
-15
-10
-5
0
5
10
15


Passage of the Soliton Through the Acoustic Track
(Range = 13.5 km, Az. Angle = 91°; c_o = 0.54 m/s)
Displacement (m)
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
0
2000
4000
6000
8000
10000
12000
14000
Range (m)
6.50 h
6.52 h
6.54 h
6.56 h
6.58 h
6.60 h
6.62 h


Change of Travel Time Bias During Soliton Passage:
Range = 13.5 km; Ocean Depth = 80 m; Source and
Receiver Depths = 30 m; Az. Angle = 91 o
t = 6.50 h
t =6.52 h
t =6.54 h
t =6.56 h
t =6.62 h
t =6.60 h
t =6.58 h
Travel Time Bias (ms)
0
-1
-2
-3
-4
-5
-6
-20
-15
-10
-5
0
5
10
15
20
Grazing Angle (deg)

Acoustic mode scattering matrix



$$p = u_m(z)\exp(i\xi_m x + iky) + \sum_n u_n(z)\int dk' \boxed{S_{nm}(k',k)} \exp\left[i\xi'_m(x-\Delta x) + ik'y\right]$$

Field transformation

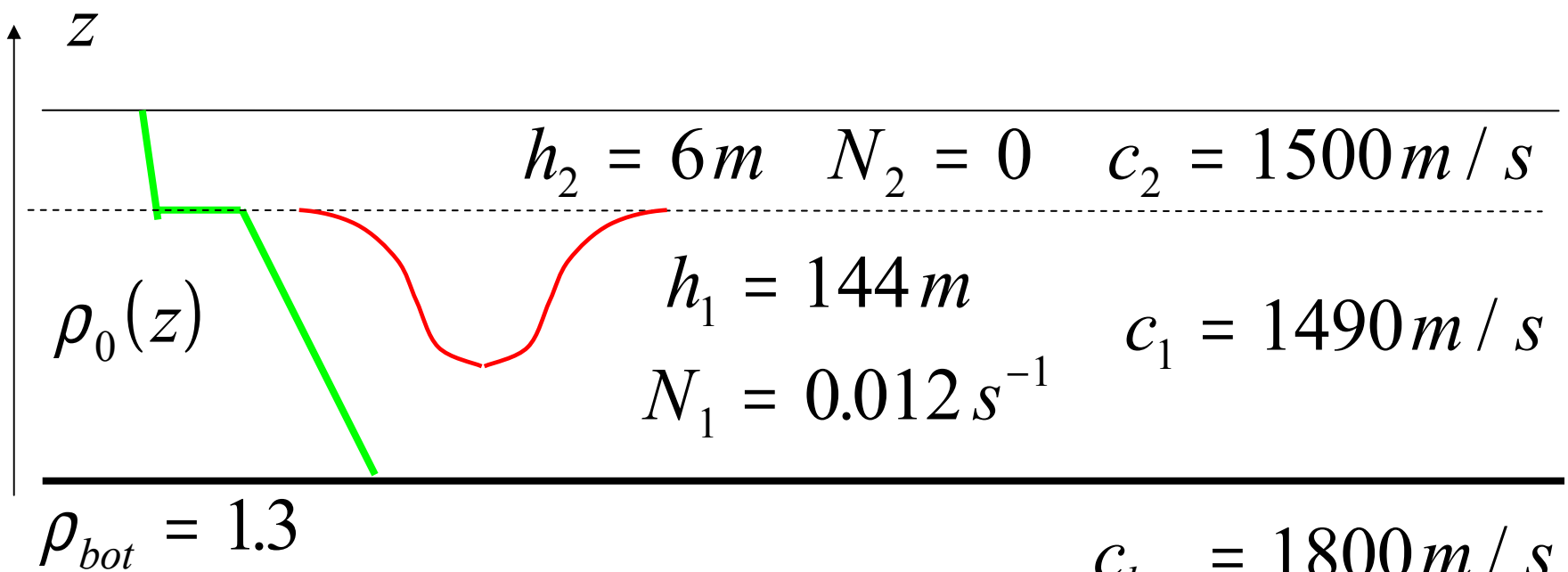
$$a_n(k, x + \Delta x) = a_n(k, x) e^{i\xi_n \Delta x} + \sum_m \int dk' S_{nm}(k, k') a_m(k', x)$$

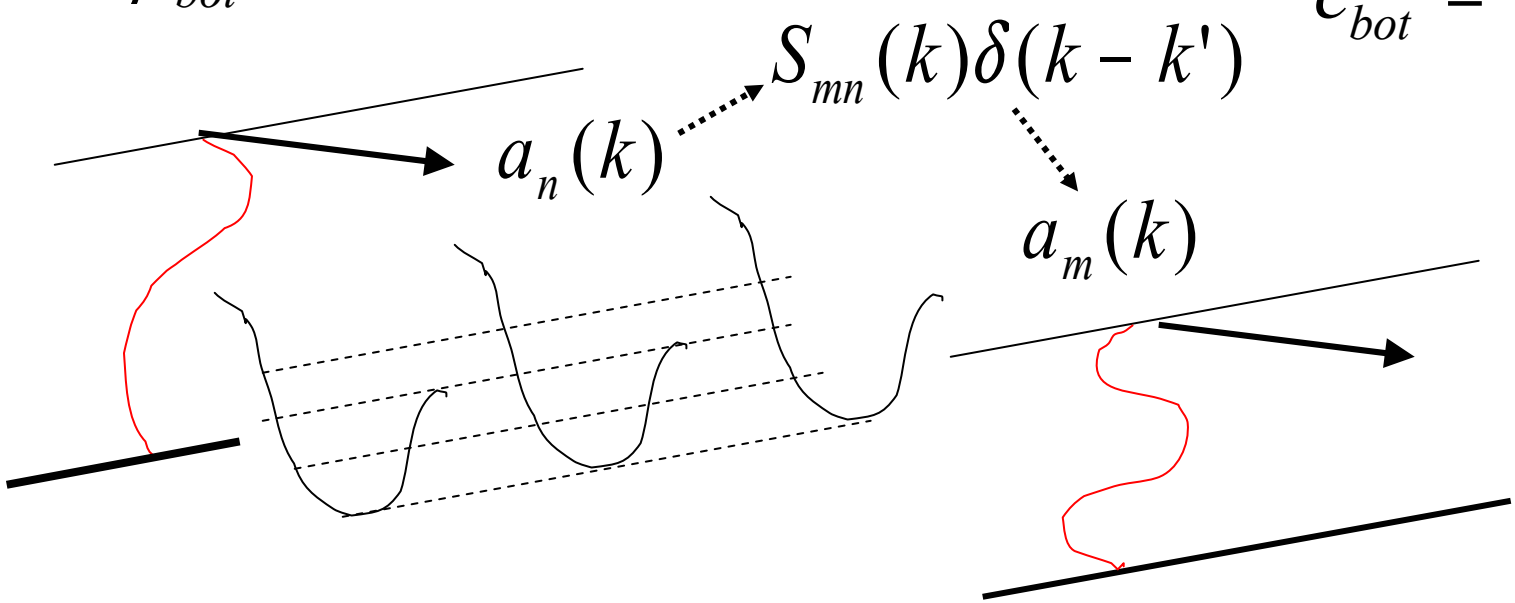
First order scattering :

$$S_{mn}(k', k) = \frac{i}{4\pi} \frac{\omega^2}{c_{00}^2} \frac{1}{\xi'_m} \int_0^{\Delta x} dx \int dy \exp\left[-i\left(\xi'_m - \xi_n\right)x - i(k - k')y\right] \cdot$$

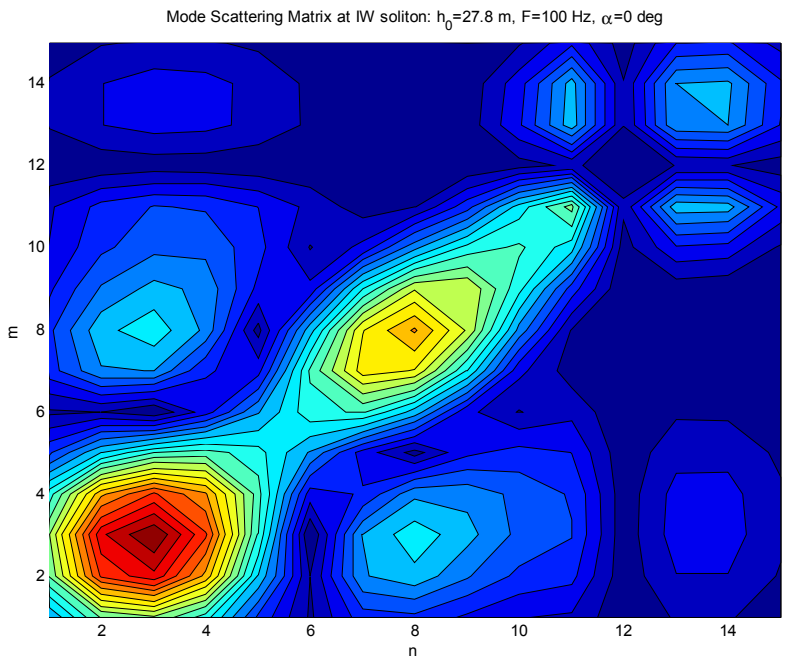
$$\cdot \int dz \underline{\Delta n^2(x, y, z)} u_m(z) u_n(z)$$

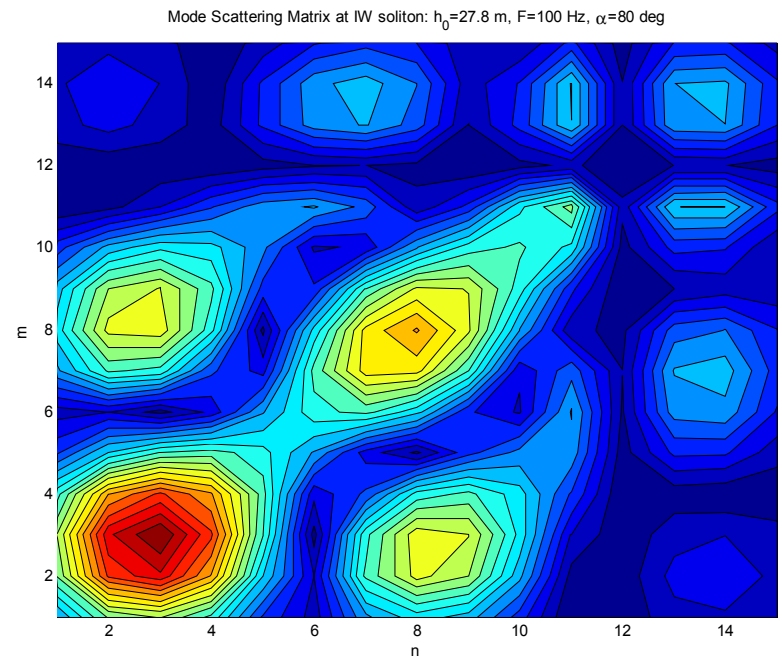
Pekeris waveguide with strong IW soliton

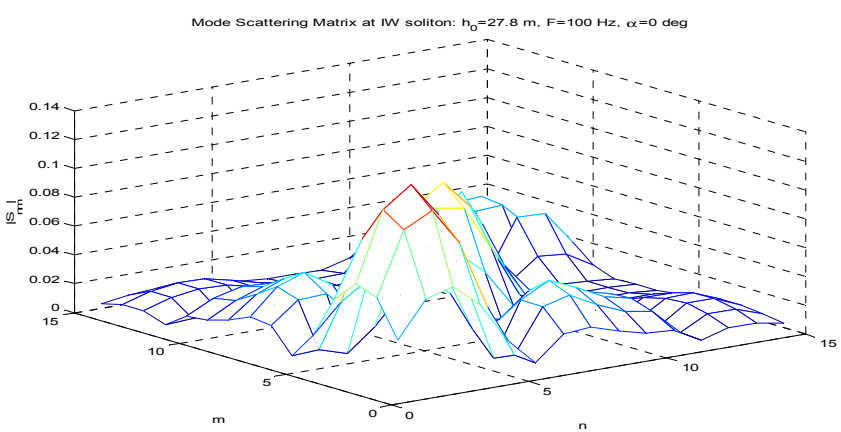


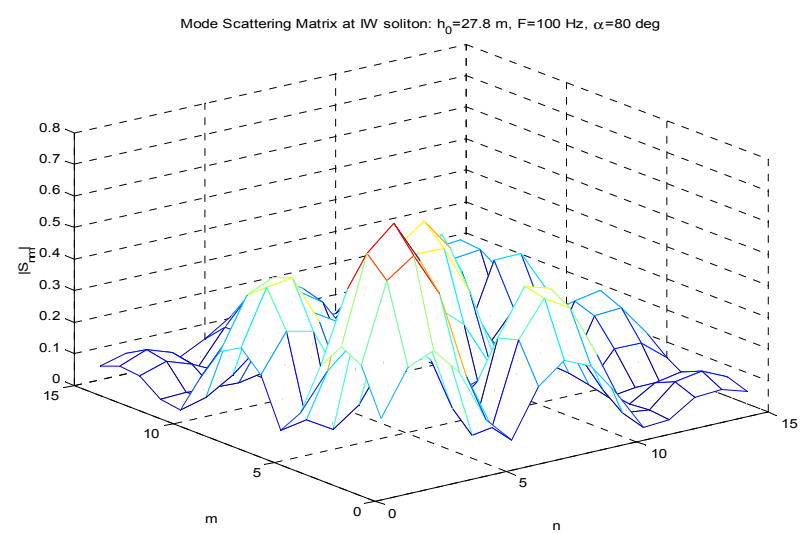


Scattering matrix at IW soliton: F=100 Hz, h=27.8 m; N=15 modes





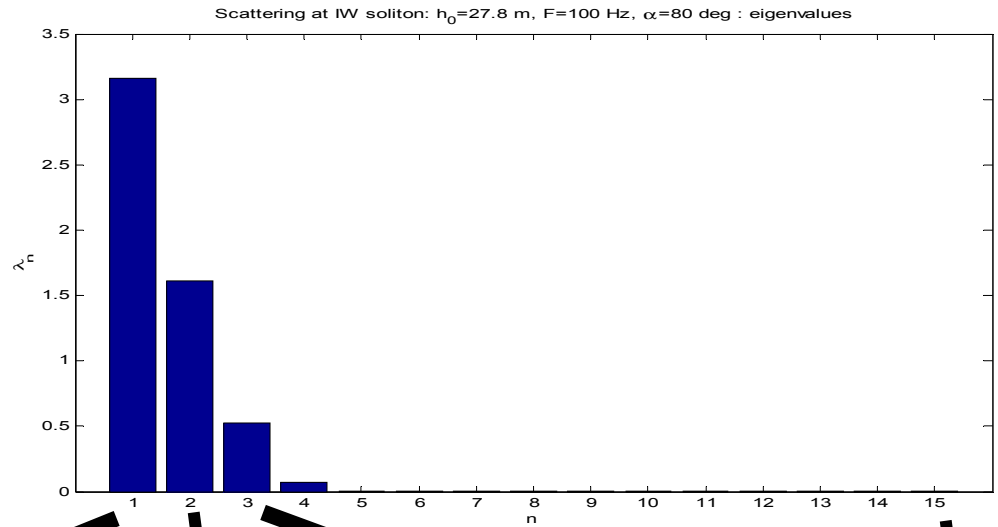


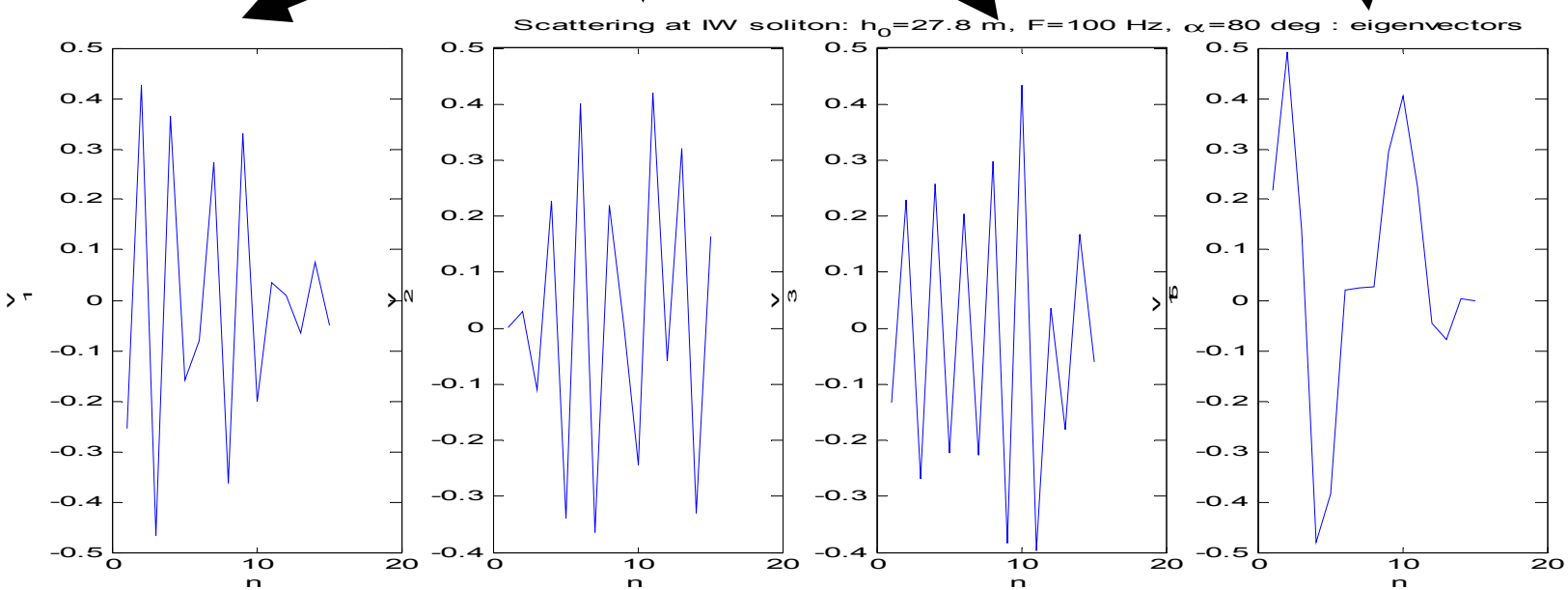


Spectrum of S-matrix and eigenvectors

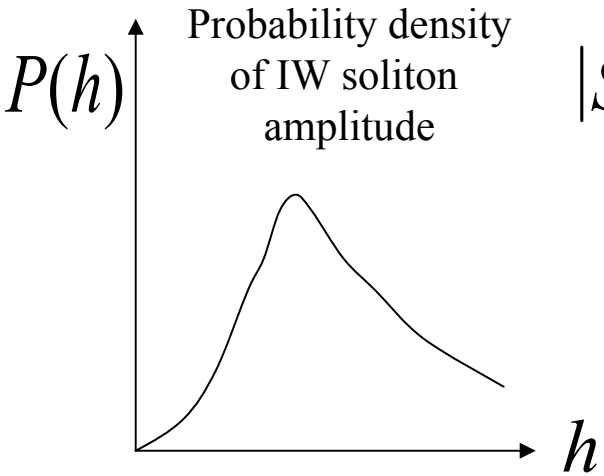
Strong scattering effects

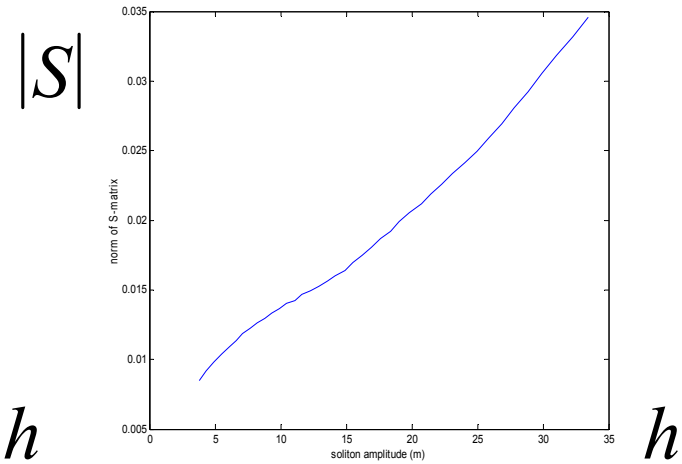
Weak scattering effects

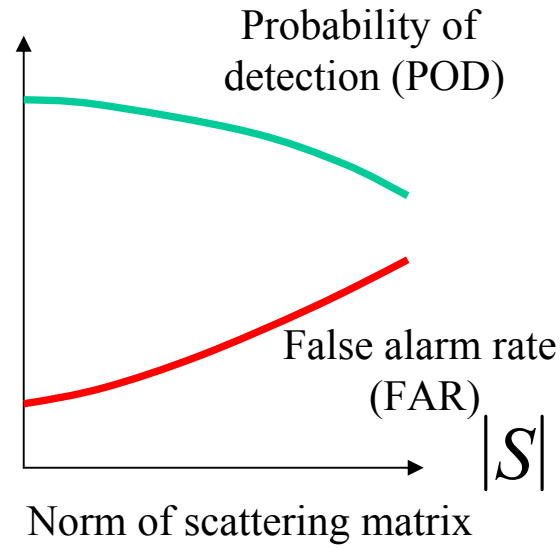




Uncertainty due to IW solitons







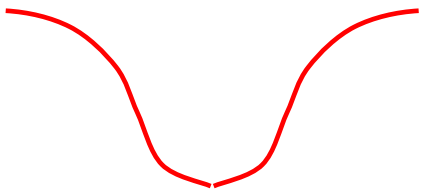
Hydrophysics | Propagation | Detection algorithm

$$\langle POD \rangle = \int POD\left(|S|(h)\right) \cdot P_{sol}(h)\, dh$$

$$\langle FAR \rangle = \int FAR\left(|S|(h)\right) \cdot P_{sol}(h)\, dh$$

Strong IW soliton for 2.5 layer model

$$\Psi_{xx}^{(2)} + \Psi_{zz}^{(2)} + \frac{N_2^2}{c^2}\Psi^{(2)} = 0$$



$h(x)$

$$\Psi_{xx}^{(1)} + \Psi_{zz}^{(1)} + \frac{N_1^2}{c^2}\Psi^{(1)} = 0$$

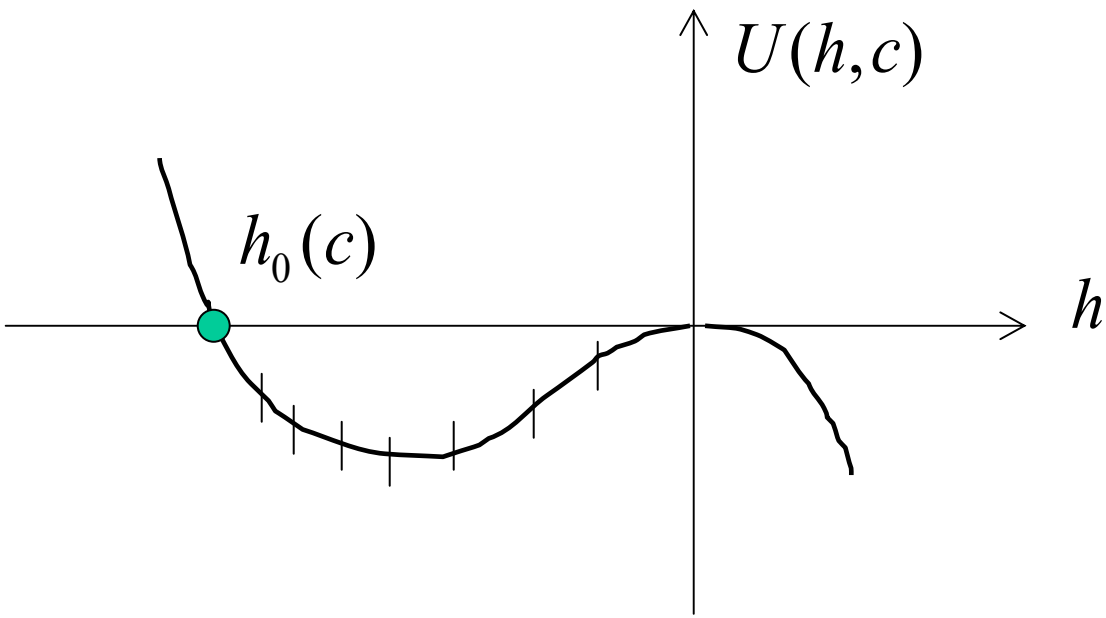
Governing equation:

$$-\frac{1}{2}\left(1+h_x^2\right)\left(c+\Psi_z^{(1)}\right)^2 + \frac{1}{2}\left(1+h_x^2\right)\left(c+\Psi_z^{(2)}\right)^2 - g\Delta\rho h = 0$$

Strong IW solitons

$$A(h)h_{xx} + B(h)h_x^2 + C(h) = 0$$

$$\frac{1}{2}\left(\frac{dh}{dx}\right)^2 + U(h) = 0$$



Weak stratification

$$\frac{N^2 H^2}{c^2} << 1$$

$$\frac{\Delta\rho_1}{\Delta\rho}\left(1+\frac{h_1}{h_2}\right) << 1 \quad , \qquad \frac{\Delta\rho_2}{\Delta\rho}\left(1+\frac{h_2}{h_1}\right) << 1$$

$N_1 = N_2 = 0 \quad \Rightarrow$ Choi and Camassa soliton

Weak stratification effect: density jump modification

$$\Delta\rho \quad \rightarrow \Delta\rho + \frac{\Delta\rho_1 + \Delta\rho_2}{3}$$

A,B,C coefficients and U-function for the weak stratification

$$A(h) = \frac{c^2}{3}\left(\frac{h_2^2}{h-h_2} - \frac{h_1^2}{h-h_1}\right)$$

$$B(h) = \frac{c^2}{6}\frac{h_1^2}{\left(h-h_1\right)^2} - \frac{c^2}{6}\frac{h_2^2}{\left(h-h_2\right)^2} + \frac{N_1^2 h_1 - N_2^2 h_2}{3}h$$

$$C(h) = -\frac{c^2}{2}\frac{h_1^2}{\left(h-h_1\right)^2} + \frac{c^2}{2}\frac{h_2^2}{\left(h-h_2\right)^2} - \left(g\Delta\rho + \frac{N_1^2 h_1 - N_2^2 h_2}{3}\right)h$$

$$U(h) = -\frac{3}{2}\frac{\frac{g\Delta\rho}{c^2}\left(h-h_2\right)\left(h-h_1\right) + h_2 - h_1}{h_2^2\left(h-h_1\right) - h_1^2\left(h-h_2\right)}h^2$$

Strong stratification

$$w_t >> N^{-1}$$

$$A(h) = \frac{c^2}{3}\frac{h_2^2}{h-h_2} - \frac{c^3}{2N_1}\left(1-\cot\theta\frac{N_1 h}{c}\right)^2\left(\theta\cot^2\theta - \cot\theta + \theta\right)$$

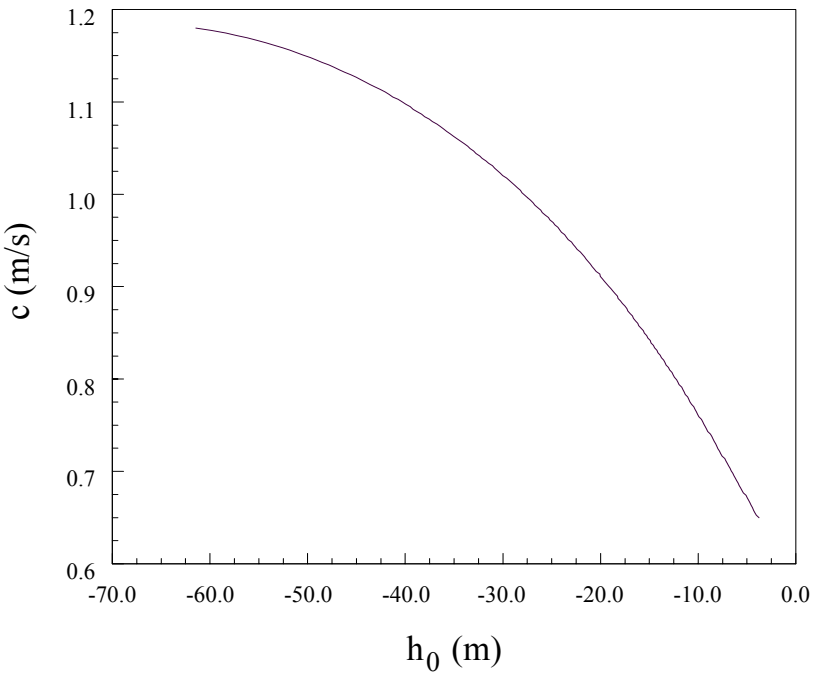
$$B(h) = -\frac{c^2}{6}\frac{h_2^2}{\left(h-h_2\right)^2} - \frac{c^2}{2}\left(1-\cot\theta\frac{N_1 h}{c}\right)^2 - \frac{c^2}{2}\left(1-\cot\theta\frac{N_1 h}{c}\right)^2\left(\theta\cot^2\theta - \cot\theta + \theta\right)\left[\left(2\cot^2\theta + 1\right)\frac{N_1 h}{c} - 2\cot\theta\right]$$

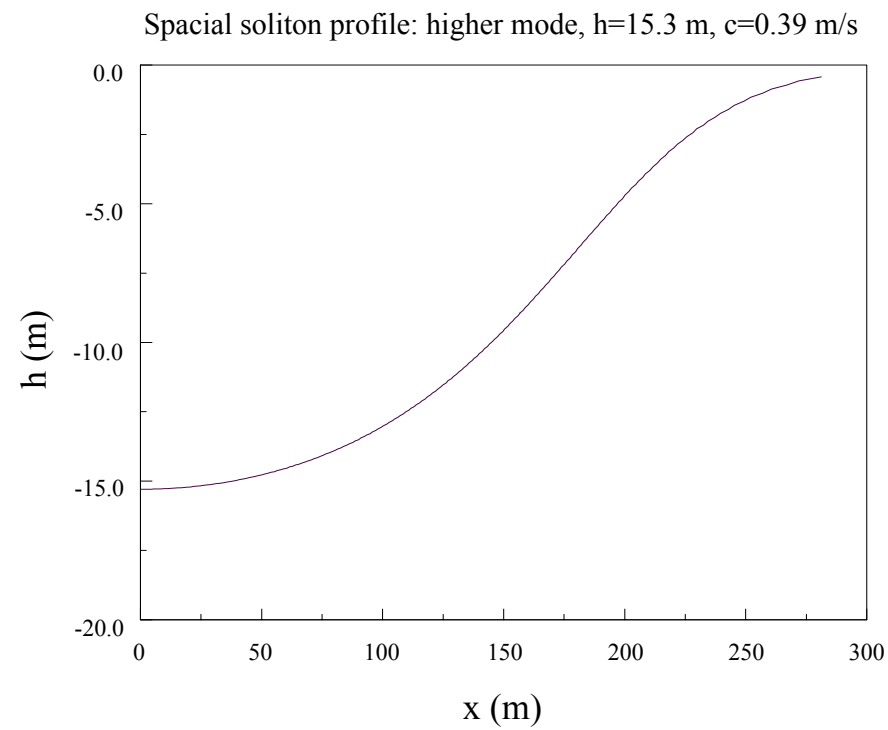
$$C(h) = \frac{c^2}{2}\frac{h_2^2}{\left(h-h_2\right)^2} - \frac{c^2}{2}\left(1-\cot\theta\frac{N_1 h}{c}\right)^2 - g\Delta\rho h$$

$$\theta = \frac{N_1}{c}\left(h - h_1\right)$$

NB: Higher mode solitons exist in strong stratification case

Soliton profile and speed vs. amplitude dependence



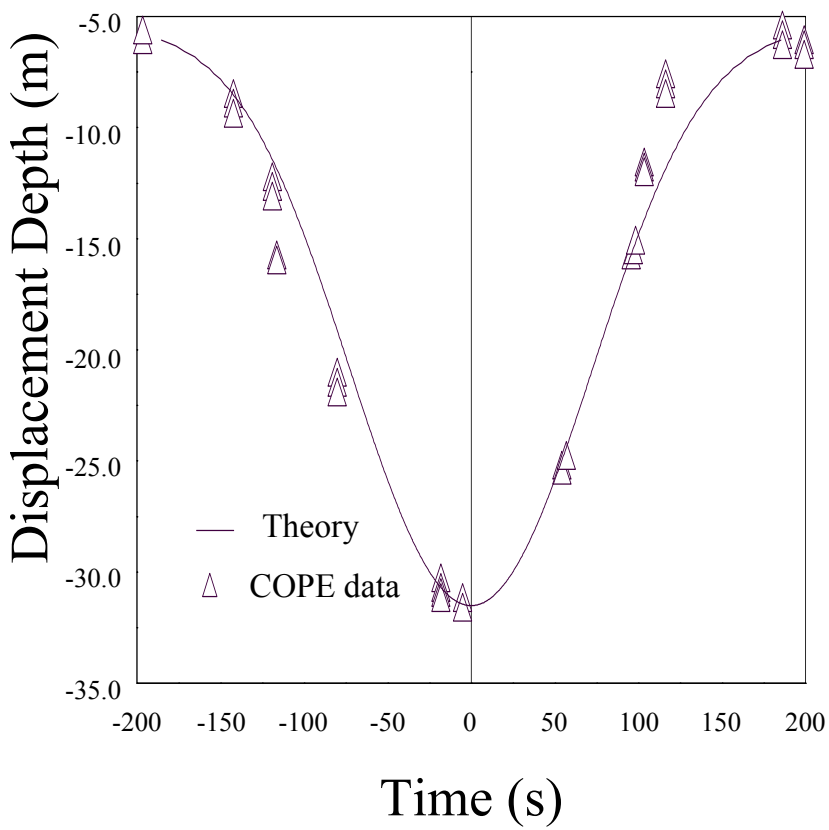


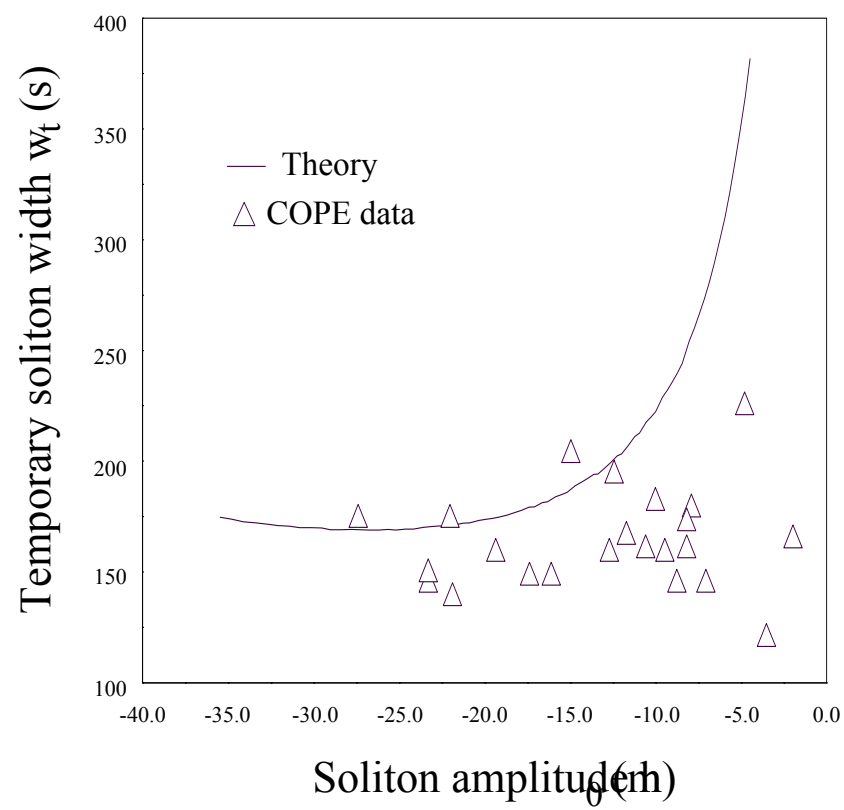
$$\frac{1}{2}\left(\frac{dh}{dx}\right)^2 + U(h,c) = 0$$

Comparison with COPE experiment

$$w_t \approx 150\,s \quad >> \quad N^{-1} \approx 80\,s$$

Internal solitary wave profile





Statistical moments of the scattering matrix and the acoustic field

$$\Psi(x,y,z) = \sum_n u_n(z) \int dk e^{iky} a_n(k,x)$$

$$a_n(k,x) = \bar{a}_n(k,x) + \Delta a_n(k,x)$$

$$S_{mn}(k,k_0) = \bar{S}_{mn}(k)\Delta x \delta(k-k_0) + \Delta S_{mn}(k,k_0)$$

$$\left\langle \Delta S_{n_1 m_1}(k-\eta/2, k_0-\eta_0/2) \Delta S^*_{n_2 m_2}(k+\eta/2, k_0+\eta_0/2) \right\rangle =$$

$$= E_{n_1 m_1, n_2 m_2}(k,k_0;\eta)\Delta x \delta(\eta-\eta_0)$$

Equations for statistical quantities

Average field:

$$\bar{a}_n(k, x + \Delta x) = \bar{a}_n(k, x)\exp\left(i\xi_n \Delta x\right) + \Delta x \sum_m \bar{S}_{nm}(k)\bar{a}_m(k, x)$$

Second order acoustic field correlator:

$$B_{nn'}(k, x; \eta) = \left\langle \Delta a_n(k - \eta/2, x)\Delta a_n^*(k + \eta/2, x)\right\rangle$$

$$\frac{B_{nn'}(k, x + \Delta x) - B_{nn'}(k, x)}{\Delta x} = B_{nn'}(k, x)\frac{\exp\left[i\left(\xi_n - \xi_{n'}\right)\Delta x\right] - 1}{\Delta x} +$$

$$+ \sum_{m'} \exp\left(i\xi_n \Delta x\right)\bar{S}^*_{n'm'}(k)B_{nm'}(k, x) + \sum_{m'} \exp\left(-i\xi_{n'}\Delta x\right)\bar{S}_{nm}(k)B_{mn'}(k, x) +$$

$$+ \Delta x \sum_{m,m'} \bar{S}_{nm}(k)\bar{S}^*_{n'm'}(k)B_{mm'}(k, x) + \sum_{m,m'} \int dk' E_{nn',mm'}(k, k')B_{mm'}(k', x)$$

Optical theorem and energy conservation

$$\xi_{n_1} e^{i\xi_{n_1}\Delta x}\overline{S}^*_{n_1 n_2}(k) + \xi_{n_2} e^{-i\xi_{n_2}\Delta x}\overline{S}_{n_2 n_1}(k) + \Delta x \sum_m \xi_m \overline{S}_{m n_1}(k)\overline{S}^*_{m n_2}(k) +$$

$$\sum_m \int dk' \xi'_m E_{mm,n_1 n_2}\left(k',k;0\right) = 0$$

$$\sum_n \int dk \xi_n \left[\left|\overline{a}_n(k,x)\right|^2 + B_{nn}\left(k,x;0\right)\right] =$$

$$= \sum_n \int dk \xi_n \left[\left|\overline{a}_n(k,x+\Delta x)\right|^2 + B_{nn}\left(k,x+\Delta x;0\right)\right]$$

Fluctuation due to continuous IW spectrum

$$\left\langle \Delta n^2(\vec{r}_1, z_1)\Delta n^2(\vec{r}_2, z_2)\right\rangle = N\left(\vec{r}_1 - \vec{r}_2; z_1, z_2\right)$$

$$N\left(\vec{r}; z_1, z_2\right) = \sum_a \int d\vec{q} e^{i\vec{q}\vec{r}} P_a\left(q_x, q_y\right)\Phi_a(q, z_1)\Phi_a(q, z_2)$$

GM spectrum:

$$P_n\left(\vec{q}\right) = \left(\frac{1}{c_0}\frac{dc_0}{dz}\right)^2 \cdot \frac{2}{\pi} E_0 \frac{\frac{2n_*}{\pi}}{n^2 + n_*^2} \frac{\left(q_* n\right) q^2}{\left[q^2 + \left(q_* n\right)^2\right]^2} \frac{1}{2\pi q}$$

$$E_0 = 4 \cdot 10^3 \frac{j}{m^2}, \quad n_* = 3, \quad k_* = \frac{\pi}{B}\frac{\omega_i}{n_0}$$

$$B \approx 10^3\, m, \quad \omega_i = 7.3 \cdot 10^{-5}\, s^{-1}\ (latitude = 30^0), \quad n_0 \approx 5.2 \cdot 10^{-3}\, s^{-1}$$

Scattering cross-section matrix

$$E_{m'm,n'n}\left(k',k;0\right)=\frac{\pi}{2}\left(\frac{\omega}{c_{00}}\right)^{4}\frac{1-e^{-i\kappa\Delta x}}{i\kappa\Delta x}\frac{1}{\xi_{m'}(k')\xi_{m}(k')}\times$$

$$\times\sum_{a}P_{a}\left(\Delta\xi,k'-k\right)N_{nm}^{(a)}\left(\sqrt{\left(\Delta\xi\right)^{2}+\left(k'-k\right)^{2}}\right)N_{n'm'}^{(a)}\left(\sqrt{\left(\Delta\xi\right)^{2}+\left(k'-k\right)^{2}}\right).$$

$$\Delta\xi=\frac{1}{2}\left(\xi_{m}(k')-\xi_{n}(k)+\xi_{m'}(k')-\xi_{n}(k)\right)$$

$$\kappa=\xi_{m}(k')-\xi_{n}(k)-\xi_{m'}(k')+\xi_{n}(k)$$

$$N_{nm}^{(a)}(q)=\int dz u_{n}(z)u_{m}(z)\Phi_{a}\left(q,z\right)$$

Diffusion approximation

$$\frac{\partial B_{nn'}}{\partial x} = D_{nn',mm'} \frac{\partial^2 B_{nn'}}{\partial k^2}$$

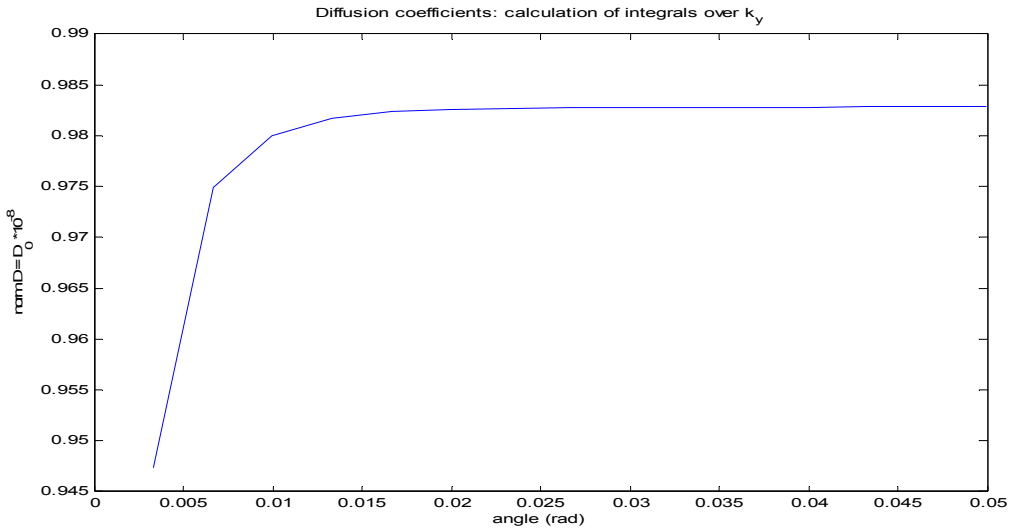
Matrix of diffusion coefficients:

$$D_{nn',mm'} = \frac{1}{2} \int dk' \, E_{nn',mm'}(k, k'; 0) k'^2$$

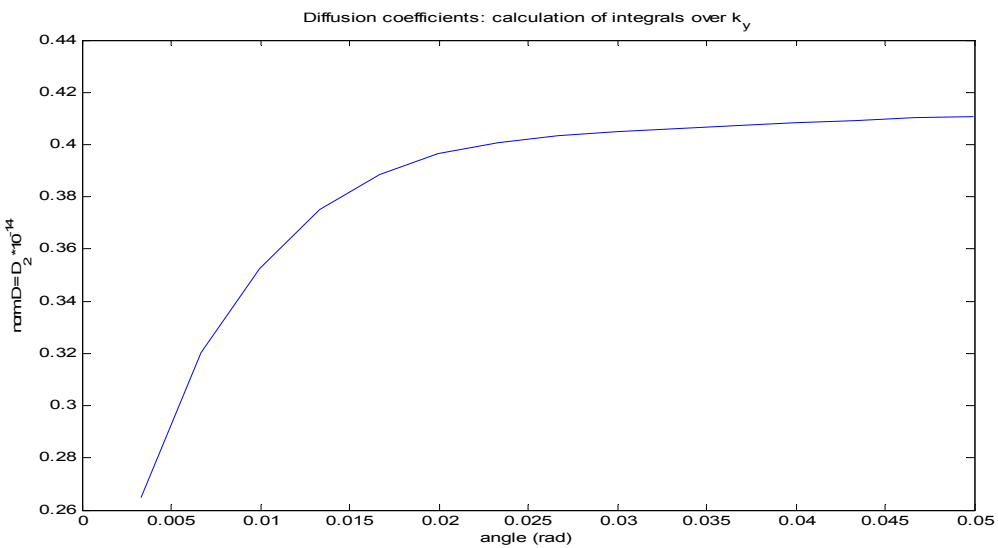
$$B \propto \frac{1}{\sqrt{x}} \exp\left(- \frac{k^2}{4Dx} \right)$$

$$\Delta \theta = \frac{\Delta k}{\omega / c_{00}} \approx \frac{\sqrt{4Dx}}{\omega / c_{00}}$$

Conversion of the integrals for GM spectrum

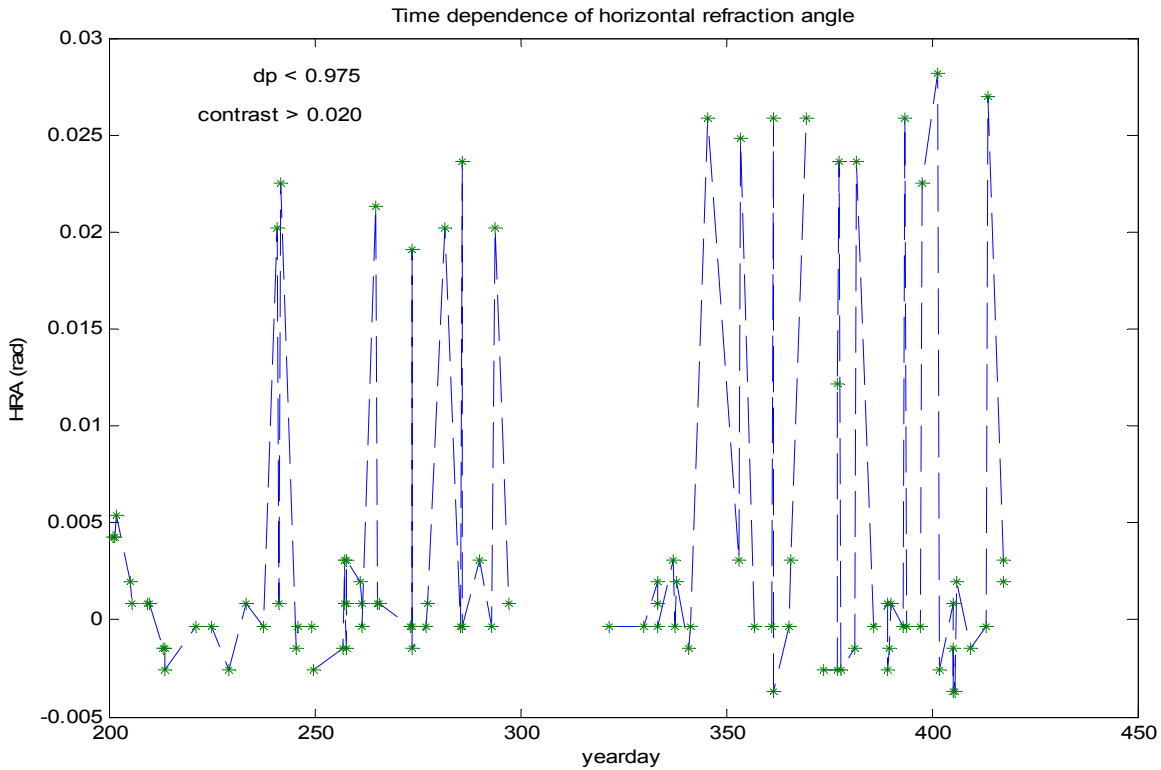


$$D_{nn',mm'} = \frac{1}{2}\int dk'\, E_{nn',mm'}(k,k';0)$$



$$D_{nn',mm'} = \frac{1}{2}\int dk'\, E_{nn',mm'}(k,k';0)k'^2$$

NPAL data



$$D \approx 10^{-13}\, m^{-3}, \quad x \approx 4 \cdot 10^{6}\, m, \quad \frac{\omega}{c_{00}} \approx 0.3 m^{-1} \quad \Rightarrow \quad \Delta\theta \approx \frac{\sqrt{4Dx}}{\omega / c_{00}} \approx 4 \cdot 10^{-3}$$

$$D \approx 0.4 \cdot 10^{-14}\, m^{-3}, \quad \Rightarrow \quad \Delta\theta \approx \frac{\sqrt{4Dx}}{\omega / c_{00}} \approx 0.8 \cdot 10^{-3}$$