Statistical and Stochastic Problems in Ocean Modeling and Prediction, Stage II

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LONG-TERM GOALS

My project addresses statistical and stochastic problems in the following fields: Lagrangian prediction (1), Lagrangian data assimilation (2), and ocean model validation (3). The long range scientific objectives of this study comprise rigorous determining limits of predictability for the Lagrangian motion in semi-enclosed seas, littoral zones, and straits on time scales of days and weeks, elaborating concrete prediction schemes, developing optimal Lagrangian data assimilation algorithms, and constructing and testing discrepancy measures (metrics) for comparison of modeled and observed time series of oceanic parameters to improve performance of numerical models.

OBJECTIVES

The objectives for the first year of research were:

- investigation of optimal predictor sampling in the Lagrangian prediction problem ,
- construction of Lagrangian data assimilation algorithms and their implementation in MICOM,
- development of Lagrangian stochastic models for studying mixing processes in the upper ocean.

APPROACH

I develop theoretical approaches to the Lagrangian prediction and Lagrangian data assimilation problem in context of statistical inference for random processes and fields covered by stochastic partial differential equations. I design computational algorithms realizing developed mathematical methods. A significant part of validating the algorithms is testing them via stochastic simulations. Such an approach implies an accurate error analysis. Together with my collaborators from Rosenstiel School of Marine and Atmospheric Research (RSMAS), we implement the algorithms in concrete ocean models such as QG and MICOM , as well as carry out a statistical analysis of different real data sets by means of new methods.

WORK COMPLETED

1. *Lagrangian prediction*.

The problem of optimal predictor sampling is to find an initial configuration of drifter positions giving the best prediction of an unobservable Lagrangian particle for a prescribed observation time. A theoretical solution of this problem has been suggested for the center of mass prediction algorithm. Numerous Monte Carlo experiments have been carried out to check the theoretical conclusions in the framework of two Lagrangian stochastic models .

2. *Lagrangian data assimilation.*

The Lagrangian assimilation method by Molcard et al (2002) was extended to the primitive equation case by using a dynamical relationship between the velocity corrections calculated from position data, and layer thickness. The method has been applied to the case of mid-latitude double-gyre circulation from a reduced-gravity Miami Isopycnic Coordinate Ocean Model (MICOM). Using a set of twin experiments, a quantitative comparison was made to an extended Kalman filter method. Also, both procedures were compared to so-called, the pseudo-Lagrangian approach, under which Lagrangian observations are essentially treated as Eulerian velocity estimates.

3. *Mixing study via Lagrangian stochastic models*.

Higher-order multi-particle Lagrangian stochastic (LS) models being a useful tool for studying mixing processes in the upper ocean, are not able to ensure incompressibility of an underlying flow. Thus, an important problem is to investigate the compressibility rate for such models. Together with Lyapunov exponent considerations, that would help to describe geometry of the material lines and surfaces in the eddy turbulence. I studied theoretically and numerically the dependence of the compressibility rate on the Lagrangian correlation time and velocity space correlation radius for the first order Markov LS model. Also, the Lyapunov exponent investigation has been continued for the second order Markov LS models aimed at understanding mixing parameters on the acceleration time scale.

RESULTS

1. It is found that there is the optimal diameter of a predictor cluster ensuring the best prediction of an unobservable particle (predictand). This conclusion is supported by both theoretical asymptotical arguments (see Fig. 1) and simulations.

The presented curves show the dependence of the prediction error on the cluster radius, r, in the isotropic case for different values of the predictand standard deviation, σ, and different values of the velocity space correlation radius, R. The optimal radius, r^*, is of order of σ. However, the difference between minimal error and the error corresponding to r in the range 0-1.5σ is very subtle. The prediction skill deteriorates sharply for $r > 2\sigma$. The prediction skill under the optimal initial configuration is slightly improving when the number of predictors is increasing from 2 to 8. Also, the optimal configurations are found in a non-isotropic case. The main optimality condition is that the initial cluster should form an ellipse of the same orientation as the predictand uncertainty ellipse.

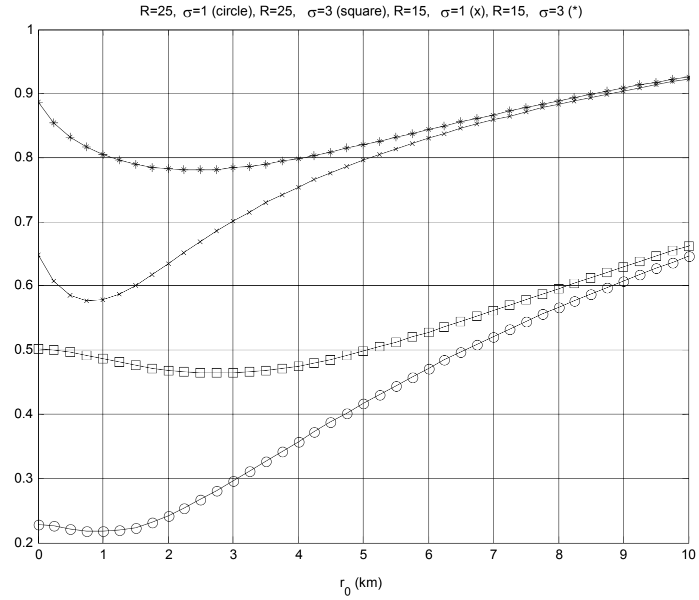

Figure 1

2. 16 experiments with three assimilation methods: Lagrangian Optimal Interpolation (LOI), pseudo-Lagrangian Optimal Interpolation (PLOI) and Kalman Filter (KF) for data sampling interval Dt =6h, 3d, 6d illustrates the superior skill of LOI (Fig.2)

The relative error in the layer thickness versus assimilation time is shown by crosses for LOI, by solid line for KF and by broken line for PLOI. All the techniques expose a nearly-exponential convergence in the assimilation time. The e-folding time has been estimated in all the experiments. The assimilation skill is significantly improving as the density of drifters is increasing. A theoretical formula for the assimilation error is found in terms of the sampling period, Lagrangian correlation time, model and observation errors. It is in a good qualitative agreement with the above numerical experiments. The error in the layer thickness assimilation is less than 10% in the most of experiments and is less than that of the velocities (15%).

3. Let α be the ratio of the Lagrangian correlation time and turnover time, which is defined as a typical vortex size over the velocity fluctuation standard deviation. The analysis of the compressibility in the framework of the first order model driven by zero-divergence forcing showed that the incompressibility rate is negative and of order α^4, while the Lyapunov exponent is positive and of order α^2 for small α (weak turbulence approximation). The conclusion is that an approximate incompressibility of the first order Markov flow can be maintained by keeping a divergence free forcing. In turn, this implies that the geometry of material line at surfaces can be effectively studied in the framework of the first order stochastic flows. As for the second order LS models, the existence of both, the top Lyapunov exponent, λ, and incompressibility rate, c, is proven. Simple systems of stochastic differential equations are obtained allowing to evaluate λ and c, as well as to investigate their asymptotics.

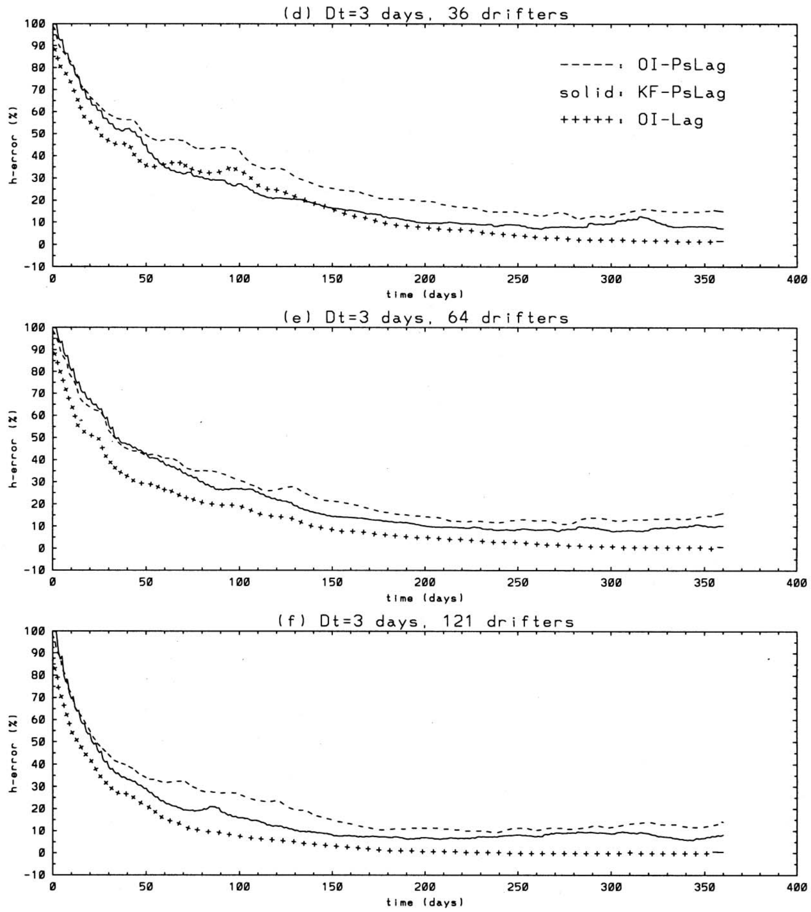

Figure 2

IMPACT/APPLICATIONS

1. The suggested optimal strategy for choosing the initial drifter configuration can be used for improving real forecasting of lost objects in the sea based on observations of drifters floating in the same area.

2. Probably, it is the first time that a truly Lagrangian assimilation algorithm is implemented in a realistic model of ocean circulation. Comparison of our algorithm with other procedures could be a helpful guidance for choosing assimilation schemes in other circulation models. The obtained results will stimulate developing more sophisticate assimilation schemes involving several time steps and non-local spatial interpolation.

3. The investigated dependence of the compressibility rate and Lyapunov exponent on the physical parameters can help to understand behavior of the passive scalar realizations rather than behavior of their statistical moments. In turn, this is could be useful for interpretation of real drifter observations.

TRANSITIONS

The developed optimal sampling procedure and assimilation algorithms were and will be used in RSMAS to test them on realistic numerical models and real data sets.

RELATED PROJECTS

1. "Predictability of Particle Trajectories in the Ocean", ONR, PI T.Ozgokmen (RSMAS),

2. "Lagrangian Data Analysis in Mesoscale Prediction and Model Validation Studies", ONR, PI A.Griffa (RSMAS)

PUBLICATIONS

1. Piterbarg, L.I., (2001): The top Lyapunov exponent for a stochastic flow modeling the upper ocean turbulence, SIAM J.Appl. Math, 62, 777-800
2. Piterbarg, L.I, A. Griffa, A. Mariano,T. Ozgokmen and E. Ryan., (2001): Predictability of the Lagrangian motion in the upper ocean, Abstracts of AGU meeting, San Francisco, Dec, 11-16, 2001
3. Piterbarg, L.I. and T.Ozgokmen, (2002): A simple prediction algorithm for the Lagrangian motion in 2D turbulent flows, SIAM J. Appl.Math, 63, 116-148
4. Molcard A., L. Piterbarg, A. Griffa, T. Ozgokmen, A. Mariano., (2002): Assimilation of Lagrangian data in Eulerian models., J. Geophys. Res., (in press)
5. Piterbarg, L.I., (2002): The Lyapunov exponent for 2D stochastic flows with memory, Stochastic Proc. Appl., submitted