

Three-Dimensional Scattering Of Internal Waves Off A Uniformly Sloping Bottom

Rudolf C. Kloosterziel
Marine Science Building 411
School of Ocean & Earth Science & Technology
University Of Hawaii Honolulu HI 96815
phone: 808-956-766 fax: 808-956-9225 email: rudolf@soest.hawaii.edu

Award#: N000149610762

LONG-TERM GOAL

Any theory based on monochromatic linear wave dynamics is too simple to properly interpret or predict anything that involves a continuum of frequencies like a pulse of wave energy. Through case studies of scattering events of realistic non-random internal wavefields an important step will be taken towards developing a capability to predict internal wavefield characteristics near sloping topography and the generation of near-slope currents.

OBJECTIVES

My objective is to determine what flows result in the vicinity of a uniformly sloping bottom when a realistic wave field encounters that slope. Present knowledge of such events are based on linear wave theory where single monochromatic waves are considered. In the case of constant buoyancy frequency N a single wave with frequency ω_i propagates at a fixed angle θ with the vertical with group velocity U_g and is either reflected forward or backward depending on whether $\omega_i < \omega_c$ or $\omega_i > \omega_c$ where ω_c is the critical frequency. At the critical frequency, where the angle θ associated with ω_i is equal to the angle of the slope, the group velocity of the outgoing wave goes to zero and wave-energy cannot propagate away. Amplitudes for the reflected wave become infinitely large and wavelengths go to zero. When a wave is reflected upslope, the propagation direction turns toward the upslope direction. These theories consider steady state situations with wavefields that extent to infinity in all directions. There are no events with a 'beginning' and an 'end'. Neither are these theories sufficient to properly interpret Oceanic data. It is my objective to determine what happens when a localized wavefield, (a wave-packet or pulse) meets a sloping bottom. I envision such a field to be generated by surface forcing of finite duration and horizontal scale.

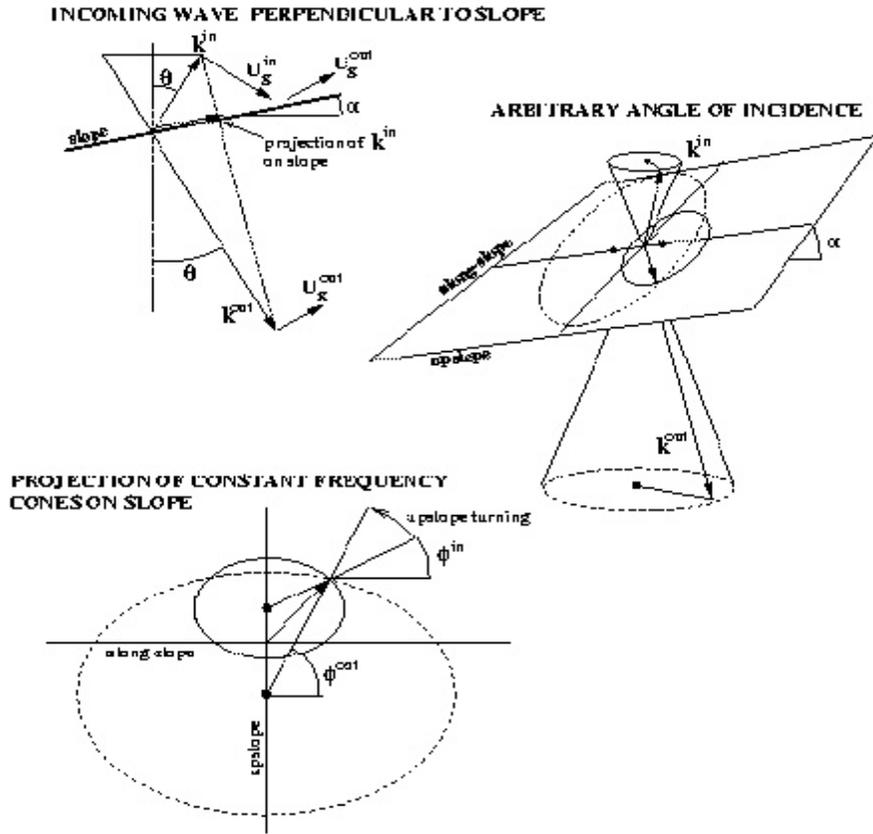


Diagram showing how to solve the reflection problem for a single monochromatic wave. This is for $\omega_i > \omega_c$. The along-slope component of the wavevector k of the incoming wave has to be matched by that of the outgoing wave. The problem reduces to a geometrical problem of finding the intersection point of two ellipses which are the projections of constant frequency cones on the slope.

APPROACH

Surface forcing is modeled by imposing a vertical velocity field at the surface which results from the divergence of a viscous Ekman layer generated by surface winds. The vertical Ekman pumping w_s at the base of the surface Ekman layer perturbs the underlying stratified Ocean. The response is investigated with the linearized dynamics, the f -plane and Boussinesq approximation and a model stratification of constant N . First the Green's function \mathbf{G}^s for the semi-infinite domain is determined. This gives the response to surface pumping infinitely concentrated at one position in space and time. The response of the fluid to arbitrary pumping w_s is then given by the convolution $\mathbf{G}^s \circ w_s$ where 'o' stands for the convolution integral over the surface and the time history of the surface pumping. The next step is to determine the Green's function \mathbf{G}^n for the response to a given normal flow u_n at the slope. If we take for u_n the normal component of the velocity field $\mathbf{u} = \mathbf{G}^s \circ w_s$ at the slope, then $-\mathbf{G}^n \circ u_n$ gives the reflected wave field. The velocity field due to both the incoming and reflected wave field is

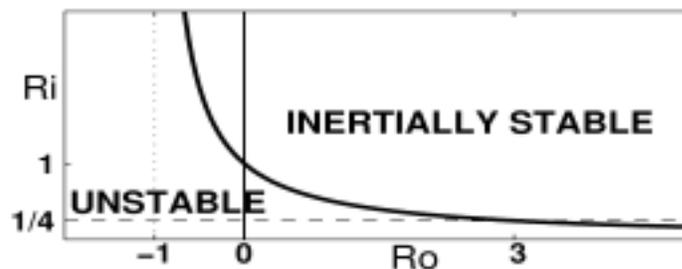
then $\mathbf{u}=\mathbf{G}^s\mathbf{ow}_s-\mathbf{G}^n\mathbf{o} u_n$. Knowing the Green's functions, the response to arbitrary forcing can be determined numerically. In special cases the response can be expressed in closed form.

WORK COMPLETED

The vector Green's function \mathbf{G}^s for the semi-infinite domain has been determined. It is such that numerical convolution with arbitrary surface forcing poses no problems. The Green's function \mathbf{G}^n has been determined. We have written an article on wavepacket propagation (Carnevale, Briscolini, Orlandi & Kloosterziel, 2001) and have submitted a paper to JFM on the suppression of Rayleigh-Taylor instability in rotating flows with viscosity and diffusion (Carnevale, Orlandi, Zhou & Kloosterziel, 2001). An article on inertial instability in rotating stratified flows is in preparation.

RESULTS

I have analyzed the response of a semi-infinite Ocean to 'switch-on' point-sources and the response to finite-sized model surface forcings. For finite-sized forcings I find that initially unbalanced geostrophic currents are created which ultimately reach a steady state through radiation of internal waves. They propagate away from the forcing region as a distinct three-dimensional pulse (a wave packet). The polarization relations for coherent linear wavefields do not hold. For two very different initial conditions (unbalanced vortices) the same fraction of the initial total energy is converted into internal wave energy and in both cases the energy spectrum of the internal wave field is the same. After the adjustment the geostrophic currents have increased in amplitude by a factor N/f . This means that if N is much larger than f , Rossby numbers increase dramatically (also the vertical shear increases). If the forcing is such that anti-cyclonic flow is generated, the final flow may be well inside the centrifugally unstable range (Kloosterziel, 2000). Centrifugal instability leads to further radiation of internal waves (see Carnevale et al., 1997). I am currently engaged in developing theory for centrifugal (inertial) instability of baroclinic vortices and currents in arbitrarily stratified rotating fluids. Instability depends not only on the sign of vorticity but also on the vertical shear (the Richardson criterion involves the magnitude of the shear).



Graph showing the inertially unstable range as a function of Richardson number and Rossby number for parallel stratified shear flows. For Rossby numbers smaller than 3 this can be a more dangerous instability than Kelvin-Helmholtz instability which only occurs when the Richardson number is smaller than 1/4. Both instabilities lead to internal wave generation.

Generalizations to curved flows suggest that anticyclonically curved meanders of currents can locally lead to internal wave generation through inertial instability. Also, boundary currents with strong anticyclonic shear will be inertially unstable and be local sources of internal waves.

For the slope it has been determined that a brief normal flow leads to unbalanced currents which unlike at the surface will not reach an equilibrium but disappear through radiation of waves.

IMPACT/APPLICATIONS

Numerical codes for simulating the response to surface forcing or the reflection of wave packets off a slope can be tested for accuracy and errors by contrasting the output with the exact results I derive. A number of paradigms based on group velocity arguments and properties of monochromatic waves do not hold for wavefields with a continuum of wavevectors and frequencies. This work will lead to a better understanding of the properties of non-random internal wave fields and currents near sloping topography.

RELATED PROJECTS

Dr. Carnevale of Scripps Institution of Oceanography, San Diego, is presently developing a code to study the reflection of internal wave packets at a slope. This code can also be used to simulate the internal wave generation due to surface pumping at an upper rigid surface. We also work on the problem of centrifugal instability of baroclinic vortices and currents which can be a source of internal waves. The theory will be complemented by numerical simulations of unstable vortices and currents. Further we work on wavepacket propagation and Rayleigh-Taylor instability.

REFERENCES

Carnevale, G.F., Briscolini, M., Kloosterziel, R.C. & Vallis, G.K. 1997 Three-dimensionally perturbed vortex tubes in a rotating flow. *J. Fluid Mech.* 341, 127-163.

PUBLICATIONS

Kloosterziel, R.C. 2000 Surface-forced internal waves and vortices in uniformly rotating and stratified fluids. *J. Fluid Mech.* 421, 39-81.

Carnevale, G.F., Briscolini, M., Orlandi, P. & Kloosterziel, R.C. 2001 Simulations of internal wave breaking and wavepacket propagation in the thermocline, `Aha Huliko`a, From stirring to mixing in a stratified Ocean, Proceedings Hawaiian Winter workshop, University of Hawaii, January 2001, ed. P. Muller and D. Henderson.