Generation and Propagation of Internal Solitary Waves on the Continental Shelf and Slope

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LONG-TERM GOALS

This project is a fundamental study of the basic dynamical mechanisms involved, and the consequent theoretical modelling approaches needed, in the generation and propagation of internal solitary waves across the continental shelf and slope.

OBJECTIVES

There are two principal objectives. The first is to develop and refine amplitude evolution equations of the Korteweg-de Vries type to the point where they can be used as validated models for the propagation of internal solitary waves. The second is to undertake a major re-examination of the generation process, using a combination of theoretical and numerical analyses, and emphasising the distinction between two-dimensional and three-dimensional mechanisms.

APPROACH

Our approach is to develop an understanding of the fundamental dynamical processes involved through a combination of theoretical analyses and numerical simulations. Our research group comprises post-doctoral fellows, research students and international collaborators who make long-term visits. We maintain contact with those making field and laboratory observations, with the aim of establishing an ongoing interactive collaboration on data interpretation, model development and validation.

WORK COMPLETED

For the first objective, the development and refinement of amplitude evolution equations of the Korteweg-de Vries type, the main focus to this point has been on understanding the role of cubic nonlinearity vis-à-vis that of quadratic nonlinearity in several contexts. First, we have obtained a correct asymptotic derivation of the coefficients for the quadratic and cubic nonlinear terms in the extended Korteweg-de Vries (eKdV) equation for background flows which allow for arbitrary density and current stratification, and importantly, allow for a free surface. Second, we have examined the solitons generated by various initial conditions in this eKdV model, and demonstrated some striking differences from the well-known situation for the KdV model. Third, we have examined the role of variable topography in deforming the solitary wave, and have determined the conditions under which the wave may break up into several wave packets, each of which may themselves generate new solitary waves. Fourth, we have examined the effect of various frictional processes on the family of solitary wave solutions of this eKdV equation, using primarily asymptotic techniques.

For the second objective, that is the utilisation of a combination of theoretical analyses and numerical simulations to re-examine the principal generation processes of internal solitary waves, we have focussed first on the development of a suite of two- and –three-dimensional numerical codes for this purpose, and secondly on the development of theoretical models of the Boussinesq-type, which can incorporate arbitrary stratification and topography which may vary in both horizontal directions. Our two-dimensional spectral codes for the Euler equations have been extended to three dimensions, and are now being tested and validated. A theoretical model suitable for describing the generation of the internal tide, which allows for general stratification and topography has been developed, and is now being analysed. Results from the linearized, non-dispersive version of this model show that it can reconcile internal ray generation mechanisms with mode scattering mechanisms. In addition, we have used a forced KP model equation to examine the transcritical generation of internal solitary waves by flow over an obstacle, in a detailed study of the effect of the obstacle shape and orientation.

RESULTS

First, we have examined various aspects of the eKdV model for the propagation of internal solitary waves. For the present discussion this is given by,

$$A_t + \Delta A_x + \mu A A_x + \nu A^2 A_x + \lambda A_{xxx} + \Gamma(A) = 0, \qquad (1)$$

where $A(x,t)$ is the amplitude of a representative isopycnal displacement and incorporates an amplification factor which takes into account the variable background, t is a time-like variable describing the evolution of a solitary wave, and x is a phase variable describing the shape of the solitary wave. The coefficients $\Delta, \mu, \nu, \lambda$ are determined by the linear long-wave modal function, which in turn depends on the background density and current stratification. The expression $\Gamma(A)$ is a dissipative term, which can take several forms. Explicit expressions are available from our work for the key coefficients μ, ν, λ for arbitrary density and current stratification, and in the presence of a free surface. One of the most important features to emerge from our work is the role of the cubic nonlinear term with coefficient ν vis-à-vis that of the quadratic nonlinear term with coefficient μ. This is immediately evident in the richer structure of the solitary wave solutions supported by (1) for the canonical case when the coefficients are all constants, and there are no dissipative or forcing terms, when compared with the corresponding family of solitary wave solutions of the KdV equation (i.e. (1) with only quadratic nonlinearity). These are shown in Figure 1, where, without any loss of generality, it has been assumed that μ, λ are both positive. Thus, when the coefficient ν of the cubic nonlinear term is negative (Figure 1a) we see that the solitary waves resemble those of the KdV equation for small amplitudes, but for large amplitudes, they are much thicker and reach a limiting amplitude of $-\mu/\nu$, known as the "thick" wave. On the other hand, when the coefficient ν of the cubic nonlinear term is positive (Figure 1b), there are two families of solitary waves. That family with positive polarity resembles the KdV family, but that with negative polarity is quite different and in particular has no small-amplitude limit; instead, there is a lower bound for an amplitude of $-2\mu/\nu$ and solutions of (1) with lower energy are represented by breathers, that is, solutions which resemble pulsating solitary waves. Given that observed internal solitary waves are often quite large, these two key differences from the familiar KdV theory are very significant.

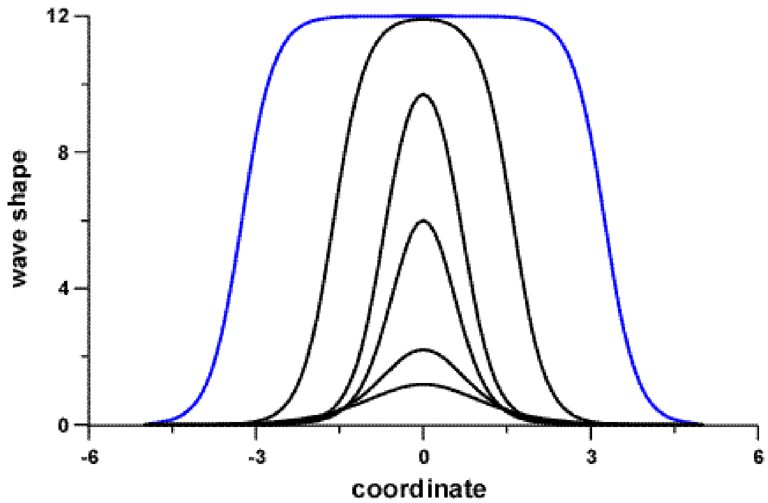

Figure 1a Solitary wave shape for negative cubic nonlinearity.

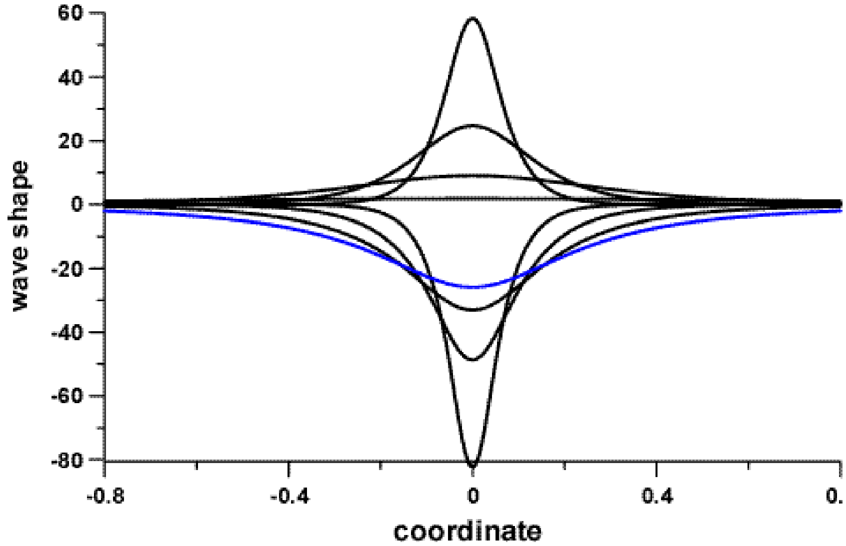

Figure 1b Solitary wave shape for positive cubic nonlinearity.

internal solitary wave across typical continental shelves, such as the NW Australian Shelf, the Malin Shelf off the west coast of Scotland, and the Artic shelf. In each case the simulations are initialized with an internal solitary wave of depression in the deep water. As the wave propagates shoreward, it deforms adiabatically, that is, it conserves its wave-action flux, until a critical point is reached, beyond which the wave breaks up into several wave packets, each of which may then generate several smaller solitary waves, sometimes of the opposite polarity. The critical points are defined by the zeros of the coefficients μ, ν of the quadratic and the cubic nonlinear terms respectively in equation (1). It is important to note that on each of these shelves, the coefficients in equation (1) undergo considerable variability across the shelf, and in particular, on the NW Shelf, several critical points were found. In a separate ongoing study, we have considered the effect of dissipation on a solitary wave, using various forms of the dissipative term $\Gamma(A)$ representing Newtonian damping, laminar or turbulent boundary layer damping, or damping due to interior turbulence. For a negative coefficient ν of the coefficient of the cubic term (with μ, λ both positive) we find that the decay of a "thick" wave can lead to the formation of secondary wave packets, while for a positive coefficient ν, the decay of a wave of negative polarity leads to the formation of breather states, which resemble a wave packet.

Second, motivated by recent satellite and *in situ* observations that show internal solitary waves emanating from point sources on the continental slope, often submarine canyons, we have focussed our present research work on the development of theoretical and numerical models which can incorporate two-dimensional variability in the bottom topography. Our three-dimensional spectral codes for the Euler equations are now developed, and are being tested and validated. These codes have the potential to provide higher spatial resolution than most current codes, and importantly are non-hydrostatic, so they can simulate both the generation process and the subsequent evolution into solitary waves. An asymptotic theoretical model suitable for describing the generation of the internal tide, which allows for general stratification and topography has been developed, and is now being analysed. The basis for the development of this model is a novel decomposition into vertical modes which allows for arbitrary topographic variation. Importantly, we retain the free-surface mode in the model, so that we are not only able to describe the forcing of the internal tide by the interaction of the barotropic tide with topgraphy, but also have the potential to determine the feedback on the barotropic tide. In its simplest form, when it is linearised and non-dispersive, we are using it to describe the generation of the internal tide, for both one-dimensional and two-dimensional topography. Our results in the former case show that this model can reconcile internal ray generation mechanisms with mode scattering mechanisms.

Another aspect of two-dimensionality in the forcing mechanism is being examined for the case when internal solitary waves are generated by transcritical flow over topography. Here we use as the model equation, the forced KP equation, so that we can make a comparison with the analogous results for a one-dimensional process described the forced KdV equation (i.e. equation (1)). Also, we have obtained some preliminary results from a fully three-dimensional Euler equation code. We have conducted a thorough study of the effect of the obstacle shape and orientation on the upstream and downstream wave trains, and find that although there is overall qualitative agreement with the results from the corresponding forced KdV equation, there are some significant differences, notably in the structure of the downstream waves.

IMPACT/APPLICATIONS

We anticipate that the results obtained will inform the scientific community about the structure of internal solitary waves, their behaviour under such environmental impacts as friction and topography, and the processes which favour their generation.

RELATED PROJECTS

"Generation of internal tides and internal solitary waves on the continental shelf" (ONR project, N00014-02-1-1004). This is essentially a continuation of this project with a greater emphasis on the role of topography.

PUBLICATIONS (2001-)

- Grimshaw, R. and Christodoulides, P. (2001). Short-wave instability in a three-layered stratified shear flow. *Q.J. Mech. Appl. Math.,* 54, 375-388.

- Aigner, A., and Grimshaw, R. (2001). Numerical simulations of the flow of a continuously-stratified fluid, incorporating inertial effects. *Fluid Dyn. Res.,* 28, 323 - 347.
- Grimshaw, R., Pelinovsky, D., Pelinovsky, E. and Talipova, T. (2001). Wave group dynamics in weakly nonlinear long-wave models. *Physica D*, 159, 35-57.
- Grimshaw, R. (2001). Internal solitary waves. *Environmental Stratified Flows, ed. R. Grimshaw, Kluwer*, Chapter 1, 1-28.
- Rottman, J.W. and Grimshaw, R. (2001). Atmospheric internal solitary waves. *Environmental Stratified Flows, ed. R. Grimshaw, Kluwer,* Chapter 3, 61-88.
- Grimshaw, R. and Skyrnnikov, Y. (2002). Long-wave instability in a three-layer stratified shear flow. *Stud. Appl. Math.,* 108, 77-88.
- Grimshaw, R.H.J., Chan, K.H. and Chow, K.W. (2002) Transcritical flow of a stratified fluid: the forced extended Korteweg-de Vries model. *Phys. Fluids*, 14, 755-774.
- Grimshaw, R., Pelinovsky, E. and Poloukhina, O. (2002) Higher-order Korteweg-de Vries models for internal solitary waves in a stratified shear flow with a free surface. *Nonlinear Processes in Geophysics*, 9, 221-235.
- Tang, Y. and Grimshaw, R. (2002). The effect of a barrier on tidally-forced flow in a density-stratified estuary. *Continental Shelf Research*, 22, 2035-2044.
- Grimshaw, R. and Pelinovsky, E. (2002). Interaction of a solitary wave with an external force in the extended Korteweg-de Vries equation. *Bifurcation and Chaos*, 12, 2409-2420.
- Grimshaw, R., Pelinovsky, E. and Talipova, T. (2002) Damping of large-amplitude solitary waves. *Wave Motion*, 37, 351-364.
- Grimshaw, R., Pelinovsky, D., Pelinovsky, E. and Slunyaev, (2002) The generation of large-amplitude solitons from an initial disturbance in the extended Korteweg-de Vries equation. *Chaos*, 12, 1070-1080.