Solitary Waves and Sill Flows

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LONG-TERM GOALS

Our goal is to elucidate the fluid dynamical processes associated with stratified flow over topography including the establishment of such flows, effects due to strong forcing and the generation of internal solitary waves.

OBJECTIVES

The key scientific objectives are (i) to determine the mechanisms responsible for streamline splitting and the role played by entrainment in forming a weakly stratified wedge of nearly stationary fluid over an obstacle as the flow adjusts to changing barotropic forcing, (ii) to examine the detailed structure of instabilities in steeply inclined shear flows in the light of recent laboratory studies, (iii) to describe the mechanism responsible for generation, trapping and release of internal solitary waves above an obstacle.

APPROACH

Observations were acquired in a collaborative study in Knight Inlet, BC using a combination of ADCP measurements, echo-sounder imaging, continuous CTD profiling, use of towed current meters and CTDs at fixed depth and air photography. The observations have been interpreted initially with the aid of two-layer internal hydraulic theory, together with analysis of interfacial instability and unsteady nonlinear wave theory.

WORK COMPLETED

The work has focused on three aspects. A detailed analysis of the mechanisms responsible for formation of the trapped wedge of weakly stratified fluid downstream of the sill crest has been completed. A summary of the observations of nonlinear internal wave formation and trapping has been completed. A detailed analysis of the formation and movement of internal hydraulic fronts has begun. This work is carried out in conjunction with D Farmer (IOS).

RESULTS

We have demonstrated that an important mechanism contributing to the formation and maintenance of a weakly stratified wedge of fluid downstream of a sill crest in hydraulically controlled conditions, or downstream of a mountain ridge in the analogous atmospheric example, is entrainment from the rapidly moving supercritical flow beneath. While this process has some of the attributes that have been identified in numerical simulations of mountain flow, our observations reveal the time dependent

evolution of the response as instabilities first form over the crest and then fill in the deepening wedge, leaving a weakly stratified zone of slowly moving fluid above the supercritical layer. The flow is observed to separate at the crest of the obstacle until the expanding wedge leads to a favourable pressure gradient suppressing the separation. Entrainment rates can be derived from the observed velocity field together with the measured rate at which successive isopycnals are incorporated into the entraining zone. It is shown that the rate of entrainment is more than sufficient to account for the observed expansion of the growing wedge of weakly stratified flow.

The upstream portion of the wedge (Figure 1) can be considered a front. In the example examined in Knight Inlet, this front is submerged beneath a fresh surface layer, but in other situations it can intersect the surface. Its location, as well as the shape of the lower boundary of the wedge, is determined by the density difference across the interface, the barotropic forcing and the topography. The consistency of this result is demonstrated through our observations. However a subtlety arises from the fact that the wedge is stratified. The density difference is not prescribed *a priori* but results from the entrainment and the development of the wedge.

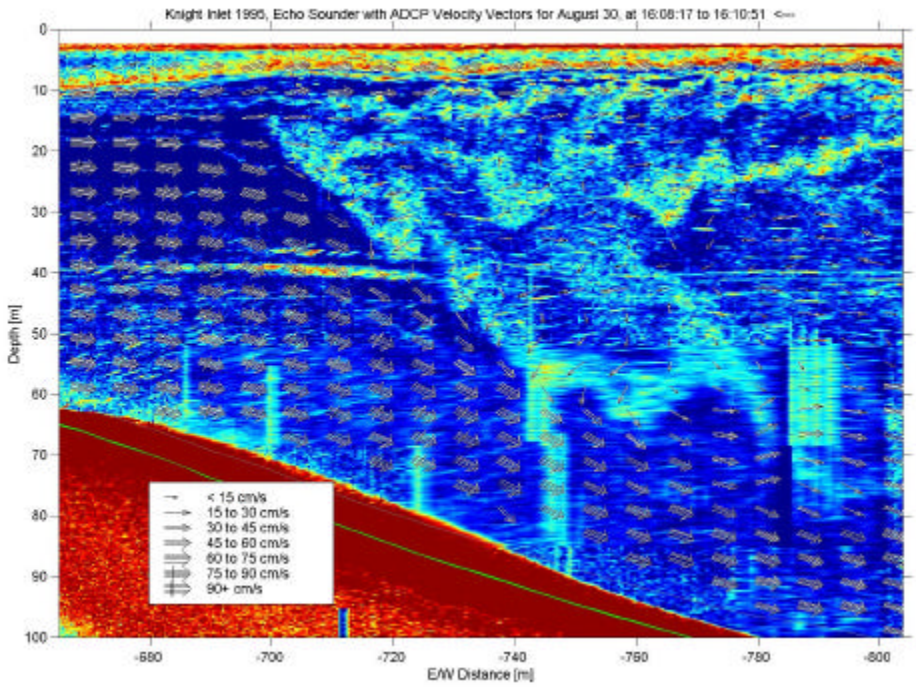

Figure 1: Detailed image of streamline bifurcation at an internal hydraulic front. Arrows indicate velocity vectors (vertical and downstream horizontal components). Instabilities are apparent in the downstream shear zone. The location of the tip moves back and forth, dependent on the density difference and the barotropic forcing, so as to maintain a densimetric Froude number of unity.

The processes described above are time dependent and can lead to the evolution of internal solitary waves. Such waves may appear spontaneously. Two possible mechanisms might account for these

waves: they may arise, as has been demonstrated in numerical calculations, in the form of an upstream influence of flow over the obstacle, or they may be due to shear flow instabilities on the interface. In the latter case, sufficiently long waves could propagate upstream if formed in the subcritical portion of the flow and analysis of the Knight Inlet data show that such instabilities do exist. It should be noted that in general, model calculations of the nonlinear evolution equations usually damp the shear flow instabilities, so that their possible role in solitary wave formation remains unresolved.

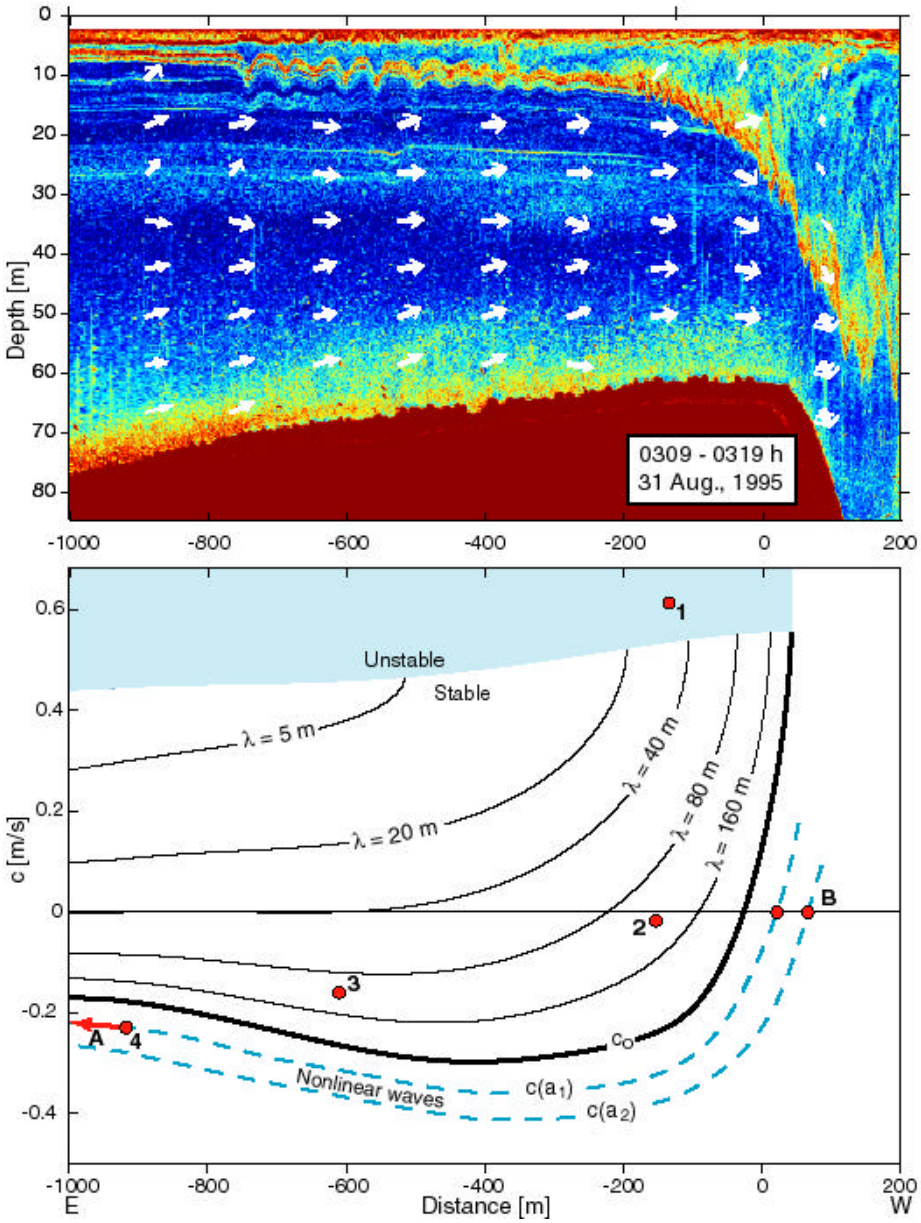

Figure 2:
Above - Image of solitary wave formation and shear flow instabilities.
Below - Dispersion relations/stability diagram for interfacial waves corresponding to observations. '1' to '4' shows possible generation sequence; B shows wave trapping.

However they are formed, interfacial waves evolving in the supercritical portion of the flow can only proceed upstream if their phase speed exceeds the local long wave speed. Waves formed just downstream of the control can become trapped, achieving a balance between nonlinear wave speed and the speed of the oncoming flow. In Knight Inlet this phenomenon is observed during flood tides, where a gently sloping lee face of the sill spreads out the developing supercritical flow, permitting nonlinear waves to grow to a size that allows them to trapped. Observations of such waves (Figure 3) show that they can be trapped for up to 2h until released by the slackening tidal current.

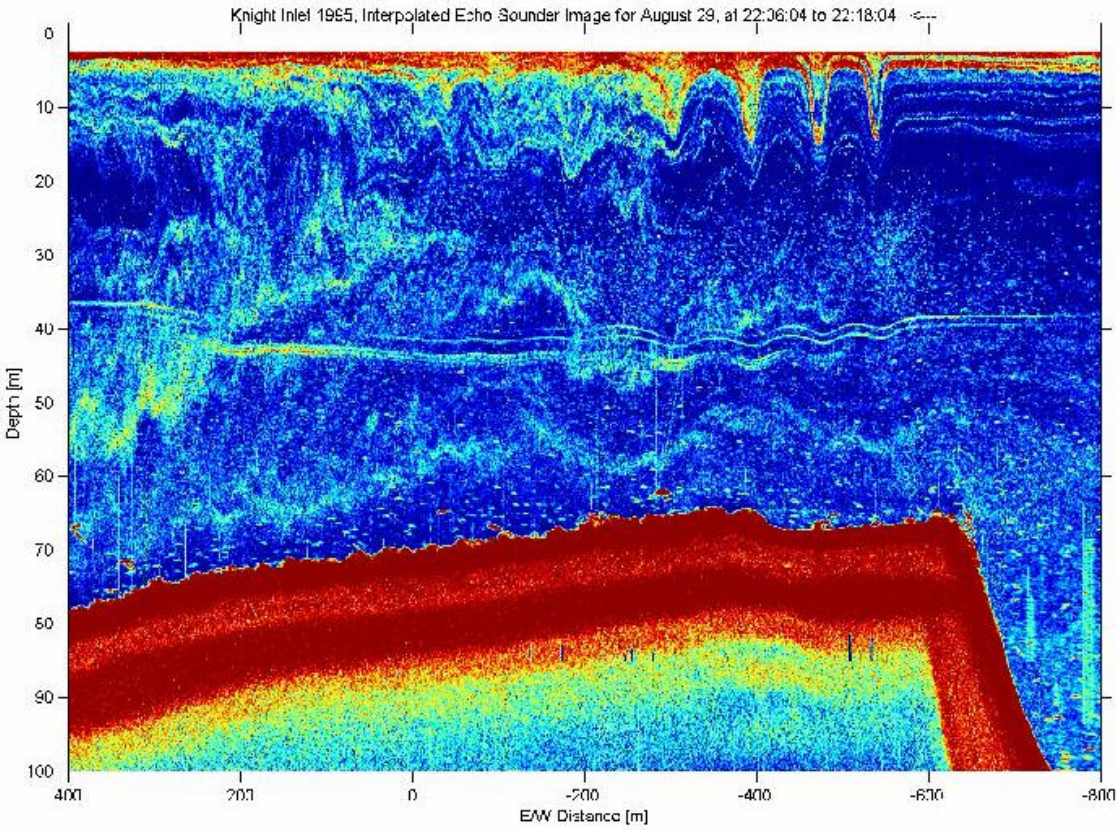

Figure 3: Trapped solitary waves in Knight Inlet. These waves form downstream of the sill crest. They therefor occur in the supercritical flow. Since the speed of a nonlinear wave exceeds the linear wave speed, it is still possible for the waves to avoid being swept downstream in the supercritical flow, provided they are close enough to the sill crest. The waves were observed to remain trapped for approximately two hours until they were finally released by the slackening tidal current.

IMPACT/APPLICATIONS

These results provide a dynamical basis for calculation of drag over obstacles, especially in cases where the forcing is unsteady, for example due to tides. The results are directly relevant to atmospheric circulation models for which the effect of form drag over mountains can account for 50% of the global value. The measurements on entrainment provide confirmation, at geophysical scales, of previous laboratory studies and also illustrate a newly described class of shear instability in accelerating flows.

Although the surface manifestation of internal solitary waves is almost ubiquitous in coastal waters, the mechanisms associated with their formation are not so well understood. Typically they may be modelled as an evolution into a train of amplitude ordered waves from a single large scale interfacial disturbance, for example as described by the inverse scattering approach for KdV models. The present observations suggest the existence of an entirely different mechanism that may be widespread. The unambiguous presence of interfacial instabilities in the flow, occurring at scales that could lead to upstream propagating waves, suggests that models which include both nonlinearity and shear flow instability will need to be considered in the interpretation of solitary wave trains observed over topography.

TRANSITIONS

The results of this work form an essential component of the larger description of sill flows and solibores in the Knight Inlet study. Our new results of flow establishment are being discussed with atmospheric scientists with a view to studying their application to parameterisation of atmospheric flow over mountains. Solitary wave generation is very widespread in the world's coastal oceans and the Knight Inlet measurements provide some of the most detailed observations of their generation and propagation.

RELATED PROJECTS

Concurrent work being carried out by G Pawlak in the laboratory on the properties of shear flow instabilities on sloping interfaces is directly related to the field studies described here. This study formed part of a collaborative program in Knight Inlet with PI's from APL-UW, SIO and IOS.

REFERENCES

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