Low-Frequency, Long-Range Sound Propagation through a Fluctuating Ocean: Analysis and Theoretical Interpretation of Existing and Future NPAL Experimental Data

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LONG-TERM GOALS

To theoretically study low-frequency, long-range sound propagation through a fluctuating ocean, including studies of 3D effects.

To compare obtained theoretical results with experimental data.

OBJECTIVES

To develop a new, modal, 3D theory of low-frequency, long-range sound propagation through an ocean with random inhomogeneities.

Based on this theory, to develop computer codes for calculating statistical moments of a sound field propagating through the ocean with internal gravity waves, internal tides, and spiciness.

To compare theoretical predictions with data obtained during the 1998-1999 and 2004 NPAL experiments.

APPROACH

Studies of the statistical characteristics of low-frequency sound waves propagating over long-ranges in a fluctuating ocean are important for many practical concerns, e.g. source detection and ranging. The main goals of our project are development of a new, modal, 3D theory of sound propagation in a fluctuating ocean, development of computer codes for processing of low-frequency sound signals recorded during the 1998-1999 and 2004 North Pacific Acoustic Laboratory (NPAL) experiments [1,2], and comparison between theoretical results and experimental data.

In studies of low-frequency, long-range sound propagation in a fluctuating ocean, a modal approach seems to be the most adequate one since, in this case, there is only a relatively small number of propagating modes. Modal approaches have been developed in a number of publications, e.g. [3] – [6].
However, they were rarely applied to practical problems due to high dimension of matrices involved. We have been developing a new, modal, 3D theory of sound propagation through a fluctuating ocean, which is based on the Chernov method for calculation of the first two statistical moments of a sound field. Our theory, in comparison to the most of the previous work, allows numerical calculation of these statistical moments in 3D.

We have also been developing computer codes for calculations of the statistical characteristics of low-frequency sound signals recorded in the NPAL experiments. In the 2004 NPAL experiment, long-range sound propagation was mainly studied in the LOAPEX sub-experiment. Two vertical line arrays (VLA) were used to record low-frequency sound signals due to a sound source suspended from the research ship Melville. The sound propagation distance varied from 50 km to 3200 km.

**WORK COMPLETED**

The following tasks were accomplished in the FY06:

Task 1. The mean field of a sound wave propagating in a fluctuating ocean was calculated. The results obtained were published in Ref. [7].

Task 2. Results obtained in development of a modal, 3D theory of sound propagation in a fluctuating ocean were summarized in Ref. [8].

Task 3. Vertical coherence of acoustic signals recorded during the 2004 NPAL experiment was studied, see Ref. [9].

**RESULTS**

In the FY06, the following results were obtained:

Task 1. The mean field of a low-frequency sound wave propagating in a fluctuating ocean is an important statistical characteristic of this wave that can be measured experimentally. Furthermore, the mean sound field contains information about a spectrum of internal waves in the ocean. However, in the literature, the mean sound field has not been studied in detail. Using the theory of multiple scattering, we derived an analytical formula for the mean sound field propagating in a fluctuating ocean [7]. The mean sound field is presented as a sum of normal modes that attenuate exponentially. The extinction coefficients of the modes were expressed in terms of the spectrum of random inhomogeneities in the ocean.

In Ref. [7], the mean sound field was calculated for both 2D and 3D geometries of sound propagation in a fluctuating ocean. The comparison between 2D and 3D results allowed us to formulate some necessary conditions of 2D approximation in ocean acoustics.

Task 2. Results obtained in the development of a modal, 3D theory of sound propagation in a fluctuating ocean were summarized in Ref. [8]. In particular, the coherence function $\Gamma$ was expressed in terms of the cross-modal correlation functions $B_{nm}(k, x, \eta)$. Here, $x$ is the propagation distance, and $k$ and $\eta$ are...
spectral coordinates corresponding to the difference and half sum of the spatial coordinates of two hydrophones. Closed equations for \( B_{nm}(k, x, \eta) \) were derived and computer codes were developed to solve these equations.

Figure 1. The dependence of the cross-modal correlation functions on the azimuthal angle. Solid lines correspond to the propagation distance \( x = 250 \) km, and dashed lines to \( x = 1000 \) km.

Figure 1 shows the cross-modal correlation functions \( B_{nm}(k_0\phi, x, \eta) \) versus the azimuthal angle \( \phi \) for the propagation distances \( x = 250 \) km (solid lines) and \( x = 1000 \) km (dashed lines). In the figure, the subplots on the main diagonal of the \( 3 \times 3 \) matrix correspond to the mode correlation functions \( B_{nn}(k_0\phi, x, \eta) \). The subplots above the main diagonal depict the real parts of the cross-modal correlation functions \( B_{nm}(k_0\phi, x, \eta) \), and those below correspond to their imaginary parts. Figure 1 allows one to study the dependence of \( B_{nm}(k_0\phi, x, \eta) \) on the propagation distance \( x \), azimuthal angle \( \phi \), and mode numbers \( n, m \). At \( x = 0 \) km the dependencies of \( B_{nm}(k_0\phi, x, \eta) \) on \( \phi \) (not shown in Fig. 2) are rectangulars with the base \(-0.005 \leq \phi \leq 0.005\) rad and different heights due to different excitation amplitudes of the modes by the source. It follows from Fig. 1 that the initial rectangular dependencies spread with distance \( x \) at different rates for different modes.
Task 3.
Computer codes were developed for processing of sound signals recorded during the 2004 NPAL experiment and calculation of the vertical coherence of these signals. Some of the results obtained are presented in Figs. 2-5.

Figure 2 shows the magnitude of the correlation coefficient $K$ of sound signals recorded by two hydrophones as a function of depths of these hydrophones. The signals were recorded on 263 day 20 h 06 min 40 s of the experiment by the lower part of the deep VLA in the depth range from 3600 to 4300 m. The distance between VLA and the ship was $r = 500$ km, source depth $z_s = 350$ m, the carrier of the sound signal $f = 68.2$ Hz, and the signal was truncated to the first 300 s. In Fig. 2, the main “ridge” with the value of the correlation coefficient $K = 1$ corresponds to the correlation of the signal recorded by a hydrophone with itself. The values of other correlation coefficients are less than 1; a nearly monotonic increase in $K$ with the increase in depth can be seen in the figure.

Figure 3 depicts the magnitude of the correlation coefficient $K$ recorded at the same time of the experiment by the upper part of shallow VLA in the depth range from 400 to 1100 m. In addition to the main ridge, there are two smaller ridges. The rest of the correlation coefficients looks like a noise. Note that for some other days of the experiment, only one of these small ridges is present. Figure 4 shows the correlation coefficient recorded on 263 day 13 h 06 min 40 s of the experiment by the upper
part of shallow VLA in the depth range from 400 to 1100 m. The distance between VLA and the ship was $r = 500$ km, source depth $z_s = 800$ m, the carrier of the sound signal $f = 75$ Hz, and the signal was truncated to the first 300 s. Small ridges are not present in the figure and, except for the main ridge, the correlation coefficients look like a noise.

Finally, Figs. 5(a) – (d) depict a time dependence of the magnitude of the correlation coefficient of sound signals recorded on 263 day 17 h 00 min 00 s of the experiment by the lower part of shallow VLA. The distance between VLA and the ship was $r = 250$ km, source depth $z_s = 350$ m, the carrier of the sound signal $f = 68.2$ Hz, and the signal was truncated to the first 30 s. Figures (a), (b), (c), and (d) correspond to the correlation coefficients between the following hydrophones: 1 and 2, 1 and 3, 1 and 4, and 1 and 5. Some regular time dependencies of the correlation coefficients can be observed in Figs. 5. These dependencies are probably caused by temporal evolution of the internal waves field.

**Figure 3.** The magnitude of the correlation coefficient versus the depths of two hydrophones of the upper part of shallow VLA for 263 day 20 h 06 min 40 s of the experiment.
Figure 4. The magnitude of the correlation coefficient versus the depths of two hydrophones of the upper part of shallow VLA for 263 day 13 h 06 min 40 s of the experiment.
**IMPACT/APPLICATIONS**

Formulas for the mean sound field and the coherence function of a sound field were derived using the theory of multiple scattering and modal, 3D theory of sound propagation in a fluctuating ocean. Computer codes were developed to calculate the cross-modal correlation functions. Computer codes were also developed for processing of sound signals recorded during 2004 NPAL experiment and calculation of the vertical coherence function.

**RELATED PROJECTS**

1. The 1998-1999 NPAL experiment, which is described in [1].

2. The 2004-2005 NPAL experiment, which is described in [2].
3. “Multiple Scattering of Sound by Internal Waves and Acoustic Characterization of Internal Wave Fields in Deep and Shallow Water”, ONR project N0001405IP20024.

REFERENCES


PUBLICATIONS

