LONG-TERM GOALS

Our long-term goal is to develop reliable tools for modeling the sensitivity of travel-time observables to sound-speed perturbations in low-frequency long-range acoustic propagation in the ocean and associated tomography experiments.

OBJECTIVES

Our primary objective is to use the wave-theoretic travel-time sensitivity kernel in order to study the effect of increasing range on the sensitivity of finite-frequency travel-time observables to sound-speed perturbations. A further objective is to compare wave-theoretic travel-time sensitivity kernels and Fresnel volumes associated with particular eigenrays, seeking connections between the ray-theoretic and wave-theoretic description of travel-time observables.

APPROACH

To study the effect of the increasing range on the sensitivity of finite-frequency travel times we use the wave-theoretic travel-time sensitivity kernel introduced by Skarsoulis and Cornuelle (2004) based upon the first Born approximation for perturbations of the Green’s function and the notion of peak arrivals (Athanassoulis and Skarsoulis 1994). Further we consider a range-independent background ocean environment, which allows for the use of the Green’s function and the sensitivity kernel in terms of normal modes. As the source-receiver distance increases we study the shape of the three-dimensional (3D) sensitivity kernel in the vertical and the horizontal seeking for the effects of increasing range in the presence of refraction.

In a further stage the wave-theoretic sensitivity kernels will be compared with the Fresnel volumes associated with the corresponding eigenrays, seeking for connections between the ray-theoretic and wave-theoretic description of travel-time observables. The Fresnel volume (first Fresnel zone)
associated with a certain eigenray is the locus of points surrounding the eigenray which cause constructive interference at the receiver, and thus defines in a ray-theoretic framework the volume that primarily affects the received signal (Kravtsov and Orlov 1990).

WORK COMPLETED

The first year of the project was mainly devoted to the study of the behavior of the wave-theoretic travel-time sensitivity kernel with increasing source-receiver range (ranges from 30 to 1500 km were considered). Further, first steps were taken to study the sensitivity of travel times to second order.

RESULTS

Bottom-related arrivals

For the calculation of waterborne arrivals patterns and associated travel-time sensitivity kernels an absorbing bottom is considered, modeled by a half space of unit density and constant sound speed, equal to the water sound speed at the water-bottom interface. From a ray-theoretic viewpoint such a bottom guides all incident acoustic energy away from the water layer. Wave-theoretic calculations, however, reveal that this is not the case for finite-frequency propagation, resulting in additional (spurious) early arrivals, earlier than the waterborne ones. Fig. 1 (upper panel) shows the arrival pattern for source-receiver range of 500 km and depth of 150 m in 2500-m deep water followed by an absorbing bottom. The sound-speed profile in the water is linear (1503 m/s at the surface, 1547 m/s at 2500-m depth). The emitted signal is a Gaussian pulse of 100-Hz central frequency and 70-Hz bandwidth (3-dB bandwidth). While the arrivals after 329.8 sec can be associated with waterborne acoustic paths, the earlier weak arrivals cannot.

The lower panels in Fig. 1 show the travel-time sensitivity kernel for two of these spurious arrivals (peaks 1 and 2, marked by crosses in the upper panel) as well as one waterborne early arrival (peak 3). While the sensitivity kernel of the latter (bottom panel) is confined in the water column, those of peak 1 and 2 penetrate the bottom half space, the earlier the arrival the deeper the penetration – the water-bottom interface at 2500-m depth is marked by the magenta line. This points to an association of these arrivals with high-order modes, whose exponentially decaying tails penetrate the upper part of the half space.

In this connection, a means to suppress the additional early arrivals is by introducing attenuation in the bottom half space and using complex eigenvalues, whose imaginary part – the part associated with attenuation – is proportional to the energy of modal tails (Jensen et al. 2000). The arrival pattern resulting from this approach is shown in Fig. 2. By comparing this figure with Fig. 1 (upper panel) it is seen that the bottom-related arrivals are drastically suppressed. The waterborne arrivals, on the other hand, are left unaffected since the attenuation mechanism is confined in the bottom half space only. In the following calculations of travel-time sensitivity kernels the bottom-related spurious arrivals are removed.
Fig. 1. Arrival pattern (top) and travel-time sensitivity kernels – vertical along-range section – for marked 3 peaks, for propagation range 500 km, source-receiver depth 150 m, linear sound-speed profile and absorbing bottom. Magenta lines indicate the water-bottom interface.

Fig. 2. Arrival pattern for propagation range of 500 km, source-receiver depth of 150 m, linear sound-speed profile and absorbing bottom with attenuation.
**Effects of increasing range**

During the first year of the project calculations of travel-time sensitivity kernels were conducted focusing on the behavior of the geometry of the 3D kernels with increasing range in horizontally stratified background environments using normal-mode propagation modeling. The focus was on waterborne early arrivals commonly used in tomographic inversions.

Fig. 3 shows vertical sections (in the source/receiver vertical plane) of the wave-theoretic travel-time sensitivity kernel corresponding to early arrivals for source-receiver separations (propagation ranges) of 30, 100, 250, 500, 1000 and 1500 km. The corresponding eigenrays are also shown (dashed lines). The background environment, the source-receiver depths and the signal characteristics are as described above. The color scales ranges from blue (negative sensitivity) to red (positive sensitivity) with green corresponding to zero sensitivity. Negative sensitivity means that a sound-speed increase will lead to a travel-time decrease – this is the anticipated behavior. Positive sensitivity on the other hand indicates that a sound-speed increase will lead to a travel-time increase, which is quite counterintuitive. Still in the wave-theoretic travel-time sensitivity kernels there are patterns of alternating negative and positive sensitivity, with the latter extending over considerable domains (Skarsoulis and Cornuelle 2004).

![Fig. 3. Vertical along-range sections of the travel-time sensitivity kernel of early arrivals in the linear-profile waveguide, for propagation ranges between 30 and 1500 km and source-receiver depth 150 m. The dashed lines indicate the geometry of the corresponding eigenrays.](image)

The wave-theoretic sensitivity kernels are concentrated about the eigenrays. For the shorter ranges the travel-time sensitivity kernels exhibit areas of near-zero sensitivity in the neighborhood of the eigenray. These areas attain their maximum extent close to the turning points and they are surrounded by a domain of negative sensitivity as the distance from the eigenray increases. Further out there are
domains of Fresnel-zone like alternating positive and negative sensitivity which cancel out leading to a net negative sensitivity. With increasing source-receiver separation the wave-theoretic sensitivity kernels are suppressed in the vertical towards the eigenray and the zero-sensitivity cores tend to disappear. This shrinkage of the sensitivity kernel in the vertical is due to refraction – in free space the cross-range extent of the zero-sensitivity core increases proportionally to the square root of the source-receiver range and the zero sensitivity on the source-receiver connecting line persists.

The upper panel of Fig. 4 shows the arrival pattern corresponding to 1000-km propagation range and the lower 3 panels show a vertical (cross-range) section of the travel-time sensitivity kernel of an early arrival (marked by a cross in the upper panel) at distances 19 km, 204 km and 500 km from the source; the arrival is the same as in Fig. 3 right-middle panel, i.e. Fig. 4 shows cross-sections of that kernel at the first, sixth and mid-range turning point. It is seen that while the sensitivity kernel remains bounded in the vertical it expands in the horizontal as we move towards the midpoint between source and receiver. This expansion in the horizontal is similar to the behavior of the Fresnel radius in free space. This similarity has clearly to do with the lack of horizontal sound-speed gradients in the considered background environment.

This is more clearly seen in Fig. 5 showing a horizontal section (top view) of the above sensitivity kernel at a depth of 1950 m, which is approximately the turning depth for the particular arrival. The traces of the acoustic paths at the ranges corresponding to the turning points are seen in this figure, and their envelopes form ellipses similar to the Fresnel zones in free space. For comparison, the first Fresnel zone assuming propagation in free space is also shown, marked by the dashed lines. As derived
by Skarsoulis and Cornuelle (2004) for the case of free space, the boundary of the first Fresnel zone is an area of positive travel-time sensitivity. This is clearly seen in Fig. 5. The agreement with the free-space Fresnel zone indicates that the horizontal cross-range behavior/extent of the travel-time sensitivity kernel at the turning depth is not affected by stratification.

*Fig. 5. Travel-time sensitivity kernel – horizontal section at 1950-m depth – for early arrival marked in previous figure, for propagation range 1000 km and source-receiver depth 150 m (linear sound-speed profile). The dashed line indicates the boundary of the first Fresnel zone assuming propagation in free space.*

Fig. 6 shows the vertical cross-range sections of the travel-time sensitivity kernels presented in Fig 3 at the midpoint between source and receiver, and it illustrates how the cross-section of the TSK behaves with increasing propagation range. All sections correspond to lower turning points. At 30 km the sensitivity kernel cross-section at mid range has a nearly circular symmetry with near-zero sensitivity in the middle. As the source-receiver range increases the TSK is squeezed in the vertical (effect of refraction) and expanded in the horizontal (similar to free-space behavior), passing from circular symmetry to ellipse-shaped and finally linear cross-section, whereas the near-zero sensitivity area in the middle disappears (it is replaced by somewhat lower sensitivity in the middle of the negative-sensitivity line).

The same type of study was repeated for an environment characterized by a bilinear sound-speed profile (1547 m/s at the surface, 1521 m/s at 1000-m depth, and 1547 m/s at 2500-m depth), forming a channel with axis at 1000 m depth in 2500 m of water. The source and receiver in this case are considered at 1000 m depth, i.e. at the channel axis, and the signal characteristics are the same as before. Fig. 7 shows vertical sections (in the source/receiver vertical plane) of the wave-theoretic travel-time sensitivity kernel corresponding to early arrivals for propagation ranges 30, 100, 250, 500, 1000 and 1500 km. The corresponding eigenrays are also shown (dashed lines). Apart from the replacement of surface reflections by upper turning points, similar comments apply as before: with increasing range the travel-time sensitivity kernels appear to converge towards the eigenrays in the vertical. The horizontal extent of the kernel is not affected by stratification, at least at the turning points, which mostly affect the integral sensitivity of travel times.
Combining the above findings it is concluded that as the range increases the travel-time sensitivity kernels take the form of a folded rug. The cross-range horizontal extent of the kernel at the turning points increases as we move from the source/receiver to their midpoint forming an elliptical shape, as in free space. This suggests that travel times are sensitive to sound-speed changes up to a distance from the eigenrays in the horizontal (at the side of the eigenrays), a distance which is maximum at the source-receiver mid range and also increases with source-receiver separation. On the other hand, travel times are sensitive to sound-speed changes only very close to the eigenrays in the vertical. Taking into account that the inhomogeneity scales in the ocean are small in the vertical and large in the horizontal, this wave-theoretic picture supports the use of ray theory for describing travel-time sensitivity in long-range propagation.

*Fig. 6. Travel-time sensitivity kernels – vertical cross-range sections at source-receiver mid range – of early arrivals in the linear-profile waveguide, for propagation ranges between 30 and 1500 km (cf. Fig. 3).*

**Second-order sensitivity of travel times**

Besides the above work on travel-time sensitivity kernels relating travel-time and sound-speed perturbations to the first order, first steps were taken towards studying the second-order behavior of peak arrival times. A second-order perturbation formula relating travel-time and pressure perturbation was derived, and further, using the second Born approximation, an expression relating travel-time and sound-speed perturbations to the second order perturbations was obtained. This expression involves quadratic forms of the first Born-approximation (terms associated with single-scattering) and also terms associated with the second Born-approximation involving single and double scattering.
IMPACT/APPLICATIONS

Our results offer support to the use of ray theory for interpretation of long-range transmissions: the convergence of the TSK in the vertical towards the eigenray and the expansion in the horizontal with increasing range indicates that wave-theoretic travel times are sensitive to sound-speed changes on the eigenrays and their horizontal (cross-range) neighborhood.

Fig. 7. Vertical along-range sections of the travel-time sensitivity kernel of early arrivals in the bilinear-profile (channeled) waveguide, for propagation ranges between 30 and 1500 km. The dashed lines indicate the geometry of the corresponding eigenrays.

RELATED PROJECTS

In the framework of NPAL (ONR contract N000140310182) Bruce Cornuelle and Matthew Dzieciuch are exploring the spatial frequency content and the stability of travel-time sensitivity kernels in range-dependent ocean environments which produce strong sensitivity of ray paths to initial conditions.

REFERENCES

