CAVITATION NOISE

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Many of the features of cavitation noise may be illustrated by consideration of the pressure field surrounding an idealized spherical cavity growing and collapsing in an incompressible fluid. The pertinent hydrodynamic relations are reviewed and extended to the computation of the frequency spectrum of the sound. Some criteria for the limits of applicability of the incompressive theory are given.

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INTRODUCTION

This paper is concerned with the sound pressure generated by a growing and collapsing cavity in a liquid, for example, a cavity which, in the flow of the liquid past a curved rigid boundary, grows in a region of low pressure and subsequently collapses in a region of higher pressure downstream. The existence, circumstances, and typical behavior of such cavities have been so fully discussed in the literature (1,2,3) as to require no further elaboration here. The discussion will be restricted to those aspects of the sound which may be elucidated by consideration of spherical cavities. In the earliest history of the study of the dynamics of cavitation, the concept of a spherical cavity was recognized as a useful idealization. Thus, Rayleigh's famous solution (4) gives the motion of an empty spherical cavity in an incompressible liquid of infinite extent with constant external pressure. More recently, Rayleigh's concept has been extended to allow for time-varying external pressure and the bubble motions so computed have been shown to correspond reasonably well with actual behavior of cavities which grow and collapse in fluid flow (5). The present work, without adding anything new in principle, extends the description to include the pressure radiated by the growing and collapsing cavity.

HYDRODYNAMIC RELATIONS

The relation between the radial motion of a growing and collapsing cavity and the pressure in the surrounding liquid have been treated extensively in the literature relating to cavitation and to underwater explosions—at least for the case of incompressive flow of the liquid (6, 7). It will be useful to take the relation valid for incompressive flow as our focal point since during the whole of the period of growth and the greater part of the collapse of the cavity the incompressive theory adequately describes not only the motion of the cavity but also the sound pressure which appears (after an interval of time) at a distance. Accordingly, we note the relation for incompressive flow (6):

$$p(r,t) - p(\infty,t) = \frac{\rho R^2\ddot{R} + 2\rho R\dot{R}^2}{r} - \frac{\rho R^4\dot{R}^2}{2r^4}. \tag{1}$$

Here, $p(r,t)$ is the pressure in the liquid at radial coordinate r and time t; ρ is the density of the liquid; R (equal to $R(t)$) is the radius of the cavity at time t; and $\dot{R}$ and $\ddot{R}$ are the first and second time derivatives, respectively, of R.

In the special case, $r = R$, Eq. (1) gives the differential equation relating the growth and collapse of the cavity to the "driving pressure" $P - p_e$:

$$\rho R\ddot{R} + \frac{3}{2}\rho\dot{R}^2 = P - p_e. \tag{2}$$

Here, P has been written for $p(R,t)$ and p_e for $p(\infty,t)$.

At a sufficiently great distance from the cavity, the second term on the right side of Eq. (1) is negligible. The remaining pressure will be called the <u>sound pressure</u>, p_s:

$$p_s = \frac{\rho R}{r}\,[R\ddot{R} + 2\dot{R}^2], \tag{3}$$

or, identically,

$$p_s = \frac{\rho}{3r}\,\frac{d^2[R^3]}{dt^2}. \tag{4}$$

Thus, the sound pressure associated with the growth and collapse may be computed from the motion of the cavity or, indirectly, from the environmental pressure which the cavity encounters.

THE CAVITY MOTION

In general, if the pressure difference $P - p_e$ is given as a function of R and t, Eq. (2) can be integrated (to find $R(t)$) only by nonanalytic methods (5,9). Exceptions are two special cases discussed by Rayleigh (4,6).

In the case of an "empty" cavity* collapsing or rebounding under the influence of a time-varying external pressure, positive and equal to P_0 at the instant of complete collapse, a series solution useful in describing the motion in the region of collapse and rebound may be obtained in the form

$$\frac{R}{R_1} = \sum_{m=1}^{\infty} A_m \left| \frac{t - t_0}{R_1[\rho/P_0]^{1/2}} \right|^{\frac{m}{5}}. \tag{5}$$

*By an "empty" cavity is meant here one for which P, the pressure in the liquid at the wall of the cavity, is substantially constant. During the collapse of vapor cavities in water, the effects of surface tension, gas contained in the cavity, viscosity, and vapor heating, are ordinarily small enough so that the cavity may be considered "empty" over that part of the collapse for which the incompressive theory is adequate.

Here, R_1 is a radius characteristic of the motion at collapse* and defined so that the limiting value approached by the kinetic energy

$$2\pi\rho R^3 \dot{R}^2$$

at collapse is equal to $4/3\ \pi R_1^3 P_0$. The time t_0 is the instant of collapse.

The coefficients A_m may be evaluated by substituting Eq. (5) in Eq. (2) and equating the coefficients of like powers of $|t - t_0|$. Thus, if the time-varying pressure difference is expressed as a power series:

$$p_e - P = P_0 \left[1 + \sum_{n=1}^{\infty} b_n \left| \frac{t - t_0}{R_1 [\rho/P_0]^{1/2}} \right| \right]. \tag{6}$$

The coefficients are given below

m	A_m	m	A_m	m	A_m
2	1.330325	18	$-0.05593\ b_2$	24	$-0.0139\ b_2$
8	-0.284733	19	$-0.02689\ b_1$		$-0.00516\ b_1^2$
13	$-0.106775\ b_1$	20	-0.009007	25	$-0.010133\ b_1$
14	-0.035848	23	$-0.034418\ b_3$	26	-0.002810

For other values of m, $(m < 26)$, $A_m = 0$.

The series converges rapidly enough to provide a useful solution of Eq. (2) for the period of collapse and the period of growth after rebound. For the purpose of describing the later stages of collapse, one or two terms of the series suffice.

THE SPECTRUM OF THE SOUND PRESSURE

Typical behavior patterns for the growth, collapse, and multiple rebound of a cavity and for the pulse of sound pressure have been shown in a previous survey (8). The spectral distribution of the energy radiated as sound may be determined by means of the Fourier transformation applied to the sound pulse. Let $p_s(r,t)$ be the sound pressure† which appears at radial coordinate r at time t. Then a Fourier transform $s(r,f)$ corresponding to the sound pressure is defined by

$$s(r,f) = \int_{-\infty}^{\infty} p_s(r,t) e^{-2\pi i f t}\, dt\ . \tag{7}$$

A spectral density $S(r,f)$ describes the distribution of the sound "energy" with respect to frequency.

$$S(r,f) = 2|s(r,f)|^2\ . \tag{8}$$

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*Near the very end of collapse or beginning of rebound, the motion is, of course, characterized only by the total energy $4/3\ \pi R_1^3 P_0$.

†The dependence of the sound pressure upon the radial coordinate r is, of course, simply the inverse relation given by Eq. (4) so that the quantity whose spectrum is independent of r is rp_s.

In order to consider the actual energy radiated acoustically, it is necessary to depart from the assumption of incompressibility of the liquid to the extent necessary to include the process of acoustic propagation. According to the usual laws of acoustics, the energy radiated in a band of frequencies df will be

$$\frac{4\pi r^2 S(r, f)\,df}{\rho c}$$

and the total radiated energy

$$\frac{4\pi}{\rho c}\int_{-\infty}^{\infty}[rp_s]^2 dt$$

equals

$$\frac{4\pi}{\rho c}\int_{0}^{\infty} r^2 S\ df\ .$$

Here, c is the velocity of propagation of sound in the liquid.

THE LIMITATION OF INCOMPRESSIVE THEORY

The use of incompressive flow theory (to determine the gross flow and the pressures) and the laws of acoustics (to account for the radiation) represents the slightest possible departure from strictly incompressive theory. A crude criterion for applicability of this procedure in estimating the sound pressure radiated by a collapsing empty cavity is that the wall velocity not exceed about one tenth of sonic velocity. Substitution of this condition ($\dot{R}/c = 0.1$) in Eq. (5) results in values of the radius and "time before collapse" as given by

$$\frac{R}{R_1} \approx 4\left[\frac{P_0}{\rho c^2}\right]^{\frac{1}{3}} \tag{9}$$

and

$$\frac{|t - t_0|}{R_1[\rho/P_0]^{\frac{1}{2}}} \approx 16\left[\frac{P_0}{\rho c^2}\right]^{\frac{5}{6}}\ . \tag{10}$$

The value of ρc^2 for water is approximately 21,000 atmospheres. Equation (9) shows that the collapse of a cavity under an external pressure of one atmosphere or more begins to be influenced by the compressibility of the water while the radius of the cavity is still as large as one-sixth of its maximum value. At that stage, the effect of the vapor and the minute amount of gas ordinarily contained in the cavity is quite negligible. Thus, whatever mechanisms are involved in the collapse and rebound, the compressibility of the liquid plays a primary role.

Equation (10) gives a rough criterion for the range of frequency for which the sound-pressure spectrum, computed according to the incompressive theory, is valid. It may be assumed that the details of the pulse of sound pressure associated with the part of the motion where compressive effects are present do not greatly affect the spectrum for frequencies several times smaller than the reciprocal of the time interval appearing in the numerator in Eq. (10). If the numerical factor is taken, conservatively, to be 4π, it follows that the required upper limit on the frequency is

$$0.005 \; \frac{[P_0/\rho]^{1/2}}{R_1} \cdot \left[\frac{\rho c^2}{P_0}\right]^{\frac{5}{6}} .$$

A number of investigators have presented theories which take into account, in some degree, the effects of compressibility and which show remarkable agreement with experiment in certain respects (9,10). A full explanation of the generation of cavitation noise awaits a solution of the flow problem involved not only during the later part of the collapse but also during the early part of the rebound.

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