VII

ON THE DEVELOPMENT OF THE THEORY OF MARINE PROPULSION

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An attempt is made in this report to survey the essential results which have been obtained in the problem of hydrodynamics of marine propulsion. In this problem the predominant interest is directed towards the flow and the interaction which arise when a body is propelled in the usual way by means of a stern propeller. The Naval Architect is interested in these phenomena since rules for the most advantageous propulsion of the system may be expected if we are sufficiently able to analyse the flow. A great deal of work has been done to find such rules by means of model testing. It is not intended in this paper to enter into this experimental work but to describe the analytical attempts which have become known in this field.

It should be mentioned from the beginning that it has not yet been possible to deal with the problem in its full generality since the assumption of a given hull is made. That is, variations of the ship form are not taken into account when establishing the minimum condition for the propulsive power. We then arrive at a partial solution of the problem which essentially means a discussion of questions related to the theory of a wake adapted propeller. The complete problem, that is considering effects from variations of both the hull and the propeller on the power input, is still far from an analytical solution. Only a very first step in this direction is known in literature [17]. Additional restrictions as well as imperfections of the theory will be mentioned later on.

The paper is divided into 3 parts. In the first one the results of the general propeller theory are reviewed, in the second one the theory of interaction between hull and propeller is briefly discussed. As a synthesis of these two parts the basis of the theory of a wake adapted propeller is developed in the final chapter. New results are not given in the paper. However, it appeared useful to collect the scattered results in order to recognize the stand of the theory which has been achieved.

I. General Propeller Theory

The central problem lies in the determination of the potential flow which is related to semi-infinite and symmetrically spaced helical vortex sheets. The sheets may be assumed of being built up by spiral vortex filaments of which both the diameter and the pitch remain constant when going aft. As follows from considerations of energy, this restriction on the shape of the vortex sheets is permissible with so-called "moderately loaded" propellers, that is, with conditions for which second and higher powers of the induced velocity may be neglected. Relative to the radial coordinate no restrictions on the shape of the sheets are imposed, which is an essential point for the following considerations.

The velocity field of the spiral filaments is determined first and that of the sheets is then obtained by an integration. Two ways are feasible to deduce the field of the filaments, viz., an integration of Laplace's equation or an application of the integral by Biot-Savart. Both ways are found in literature, the first one in papers by Kawada [1] and Lerbs [6], the second one in papers by Maikapar [2], Schubert [3], Guilloton [4], Strscheletzky [5] and Lerbs [6]. Of main interest are the axial and tangential components which are induced on a propeller blade. If this blade is replaced by a lifting line for the present, the components of the velocity induced on the radius $x = r/R$ of this line are represented by the following expressions:

$$d\left(\frac{w_a}{v}\right) = \tfrac{1}{2} i_a \frac{dG}{dx_0} \cdot \frac{dx_0}{x - x_0} \tag{1}$$

and correspondingly for the tangential component if the subscript a is replaced by t. This element of velocity is generated on the point of reference x by a semi-infinite spiral vortex line of radius $x_0 = r_0/\mathrm{R}$ and of circulation dG/dx_0. By $G = \Gamma/\pi D v$ the non-dimensional bound circulation on the radius x_0 is denoted. The "induction factors" i represent the ratio of the velocities induced from a spiral vortex line to that of a straight vortex line. They are introduced to make possible a finite representation of the element of the induced velocity if $x \to x_0$.

On the basis of the integral by Biot-Savart the induction factors may be ascertained in the following way [6]. As a first step, the expressions for dw_a and dw_t as established from Biot-Savart's law may be considerably simplified since both of these expressions may be reduced to a derivative of an integral over $1/T$, T being the distance between an element of the spiral vortex line and the point of reference. One finds that

$$d\,w_a = -\frac{1}{4\pi}\frac{d\Gamma}{dr_0}\sum_{n=1}^{z} r_0 \frac{\partial}{\partial r_0}\int_0^\infty \left(\frac{1}{T}\right) d\alpha \tag{2}$$

$$d\,w_t = \frac{1}{4\pi}\frac{d\Gamma}{dr_0}\frac{k_0}{r}\sum_{n=1}^{z}\left[\frac{1}{k_0} + r_0\frac{\partial}{\partial r_0}\int_0^\infty\left(\frac{1}{T}\right) d\alpha\right] \tag{3}$$

where $k_0 = r_0 \tan \beta_i$, β_i being the pitch angle of the vortex line. The integral may be solved if $1/T$ is expressed first by an integral over the Bessel function J_0 applying the Lipschitz integral and if J_0 is then developed into a series of Bessel functions of the first kind by means of Neumann's addition theorem. In this way one is finally led to the Hankel integral which makes possible the following representation of the induction factors:

Exterior field ($x > x_0$)

$$i_a = 2\left(\frac{x}{x_0} - 1\right)\left(\frac{z}{\tan \beta_i}\right)^2 A \tag{4}$$

$$i_t = \left(1 - \frac{x_0}{x}\right) z \left(1 + 2\frac{z}{\tan \beta_i} A\right) \tag{5}$$

$$A = \sum_{m=1}^{\infty} m I'_{mz}\left(\frac{mz}{\tan \beta_i}\right) . K_{mz}\left(\frac{mz}{\tan \beta_i}\frac{x}{x_0}\right) \tag{6}$$

Interior field ($x < x_0$)

$$i_a = \left(1 - \frac{x}{x_0}\right)\frac{z}{\tan\beta_i}\left(1 - 2\frac{z}{\tan\beta_i}B\right) \tag{7}$$

$$i_t = 2\left(1 - \frac{x_0}{x}\right)\frac{z^2}{\tan\beta_i}B \tag{8}$$

$$B = \sum_{m=1}^{\infty} m I_{mz}\left(\frac{mz}{\tan\beta_i}\frac{x}{x_0}\right)K'_{mz}\left(\frac{mz}{\tan\beta_i}\right) \tag{9}$$

One recognizes that the induction factors depend on 3 variables, viz., on the ratio x_0/x, the number of blades z and the pitch angle β_i. For an application, it is important to note that these factors are independent of the circulation and are solely determined by the geometry of the vortex system. Numerical calculations of the factors have been carried out for $z = 3$ to 5 [6].

It is now possible to solve a fairly general propeller problem, viz., to determine the velocity components which are induced at any radius x of a bound vortex line if the bound circulation is an arbitrarily given function compatible with the end conditions at both the hub and the tip. This problem requires an integration of (1). This integration has two difficulties, firstly that the integral (1) is an improper integral and secondly that the induction factors depend on the unknown induced velocity components by way of the pitch angle β_i. The first difficulty is removed by expanding both the given circulation function and the induction factors into Fourier series. It is then possible to determine the principal value of the integral [3, 6]. The second difficulty requires successive approximations starting in the first step with both w_a and w_t equal zero to determine a first approximation for β_i. The convergence of this process is very rapid for functions $G(x)$ which are met in practical applications. Having determined the components of the induced velocity the force components which correspond to $G(x)$ follow immediately from the law of Kutta-Joukowsky.

It should be mentioned that the analysis of a free-running propeller with minimum kinetic energy within the slipstream for a given thrust arises as a special case of the afore written general equations. In this case the vortex sheets are of a true helical shape corresponding to a rule by Betz [7]. From this rule the relation

$$(w_a/v) + (\lambda_i/x)(w_t/v) = w^*/v \tag{10}$$

follows, within which relation the quantities λ_i and w^* are independent of the radius. Introducing expressions (1), an integro-differential equation for the bound circulation $G(x)$ of the free-running optimum propeller is obtained:

$$\int_{x_n}^{1}\frac{dG}{dx_0}\left(i_a + \frac{\lambda_i}{x}i_t\right)\frac{dx_0}{x - x_0} = 2\frac{w^*}{v} \tag{11}$$

This equation permits a numerical calculation of the Goldstein factor which plays an important role in the application of propeller theory. From Stoke's law, this factor is defined by the following relation:

$$\kappa = (zG)\Big/\left(2x\frac{w_t}{v}\right) \tag{12}$$

From both (11) and (1) the function κ is obtained in a way which is independent of Goldstein's former analysis carried out on a basis of Betz's displacement theorem [8].

Difficulties with slow convergence which arise with Goldstein's series expansions for great values of λ_i are avoided in numerical computations when applying the integral equation.

The relations mentioned so far hold both for an airplane and for a marine propeller. For the latter, however, the replacement of the blade by a line vortex does not suffice because its aspect ratio is usually small. This arises from the requirement to avoid the onset of cavitation from which condition an upper limit for the lift coefficient follows. Replacing a blade by a lifting surface instead of by a lifting line introduces a boundary value problem, viz., to determine the shape and the geometric angle of attack of an infinitely thin section such that a prescribed pressure distribution is realized within the propeller flow. There are two ways known in literature to treat this problem. In a direct way, the velocity components induced from the free and bound vortex sheets are calculated at several stations of the chord length and the boundary condition is satisfied at these stations [4, 5]. These calculations are fairly lengthy also when using precalculated functions. In a faster working approximate method one starts from a skeleton which generates the prescribed pressure distribution in 2-dimensional flow. The propeller flow requires corrections on both the shape and the angle of attack of this skeleton. These corrections follow from the curvature of the propeller flow at the half-way point, which is known from papers by Ludwieg and Ginzel [9], and from the boundary condition at the ¾-point. This latter means an application of Weissinger's approximate lifting surface theory to the propeller flow [10].

Marine propellers also differ from airplane propellers relative to the ratio of the hub diameter to the propeller diameter which is greater for marine propellers. The question arises whether or not the boundary condition on the hub requires corrections on the components of the induced velocity in the case of marine propellers since this boundary condition is not taken into account within equations (4) to (9) for the induction factors. Attempts to determine the order of magnitude of these corrections are made for the free vortex sheets in a paper by Lerbs [6] and for the bound vortices in a paper by Ackeret [11]. Numerical calculations have shown that this correction may still be neglected for a hub-diameter ratio of 0.2.

II. THEORY OF INTERACTION BETWEEN HULL AND PROPELLER

The older method to explore the interaction is based on the laws of conservation of energy and of momentum. The outstanding papers are those by Fresenius [12] and by Horn [13]. From these investigations it became apparent that there is a marked difference in the effects of a viscous and of a non-viscous fluid on the propulsive power. It is shown that a positive viscous wake has a favorable effect on the propulsive coefficient and that a positive displacement wake is unfavorable. The reason for the opposite behaviour of these two components of the wake lies in the energy of the obsolute motion which is left far behind the propeller body. In the case of a viscous fluid the absolute motion consists of 2 parts of opposite direction, viz., the viscous wake of the body and the induced velocity of the propeller race, whereas in the case of potential flow only the propeller field is present. From these considerations it appears that a wake adapted propeller should be designed such that the remaining absolute motion becomes as small as possible. This requires a large axial component of the induced velocity and, therefore, a large thrust element to be generated on those radii on which a large viscous wake is found.

These general statements on the interaction have been considerably extended and refined by Dickmann [14]. In addition to the afore mentioned effects on the propulsive coefficient an influence of the free surface is also considered. From the interference of the wave systems of the hull and of the propeller it is shown that an improvement of the propulsive coefficient takes place if the propeller is placed below a hump of the hull's wave system.

For the purpose of this paper the relation between interaction force and wake is of predominant interest. It follows from the foregoing discussion that the wake and correspondingly the interaction force consist of three parts, viz., a viscous, displacement and wave part, the latter two being potential phenomena. These three parts are mutually dependent, that is, writing the total wake or the interaction force as a sum of the three components each one will be different from that which is obtained when the two other ones are neglected. For instance, calculating the displacement wake from potential flow, effects from viscosity will change the result since, so to speak, the hull is changed by the displacement thickness of the boundary layer.

It is to be expected that the relation between wake and interaction force is represented by different laws for the different components. Our knowledge of these laws is still fairly limited and is advanced best for the case of a deeply submerged body of revolution in non-viscous flow. For this case, Dickmann has introduced singularity methods. That is, the body as well as the propeller are replaced by proper singularities from which follow both the perturbation velocity created by the hull, which is the displacement wake, and the interaction force between hull and propeller, the latter by an application of Lagally's law. This concept has proven very fruitful and has made accessible the interaction problem to a detailed analytical treatment [14 to 20].

The singularities which replace the body with respect to its action on the flow are known to be either axially distributed sources and sinks or surface distributions. The absolute flow created by these singularities in the plane of the propeller is denoted the "nominal" displacement wake.

To replace the propeller by simple singularities infinitely many blades are assumed. This reduces the vortex system of the propeller to cylindrical sheets of ring vortices and straight line vortices. The latter are responsible for the tangential flow and may be neglected since this flow component is of minor interest in these considerations. To further simplify the problem the bound circulation is assumed independent of the radius. What then remains of the vortex system is a cylindrical sheet of ring vortices in the boundary of the slipstream. Relative to the inflow this sheet of ring vortices is equivalent to an uniform distribution of sinks on the propeller disc which may be replaced by a point sink in the propeller center if the action at large distances ahead is considered. It should be mentioned that the equivalence of the propeller inflow and the flow ahead of a sink disc, which has been obtained in the foregoing discussion employing ideas of vortex theory, may also be directly deduced as a first approximation from the fundamental equations of hydrodynamics as shown by Burgers [21].

On the basis of Lagally's law the following fundamental relation for the interaction force ΔR between the body singularities and the propeller point sink is obtained:

$$t_d = \Delta R/T = \rho E_P u_d/T \tag{13}$$

From the potential of the propeller singularity and from considerations of momentum it follows that

$$E_P/F_P = 2w_a \tag{14}$$

so that

$$t_d = 2w_d/(1 + \sqrt{1 + C_T}) \tag{15}$$

The notation is as follows:

t_d = displacement thrust deduction coefficient = $\Delta R/T$, T = thrust, Ep = input of the propeller sink which is related to the loading coefficient of the propeller, viz., $C_T = T/\frac{\rho}{2} v_s^2 F_p$, v_s = speed of the body, F_p = propeller disc area, u_d = velocity induced from the body singularities at the propeller sink, $w_d = u_d/v_s$ = displacement wake fraction.

For a numerical application of (15) the difficulty arises that the body singularities should include the image of the propeller singularity into the body. The absolute flow generated by the complete system of body singularities in the propeller plane is called the "effective wake" and it is this quantity which should be introduced into (15) instead of the nominal wake. This complicates an application of (15) since the images are difficult to determine for a general body of revolution or, from the experimental point of view, since only the nominal displacement wake can be measured. It is therefore important to know the general structure of the relation between effective and nominal wake. For this purpose, it suffices to assume a simply shaped body. Considering the interaction of a sphere with a point sink, Martinek and Yeh have deduced the following relation [17]:

$$w_{d,e} = w_{d,n} \left\} 1 + \frac{E_P}{36\pi v_S} \left(\frac{w'_{d,n}}{w_{d,n}} \right)^2 [{w_{d,n}}^{2/3} - 1]^{-2} \right\{ \ (7) \qquad (16)$$

By the subscripts *e* and *n* the effective and nominal displacement wakes are denoted, respectively, and by $w'_{d,n}$ the derivative of the nominal wake in the axial direction. From this expression, it follows that the effective wake is greater than the nominal wake which is in accord with experimental experience.

A numerical evaluation of the sphere-sink interaction [18] shows that the interaction force decreases both when the radius of the sphere decreases, the distance of the sink from the sphere being constant, and when the distance increases, the radius held constant. However, the decrease of the force is shown to be much more rapid when the distance increases than when the radius decreases. From this it is concluded that it will be more favorable for the propulsion of a deeply submerged body to move the propeller away from the stern of the body than to refine the stern. This holds in a non-viscous fluid. In viscous flow the opposite effects of displacement and frictional wake on the propulsive power determine an optimum position of the propeller.

An obvious extension of the theory is possible when introducing a sink disc for the propeller instead of a point sink and when assuming that the input of the disc depends on the radial coordinate corresponding to the thrust distribution of a propeller. From Lagally's law the element of the interaction force $d(\Delta R)$ which is generated on an annular element of the disc is represented by the relation

$$d(\Delta R) = \rho u_{d,e}\, dE_P = \rho u_{d,e} e_P dF_P \qquad (17)$$

where both the effective displacement wake and the surface sink distribution e_p depend on the radial coordinate *x* of the disc. The dependence of $d(\Delta R)$ on radius necessitates definition of a local thrust deduction coefficient, viz.,

$$t(x) = d(\Delta R)/dT \qquad (18)$$

Expressing the surface density e_p by the thrust distribution dC_T/dx by means of the energy and momentum equations the local displacement thrust deduction coefficient becomes

$$t_d(x) = d\,(\Delta R)/dT = 2w_{d,e} \Bigg/ \left(1 + \sqrt{1 + \frac{1}{2x} \frac{dC_T}{dx}} \right) \qquad (19)$$

We now face the problem of determining the image of a sink disc of variable density into a body of revolution in order to ascertain the effective wake. For this problem, solutions have become known recently. In a paper by Martinek and Yeh [18] certain surface singularities are studied in the presence of both a sphere and a prolate spheroid, among them the sink disc. For the spheroid the solution is given in terms of Legendre functions using spheroidal coordinates. In a subsequent paper by Hunziker [19] the restriction to bodies of revolution of special shape is abandoned and the problem of rotational symmetry is formulated generally. Introducing a distribution of sources of strength e_H over the hull surface H, the boundary condition is expressed by the following integral equation:

$$e_H(x_1,x_2,x_3) + \frac{1}{2\pi}\iint_H \frac{\cos(r,n)}{r^2} e_H(\xi_1,\xi_2,\xi_3)\, dH = q(x_1,x_2,x_3) \tag{20}$$

$$q(x_1,x_2,x_3) = 2\left[v_S \cos(n,x_3) + \left(\frac{\partial\phi_P}{\partial_n}\right)_H\right] \tag{21}$$

$$\phi_P = \frac{1}{4\pi}\iint_{F_P} \frac{e_P}{r_P} dF_P \tag{22}$$

$$r = [(x_1 - \xi_1)^2 + (x_2 - \xi_2)^2 + (x_3 - \xi_3)^2]^{\frac{1}{2}} \tag{23}$$

$$r_P = [(x_1 - x_{1P})^2 + (x_2 - x_{2P})^2 + (x_3 - x_{3P})^2]^{\frac{1}{2}} \tag{24}$$

By x .... the coordinates of the point of reference within the hull are designated, by ξ .... the running coordinates of the hull and by x_P .... the coordinates of the propeller disc.

The solution of this Fredholm equation of the second kind is known to be feasible by means of the method of iteration of kernels.

Having determined e_H, the effective wake in the plane of the propeller may be ascertained from the potential of the source distribution. The paper includes a numerical example for a body of revolution which has been tested formerly by Weitbrecht [22]. The thrust deduction coefficient obtained experimentally amounts to 0.128 and that by computation to 0.119. The difference lies in the right direction and the order of magnitude of the differences is in agreement with the generally held opinion, viz., that the frictional thrust deduction is a small quantity in the case of rotational symmetry.

In papers by Korvin-Kroukowsky [20] the interaction theory is developed on a basis of the singularity concept in the form of a computational procedure. The boundary condition is satisfied at a certain number of arbitrarily chosen control points on the surface of the body which leads to a set of linear algebraic equations for the strength of axially distributed sources and sinks. For the propeller, infinitely many blades are assumed and the radial distribution of its bound circulation is replaced by a rectangular one. Correspondingly, two free vortex sheets of different diameters each consisting of a semi-infinite row of ring vortices are introduced. Their velocity field is derived from Laplace's equation. Taking into account the velocities induced from the propeller vortex system when satisfying the boundary condition at the control points on the hull modifies the strength of the body singularities. The modification approximately corresponds to the image of the propeller system. From the modified singularities of the body the effective wake follows. The thrust deduction force is determined in two ways, vis., on the basis of Lagally's law from the perturbation velocities of the propeller at the body sinks and, alternatively, using surface pressure integration. Numerical results indicate that, for the example treated, the wake fraction is not sensibly affected by the action of the propeller. These results show further that the thrust deduction force decreases rapidly on the body when going forward from the stern. The first four sinks nearest to the stern situated within 12% of the body length already contribute 90% to the total force.

As far as a non-viscous fluid is concerned singularity methods have proven successful to understand the body-propeller interaction and to express this interaction analytically. The results which are known today are restricted to axi-symmetrical flow. The next step should be to explore the effect of a circumferentially non-uniform wake on the interaction force. Replacing the propeller by a vortex system, additional vortex sheets built up from radial filaments become necessary to account for the change of bound circulation in a circumferentially non-uniform inflow. These additional sheets generate additional perturbation velocities on the body singularities which affect the interaction force. Studies are indicated by Martinek and Yeh on a general ellipsoid and by Korvin-Kroukowsky on 3-dimensional bodies. It is to hope that these studies will shed some light on the question whether the increase of the interaction force in a non-homogeneous wake may be explained from non-viscous flow or whether the frictional wake is responsible. The first opinion has been expressed by Dickmann [14] on the basis of first order estimates; the latter by van Manen [23] on the basis of tests on ship models conducted by van Lammeren. In these tests it has been found that the potential nominal wake is fairly evenly distributed over the propeller disc. From this is concluded that a circumferentially non-uniform wake arises essentially from frictional wake which would then be responsible for the increase of the thrust deduction coefficient.

We will now briefly consider a viscous fluid. Unfortunately, the methods to establish expressions for the frictional parts of both the wake and the interaction force cannot yet be regarded as sufficiently complete. What has been done essentially in the afore mentioned papers by Dickmann [14] and Martinek and Yeh [17] is to utilize the laws of the boundary layer of a flat plate. That is, the increment of the frictional force is determined from the increment of the free stream velocity which takes place from the action of the propeller race. The modifications of the boundary layer of a flat plate which arise from the pressure fields of both the body and the propeller and the modification of the displacement flow from the displacement thickness are left out of consideration. With these assumptions the frictional part of the interaction force is found to be a small quantity as compared to the displacement part. Somewhat more detailed are the calculations made by Korvin-Kroukowsky [20]. In this paper velocity profiles as measured experimentally on a body of revolution are introduced into the law of momentum. Further, the effect of the propeller flow on the boundary layer is estimated. Again, the result is obtained that the frictional thrust deduction is small and is essentially independent of the frictional wake fraction. However, these results refer to axi-symmetrical flow. Trying to estimate the influence of a non-homogeneous frictional wake on the interaction force on a basis of boundary layer theory great difficulties are encountered and it has not yet been possible to establish an order of magnitude of this effect. In this situation one is compelled for the time being when designing a wake-adapted propeller to use semi-empirical relations for the dependence of the local thrust deduction coefficient on the wake fraction, the latter being known as a function of the radius from experiment. Either, the assumption is made that the local thrust deduction coefficient is independent of the wake fraction, or the following relation is employed which has been given by van Manen [23]:

$$[1 - t(x)]/[1 - t_0] = \{1 - w(x)]/[1 - w_0]\}^{0.25} \tag{25}$$

The quantities t_0 and w_0 are average values as determined by analysis of a model propulsion test. The relation is an interpolation formula obtained from numerical results for $t(x)$. Denoting the thrust deduction coefficients which are related to the elements of an annual ring of the propeller disc by $t(x, \varphi)$, $t(x)$ is considered the average value of $t(x, \varphi)$. To calculate $t(x, \varphi)$ two essential assumptions are made. Starting from the axi-symmetrical case, it is assumed that the local frictional thrust deduction is proportional to the local frictional wake fraction. Secondly, when the loading changes because of a circumferentially non-uniform wake, that the frictional thrust deduction is inversely proportional to the frictional wake. This means that

$$\begin{aligned} t_f(x,\varphi) &\approx t_f(x)/w_f(x,\varphi) \\ &\approx w_f(x)/w_f(x,\varphi) \end{aligned} \tag{26}$$

is written. Whether or not these assumptions are physically sound is unproven. It should be mentioned that it makes only little difference for a propeller design whether the assumption $t = \text{const}$ is introduced or whether the function $t(x)$ as follows from this relation is utilized.

In spite of the imperfections of the existing theory of hull-propeller interaction it has proven of great value in explaining experimental findings on the thrust deduction coefficient which cannot be understood otherwise. In recent work on the scale effect of propulsion coefficients by van Lammeren, van Manen and Lap [24] the result has been obtained that the thrust deduction coefficient decreases both when the dimensions of the model are decreased and when the roughness of the model surface is increased for the same model. The explanation follows from the afore mentioned interplay of the displacement thickness of the boundary layer with the singularities of the body which may be axially distributed sources and sinks. To describe properly the displacement flow in a viscous fluid, an "effective" hull should be introduced which is the hull of the body augmented by the displacement thickness. The strength of the singularities depends essentially on the curvature of the hull such that the singularities become feebler if the curvature becomes smaller. This happens with the effective hull if the displacement thickness becomes greater either because of a smaller Reynolds number or because of increased roughness. Reducing the input of the singularities decreases the interaction force from Lagally's law which gives the explanation for the afore mentioned experimental results.

III. THEORY OF THE WAKE ADAPTED PROPELLER

The basis of the available theory is a steady relative flow, i.e., the wake is assumed to depend only on the radius. The problem is to ascertain the bound circulation such that the useful power of the system "ship and screw" becomes a maximum value for given quantities of power input, advance ratio and wake distribution. As a generalization of the optimum condition of a free-running propeller, the condition for a wake adapted propeller may be written in the form

$$\tan \beta_i = F(x)/k \tag{27}$$

where k is independent of the radius. The optimum function $F(x)$ is still to be determined. In the case of a free running propeller, $F(x) = \lambda/x$.

Combining the optimum condition with the geometrical relation

$$\tan \beta_s = \left[(1 - w) + \frac{w_a}{v_s} \right] \Bigg/ \left[\frac{x}{\lambda_s} - \frac{w_t}{v_s} \right], \qquad \lambda_s = v_s/R\omega \tag{28}$$

which follows immediately from the diagram of the relative flow, and expressing the components of the induced velocity by their respective induction factors, an integro-differential equation for the bound circulation of the optimum wake propeller has been deduced by Lerbs [6]:

$$\int_{x_n}^{1} \frac{dG_S}{dx_0} [k i_a + F(x) i_t] \frac{dx_0}{x - x_0} = 2 \left\{ F(x) \frac{x}{\lambda_S} - k[1 - w(x)] \right\} \tag{29}$$

$$G_s = \Gamma/\pi D v_s$$

With $w = 0$ and with $F(x) = \lambda/x$ this equation passes over into the equation (10) for a free-running optimum propeller. An approximate solution is obtained if both G_S and the induction factors are developed into Fourier series. This leads to a system of linear algebraic equations for the coefficients of the bound circulation. For a practical application in propeller design work an assumption relative to the geometry of the relative flow is introduced which simplifies the calculations considerably without losing too much accuracy. This is explained in detail in a paper by Eckhard and Morgan [25].

It remains to determine the optimum function $F(x)$. This function follows from a rule which has been expressed first by Helmbold [26], viz., that the propulsive coefficient of an element of the propeller which is related to an increment of the bound circulation at any radius be independent of the radius for optimum conditions. There are several attempts found in literature to establish $F(x)$ on a basis of this rule, see [27], [23] and [28], from which the form

$$F(x) = \frac{\lambda_s}{x}\,\phi\,[w(x),\, t(x)] \tag{30}$$

follows. In the opinion of the author, however, the assumptions made in the afore mentioned papers include oversimplifications so that the results can not yet be considered final.

This is about the status to which the theory of marine propulsion is developed today. Improvements which are desirable have been mentioned in the paper. It is essentially the theory of interaction which needs to be extended whereas the propeller theory has been developed in a sufficiently general form, in the opinion of the author. In particular, the effects of non-homogeneties of the wake, including effects from the rudder need further considerations. This point is essential. In addition, the hull-propeller interaction effect of the unsteady relative propeller flow arising from a finite number of blades in a non-homogenous wake should be investigated.

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DISCUSSION

A. J. Tachmindji

Dr. Lerbs has given us a very good summary of the fundamental and basic work that has been done in the last few years in the development of propeller theory. His personal work has contributed materially in the investigation of vortex theory when the circulation distribution is considered arbitrary, and when the induced velocities can be expressed in terms of induction factors. By means of this method it has been possible to compare the validity of the circulation theory as derived by Betz's and Goldstein's work for lightly loaded propellers where the condition of normality of the induced velocity is assumed to hold true. The induction factors method, therefore, gives a means of evaluating the applicability of the lightly loaded propeller theory to moderately loaded propellers. Furthermore, it also allows the evaluation of such effects as wake distribution and hull interaction on the loading distribution of the propeller.

The paper has given an excellent outline of the work conducted on propeller-hull interaction, a problem which is of primary concern to the propeller designer. It is by increasing development in such areas that a more fundamental understanding of propeller action will be accomplished.

The increasing use of propellers with relatively large hub diameters has emphasized the need of a solution for the circulation distribution for these cases. Such a circulation distribution may be considerably different than the one obtained by considerations of zero hub diameter. Through the use of corrective induction factors, Dr. Lerbs has investigated the effect of the presence of the hub. The problem has been pursued further at the David Taylor Model Basin and it was possible to extend Goldstein's work and derive the potential for the optimum circulation distribution[1] to include the effect of a finite hub. Similar work has also been conducted by McCormick who, however, solved the problem for simplified boundary conditions at the hub radius. Figure 1 shows a plot of the Goldstein factor *K*, which is defined as the ratio of the circulation for a propeller having a finite number of blades to the circulation for one having an infinite number of blades, plotted against the propeller radius. This has been obtained for a four bladed propeller operating at a $\lambda_i = 0.25$ and shows the effect of hub diameters of 0.2, 0.3 and 0.4 as compared to a propeller having zero hub. Points calculated by means of the induction factors are also shown.

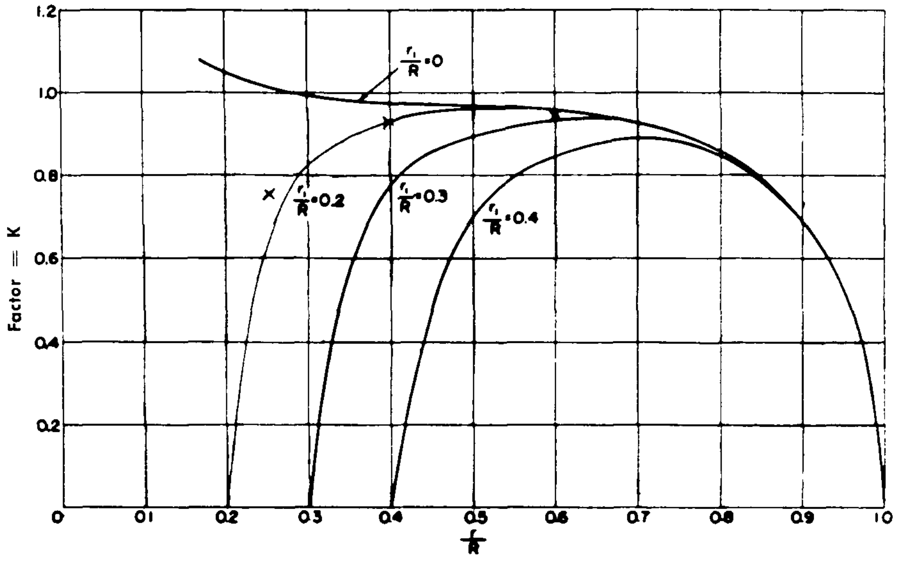

Figure 1 - Effect of Hub Diameter for $\mu_0 = 4$, $p = 4$

Determined by Induction Factors for $\frac{r_1}{R} = 0.2$

Figure 2 shows the actual circulation distribution for the same propeller and the same cases. These calculations have been performed by using approximations to the solution but it is planned to calculate the exact solutions at DTMB.

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[1] DTMB Report 1051

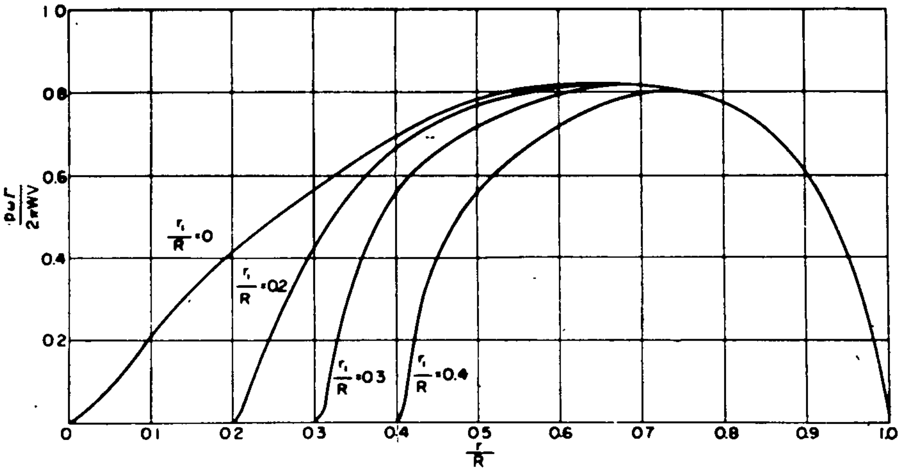


Figure 2 - Effect of Hub Diameter on Distribution of Circulation for $\mu_0 = 4$, $p = 4$

Induction factors have also been used for the development of the theory for wake adapted contrarotating propellers. Although considerable effort has been expended, particularly by Lieber, Yen and Spiegel, in trying to determine the potential function for contrarotating systems, calculations cannot yet be performed by the theory. The use of induction factors is presently the only method available without going into arbitrary assumptions on the applicability of Goldstein's work for such propellers. It would be interesting to hear Dr. Lerbs' opinion as to the magnitude of the time dependent effects on contrarotating propellers and the need of theoretical work in this subject.

A. Silverleaf

I wish to add a few footnotes to Professor Lerbs' lecture rather than comment directly on the theory he summarised. One of the duties of the propulsion and cavitation group at Ship Division, N.P.L., is to design propellers which will be built and fitted to ships, and we are therefore concerned with both propeller theory and its practical consequences. The theoretical developments of the last ten years have undoubtedly been of very great value as a firm guide to practical design, and, as Mr. Tachmindji mentioned, Prof. Lerbs' own part in this has been a major one. However, it is necessary to check the consequences of using these theoretical design methods, and, whether we like it or not, at present the only reasonably precise way of doing so is by experiments with models. This produces some disconcerting features and also presents some difficulties. For instance, propellers designed by modern theoretical methods quite often have a knack of being slightly less efficient than those designed by what may be called rather 'slap-happy' methods. Further, these 'empirical' propellers are often less sensitive to variations from their designed operating conditions which, of course, occur owing to changes in loading, and this adds to the difficulties of using theoretical methods in designing propellers for ships which have to operate under widely varying conditions.

Many of the problems in the model experiment techniques can be grouped together under the general heading of scale effects. Thus we have found at N.P.L. that propeller models often suffer heavily from laminar boundary layer flow, affecting their performance under both non-cavitating and cavitating conditions, while differences in free stream turbulence levels between model and full-scale must also be taken into account. Another problem is that of boundary wall interference effects in propeller water tunnels; this is susceptible to much more detailed mathematical study than it has yet been given. From this point of view it is possible that the effective length of a marine propeller is much greater than its physical dimensions would suggest, and is perhaps related to the axial dimension of the non-parallel induced flow and wake.

I am a little disappointed that Prof. Lerbs did not say a little more about the future of marine propulsion. Undoubtedly machinery powers are going to increase—as we have been reminded earlier today—thus the powers to be transmitted by orthodox propellers will also increase. Like Mr. Tachmindji, I believe that for this reason contra-rotating propellers may have quite a lively marine future, particularly as recent developments have been made in their theory. While they may not be applied to spectacular ocean liners, they may well be of value in future cross-channel type ships. It is also clear that we need, and shall increasingly need, further developments in the theory of fully cavitating propellers. Much work is at present going on in this field, of course, and opportunities for merchant ship applications will probably soon occur. However, there are many of us who believe that orthodox propellers will not be entirely adequate for future really advanced conditions. The overlapping problems of heavier loadings, the incidence and effects of cavitation, and propeller induced vibration may well force us to consider rather less orthodox propulsion methods. Although Prof. Lerbs uses the word 'propulsion' rather than 'propellers' in his title, he has in fact confined his comments to orthodox propellers; some remarks on other propulsion devices would have been a valuable addition.

The equipment needed for model experiments with these advanced designs will be considerably different from that used now. In this country and in Britain we are attempting to meet this need by building much larger water tunnels than we have at present, but I am perturbed that we are embarking on these expensive pieces of equipment with such an inadequate knowledge of how propellers perform in them, their limitations, and the relation between tunnel results and full-scale performance.

Finally, I should like to register a mild disagreement with the apologies made this morning for the naval architect's laggard approach to his problems compared with the aeronautical engineer. I believe that the ship designer has not a great deal of which to be ashamed, and I would suggest that Prof. Lerbs' exposition of the theory of broad bladed propellers is an excellent example of one field in which the marine approach has been at least as thorough as anything comparable in aircraft work.

T. Y. Wu

I wish to compliment Professor Lerbs on his interesting lecture which presents some general and instructive viewpoints. As was described in his lecture, the vortex theory for propeller problems has been developed to account for the flow conditions where both the inflow and the bound vortices are arbitrary functions of the radius. This contribution then narrows the gap between theory and application, for the marine propellers in general operate with a nonuniform inflow. Concerning the theory itself, I wonder whether a general expression has been obtained for the optimum configuration of the trailing vortex sheets when the propeller operates at the highest efficiency with a given inflow and loading distribution. I am also interested to know the feasibility of generalizing the theory to include two additional factors: 1. the effect of heavy loading due to the contraction of the slipstream and 2. the case of general nonuniform inflow, such as a shear flow with no axial symmetry, as is typical for stern propellers.

In the problem of the interaction between hull and propeller, some further improvements perhaps can be visualized. In practice, most conventional ships are likely too "fat" for the thin ship theory to remain a good approximation, especially for those ships with blunt stern section for which the interaction problem is concerned. The idea thus seems inviting that improvements could be obtained by using the slender body theory to represent the hull with the added displacement thickness of the boundary layer and by using a simplified vortex distribution for the propeller disc. In the case where the boundary layer separates from the hull in front of the propeller, it would be of interest to study the interaction under that adverse condition and the effect on separation phenomena due to the propeller action.

S. F. Hoerner

It has been mentioned here this morning several times that there is a lack of understanding and contact between "mathematicians" and the naval architects. Now, as one who has been on the inside of aviation as well as naval architecture, I would like to tell you that that situation prevails in the field of aviation just the same.

There are highly skilled mathematicians working on theory, and there is the engineer who tries to apply the results. Usually, the engineer is lost. He does not fully understand the language of theory.

Since the advent of modern computers there is also the idea that those computers would solve all problems. This is not so at all. The computer is a "perfect" idiot; the scientific term is "idiot-savant." Now, the man who handles the computer must provide the brains to put the right information into it, and to give an adequate interpretation of what is coming out of it.

In the field of propellers, this is also true to a certain extent, and this has been realized in aviation for some time (for some twenty-five years). To help the engineer, methods have been evolved known as "polar methods," replacing the propeller blades by "equivalent" wings. The characteristics of the propeller are then reduced to those of wings.

Such an interpretation is much more understandable to the engineer.

I would suggest that for the benefit of the engineer, we would revive these methods a little bit. This does not mean, however, that I would critize the efforts of men such as Dr. Lerbs. He is in his perfect right to develop an accurate theory.

R. R. Hunziker

I would like to comment on the subject of "thrust deduction" and interaction phenomena of propeller and hull in deeply submerged submarines, a subject which was a part of the extensive and sound lecture of Dr. Lerbs [1]. Dr. Lerbs started from the general propeller theory and considered later the hull-propeller system, pointing out the interaction character of the hydrodynamic phenomena. He mentioned the use of Lagally's theorem [2], as starting point of the theory, and the introduction of a sink-disk as a "first approximation" for the propeller. I would refer to my recent theoretical work investigating the interaction phenomena of the hull-propeller system.

Details may be found in my last paper [3] (30 August 1956) which supersedes my earlier paper cited by Dr. Lerbs.

First, I have dealt with the construction of the hull-propeller velocity field that satisfies the hydrodynamic equations in the form of Oseen [4], as an extension of Burger's [5] analysis for the open water propeller.

Departing from the Oseen equations, I have proved that in the steady flow hypothesis (used also by Burgers), the velocity field in the first approximation contains a system of helical vortices shed by a propeller of an infinite number of blades.

In the exterior of the slipstream and around the body, the field has zero tangential component, and is given by the superposition of the flow for the body alone, plus the axial and radial fields of a circular sink-disk over the propeller circle, plus an interaction harmonic field with which the hydrodynamic boundary condition remains satisfied over the body. This idea of the interaction field, characterizing the effective wake over the propeller circle, was introduced by Helmbold in 1938 [Ref. 6 p. 354 Eq. 5a]. In the interior of the slipstream the flow is rotational and at infinity has a net augmentation of the axial asymptotic velocity U, which is equal to the strength c of the simple layer over the disk. It is not too much to observe that the constructed velocity field which is singular over the propeller disk, implies a removal of the D'Alembert paradox for the body.

As the axial component of the pressure integral over the body is characterized by the velocity field and Bernouilli theorem, it is legitimate to apply Lagally's exterior theorem [2] to calculate this force of resistance in the potential flow that comprises the body. In this way, a general expression for the "thrust deduction fraction" is derived:

$$\Theta = \frac{\Delta T}{|T|} = \frac{\psi_1 + \left(\dfrac{c}{U}\right)\psi_2}{1 + \frac{1}{2}\left(\dfrac{c}{U}\right) - (\rho c U A)^{-1}\Delta P} \tag{31}$$

where T is the thrust force over the propeller support. $U\psi_1$, is the average of the normal velocity component over the propeller circle, for the body without propeller ($U\psi_1$, the "nominal wake"), and $c\psi_2$ is the average of the normal velocity component of the interaction potential. The density is denoted by ρ, the propeller circle area by A, and ΔP is the increment in the integral of the pressures over the intersection of the rotational slipstream with the plane at infinity.

ΔP is a consequence of the rotation of the water in the slipstream and introduces a dependence of Θ on the torque. Since in general the angular velocities impressed on the particles of the slipstream are small compared with the angular velocity of the propeller [7, 8, 9] it is assumed $\Delta P = 0$. In the limiting case of very light loadings ($c \to 0$), it is $\Theta \to \psi_1$, and the thrust deduction may be approximated by $\Theta = \psi_1$, which is a generalization of Fresenius's expression [10]. The expression for Θ (generalization of Helmbold formula [6]) for the moored condition, with $\left(\frac{c}{U}\right) \to \infty$, must be taken with reservation due to the asymptotic character of the first approximation to the solution of Oseen equations.

Therefore, the consideration that $\lim\limits_{U \to 0} \Theta = \Theta_0 = 2\psi_2$ is the thrust deduction coefficient for $U \to 0$ (moored condition) must be taken as a conjecture of empirical validity.

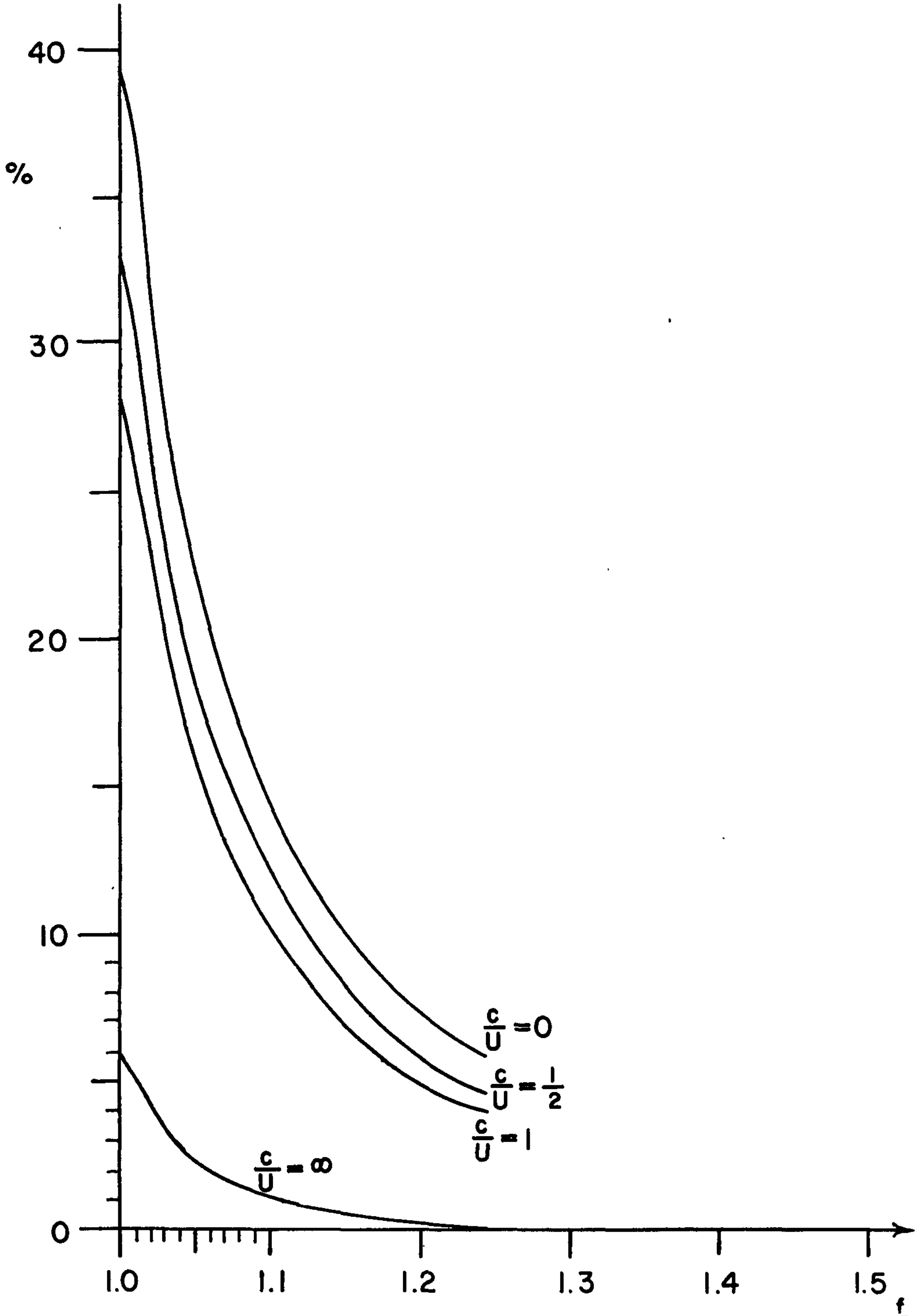
The hypothesis of a boundary layer with extreme backward separation corresponding to a favorable distribution of pressures due to the propeller inflow, seems to accord with bodies of reasonable shape such as torpedoes and streamlined submarines.

The solution of the corresponding harmonic problem for the case of the ovoid ellipsoidal hull was developed in Legendre functions and ψ_1 and ψ_2 have been tabulated for different configurations [3]. The results indicate that the thrust deduction fraction exhibits a rapid diminution with the increase of the distance of the propeller to the hull and with the increase of propeller diameter. (See Fig. 1.)

These theoretical results are slightly lower than the experimental values for the ovoid hulls tested by Weitbrecht [11]. This confirms the assertions of Dickmann [12] and van Manen [13] that for the case of radial symmetry the "frictional thrust deduction" is small. The theoretical values of the figure correspond to one of the cases investigated experimentally by Weitbrecht and show a typical variation of Θ with the distance ratio $\frac{f}{a}$ of propeller to hull, and for different rations $\frac{c}{U}$. (a, b, ellipsoid axes, R radius of propeller.)

Dr. Lerbs [1] has pointed out that "imperfections of the theory of interaction exist relative to viscous flow and to time-dependent flow on the propeller which arises with a finite number of blades in a non-homogeneous wake." A possible source of small discrepancies with experiments [11] is that the hull is not exactly our prolate ellipsoid.

Experiments with such prolate hulls may be performed and it is very reasonable to expect that the results would be also in good agreement.



40
%
30
20
10
0
1.0
1.1
1.2
1.3
1.4
1.5
$\frac{f}{a}$
$\frac{c}{U}=0$
$\frac{c}{U}=\frac{1}{2}$
$\frac{c}{U}=1$
$\frac{c}{U}=\infty$

Another more important source may be the separation of the flow, due to viscosity, in the hull after-body. Further, the simplifying assumption $\Delta P = 0$ gives lower values for the theoretical thrust deduction fraction.

Besides the proof of the hydrodynamic equivalence of the infinite blades propeller to steady flow, the approximate steadiness of the flow, depends essentially on the angular velocity Ω of the propeller, since the periodicity of the flow field is $\propto (2\pi\Omega z)^{-1}$, where z is the number of blades [8]. The thrust of the propeller and the thrust deduction should be quasi-non-periodic, if Ω is great enough.

Because our results are in substantial agreement with the observed values of time average thrust deduction [10], it is very plausible to expect that a more involved time-dependent theory, that is, for a finite number of blades, would not introduce differences significant for practical purposes.

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J. S. Florio

The purpose of my comment is to emphasize some critical aspects of the theory of the "thrust deduction" set forth by Dr. Lerbs [1] and commented on by Dr. Hunziker [2], as essentially a theory of hydrodynamical interaction, and particularly the generalized formula of the Helmbold type [3], which is a definitive conclusion, on some established hypotheses, following the line of thought mentioned by Dr. Lerbs.

The primitive Helmbold formula was derived directly from the principle of conservation of energy [4], and was the most important contribution, without the application of the Lagally theorem. In the other development of the theory (Dickmann) [5, 3] the starting point is the fundamental "exterior theorem" of Lagally [6], based on the Bernouilli equation which, as is well known, expresses the same Conservation Principle; this theorem, which goes beyond the earlier work by Cisotti on the "D'Alembert Paradox," taking account of continuous distributions of singularities of a general field (ideal fluid) expresses the corresponding resistance force and torque. Betz and Dickmann [5] applied partially Lagally's theorem only to calculate the force exerted in some special cases of submerged bodies. Some authors have used the particular forms of Lagally's theorem, for the case of point singularities, as developed by Betz, but sometimes with improper interpretations that exceed its true physical meaning, or ignoring the true power of the theorem, which avoids some usual considerations—[Dickmann: "Kräfte auf seine Senke," "Widerstand der Senke," [5] etc., (pp. 468, 649, etc.). Also in references [7] and [8].] It is evident that forces are exerted only when there are bodies which experience pressure, and no more. There are not "forces acting between singularities." These considerations are not justified in any strict field theory of hydrodynamic phenomena.

The new formulas given [3] for the "thrust deduction" phenomenon, as essentially an interaction phenomenon, not only signify the correct application of Lagally's theorem in its original form, but also arise from the direct construction of the total velocity field satisfying Oseen's equations, as extension of Burgers' solution for the open-water propeller [9], containing an "interaction field."

Dr. Lerbs, in his interesting article entitled "Marine Propulsion" [10] has well observed that (for infinite bladed propeller) ". . . the tangential component of the induced velocity is of minor interest in these considerations." Indeed, in reference to the equivalence of the velocity field to a flow with infinite helical vortices over the slipstream, there is an interesting result. It has been proven [3] that even with the vortices over a non-cylindrical slipstream, the total velocity field has zero tangential component (in cylindrical coordinates) around bodies with a plane of symmetry. This result is valid in the "first approximation" in Oseen theory.

In this velocity field, the singularities are those for the flow grad $(U\phi_j{}^*)$ around the body alone, plus a simple layer over the propeller circle with potential ϕ_1, plus a simple layer over the hull. The latter corresponds to the "interaction field" grad $(c\phi_i{}^*)$ with which the hydrodynamic boundary conditions remain satisfied over the body, subjected also to the propeller's inflow, grad ϕ_1. This modification of the field around the hull is the cause of a resistance to the advancement of the hull. This determines with precision the meaning of the term "interaction."

The whole field, which has a region of singularities, satisfied the conditions for which the D'Alembert paradox is removed; and the hull is subjected to a hydrodynamic resistance (or a "thrust deduction," if a reduction of the thrust of the propeller is referred to by this resistance), which is expressed, as derived directly from Lagally's theorem, by [3]

$$\Delta T = -\rho c \iint_s \left(\frac{\partial \phi^*}{\partial x_3} \right)_s ds \tag{32}$$

where $\phi^* = U\phi_j{}^* + c\phi_i{}^*$. The total velocity potential is $\phi = Ux_3 + U\phi_j{}^* + c\phi_i{}^* + \phi_1$. The propeller thrust over the support is: $T = -\rho c(U + c/2)A + \Delta P$, where ΔP is an increment due to the rotation of water in the slipstream, T depending only on the values at infinity, U and $U + c$, of the velocity field in the exterior and interior to the slipstream respectively. The net thrust $T + \Delta T$ of the propeller, at a certain steady velocity of advancement, $-U$, is assumed to be equilibrated by the frictional resistance transferred to the hull by the boundary layer. The substantial agreement with observed values of the time average thrust deduction shows the plausibility of the theory (ref. [3] p. 40).

Without further comment, Dr. Lerbs has mentioned [1] (also ref. [10], p. 283), a type of approximation for the velocity field of the propeller. It consists in an unjustified conjecture replacing the helical free vortex sheets, that corresponds to a non-steady theory, by an infinite number of equidistant vortex disks along the propeller axis. Besides the latter field is computed as steady, we have to recognize that this arbitrary geometrical assumption cannot give a representation of the propeller field with its natural slipstream. Otherwise, in the steady case of the infinite number of blades, the mentioned "representation" fails completely since the actual flow around the body is uniquely characterized only as the field of a sink disk plus a perturbation potential field grad ϕ^*. This results as a steady solution of Oseen's equations in the first approximation, which contains a continuous (never discrete) ring vortex distribution over the slipstream. No other distribution is compatible with the motion equations, and a discrete distribution is not an approximation. For a steady flow theory it is not necessary to consider more than a general simple layer distribution over the propeller circle. For a non-steady theory it is at least necessary to compute a field compatible with the existence of a finite number of helical rotating free vortex sheets.

The great importance of the theory and formulae mentioned by Dr. Lerbs and here commented, lies in its broad applicability, especially the results and general formulae of ref. [3].

A solution for general ellipsoids is at present in development as extension of the above results [3, 12].

I think that the extension of the theory, formulae, and method of calculus for submerged hull with two propellers is immediate, with the appropriate "interaction potential" for the total field.

The interaction theory of "thrust deduction" for the surface ship is in development [11, 12], considering the free surface of water. The "interaction field" has a quite different form. The velocity field for the hull-propeller combination is characterized as boundary value problem with a linear boundary condition on the free surface. The perturbation potential is a modified Michell's potential that includes an interaction term due to the propellers inflow [12]. The essential result is that the "Potential thrust deduction" appears as an augmentation of the wave resistance.

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J. Martinek and G. C. K. Yeh

We wish to congratulate Dr. Lerbs for his fine and comprehensive presentation of a subject to which he has contributed so much in the past and in which he has pioneered to such an extent that classical hydrodynamics has again been established as a powerful tool for solving so complicated a problem as marine propulsion. We are referring particularly to the theory of interaction between hull and propeller and the theory of wake-adapted propeller. Not all in the audience may have been aware of the fact that the researches in the last hundred years in hydro- and aerodynamics were essentially concerned with the motion of bodies where the disturbance on external forces due to the presence of the bodies was usually negligible. If we consider, however, the fact that in the case of a self-propelled vessel such disturbance is of first order, we need an analysis to study the interaction of flows and bodies. To Dr. Lerbs must go our deep appreciation, for it was he who introduced us to these subjects and whose wise counsel and encouragement were vital for whatever improvement and understanding we have been able to achieve on these problems.

We should like to mention now some of our recent results and their interrelation with Dr. Lerbs' work, by which they were inspired.

a. The application of the sphere theorem by the discussors to the problem of hull-propeller interaction is of course an approximation. Nevertheless, it was the simplest conceivable model of interaction available and it was applied with the objective of searching for a relationship which can utilize model testings. The reward was indeed surprising, as a relationship could be found which in order to obtain the interaction force requires only measurements of the flow field at the domain of the propeller location of a towed vessel and the free-running propeller itself, which complies with the conventional testing practice. This result encouraged our endeavor to study more complicated bodies such as oblate and prolate spheroids, and ellipsoids, as well as general bodies of revolution, with the objective of finding similar laws which may govern the problem of interaction. Indeed, in our Final Report II [4] a theorem has been developed for the prolate spheroid which was not directly stated, but was indirectly a result of the analysis *, as was found later by inspection. This theorem opens the way to finding a general method which will lend itself to the evaluation of the disturbance potentials and velocities for those cases where bodies of more complicated shapes are encountered. It indicates that the customary testing procedure is essentially sound and needs only a slight modification. Most important of all, it becomes apparent that more extended and refined measurements at the propeller disc of a self-propelled model, which to date have defied any such attempt, are not necessary. For the prolate spheroid we can demonstrate this in a few lines. The potential function of the totality of singularities which includes discrete ones as well as distributions in an infinite ideal fluid domain can always be expressed in the form

$$\phi_0 = \sum_{n=0}^{\infty}\sum_{m=0}^{n} [\cos(m\varphi) + A_{mn}\sin(m\varphi)]P_n{}^m(\eta)\begin{cases} B_{mn}Q_n{}^m(\xi) \\ C_{mn}P_n{}^m(\xi) \end{cases} \text{for} \begin{cases} \xi > \xi_0 \\ \xi < \xi_0 \end{cases} \tag{33}$$

(ξ_o is ξ of the singularity)
The disturbance potential ϕ_1 due to the presence of a prolate spheroid $\xi = \xi_1 < \xi_0$ is a solution of the Laplace equation and can be written as

$$\phi_1 = \sum_{n=0}^{\infty}\sum_{m=0}^{n} [\cos(m\varphi) + D_{mn}(m\varphi)]E_{mn}P_n{}^m(\eta)Q_n{}^m(\xi) \quad \text{for } \xi \geq \xi_1 \tag{34}$$

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* of finding an exact solution for the interaction potential due to the action of an infinite-bladed propeller in the presence of a hull of prolate spheroidal form.

The general exterior theorem [6] of the prolate spheroid reads then

$$\left.\begin{aligned} D_{mn} &= A_{mn} \\ E_{mn} &= -\frac{P_n{}^{m1}(\xi_1)}{Q_n{}^{m1}(\xi_1)}\, C_{mn} \end{aligned}\right\} \tag{35}$$

which is not confined to the case of axial symmetry. Hence we see that the remarks above referring to the problem of testing are substantiated. Furthermore, numerical examples of such spheroids compared with Weitbrecht's tests show excellent agreement. Theorems for more general bodies have been found and are now in the process of publication. In conclusion, we can say that the vital part of the interaction force or, more generally, thrust deduction, namely the interaction potential, has been successfully clarified and established.

b. The topics and formulations discussed in a. are still confined to two restricted cases: the propeller is representable by a singularity distribution and, second, the hull form is supposed to be completely given. In practice, however, we like to know after the stage of preliminary design what minor changes in the hull form and the propeller characteristics could be effected in order to obtain an optimum combination of hull and propeller in the sense that for a given constant input power a maximum useful power can be obtained. This requirement was first stated by Dr. Lerbs [2] and expressed in terms of the local thrust deduction coefficients and the local induced velocity components. The formulation of this optimum problem whereby the average thrust deduction coefficients and the average induced axial velocity components were expressed in terms of the hull form parameters, and the disturbance velocity has been given in Final Report I [3]. At that time the general method of a. was not known and the sphere was used as a substitute. Nor has the thrust deduction formula been in its general form, as it was first derived by R. Hunziker [7]. This problem can now be analyzed where the mathematical work can be performed in a straightforward manner.

c. Finally, the general method applied recently by R. Hunziker (see Discussion by Dr. Hunziker) has confirmed the assumption that the essential information necessary for the thrust deduction coefficient is the disturbance potential of the body in the presence of the propeller in an ideal fluid flow, an assumption which was adopted *a priori*, without proof, by the discussors in their previous work.

In conclusion, we can say that some of the criticism expressed by other discussors on the fruitful use of the propeller theory will definitely be removed if the state of the art of wake-adapted propellers as already available is applied to its fullest extent to deeply submerged vessels.

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J. J. Eisenhuth

The paper by Dr. Lerbs was indeed very interesting and touched on some important aspects of the theory of marine propulsion. The comments which are about to be made are only intended to bring to the attention of the readers some other comparatively recent work that has been done in this general field, specifically in the field of torpedo propulsion. It is felt that certain phases of this work might be of interest to the ship propeller designer. The torpedo propeller work that will be mentioned here will reflect primarily the writer's direct familiarity with what was done at the Ordnance Research Laboratory of the Pennsylvania State University. The phases that will be touched upon will not be all-inclusive since some of the work has been of a classified nature.

Mention should be first made of work done on the theory of optimum propellers. In Reference [1], F. Lane describes a variational method applied to the problem of optimum wake-operating propellers. His method simply involves the variational problem of minimizing the integral of power input while holding constant the useful or thrust work integral. The solution of the problem establishes the condition of optimum distribution of bond circulation along the blade for minimum power loss at any required value of thrust or useful power.

On the problem of representing a blade of finite chord with a bound vortex or lifting line, Reference [2] presents an interesting way of handling an otherwise very complicated calculation. In this reference B. W. McCormick applies an electromagnetic analogy to the Ginzel-Ludwieg theory for the correction of cambers of wide-bladed propellers. McCormick has also reported on the investigation of such subjects as the effects of a finite hub on the optimum propeller and the minimum pressure in a trailing vortex system (Reference [3 and 4]).

In Reference [3], Goldstein's analysis of a propeller satisfying the Betz condition is extended to the case where hub radius is appreciable relative to the propeller radius. The boundary condition which is satisfied is that of an infinitely long cylinder representing the hub. It is shown that for a trailing vortex sheet of given pitch, the insertion of a hub increases the value of the bound circulation of the line vortex generating the sheet. This change is more pronounced the smaller the number of blades.

In Reference [4], a report is made on a fairly extensive study made on the subject of tip vortex cavitation. In that study, the problem of predicting the minimum pressure in the trailing vortex system of a planar lifting surface was investigated both theoretically and experimentally. A semiempirical method of making these minimum pressure predictions was finally evolved, and later applied successfully to the design of torpedo propellers.

The items mentioned thus far are summarized in Reference [5]. This reference was more or less intended as a handbook for the design of torpedo propellers.

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R. W. L. Gawn

The development of theory to the stage that many propeller designers have been encouraged to apply the methods to practice has really only materialised during the last decade or so, although the basic circulation theory was propounded long ago. This is not at all surprising, since detailed corrections are involved for blade width, number of blades and other characteristics and it was necessary to establish these with the degree of accuracy essential to the propeller designer. In fact no more than a few years ago the induced velocities on the lifting line could only be reliably determined for an optimum distribution of circulation. Dr. Lerbs extended this to an arbitrary distribution and later evolved a method for determining the pitch corrections to take account of lifting surface effect. Such details generally are more important for propellers with wide blades as the theory is then less accurate and progressively so with increase of width. Dr. Lerbs' approach has been an invaluable guide in propeller design at the Admiralty Experiment Works, Haslar, and this opportunity of paying tribute to his admirable work is warmly welcomed.

There are, of course, many major aspects of design that the theory of propulsion does not cover. For example, full regard must be paid to strength, stiffness, manoeuvring qualities, propeller induced vibration and erosion, as well as propulsive efficiency over the operational speed range of the ship. The propeller must be suited to the hull and to the machinery. Many admirable researches have already been devoted to the various aspects mentioned but there seems ample scope for further development of the theory and it is suggested to Dr. Lerbs that among other important matters calling for further elucidation are cavitation, distribution of pressure, the interaction between hull and propeller and non axial flow to cover inclinations up to at least 15 degrees.

The designer has for long years been fortified by charts showing the variation of thrust, torque and efficiency of propellers over a wide range of slip obtained from tests of a series of model propellers in which one or more basic parameters of design have been systematically varied. One of the latest results of such tests published by myself three years ago covered the full range of pitch ratio and blade area ratio of practice. Each propeller was of constant pitch and segmental blade section which is a type that has long been recognized as efficient. One contribution of theory is to indicate the modifications that can be made to this comparatively simple shape with possible advantage albeit small. Dr. Lerbs' theory has been of great help in modifying the shape to suppress tip vortex or other type of cavitation to a higher speed and generally the indications of theory have been confirmed by tests of the model so modified in a Cavitation Tunnel. It frequently happens however that the propeller design so developed requires some further adjustment, to suit the particular hull and machinery and this is determined from the results of speed trials of a previous comparable type of ship. The final propeller design is thus a blend of theory with model experiment and ship trial results to which theory is making a contribution of increasing importance.

For completeness propeller charts should cover variations of characteristics other than the two mentioned above, namely pitch grading, blade section, camber, blade outline, rake and skew. Some information on these variables has been published from time to time but there is a need for a really systematic programme of tests. A complete programme would be a formidable undertaking were it not for the fact that the development of theory points the way to a strict limitation of the range of useful variation of the characteristics to such a degree that the requirements for experiment can be reduced to a scope that do not involve an excessive demand on the limited test facilities afforded by ship tanks and water tunnels.

H. W. Lerbs

I should like to thank the gentlemen who contributed discussions, for enhancing the value of the paper and for indicating interesting related problems which fell beyond the intentionally limited scope of this treatise on the basic theory.