THE EFFECT OF SIZE ON THE SEAWORTHINESS OF HYDROFOIL CRAFT

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The pitching movements and the vertical accelerations of hydrofoil craft with surface-piercing foils are studied in a very simplified way to show the effect of size as a function of wave parameters, and speed or Froude number. Considerations are given to variations of static pitch angle, heave, natural frequency, and damping ratio—the latter based on a linearized equation of motion. The analysis indicates that at wave lengths of 2/3 and 3/2 of the length of the hydrofoil craft, extreme values of vertical accelerations are to be expected, which decrease rapidly with increasing length of the craft, or decreasing speed, which leads to the conclusion that the seaworthiness increases with the length of the hydrofoil craft.

INTRODUCTION

The seaworthiness of hydrofoil craft is sometimes doubted, while the comfort offered to the passengers is often considered to be insufficient, when going on a seaway [1-7]. The purpose of this paper is to give in the most simple way some insight into the behavior of hydrofoil craft on sea, and to determine the most unfavorable conditions, which are compared for craft of different sizes.

This, of course, needs some explanation. The high cost involved in constructing big hydrofoil craft makes it necessary to obtain experience regarding seaworthiness, stability, and comfort, as well as structural loadings and other technical aspects, by means of scale models or small prototypes. It is therefore desirable to prove that the results obtained in this way can be interpreted for craft of greater size.

Today a hydrofoil craft does not present problems on relatively smooth water. Its behavior under unfavorable conditions is thus decisive for its suitability for passenger transport at sea.

Therefore it is necessary to gather the existing knowledge which may contribute to a prediction of the properties of a big hydrofoil craft, based on the known performance and behavior of a small craft, so that a project for a seaworthy passenger ferry can be started in full confidence.

The following considerations and calculations have been formulated in a general way, making them valid for a large number of existing hydrofoil systems. However, they are not valid for systems which cannot be compared to a spring-and-mass system. Thus they do not apply to hydrofoil craft with fully submerged foils, lacking a definite position of equilibrium relative to the water surface. Their stability depends only on a human or electric brain, controlling the lift developed by the different foils. The behavior in a seaway of this latter category depends merely on the degree of intelligence of its governor.

Furthermore, only the most unfavorable conditions have been considered. Going against the waves, the vertical accelerations were found to be critical; when going with the waves, this was the case with the maximum negative pitch angle.

Both critical values depend on design characteristics of the different hydrofoil systems, as there are: the lift reserve of the forward foils, the natural frequencies for pitch and heave, the damping ratio, etc. These design characteristics, being independent of size, will not be discussed in this paper.

The many purposely introduced simplifications, of course, create deviations. Thus it is necessary to apply the results only to two or more craft of the same hydrofoil system, differing essentially in a scale factor only, for it is only in that case that the deviations are in the same sense for all units, so that they will largely compensate each other.

THE SEAWAY

The waves are assumed to be of sinusoidal shape, where the waveheight H is equal to σ times the wavelength λ, and σ indicates the rate of steepness of the waves. The craft's speed V forms an angle γ with the vector of the speed of wave propagation c. Then the forced frequency or excitation frequency of the forces, trying to disturb the craft from its equilibrium position, is

$$\omega = 2\pi \left| \frac{c - V\cos\gamma}{\lambda} \right|, \quad \text{where} \quad c = \sqrt{\frac{g\lambda}{2\pi}} \tag{1}$$

The amplitude of the static pitch angle on a given wave pattern can be determined as follows: The maximum static pitch angle ψ_0 is attained in the position shown in Fig. 1. Its value is

$$\psi_0 = \frac{\sigma\lambda}{\ell} \sin \left(\pi \frac{\ell \cos\gamma}{\lambda} \right) \tag{2}$$

which can never exceed $\pi\sigma$.

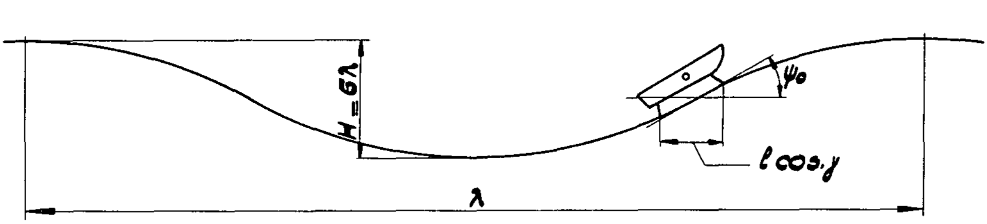

Fig. 1. Maximum static pitch angle

Similarly, the amplitude of the static roll angle is found to be

$$\varphi_0 = \frac{\sigma\lambda}{b} \sin\left(\pi \frac{b \sin\gamma}{\lambda}\right) . \tag{3}$$

The vertical movements of the center-of-gravity of a hydrofoil craft can also be determined: The dimensionless static amplitude of the center of gravity is (see Fig. 2)

$$\frac{Z_0}{\ell} = \frac{\sigma\lambda}{2\ell} \cos\left(\pi \frac{\ell \cos\gamma}{\lambda}\right) . \tag{4}$$

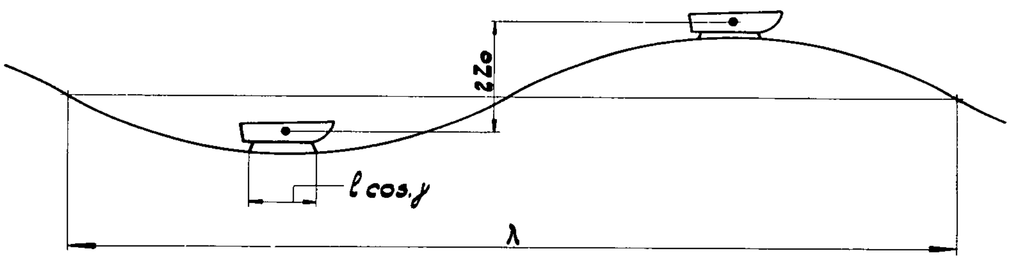

Fig. 2. Static amplitude of the center of gravity

Both values are shown in Fig. 3 as a function of wavelength divided by craft length. The scales for ψ_0 and Z_0/ℓ are valid for $\sigma = 0.05$. For other values of σ the scales have to be adjusted.

The roll angle φ will not be discussed in more detail, because experience has shown that rolling is not a critical factor. Moreover it can be treated in the same way as the pitch angle.

Figure 3 shows both the static pitch amplitude ψ_0 and the dimensionless static vertical amplitude of the center of gravity Z_0/ℓ as a function of the relative wavelength $\lambda/\ell\cos\gamma$. If

$$\frac{\lambda}{\ell \cos\gamma} = \frac{2}{2}, \frac{2}{4}, \frac{2}{6}, \text{ etc., then } \psi_0 = 0$$

but Z_0/ℓ shows an extreme value, while if

$$\frac{\lambda}{\ell \cos\gamma} = \frac{2}{1}, \frac{2}{3}, \frac{2}{5}, \text{ etc., then } Z_0/\ell = 0$$

and ψ_0 attains its extreme values. The effect of these parameters on the craft's behavior will be discussed later.

It should be noted that the values of the excitation frequency resulting from Eq. (1) should be regarded with some reserve. Practice has shown results to be sufficiently accurate when going against or with the waves, but when going along the waves higher values of ω are actually found due to the irregularity of the seaway [8].

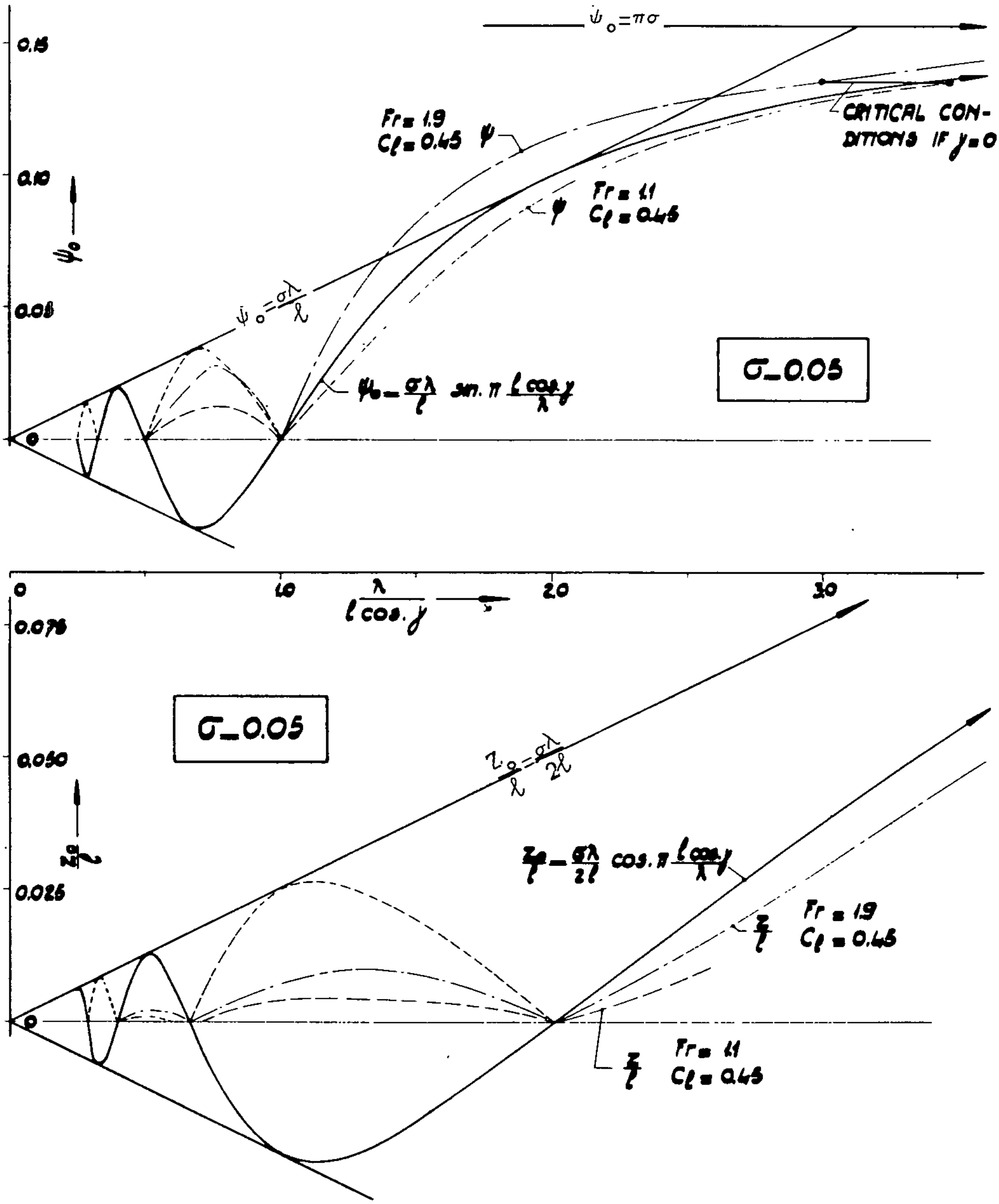

Fig. 3. Wave characteristics as shown in Figs. 1 and 2 (a list of symbols is given at the end of the paper)

THE HYDROFOIL CRAFT

The seaworthiness of a craft might be specified as a combination of characteristics which guarantee safety and comfort for passengers and crew while traveling at sea. Some of those characteristics are: static stability, dynamic stability, constructional properties, proper means of navigation, and lifesaving equipment. Variation of size of a certain type of hydrofoil craft will not affect its static stability. Constructional properties, proper means of navigation, and lifesaving equipment are (although very important) not to be discussed.

Hence the following will be limited to the dynamic stability and its influence on the comfort of hydrofoil craft. The main factors which define the movements are: static amplitude, (already discussed), excitation frequency, (already discussed), natural frequency, and damping ratio.

The motions of hydrofoil craft are defined by a number of basic equations. There is equilibrium when the weight of the craft equals the combined lift forces of the foils:

$$mg = \Sigma L \ .$$

The lift force of a foil is proportional to dynamic pressure, lift coefficient, and submerged foil area:

$$L = \frac{1}{2}\rho V^2 C_l F \ . \tag{6}$$

A deviation in pitch from the equilibrium position creates an extra lift ΔL which tries to restore the craft to its original position. The restoring moment is called R. It will be clear that: $\Delta L :: mg$ since $L :: mg$ (Eq. (5)), so $R :: mgl$. Further, for the moment of inertia, $I :: ml^2$ if the mass distribution can be supposed to be similar. Thus the natural frequency of the craft can be calculated:

$$w_{n_p} = \sqrt{\frac{R}{I}} \ :: \sqrt{\frac{mgl}{ml^2}}$$

where the index p is used for pitch, or

$$w_{n_p} = k_{w_p} \sqrt{\frac{g}{\ell}} \ . \tag{7}$$

In an analogous way the natural frequency for heaving motions can be found to be

$$w_{n_h} = k_{w_h} \sqrt{\frac{g}{\ell}} \ . \tag{8}$$

The dimensionless damping ratio in case of pitch, d_p, is the damping N divided by the critical damping:

$$d_p = \frac{N}{2\sqrt{IR}} \text{ where } N :: D_p \ell \frac{1}{\dot{\psi}} \ . \tag{9}$$

In this equation D_p is the damping force:

$$D_p = \frac{1}{2}\rho V^2 \Delta C_l F_d \tag{10}$$

in which $\Delta C_l = W/V$ and $W :: \ell\dot{\psi}$ so $N :: V\ell^2 F_d$.

The foil area F_d, contributing to the damping, is of course proportional to the foil area F:

$$N :: V\ell^2 F \ . \tag{11}$$

It was already stated that $I :: ml^2$ and $R :: mgl$, so

$$d_p :: \left(\frac{V\ell^2 F}{\sqrt{ml^2 mgl}} = \frac{VF\sqrt{\ell}}{m\sqrt{g}}\right). \tag{12}$$

Combining Eqs. (5) and (6) with (12) leads to

$$d_p = \frac{k_{d_p}}{C_l Fr} \, . \tag{13}$$

Similarly it can be found that in case of heave

$$d_h = \frac{k_{d_h}}{C_l Fr} \tag{14}$$

Consequently the damping ratio is inversely proportional to the lift coefficient and the Froude number for both types of motion.

This result might seem unexpected, the damping being proportional to speed, while the damping ratio is inversely proportional to the Froude number. The explanation is simple: an increase of speed of 10 percent corresponds with a decrease of submerged foil area of 20 percent, if the lift coefficient remains constant (see Eq. (6)). For similar hydrofoil craft of differing size, k_{w_h}, k_{w_p}, k_{d_p}, and k_{d_h} are constant values. They may vary, however, for different hydrofoil systems.

All elements necessary to calculate the motions of a hydrofoil craft are now available if the craft may be considered as a damped linearized spring and mass system. The dynamic amplitudes are then determined by the equations

$$\psi/\psi_0 = \frac{1}{\sqrt{(1 - w/w_{n_p}^2)^2 + 4{d_p}^2 w/w_{n_p}^2}} \tag{15}$$

$$Z/Z_0 = \frac{1}{\sqrt{(1 - w/w_{n_h}^2)^2 + 4{d_h}^2 w/w_{n_h}^2}} \, . \tag{16}$$

These equations have been plotted in Fig. 4 for various damping ratios.

It is now possible to determine the vertical accelerations at a certain point of the craft. This point has been chosen near the bow at a distance from the center of gravity of $l/3$, no passengers or crew being expected to be carried at a more forward point. The resulting accelerations have been calculated by adding vectorially the vertical acceleration caused by a variation of the pitch ψ and the vertical acceleration caused by a variation of heave Z. This is based on the assumption that pitch and heave are entirely independent of each other, which, although not correct, gives very acceptable results, as has been shown by Abkowitz [9].

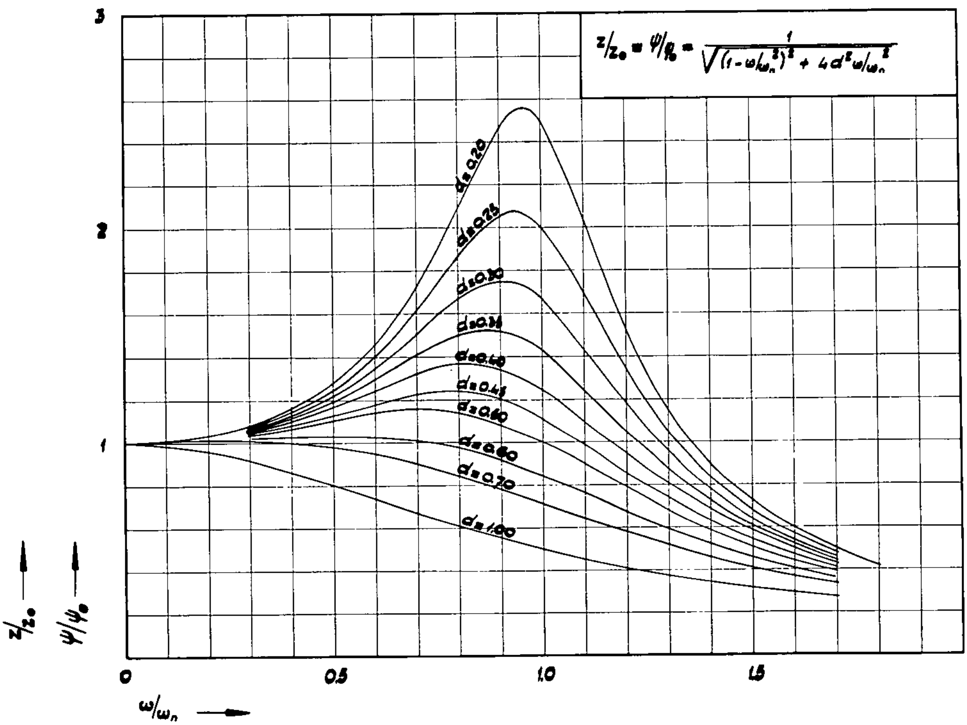

Fig. 4. Amplitudes of forced vibrations for various degrees of damping

BEHAVIOR OF THE CRAFT*

Concerning the course of a hydrofoil craft relative to the waves, experience has shown two conditions to be of decisive importance for its seaworthiness. The first occurs if $\gamma = O$, i.e., on a following sea. In that case, the extreme value of ψ plays a dominant role.

Figure 5 shows the influence of the orbital motion of water particles in a following sea. This orbital motion creates a moment which tries to pull the nose of the craft down.

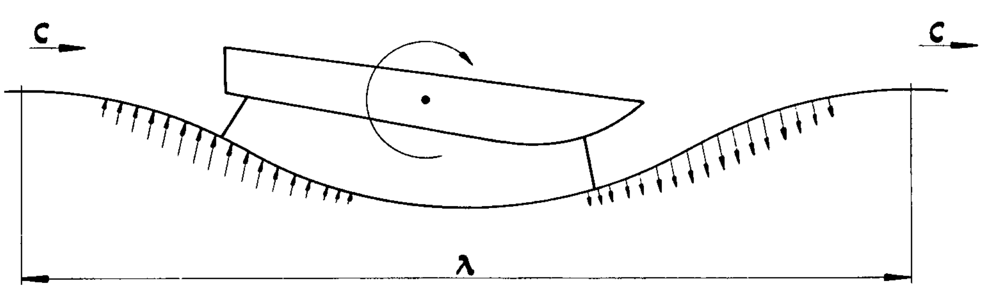

Fig. 5. Influence of the orbital motion of water particles in a following sea

As the pitch increases with increasing wave height, it will be clear that at a certain wave height the combination of a high value of negative pitch, combined with the orbital effect, makes the bow touch water. This phenomenon introduces a braking effect which causes the craft to slow down, so this wave height represents the limit up to which the craft can remain foilborne on a following sea. The lift reserve of the bow foils of a certain hydrofoil system evidently determines the limiting wave height.

However, in Fig. 3 the dynamic pitch amplitudes of two similar hydrofoil craft differing only in size have been indicated as functions of the relative wavelength. The critical condition for the craft with the lower Froude number is seen to occur at a greater relative wavelength. Consequently, of two similar craft the bigger one (having the lower Froude number) will encounter its critical condition on a following sea on relatively higher waves.

When going along the waves, i.e., when $\gamma = \pi/2$, a hydrofoil craft will attain its greatest roll angle φ, However, experience has shown that this is not a critical condition at all; therefore it will not be discussed in more detail.

When the craft is going against the waves, i.e., $\gamma = \pi$, the pitch amplitude is not critical, because the orbital moment will help to surmount the next wave, but the maximum value of the excitation frequency is obtained. Therefore this can be called a critical condition as far as vertical accelerations are concerned, these accelerations being proportional to the square of the excitation frequency. While discussing vertical accelerations, γ will henceforth be assumed to be equal to π.

Applying the foregoing theory to an example, a comparison is made between an existing small hydrofoil craft (l = 6.73 m, V = 30 knots) and a designed bigger craft (l = 36 m, V = 45 knots). Of the existing craft (Fig. 6), the natural frequencies for pitch and heave and the damping ratio are known. With the aid of Figs. 3 and 4 a curve could be obtained indicating the vertical acceleration at the bow as a function of the relative wavelength. The natural frequencies and the damping ratio of the designed bigger craft have been calculated according to Eqs. (7), (8), (13), and (14), which made it possible to establish an equivalent curve for the bigger craft. Both curves are shown in Fig. 7.

It will be noted that there are two critical values of the relative wavelength λ / l where the vertical acceleration at the bow $\ddot{Z}_b/g$ attains peak values. These critical wavelengths correspond with 2/3 and 3/2 times the craft's length.

Contrary to what might be expected, Fig. 7 shows that critical values of acceleration are not to be expected at a relative wavelength of more than two. This means that vertical acceleration decreases with increasing wavelength. In other words, the comfort of the small craft will be much better on waves with a length of 20 m and a height of 1 m, than on waves with a length of 10 m and a height of 0.5 m.

Similarly, the big craft will be more comfortable on waves of 100-m length and 5-m height, than on waves of 50-m length and a height of 2.5 m, although the comfort in the latter case is much better than the best that can be obtained with the small craft.

In reality the wave pattern is never found to be so regular as the theory supposes it to be. On a seaway, wavelengths vary considerably, which tends to level off the extreme accelerations. Furthermore small waves are always superimposed on the longer waves, which will tend to raise the maximum accelerations to be found for values of λ/l higher than two. These two effects have been taken into account in the dashed curves shown in Fig. 7.

Fig. 6. Small hydrofoil craft

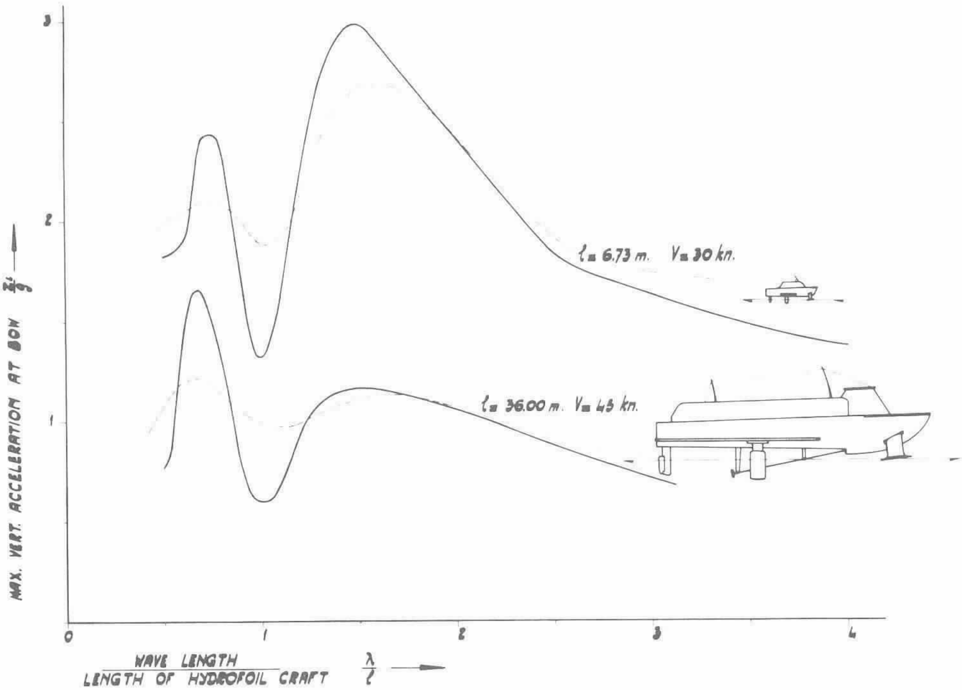

Fig. 7. Maximum vertical acceleration at the bow of a small hydrofoil craft and a larger hydrofoil craft

As the behavior of a hydrofoil craft in the most unfavorable conditions is of decisive importance for its seaworthiness and comfort, the critical accelerations at $\lambda/l = 2/3$ and $\lambda/l = 3/2$ have been plotted in Fig. 8 as a function of craft length. This graph is valid for one speed only, i.e. 30 knots, which makes it possible also to show the variation of the Froude number with the craft length. The vertical acceleration at the bow for $\lambda/l = 1$ is also shown.

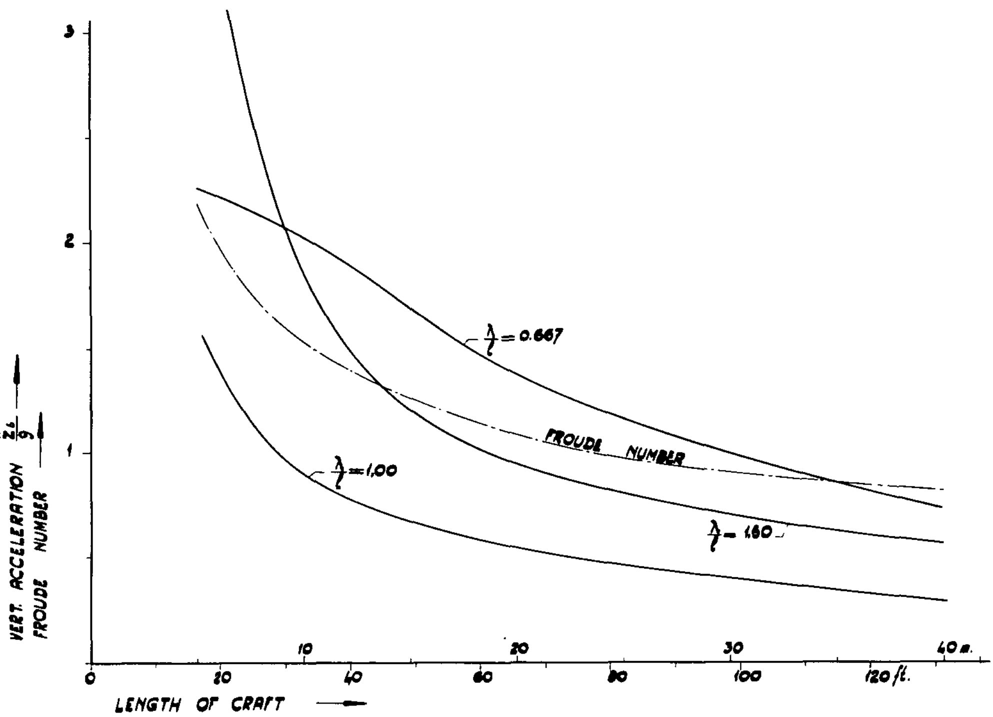

Fig. 8. Maximum vertical acceleration at the bow for critical values of λ/l as a function of craft length l at a constant speed of 30 knots

Of these curves the one for $\lambda/l = 3/2$ is the most important, because the critical accelerations at $\lambda/l = 2/3$ occur at relatively higher frequencies and can therefore be more easily attenuated or even eliminated by means of foil suspension or similar improvements.

The influence of the speed of the craft is shown in Fig. 9, where the curve for $\lambda/l = 3/2$ has been drawn for five different speeds. Evidently the vertical acceleration is approximately proportional to the square of the speed for a given craft length.

Thus it can be stated that vertical accelerations decrease rapidly with increasing length of the hydrofoil craft, which means that the comfort, depending on the frequency of variation of the accelerations (jerk), improves even more rapidly.

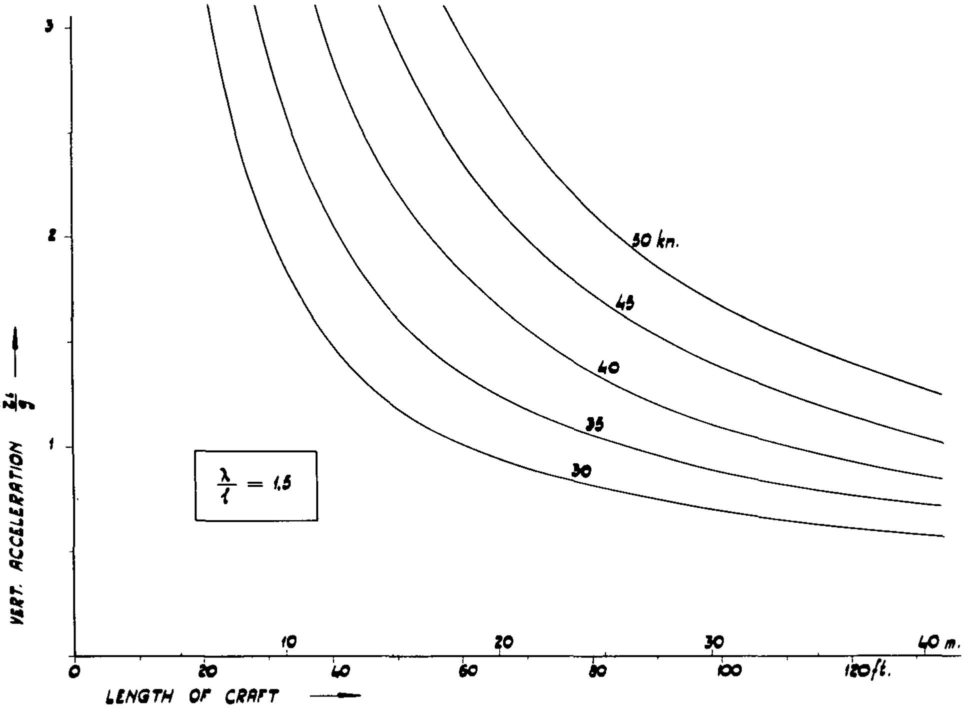

Fig. 9. Influence of the speed on the maximum vertical acceleration at the bow as a function of craft length for the critical value $\lambda/l = 1.5$

CONCLUSIONS

Assuming that the wavelength and height are proportional to the length of the hydrofoil craft, in other words, that a craft of double length is running on waves which are twice as long and high, it may be concluded that:

1. On a following sea, where maximum pitch is the critical factor, the seaworthiness is nearly unaffected by size. Actually the seaworthiness improves slightly with decreasing Froude number, i.e., increasing length.

2. When going against the waves, vertical accelerations are critical, which, with increasing length of the craft, decrease more than proportionally with the Froude number.

Consequently the seaworthiness increases with the length of the craft, the more so when the comfort is taken into consideration.

LIST OF SYMBOLS

a = angle of attack (effective)
b = maximum span of hydrofoils
C_l = lift coefficient
c = speed of wave propagation

γ = course angle relative to wave propagation
D = damping force
d = damping ratio
F = submerged foil area (effective)
Fr = Froude number
F_d = submerged foil area, contributing to damping
φ_0 = static roll amplitude (radians)
φ = roll amplitude (radians)
g = acceleration of gravity
H = waveheight
h = index for heave
I = moment of inertia
k = design constant
L = lift force
l = craft's length
λ = wavelength
m = mass of the craft
N = damping
w = excitation frequency
w_n = natural frequency
p = index for pitch
ψ_0 = static pitch amplitude (radians)
ψ = pitch amplitude (radians)
R = redressing moment (per radian)
ρ = density
σ = steepness of waves
V = craft's speed
w = relative vertical speed
Z_0 = static heave amplitude
Z = heave amplitude.

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DISCUSSION

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I want to know if the results just depend upon the assumption that the system is a simple spring-mass system. It appears that hydrofoils alone as simple spring-mass systems are overdamped in free motion as a single-degree-of-freedom. You have no such thing as a natural frequency in many cases. Also, I would like to know if the result that vertical accelerations are maximum at ratios of wavelength to craft length equal to two-thirds and three-halves is true for all cases or just for the particular designs chosen for illustration.

A. Hadjidakis

I said at the beginning of my lecture that what I have done was done in a very general way and in a very simple way, because, otherwise, one would need a computer and would have to go into very difficult calculations. I have chosen this very simple spring and mass system to get some quick results and to make it clear that in any case the seaworthiness of bigger craft would be better than that of small craft. I meant, indeed, in all cases, except fully submerged foil systems.

I might say now concerning what one should take for the characteristic length of the craft; one had perhaps better take the distance of the extreme foils, or something like that, to obtain better results. I do not pretend the diagrams I have shown are very accurate, but the main thing I wanted to prove is the direction wherein we are going if we make craft bigger. What you said about the damping ratios, that hydrofoil craft are overdamped, I do not agree. Damping, as far as heave is concerned, is very high indeed, but damping on pitch depends on what sort of hydrofoil system you have. With our one-foil system, on very small craft we find damping ratios as small as 0.4, so in that case one can feel on different wavelengths what is approximately the critical frequency of the craft. If you look at Fig. 4 it will be clear that this critical frequency is not a well-defined thing; you cannot measure it very exactly.

P. Kaplan

You said that in following seas the important characteristic of seaworthiness is pitching. I think that heave is rather important, and that you get large possibilities of settling in the water just due to that motion as well.

A. Hadjidakis

I don't agree, because although the heave is there, of course, the pitch will be more important, making the craft nosedive. In that case the angle of attack of the foils decreases, which means that lift decreases too (Fig. 5). Quite a big lift reserve on the forward foils is then needed to take the craft up again for the next wave, because otherwise the nose will touch that wave and then what we call seaworthiness in that particular case is finished and one cannot go at full speed. That is what I meant; I insist on the pitch being the most important factor on a following sea because, if you would go down horizontally, which is the case when heave is the main factor, you still have the same lift-coefficient, and it would be a very poor design if the craft wouldn't go up then in time.

P. Kaplan

I want to know just how you get better performance from a small craft to a large one. You did not just increase the size, you also changed the speed. Now with Froude scaling we get the same relative values. Is there any particular way in which you did this or do you see any particular optimum way of carrying out this change in size and speed to obtain an improvement in the craft?

A. Hadjidakis

The speed of 45 knots was chosen for the big craft with a capacity of about 400 passengers because of economic and efficiency reasons, and in my opinion it is not necessary to have higher speeds. The owner of the passenger-carrying craft will generally not gain more money by a somewhat higher speed and I think the passengers will not even realize the small time gains they will have in that case. Hence the number of 45 knots was not chosen for some physical reason. There is, however, another aspect to this matter. We have experience with all sorts of craft of different sizes and the biggest one could carry about 70 passengers. We found weight is not going up as the cube of the length of the craft, but somewhat less than that. It is logical to give a bigger craft a higher speed, which helps a lot in keeping the foil dimensions and weight down. In our designs the foil weight is about 5 to 8 percent of the total displacement weight.

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