EXCITATION OF ACOUSTIC RESONATORS BY FLOW

Uno Ingard and Lee W. Dean, III
Physics Department
Massachusetts Institute of Technology

INTRODUCTION

The study of excitation of cavities by wind or jets of air has a long history. All wind instruments and their antecedents consist of cavities excited by a jet or a larger flow of air which is blown through them or across their openings. The effects on the frequency and quality of the tone, which are produced by varying the dimensions of the cavity or the strength of the air stream, have been studied for centuries.

Helmholtz (1) in 1868 published what appears to be the first significant paper on the actual mechanism of excitation. He describes the boundary between two layers of fluid moving with different velocities as being composed of parallel vortex threads. If these threads initially lie in one plane — so that the boundary between the two regions of fluid is flat — they will remain so in unstable equilibrium. However, if one is disturbed from its position, it will continue to move out of line so as to cause the dividing surface of hump and eventually curl over. Helmholtz concludes that the "vis viva" of the oscillations which is lost by radiation is replaced by energy from the blast of air blowing across or through the mouth. This blast is directed slightly into or out of the pipe in proper phase for reinforcement by movement of the vortex threads at the boundaries.

The understanding of the mechanism of excitation has advanced only slightly since this early paper by Helmholtz. P. M. Morse (2) in 1948 indicates that there is a nonlinear coupling between the driver, or air jet, and the pipe. He also points out that the presence of a pressure node at the opening of the pipe identifies the excitation as a pressure fluctuation because the system aligns itself so as to extract a maximum of energy from the jet. His concluding remark — "A great deal of experimental and theoretical work is needed before we can say we understand thoroughly the behavior of any of the wind instruments" — points not only to the lack of a good theoretical model for the mechanism of excitation, but also to the general lack of experimental evidence. This report describes some studies of the excitation of a pipe by flow past the open end of the pipe. These studies are restricted to one particular pipe geometry, but most other parameters are considered, including the internal damping of the pipe resonator.

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It appears that one can distinguish two different regions or mechanisms of oscillation of the pipe, a linear and a nonlinear region. The linear region corresponds to low flow speeds in which the acoustic oscillations in the resonator do not affect the flow about it. The pressure fluctuations about the resonator are then determined solely by the geometry of the pipe at the mouth and the nature of the incident stream, and they are independent of, for example, the length of the pipe. The acoustic oscillations in the pipe are then linearly related to the pressure fluctuations in the flow which are already present in the incident stream or which are caused by the turbulence generated when the stream strikes the pipe. As the flow speed increases, there is a transition to more violent nonlinear oscillations in which the sound field in the pipe indeed reacts back on the flow.

The sound generated in the linear region is generally very weak and can be considered as background noise for the purposes of this report, which is concerned mainly with the nonlinear oscillations.

Blokhintzev (3) has investigated the behavior of a pipe resonator in the linear region and finds on the basis of dimensional arguments that the sound pressure in the resonator is proportional to the square of the flow speed. His theory does not explain any of the features of the nonlinear oscillations. There are several discrete modes of nonlinear oscillations, of which only the fundamental is considered in this report.

This study starts with an investigation of the flow about the mouth of the resonator involving both schlieren photography and hot-wire measurements. The acoustic response of the resonator is then studied as a function of flow speed and the angle of attack of the flow. Finally, a study is made of the effect of internal damping of the resonator on its acoustic output, with particular emphasis on a determination of the critical damping beyond which the nonlinear oscillations no longer can be sustained.

STUDIES OF THE FLOW PATTERN ABOUT THE RESONATOR AND THE MECHANISM OF OSCILLATION

Schlieren Studies

The pipe, 2 cm in diameter and 30 cm long, was placed in an air flow the speed of which could be varied from 0 to about 4000 cm/sec. The air stream was uniform over an area of about 3 × 3 cm. The mouth of the resonator was introduced into this region.

To obtain some over-all ideas concerning this oscillation mechanism of the flow, exploratory experiments were made which were designed to study the flow pattern about the opening of the resonator. After several attempts involving hot-wire measurements, a schlieren system was arranged which made possible direct visual observation of the flow field (Fig. 1). It utilizes two optical systems. One, consisting of the light source, condenser, flat mirror, concave mirror, and knife edge, is arranged so that the light source is imaged in the plane of the knife edge. The other, consisting of the camera, is arranged so that the field of view, located directly in front of the mirror, is focused onto a screen or photographic plate. Each point in the field receives light from every point of the source. Therefore, the field, as seen from the camera, darkens uniformly when the knife edge cuts off part of the image of the source. Local density gradients in the field bend the light rays, causing a larger or smaller number of them to be intercepted by the knife edge with the result that the field appears locally lighter or darker. In practice, the knife edge is moved back and forth until the field is uniformly illuminated.

This system using a concave mirror has three main advantages over one in which the mirror is replaced by a lens and the light source is placed to the right of the lens in line with the camera. First, this system is more sensitive because light passes

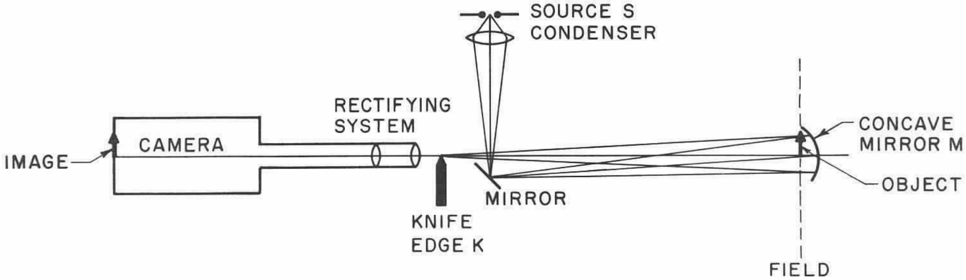

Fig. 1 - The schlieren system

through the field twice. Second, the size of the field is limited to the size of the lens or mirror; large mirrors are easier to make. Third, schlieren systems are extremely sensitive to chromatic dispersion, which is avoided by use of a mirror.

In order to increase the contrast, the air flow was heated by loops of a heated wire placed in the air stream. In the direct visual observations, stroboscopic illumination was used — obtained by means of a steady light source followed by a rotating disc chopper. The photograph of the flow pattern shown in Fig. 2 was obtained using a high-intensity flash source. This picture does not show the field as clearly as direct observation under stroboscopic illumination. However, it does indicate the general features of the flow field.

Mechanism of Oscillation

The flow field about the mouth of the resonator appears to be similar to the flow about a knife-edge oscillator. When the flow strikes the downstream edge of the

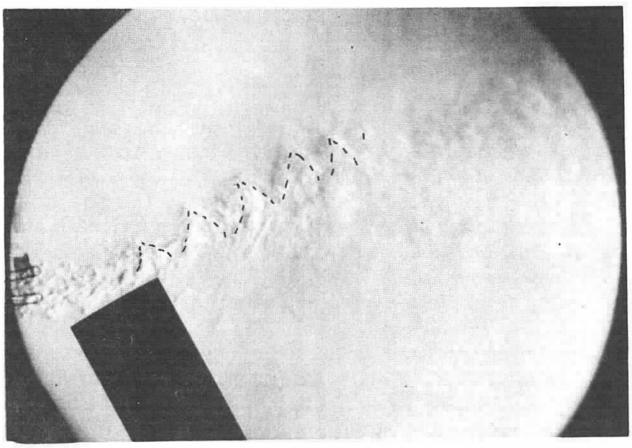

Fig. 2 - Flash photograph of the flow past the mouth of the resonator

resonator, eddies are shed at a rate corresponding to the resonance frequency of the resonator. There is a class of hydrodynamic oscillations in which a disturbance produced at an edge travels downstream. On meeting a second edge, a sound signal is sent back which triggers a second disturbance. This causes reinforcement at a particular frequency, the period of which is approximately $d/U + d/c$, where U is the velocity of the flow, c the speed of sound, and d the distance between the edges. Ordinarily, d/U is much greater than d/c so that the frequency of oscillation is of the order of U/d. The jet-edge oscillator is a striking example of such a system (4,5). In addition to this hydrodynamic "feedback," there may also be an acoustic coupling resulting from a reflection of the sound wave from a boundary in the neighborhood of the edge system. If the reflector is a distance L away, the sound signal will return after a time $2L/c$.

In general, one might expect that both of the feedback mechanisms described above might be of importance in hydrodynamic nonlinear oscillators. Furthermore, it is expected that in the case of such a dual feedback the oscillations should be particularly strong if d/U and $2L/c$ are the same.

In the present pipe-resonator oscillator, the major feedback mechanism seems to be acoustic, as indicated by the acoustic characteristics discussed in the next section. The two edges referred to above are represented by the edges of the pipe opening. The acoustic reflector is the rigid termination at the end of the pipe. The frequency of oscillations is found to be approximately equal to $c/2L$ for a relatively large range of flow velocities, where L is the length of the pipe. The characteristic time d/U corresponding to the hydrodynamic coupling between the two edges seems to be of less importance in the pipe-resonator. Similar acousto-hydrodynamic oscillators have been discussed by Anderson (6) and von Gierke (7).

Hot-Wire Measurements of the Flow Gradient in the Mouth of the Resonator

In addition to the schlieren studies mentioned above, some measurements were made of the flow in the opening of the resonator by means of a hot wire. The variation in the flow speed as one goes from the outside into the pipe is shown in Fig. 3. The velocity profile has a strong gradient, which can be shown to promote transfer of energy from steady to oscillatory flow (8). This is further evidence to support the picture of a nonlinear mechanism producing oscillations at the mouth of the resonator.

ACOUSTIC MEASUREMENTS

The sound field generated by the flow as it passes the pipe was measured at a distance of 15 cm from the mouth of the resonator. Some measurements were also made when the microphone was inserted at the bottom of the resonator. However, the sound-pressure levels inside the pipe often exceeded 140 db, the upper limit of the dynamic range of the microphone without special acoustic attenuating devices. Therefore, most of the measurements were taken outside the resonator, as indicated in Fig. 4. Both the sound-pressure amplitude and the frequency were measured on a General Radio Wave Analyzer with a 4-cps bandwidth.

The air supply consisted of a compressor and regulator set to deliver air at a constant pressure of 30 psi gauge. The volume flow was measured on a Flowrater. The air then passed through a settling tank, and issued from the tank into the room through a pipe 3 cm in diameter and about 10 cm long. The air velocity was measured by a hot-wire anemometer and was found to be uniform over an area of about 3×3 cm in front of the pipe. It was in this region that the mouth of the resonator was placed.

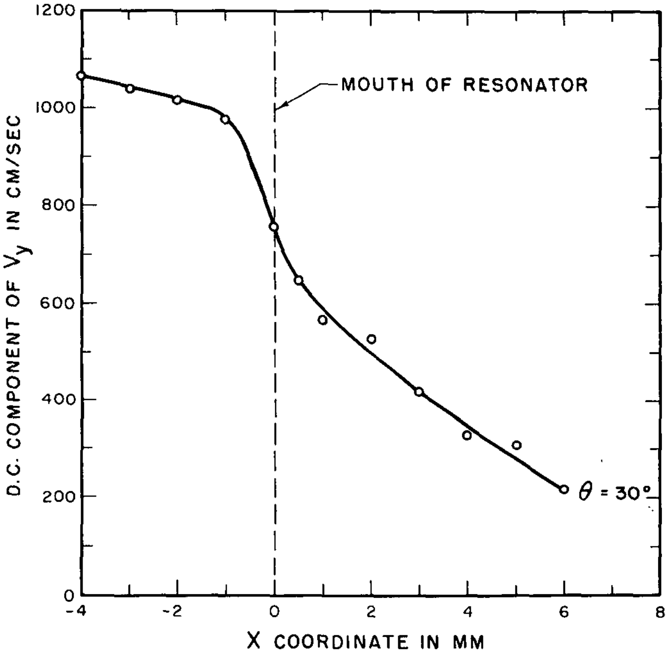

Fig. 3 - The variation of flow speed along the axis of the resonator

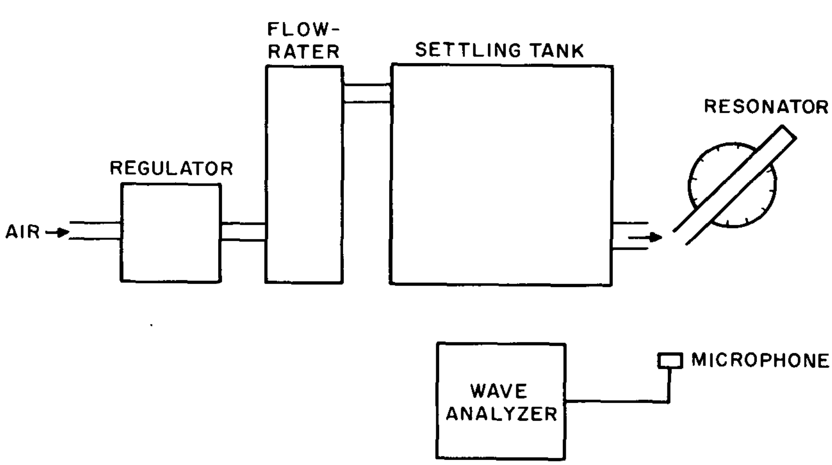

Fig. 4 - The experimental setup for acoustic measurements

Sound Pressure as a Function of Flow Speed and the Angle of Attack

Measurements of the sound pressure were made for flow speeds up to about 3500 cm/sec and angles of attack from 0 to 50 degrees, as defined in Fig. 5.

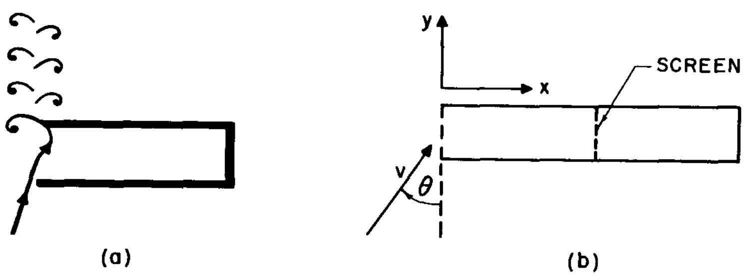

Fig. 5 - Resonator pipe (with damping screen)

In Fig. 6 are shown results of such measurements. There is a critical angle of about 15 degrees and a critical air speed of about 1000 cm/sec below which no oscillations are observed. Not only is there a lower limit of the flow speed but also an upper one at which oscillations cease. This upper limit increases with the angle of attack. With the flow equipment available in the present experiment, the highest flow speed was about 3500 cm/sec. The largest angle of attack at which oscillations could be sustained at this speed was about 50 degrees.

The envelope of the pressure versus flow-speed curves for the various angles of attack indicates a sound pressure which is approximately a linear function of the flow speed. At the lowest measured sound pressures shown in Fig. 6, it is difficult to decide whether the oscillations are nonlinear with an oscillating flow pattern described above or if they result from the turbulence already present in the incident air stream.

One interesting aspect of the problem is that the air flow must be directed into the resonator for oscillations to occur. In fact, the more the air stream is directed into the resonator, the stronger the oscillations become. It may be that the disturbances produced at the mouth of the resonator by the sound field direct the main flow into and out of the tube. The oscillations would then be driven by a switching of the air flow rather than by vortexes which would be generated regardless of the angle of attack. This aspect of the problem could be clarified by more detailed observations of the flow field in the vicinity of the mouth of the resonator.

Frequency of Oscillation vs Flow Speed

As already mentioned, the frequency of oscillation of the tube stays almost constant, independent of the flow speed. Some measurements of the slight frequency variation that does occur are shown in Fig. 7. This result is for an angle of attack of the flow of 30 degrees. It is seen how closely the oscillations are held to a single frequency for a wide range of the flow speeds. This result supports the observation that in most of the flow range the coupling between the flow and the sound wave seems

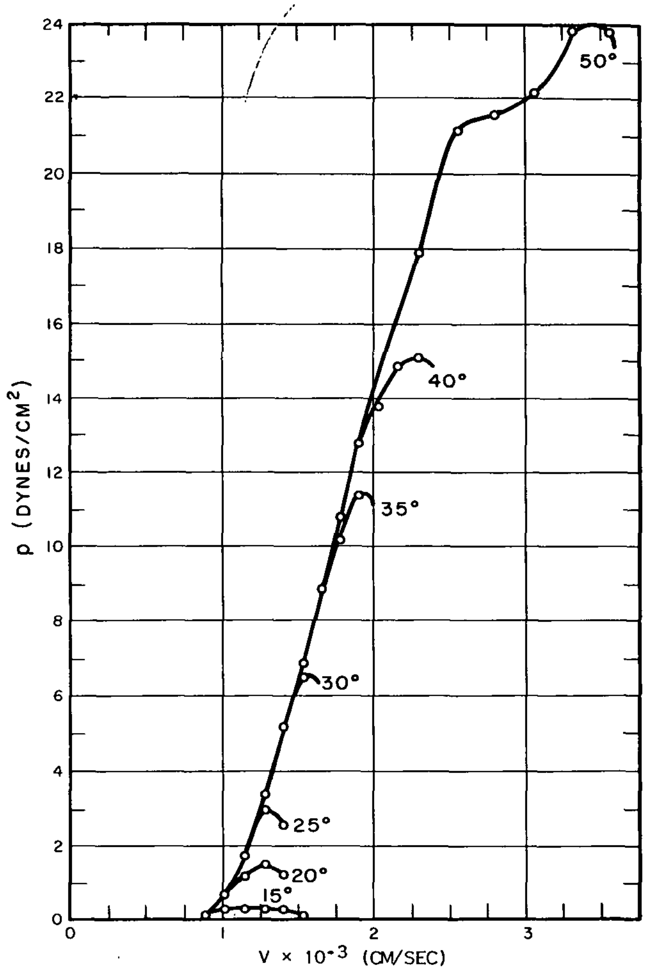

Fig. 6 - Measured sound pressure at a distance 15 cm from the mouth

to be dominant in the mechanism of oscillation. On the other hand, in a purely hydrodynamic self-oscillator, such as the jet-edge system, the hydrodynamic boundary conditions determine the characteristic frequencies of oscillations which are of the order of U/D, where U is the flow speed and D is a characteristic length of the system.

The Effect of Damping in the Resonator

A factor which affects the oscillations of the resonator to a considerable degree is damping. The damping in the pipe system consists of radiation damping and the viscous and heat conduction losses at the walls of the resonator. The damping in a system is often described in terms of the Q value which for $Q \gg 1$ equals $\omega_0/\Delta\omega$, where ω_0 is the resonance frequency and $\Delta\omega$ is the half-power bandwidth of the resonator.

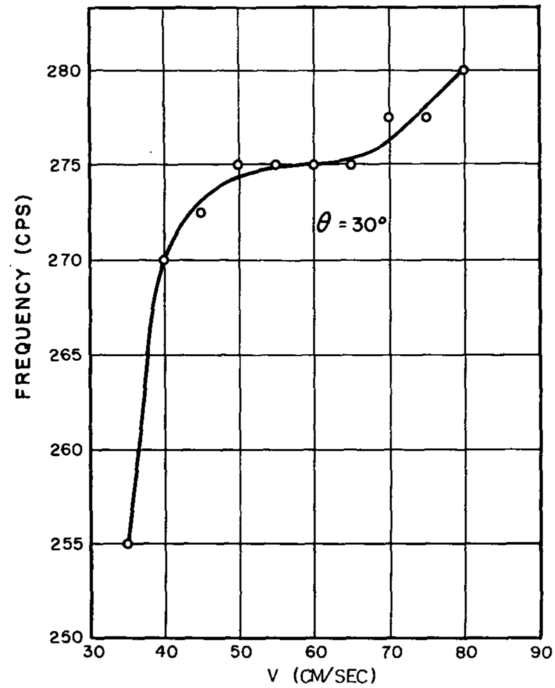

Fig. 7 - Variation of the frequency of oscillation with flow speed

The largest value which ordinarily can be obtained with acoustic resonators in air under standard conditions is of the order of 50. The resonator tube used in the experiments described above was about 42.

Variation of the Q of the Resonator - It is of considerable practical importance to investigate the effect of the damping on the flow-excited oscillations. In order to vary the damping in the resonator, a damping element in the form of a fine screen was introduced into the tube, as indicated in Fig. 5. The particle velocity is zero at the rigid termination of the tube. If the screen is placed close to the wall, the damping effect of the screen is clearly negligible. The damping effect increases as it is moved toward the mouth of the tube. In fact, if the screen does not change the original flow distribution $\cos(\pi x/2L)$ appreciably, it follows that the Q value will vary with the position of the screen as

$$Q \cong \frac{Q_0}{1 + (R_f/R)\left(\cos \frac{\pi x}{2L}\right)^2} \tag{1}$$

where x is measured from the open end of the tube, R_f is the screen resistance, and R is the resistance caused by the radiation, viscous, and heat-conduction losses.

Using a fine-mesh screen (open area 29 percent, 306 holes/cm^2, diameter of strands 0.101 mm), the Q value of the resonator may be varied from the maximum value of 42 down to a value of about 19 by moving the screen from the bottom to the mouth of the tube. This is shown by the top curve in Fig. 8. The second curve in the figure shows the measured Q value in the case when three screens are introduced into the tube. In that case, the Q value was brought down to a value of about 8. The curves obtained in this way are in quite good agreement with Eq. (1).

It is interesting to note in passing that this method of introducing damping into the tube may readily be used for the measurement of small flow resistances.

In the measurements of the Q value as given in Fig. 8, the microphone itself formed the rigid termination of the tube. The frequency response of the pipe to an incident sound wave was determined and the Q value was evaluated as the ratio $\omega_0/\Delta\omega$. Here $\Delta\omega$ is the "width" of the pressure-frequency curve at the points where the pressure is 6 db below the maximum pressure.

The Effect of Q on the Flow-Excited Oscillations - Varying the Q value of the resonator as described above, a series of measurements was made of the strength of the flow-excited oscillations of the resonator pipe as a function of the Q value. The angle of attack of the flow was kept constant at 35 degrees. The sound pressure was measured at the bottom of the resonator, but the experimental setup was otherwise

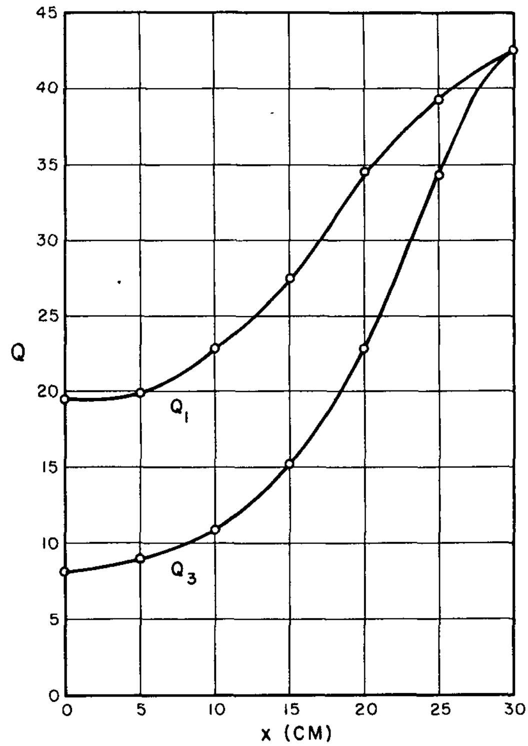

Fig. 8 - Variation of the Q of the resonator with the position of the screen (see Fig. 5). Q_1 corresponds to one screen, Q_3 to a combination of three screens.

identical to that described above. The flow speed was varied as before. The results of these measurements are shown in Fig. 9. In this figure, the value 100 on the velocity scale corresponds to a speed of 1150 cm/sec. At the lower wind speeds, the sound pressure increases quite rapidly at first and then levels off so that it is approximately proportional to Q for large values of Q. At the higher wind speeds, oscillations were obtained only for large values of Q. For example, with a wind speed corresponding to 120 in the figure, a decrease of the Q to about 34 stopped the oscillations completely. At lower wind speeds the limiting Q value, below which no oscillations can be sustained, is not defined as sharply.

As indicated already in Fig. 6, the resonator can be kept in oscillation only in a certain flow speed range. This range depends on the Q value of the resonator as illustrated in Fig. 10, where the upper and lower flow speeds, which define the oscillation range, are plotted as a function of Q. The curve refers to an angle of attack of about 35 degrees, and the sound pressure was measured with the microphone forming the termination of the resonator.

As far as the lower speed limit is concerned, it is often difficult to decide when the oscillations are of the nonlinear type and when they are excited (linearly) by

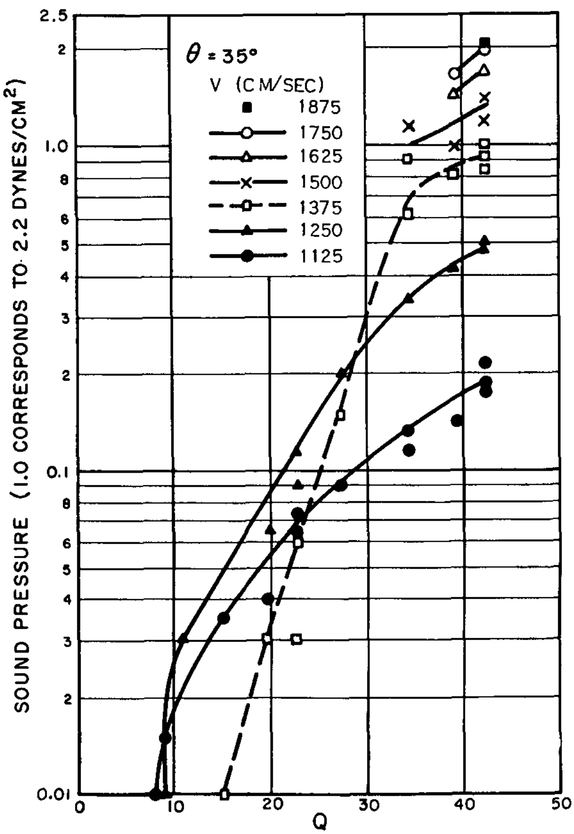

Fig. 9 - Variation of the sound pressure with the Q of the resonator

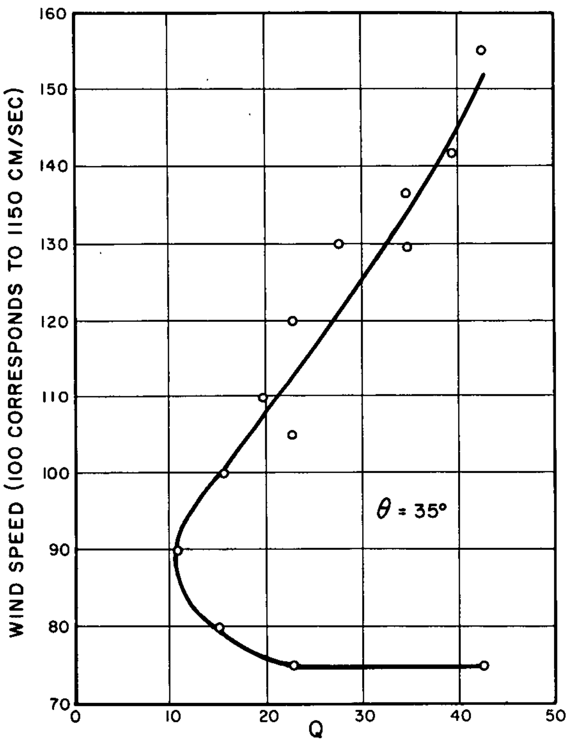

Fig. 10 - The upper and lower limits of the flow speed between which the nonlinear oscillations of the resonator can be sustained

turbulence in the incident flow. Although presumably there is a smooth transition between these two kinds of oscillations, it seems reasonable in the present case to use a sound pressure of 0.02 dynes/cm^2 as representing the dividing line between the two regions. Above this pressure the oscillations were definitely of the nonlinear type.

As the Q decreases, the range of oscillations clearly decreases until it narrows to a point at a Q value of about 11. Below this Q value, only the weak "background" oscillations will remain, but the self-sustained nonlinear oscillations can be considered to be completely stopped at the particular angle of incidence of 35 degrees. The existence of a limiting Q value, below which the resonator cannot be brought into oscillations by flow, should be of particular interest in connection with noise control in cases where some of the noise is produced by air rushing past cavities of various kinds.

The shape of the curve in Fig. 10 is also interesting inasmuch as it indicates a rapid increase of the range of oscillation in the region of high Q values. A small increase of Q in this region would produce a relatively large increase in the oscillation range. This same characteristic feature is probably typical for all acousto-hydrodynamic oscillators.

REFERENCES

1. Helmholtz, H.L.F., "Sensations of Tone," New York:Dover, 2nd English edition, translated by A. J. Ellis, 1954, pp. 394-397

2. Morse, P.M., "Vibration and Sound," New York:McGraw-Hill, 1948, pp. 244-254

3. Blokhintzev, D.I., "Acoustics of a Nonhomogeneous Moving Medium," NACA TM 1399, translated by S. Reiss, 1956

4. Curle, N., "The Mechanics of Edge Tones," Proc. Roy. Soc. A216:412 (1953)

5. Nyborg, W.L., et al., "Acoustical Characteristics of Jet-Edge and Jet-Edge-Resonator Systems," J. Acoust. Soc. Amer., 24 (No. 3):293-304 (1952)

6. Anderson, A.B.C., "Vortex-Ring Structure-Transition in a Jet Emitting Discrete Acoustic Frequencies," J. Acoust. Soc. Amer. 28 (No. 5):914-921 (1956)

7. Von Gierke, H., "Über Schnidentöne an kreisrunden Gasstrahlen und ebenen Lamellen," Z. angew. Phys. 2:97 (1950)

8. Lin, C.C., "Theory of Hydrodynamic Stability," New York:Cambridge University Press, 1955, pp. 52-60

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DISCUSSION

M. Strasberg (David Taylor Model Basin)

It might be worth describing a possible phenomenological explanation for the excitation. It is based on certain observed facts, namely, the sound only occurs when the speed is in certain ranges; there are jumps in these ranges of speed; and, another fact which was not mentioned, but which I believe Prof. Ingard has observed, that the radiation impedance seen by the mouth of a resonator is a function of the air velocity past the mouth. This means that the flow past the mouth of the resonator provides a possible mechanism for varying the acoustic impedance seen by the resonator.

Now one can assume that there is some instability in the flow, causing a variation in velocity of flow past the mouth. This instability is inherent in the flow and has nothing to do with the resonator; it would occur even for a hole without a resonator. The flow instability results in a periodic fluctuation in the mass reactance seen by the resonator. Now, for a mechanical or acoustic system which is linear, if there is a periodic variation in one of the parameters of the system, and if the variation occurs at the right frequency and amplitude, it is known that an instability of the system can cause self-maintained oscillations.

More specifically, suppose one has a simple mechanical system, whose differential equation can be represented by:

$$(d^2y/dt^2) + w_0^2(1 + a \cos w_1 t)\ y = 0.$$

This is called a Mathieu linear differential equation and is discussed by T. Brooke Benjamin in his comment on Prof. Meyer's paper in this Proceedings.

In the present situation it may be possible that these fluctuations of the flow past the mouth of the resonator, which occur even if the resonator were not there, cause such a periodic variation in the frequency, and if the amplitude a of this fluctuation is large enough, instability can occur and the system will oscillate naturally.

If the instability inherent in the flow past the neck is at twice the frequency of the resonator, i.e., if $w_1 = 2w_0$, oscillations will occur at small amplitude a. If there were no damping, zero amplitude would do it, but with damping a certain finite amplitude is needed if instability occurs at twice the frequency of the resonator. If it occurs at a frequency that is equal to that of the resonator, oscillations will also occur at relatively small amplitude; in fact, oscillations will occur at small values of the amplitude whenever w_0 is an integral or half-integral number of times w_1.

This doesn't explain the details, but it is a possible way of looking at what happens. There is one way that we could ascertain whether this is a reasonable explanation, and that is to actually observe the flow past the hole in a plate in the absence of the resonator and see if it is essentially the same as with the resonator.

M. C. Harrington (David Taylor Model Basin)

Some experiments which have been done in the David Taylor Model Basin are related to those mentioned above.* In this case, the flow passed over a plate in which the aperture was located. The resonant cavity was situated behind this plate. It has been mentioned in some of the previous papers that the increase in the frequency of resonance with increase in the speed of air flow was relatively small. In our experiments, however, we found that the frequency increased considerably, in some cases by 30 or 40 percent of the initial frequency with which the resonator started to oscillate. The frequency approached a final value asymptotically and as the air speed was further increased the oscillation died out abruptly. This increase in frequency of resonance may be explained by a decrease in the equivalent acoustic mass reactance of the air in the opening with increasing turbulence as the air speed increases. This interpretation is consistent with the results reported by McAuliffe† and Ingard‡ in which the resonators were driven by an independent source of sound of known frequency and intensity. There it was found that the resonant frequency increased (within limits) with increasing air flow through or past the opening.

In the Model Basin experiments when the air speed was doubled, provided that this was within the range of speeds available, the oscillator began again to resonate at the same frequency and the regime was repeated. If the actuation of resonance can be explained in terms of vortices, there would appear to be a double set of vortices in the second case. Occasionally resonance was actuated again at triple the initial speed.

*M.C. Harrington, "Excitation of Cavity Resonance by Air Flow, Abstract J4, Jour. Acoust. Soc. Am. 29:187 (1957).

†C.E. McAuliffe, "The Influence of High Speed Air Flow on the Behavior of Acoustic Elements," M.S. Thesis, M.I.T. 1950.

‡K.U. Ingard, "On the Theory and Design of Acoustic Resonators," Jour. Acoust. Soc. Am., 25:1037-1061 (1953).

E. E. Covert (Massachusetts Institute of Technology)

At MIT, we have been conducting a series of tests on cavities that are similar in nature to those that Dr. Harrington just reported on. They are essentially holes in the bottom of a wind tunnel with a solid wall. There are several shapes: triangular, circular and rectangular, and we have been running them from Mach number of 1.5 up to 3. These cavities seem to vibrate at the Helmholtz room frequencies. If they are deep and long they seem to vibrate in the up and down mode. If they are shallow they seem to slosh back and forth, much in the same manner as reported by Krishnamurty of the California Institute of Technology.

We have observed that for the most part, for the rectangular boxes, the behavior is independent of the cross-wide characteristic length. In other words, the behavior is two-dimensional, as you would expect. However, at the higher Mach numbers a narrow box seems always to sing out fairly loud and clear at about 165 decibels. The wider boxes are not easy to excite and we don't know why at the moment.

Another point of interest is that these can be correlated in terms of a Strouhal number if one uses the correct velocity of sound within the box and the Mach number of the free stream. They exhibit also the same edge-tone type behavior that everyone else has remarked on, where it will take off at a certain frequency and at some speed will jump to a higher frequency. We have taken some observations with rectangular boxes that had glass walls, and it seems that the sound is radiated from the upstream edge of the box. We get nice circular waves moving down into the box. Of course, this gives us a shock wave at the leading edge and we get nice waves going out.

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