X

HYDRODYNAMIC SOURCES OF SOUND

H. M. Fitzpatrick and M. Strasberg
David Taylor Model Basin

ABSTRACT

The paper describes the various flow phenomena that result in underwater sound in flowing water, and reviews the recent experimental and theoretical investigations of the sound sources. Four distinct sources of hydrodynamic noise are discussed: entrained air bubbles; vaporous cavitation; surface disturbances; and unsteady flows. For each of these, brief descriptions are given of 1. the physical mechanisms involved in the noise generation; 2. the available experimental data; 3. the theoretical methods of analysis; and 4. the relations between the flow parameters and the amplitude and spectral distribution of the sound.

I. INTRODUCTION

Sounds associated with flowing air and flowing water are everyday experiences. Examples are the splash of water and the proverbial babble of the brook, the howl of the wind, the hiss of escaping air, and indeed all the sibilant and aspirate sounds of speech itself.

Despite their familiarity, sounds from flowing fluids have received comparatively little scientific attention. Although most text books on acoustics discuss at length the sounds produced by vibrating bodies, the equally-common sounds associated with flow are usually ignored.

Within the past decade, however, interest in hydrodynamic and aerodynamic sources of sound has increased. At the same time, a theoretical foundation for dealing with these acoustical problems has been provided by recent developments in fluid dynamics, particularly in the study of turbulence. As a result, important advances in our understanding of hydrodynamic noise have been made since Folsom, Howe and O'Brien [1] reviewed the subject in 1942.

The study of hydrodynamic noise involves a combination of hydrodynamics and acoustics. These two subjects were closely allied in the classical physics of the nineteenth century; Stokes, Helmholz, Rayleigh, and Lamb, for example, contributed to both. But in recent years they have grown apart; acoustics becoming associated with electronics, while hydrodynamics concentrated on problems of steady motion. As a result, many workers in each of the fields are scarcely aware of developments in the other. The study of hydrodynamic noise requires a reunion of the two disciplines.

The object of this paper is to describe the various flow phenomena that can result in sound, and to review the recent theoretical and experimental investigations that have led to a partial understanding of the relations between the flow parameters and the intensity and spectral distribution of the resulting sound. Because this symposium is concerned with naval problems, it is appropriate to limit the discussion to underwater sounds associated with the flow of water. The discussion will also be limited to those sounds which can be considered to be "noise" in the sense that the sound is either undesirable or else an incidental concomitant of the flow.*

* The generation of sound by underwater sirens and other devices specifically designed to act as sound sources is discussed by Bouyoucos.(2)

Underwater sounds are ordinarily detected by devices sensitive to pressure fluctuations. These devices are called "hydrophones," and they are simply waterproofed microphones. Accordingly, the physical quantity of interest in hydrodynamic noise is generally the fluctuating pressure associated with the hydrodynamic flow. The magnitudes of the fluctuations of interest are very small compared with the pressures ordinarily met in steady hydrodynamics: a pressure fluctuation of only 100 dyne/cm^2 (a head of about 0.04 inch of water) is an intense sound in water. The power associated with these fluctuations is also very small: the entire underwater sound output of a noisy ship is only a few watts. It is thus apparent that hydrodynamic noise may stem from second-order effects which have no influence at all on the more obvious characteristics of the flow.

It is convenient to classify the various forms of hydrodynamic noise according to the grosser flow phenomena with which they are associated. Four categories will be considered here: 1. Air bubbles entrained in water, 2. Vaporous cavitation, 3. Surface disturbances such as splashes, and 4. Unsteady flows such as vortex shedding and turbulence. Each of these phenomena requires somewhat different methods of analysis; accordingly they will be discussed in separate sections, the order being chosen merely for convenience of exposition.

II. Entrained Gas Bubbles

Perhaps the most commonplace of all the sounds in water are those associated with bubbles. Sound is generated when bubbles form, when they divide or unite, and when they stream past an obstacle in a flow or through a constriction in a pipe.

The sound of bubbles forming at a nozzle was investigated by Minnaert [3] in 1933, and later by Meyer and Tamm [4], who showed that the sound was associated with volume pulsations of the bubble, and that in these pulsations the bubble behaved like a simple oscillating system with damping. The frequency of the pulsation was calculated by Minnaert and the damping by Pfriem [5] among others.

When bubbles are observed as they rise through a liquid, large oscillations in their shape are apparent. It is natural to ask whether the sound associated with these shape changes is significant relative to the sound from the volume pulsation. This question can be answered by representing the vibration of the bubble wall by a sum of spherical surface harmonics (see Lamb [6] §294)

$$R(\theta,\phi,t) = R_0 + \Sigma_n A_n S_n(\theta,\phi) e^{2\pi i f_n t}, \tag{1}$$

where R is the instantaneous radial coordinate of a point on the bubble wall, as a function of the spherical angles θ and ϕ, and time t; R_0 is the mean radius of the bubble; $S_n(\theta,\phi)$ is the surface harmonic of order n giving the variation with the angles; $i = \sqrt{-1}$; and A_n and f_n are the amplitude and frequency of oscillation for the n-th order. The zeroth order, with $S_0 = 1$, corresponds to volume pulsation, the first order to translational oscillation, and the higher orders to oscillation in shape with n nodal lines.

If the amplitude of oscillation is small, i.e., if $A_n \ll R_0$, the oscillations of each mode contribute independently to the sound pressure. Far from the bubble, the sound pressure amplitude p_n associated with oscillation in the n-th mode at frequency f is given by

$$p_n = \frac{\rho A_n R_0^2 (2\pi f)^2 e^{-ikr} (ikR_0)^n S_n}{r(n+1)\,[1\cdot 3\cdot 5 \cdots (2n-1)]}, \tag{2}$$

where r is distance from the bubble, and $k = 2\pi f/c$. The derivation of this equation assumes that the bubble is small, so that $kR_0 \ll 1$, and that the distance is large so that $kr \gg 1$.

Each mode has a natural frequency of free oscillation. For volume pulsation, the natural frequency is given by

$$f_0 = (3\gamma P_0/\rho)^{\frac{1}{2}}/2\pi R_0, \tag{3a}$$

where ρ is the density of the liquid, and P_0 and γ are the mean pressure and specific-heat ratio for the gas in the bubble. The natural frequencies of shape oscillation depend on the surface tension T in accordance with the relation (see Lamb [6] §275)

$$f_n = [(n^2 - 1)(n + 2)T/\rho R_0]^{\frac{1}{2}}/2\pi R_0. \tag{3b}$$

By substitution of the natural frequencies given by Eqs. (3) into Eq. (2), it can be shown that the sound pressure associated with free oscillations is negligible except for the zeroth mode. For example, at the relatively large amplitude of $A_n = \frac{1}{4} R_0$ (with $R_0 = \frac{1}{3}$ *cm*, $P_0 = 1$ *atmos*) the sound pressures are about 3×10^4 and only 9×10^{-4} *dyne/cm*^{2} in the zeroth and second mode respectively, at r = 100 *cm*. Accordingly, only volume pulsations are of importance in calculations of the sound radiated by bubbles.

For volume pulsation, the instantaneous sound pressure $p_s(t)$ and the instantaneous bubble volume $V(t)$ are related simply by

$$p_s(t) = \rho \ddot{V}(t')/4\pi r, \tag{4}$$

where the two dots indicate the second time derivative, and $t' = t - r/c$.

Sound of bubble formation.—When a bubble forms on a nozzle in water, a pulse of sound is emitted just as the bubble leaves the nozzle. An oscillogram of a typical sound pulse is shown in Fig. 1, together with synchronized frames from a

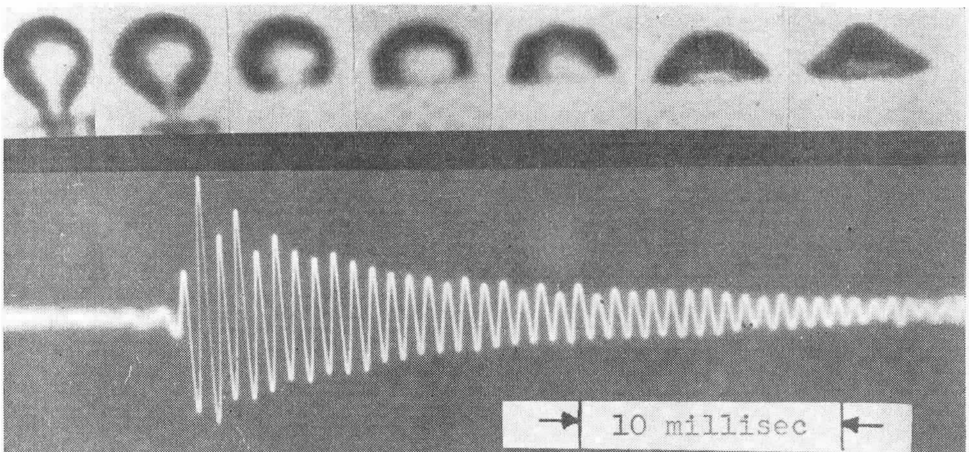


Figure 1. Oscillogram of the sound pulse from an air bubble formed at a nozzle, with synchronized photographs of the bubble. The time each photograph was taken corresponds to the point below the bubble on the oscillogram.

motion picture of the bubble. Although the oscillations of the bubble shape are apparent in the motion picture, the volume pulsations generating the sound are not visible because of their small amplitude.

If the pulsations are of small amplitude, the instantaneous volume of the bubble pulsates in accordance with the linear differential equation

$$(\rho/4\pi R_0)\ddot{V} + D\dot{V} + (\gamma P_0/V_0)(V - V_0) = 0, \tag{5}$$

where V_0 is the mean volume of the bubble; D is a constant which accounts for the damping; and the dots indicate time derivatives.* The volume oscillates sinusoidally with decaying amplitude about its mean value. Accordingly, the instantaneous sound pressure is also a decaying sinusoid represented by

$$p_s(t) = p_0 e^{-\pi\delta f_0 t} \cos(2\pi f_0 t - \vartheta), \tag{6}$$

where f_0 is the natural frequency of pulsation as given by Eq. (3a), and $\pi\delta$ is the natural-logarithmic decrement of the oscillation. The amplitude p_o and phase constant ϑ depend on the initial conditions exciting the bubble. (For an air bubble in water at atmospheric pressure, $f_0 R_0 = 330$ *cm/sec;* and $\pi\delta = 0.0045 + 0.0014(f_0 sec)^{1/2}$, the two terms resulting from radiation and thermal dissipation, respectively.)

For a bubble leaving a nozzle, the constants which depend on the initial conditions can be evaluated from the radial velocity attained by the growing bubble just before it separates from the nozzle. The sound pressure was calculated in this way by one of us [7] using values of the radial velocity measured on motion pictures of the bubble. The calculated sound pressures agree quite well with the measured values. The frequency and decay rate also agree with the theoretical values. Accordingly, it can be concluded that the generation of sound at bubble formation is well understood.

When bubbles split or unite, a pulse of sound is generated just as at bubble formation. In this case the excitation is caused by a change in the pressure inside the bubble: the single larger bubble has less surface tension pressure. If a bubble splits into two smaller ones of equal size, the peak sound pressure of the pulse at distance r can be shown to be of the order of $0.5T/r$, independently of the bubble size. This value is considerably less than the pressure generated at bubble formation.

Sound from entrained bubbles.—Probably the most important sounds result from the flow of entrained bubbles past a body in a stream. The bubbles experience a transient pressure as they move through the hydrodynamic pressure field around the body. The transient pressure causes the bubbles to pulsate and radiate sound.

The pulsations are described by a differential equation like Eq.(5) but with a forcing term on the right side equal to $P_0 - p_e(t)$, where $p_e(t)$ is the instantaneous environmental pressure, i.e., the pressure that would exist in the liquid at the bubble location if the bubble were absent. The solution of this equation is conveniently expressed in terms of Fourier transforms.

If a Fourier transform $h(f)$ of the environmental pressure is defined by

$$h(f) = \int_{-\infty}^{\infty} [p_e(t) - P_0] e^{-2\pi i f t}\, dt, \tag{7}$$

then the transform $s(f)$ of the sound pressure radiated by the bubble can be calculated from

$$s(f) = \frac{(R_0/r)e^{-ikr}}{(f_0/f)^2 - 1 + id} h(f); \tag{8}$$

where the entire fraction is the response characteristic of the bubble. The spectral density $E(f)$ of the sound energy is related to these transforms by

$$E(f) = (8\pi r^2/\rho c)\, |s(f)|^2, \tag{9}$$

$E(f)$ being defined so that the energy radiated by the bubble in a narrow frequency band of width Δf is $\Delta f\, E(f)$, and the total sound energy radiated over the entire frequency range in all directions is $\int_0^\infty E(f)\, df$. If a large number of bubbles radiate similar transients at random intervals with average repetition frequency N, the spectral

* When written in terms of the volume, Eq.(5) gives an approximate description of the pulsations even for a non-spherical bubble, if R_0 is taken as the radius of a spherical bubble of equal volume. Eq.(4), however, is exact regardless of the bubble shape.

density of the sound power is $N\,E(f)$, and the *rms* sound pressure in a band of width Δf is $(2N\Delta f)^{1/2}\,s(f)$.

The sound energy tends to be concentrated at the natural frequency f_0 of the bubble, because the denominator in Eq. (8) is a minimum when $f = f_0$. However, in many practical situations the fluctuating environmental pressure is such that $h(f)$ differs from zero only at low frequencies, much below the natural frequency f_0 of the bubble. In this case, the instantaneous sound pressure $p_s(t)$ can be expressed directly in terms of environmental pressure $p_e(t')$ at earlier time $t' = t - r/c$ by

$$p_s(t) = -\,(R_0/r)\,(2\pi f_0)^{-2}\ddot{p}_e(t'). \tag{10}$$

The calculation of the sound radiated by motion about a particular body requires a knowledge of the environmental pressure at the moving bubble as a function of time. This can be obtained from the known pressure and velocity field around the body. If, for simplicity, it is assumed that the bubble follows a streamline, its position as a function of time is determined by integrating the velocity along a streamline, and the pressure is then determined from the position. This procedure, although straightforward in principle, usually results in expressions which cannot be expressed in closed form.

These calculations have been carried out by one of us [7] for the flow past a circular cylinder, with the bubble moving around the surface of the cylinder. The relatively complicated result for the sound pressure is

$$p_s(t) = \frac{32\rho^2 U_0^4 R_0^3}{3\gamma P_0 R_c^2 r}\left[\operatorname{sech}^2\left(\frac{2U_0}{R_c}t'\right) - \tfrac{3}{2}\operatorname{sech}^4\left(\frac{2U_0}{R_c}t'\right)\right] \tag{11}$$

where U_0 is the free-stream velocity; R_c is the radius of the cylinder; t is measured relative to the instant the bubble passes the median plane; and it is assumed that $(2U_0/R_c) \ll f_0$. The waveform of the sound pressure is shown in Fig. 2. The sound pressure has a negative peak at $t' = 0$ and two smaller positive peaks at $t' = \pm 0.6 R_c/U_0$. The peak pressure can be quite high: for the conditions $U_0 = 5$ *meters/sec*, $R_0 = 1/3$ *cm*, $R_c = 10$ *cm*, the negative peak from an individual bubble is about 16 *dyne/cm*^{2} at 1 meter. No experimental work has yet been performed to verify these calculations.

For bubbles streaming past a cylinder, the sound pressure should increase with the fourth power of the velocity, as indicated by Eq.(11). In fact, the sound pressure should increase as U_0^4 for all similar forms of bubble motion, as is apparent from a dimensional consideration of Eq.(10): the environmental pressure fluctuations increase as U_0^2 and the second time derivative introduces another factor $(U_0/L)^2$, L being a characteristic length.

Large-amplitude pulsations.—The rapid increase of sound pressure with increasing velocity leads to a consideration of the limitations of the linear theory on which the above results are based.

The large-amplitude free pulsations of gas bubbles have been studied extensively in connection with underwater explosions [8]. It is known that the radiated pressure loses its sinusoidal form with increasing amplitudes, the negative excursion becoming flatter while the positive excursion develops a sharp peak. The same effect occurs in forced large-amplitude pulsations in response to a widely fluctuating environmental pressure.

As a first step toward a more-exact description of large-amplitude pulsations, the radial pulsation is described by a non-linear equation

$$\rho(R\ddot{R} + \tfrac{3}{2}\dot{R}^2) = P - p_e(t), \tag{12}$$

where R is the instantaneous radius of the bubble, and $p_e(t)$ is again the environmental pressure that would exist at the bubble location in the bubble's absence. Also, P is the actual pressure in the water at the bubble wall; for a bubble filled with gas obeying the

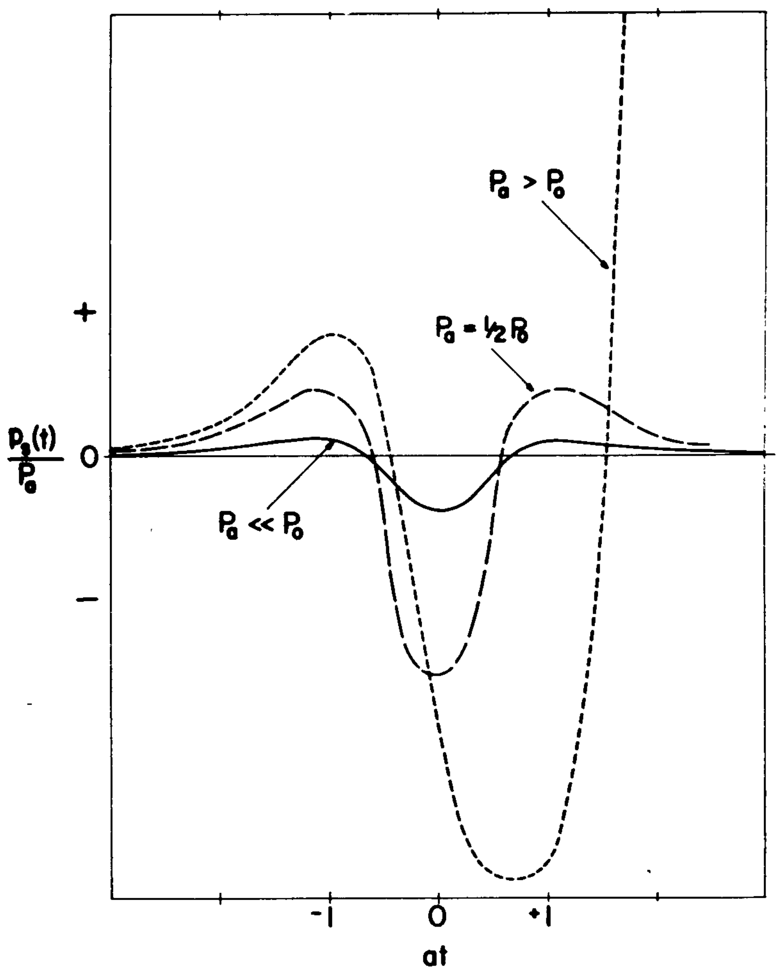


Figure. 2... The sound pressure radiated by a bubble in response to an environmental pressure given by $p_e(t) = P_0 - P_a \operatorname{sech}^2 at$, which is similar to the pressure on a bubble moving around the surface of a cylinder in a flow. The curves indicate the development of non-linearity for successively smaller values of P_0 relative to P_a. The solid curve represents the linear range.

adiabatic gas law, $P = P_g(R_0/R)^{3\gamma} - (2T/R)$, where R_0 is the equilibrium radius of the bubble at the mean gas pressure $P_g = P_0 + 2T/R_0$; and T is the surface tension.*

The non-linear effects can be investigated most easily if the environmental pressure is assumed to vary in some definite way. For comparison with the linear theory, consider the fluctuation given by $p_e(t) = P_0 - P_a sech^2 at$, which results in sound with a waveform like that of Eq. (11) if $P_a \ll P_0$. The changes in the waveform associated with larger values of P_a are illustrated qualitatively in Fig. 2. For $P_a \ll P_0$, Eq. (12) becomes linear and the waveform of the radiated pressure (solid curve) is just that given by Eq. (11). As P_a is made larger, non-linearity occurs first in the term $(R_0/R)^3$ in Eq.(12), and the negative peak in the sound pressure grows rapidly (dashed curve).

* All pressures are measured relative to the vapor pressure of the water, in order to avoid a repetitive constant.

If $P_a > P_0$, so that $p_e(t)$ is actually negative at $t = 0$, the bubble becomes statically unstable and the non-linear time-derivative terms in Eq.(12) control the radial motion of the bubble.** As $p_e(t)$ goes negative, the bubble grows to many times its equilibrium size and then, when $p_e(t)$ returns to positive values, the bubble suddenly collapses and radiates a very large positive pulse (dotted curve). Noltingk and Neppiras [9] have calculated the size to which the bubble grows and the magnitude of the pressure pulse as a function of the ratio P_a/P_0.

During its growth, the bubble becomes filled primarily with vapor. The large growth and subsequent rapid collapse of the bubble are characteristic of the behavior of vaporous cavitation bubbles. Indeed, the transition from small to large amplitude pulsations is just the transition to cavitation, with the bubble acting as a cavitation nucleus. The sound associated with these large-amplitude pulsations will be discussed in greater detail in the following section.

III. Cavitation

The origin of cavitation noise.—The earliest investigators of cavitation were aware of the noise it makes. It was probably the loud hissing sound which first attracted the attention of Osborne Reynolds [10] to the occurrence of cavitation in water flowing through a constricted tube. He recognized the cause of the sound to be the "boiling" of the water. During the First World War, it was known that cavitation of ships' propellers radiated sound which could be heard underwater for great distances [11].

The sound is generated as a result of the growth and collapse of vapor cavities. A cavity, beginning as a microscopic nucleus, grows when its environmental pressure becomes sufficiently negative and collapses when the pressure is restored. Such behavior, with accompanying noise, is to be expected wherever nuclei in the water are subjected to sufficiently extreme transient reductions in their environmental pressure.

The sketch, Figure 3, depicts one common occurrence, the growth and collapse of individual cavities in the pressure field produced by flow past a curved boundary such as that of a propeller, a strut or similar appendage, or past a contraction in a conduit. The growth begins after a nucleus of some sort enters the region of low pressure and the collapse takes place when the cavity is carried downstream into a region of higher pressure. Various experimenters have employed high-speed photography to demonstrate the existence of individual cavities, roughly spherical, which grow and collapse in the manner indicated [12, 13, 14, 15].

Cavitation noise may be produced also in turbulent shear flows in pipes, jets, or boundary layers; by water hammer and by acoustic radiation; and even as a concomitant of "steady" cavities. In each instance, the sound is related directly to the kinematics of the individual transient cavities. The latter may appear as elongated "cores" at the centers of vortices; in shear flows, they may be rather amorphous and may undergo rapid distortion. Where the velocity gradients are mild, the cavities tend to a spherical shape. Nearly all analytical treatments of the hydrodynamical problems assume a spherical cavity. This is not completely unrealistic: the validity of many of the relations derived is not seriously affected by departures from spherical symmetry.

Interest in the inception, growth, collapse, and rebound of vapor cavities stems from various fields of interest: boiling of liquids, detergent and chemical effects of ultrasonics, absorption of sound in water, efficiency of hydraulic machinery, and pitting and corrosion of structural materials. A great amount of research has been

** If the equilibrium size of the bubble is very small, the condition for instability is not simply $P_a > P_0$, but rather $P_a > P_0 + (4\sqrt{3}\ T/9R_0)\cdot[1 + (P_0R_0/2T)]^{-1/2}$. If P_a exceeds this value by an appreciable amount, the size to which the bubble grows before collapsing is relatively independent of its original size R_0.

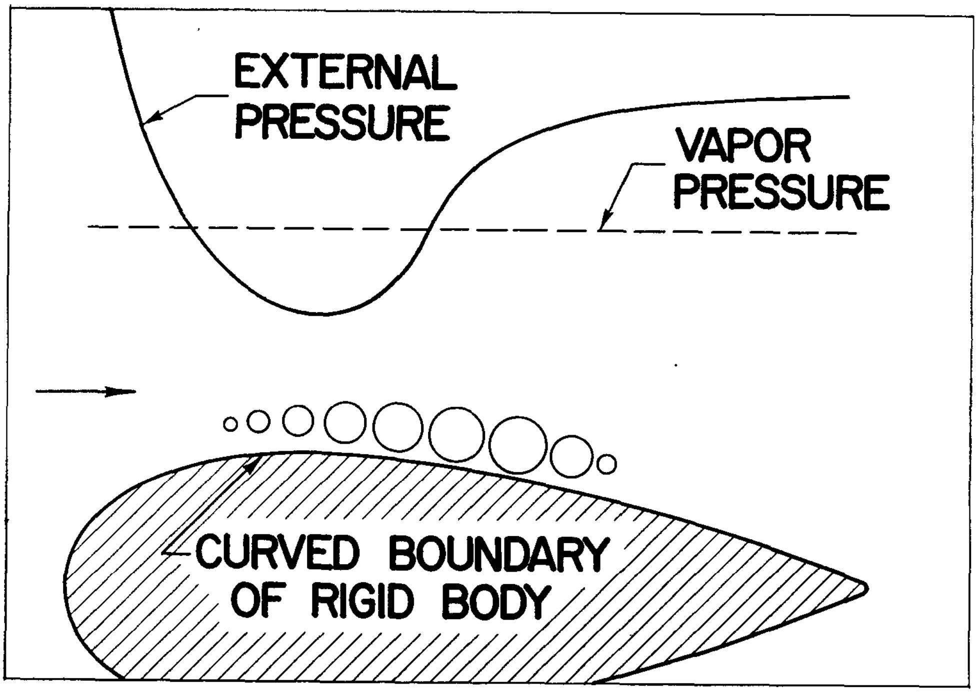


Figure 3. Sketch indicating growth and collapse of a vapor cavity.

directed toward each of the aspects mentioned. For our purpose, here, it will be sufficient to select a few examples of theory, experiment, and computation best suited to illustrate the way in which cavitation acts as a source of sound. Those interested in more general aspects of the various phenomena touched upon will find extensive discussions in bibliographical material relating to cavitation [16].

Calculation of the sound pressure: acoustic theory.—In the previous section, the sound pressure generated by a gas bubble was related to the fluctuating environmental pressure encountered by the bubble. It is of interest to ask whether, in the case of a transient vapor cavity, a corresponding relation can be given.

One of the characteristics of the behavior of transient vapor cavities is the violence of the collapse. In that part of the motion, the inward radial velocity of the fluid in the immediate vicinity of the cavity may exceed the velocity of sound in the liquid and pressures comparable to the modulus of compressibility of the liquid may be developed. The description of the motion in the final stages of collapse therefore requires consideration of the compression of the liquid. But during the period of initial growth and most of the period of collapse, the incompressive theory yields a correct description of the motion of the cavity, and the sound pressure (at sufficiently great distance $r \gg R$) is given accurately by Eq. (4) in Section 2.

If the temporal variation of the environmental pressure encountered by the cavity is known, the resulting motion, or at least the incompressive stages thereof, may be computed from Eq. (12). The roles of surface tension and of any minute quantity of gas which may be contained in the cavity are important only when the cavity is very small. For the purpose of the present discussion it is sufficient to consider that their combined effect is simply to forestall the inception of growth of the nucleus until the environmental pressure has reached a certain value (smaller than the vapor pressure and, in fact, typically negative) characteristic of the individual nucleus (See Section 2). Once the growth has begun, the pressure P at the cavity becomes essentially zero*. The example of Figure 4 rather exaggerates the ordinary extent of the delay of the inception of growth.

Of special interest is the computation of the motion for situations like that depicted in Figure 3. If the temporal variation of the environmental pressure

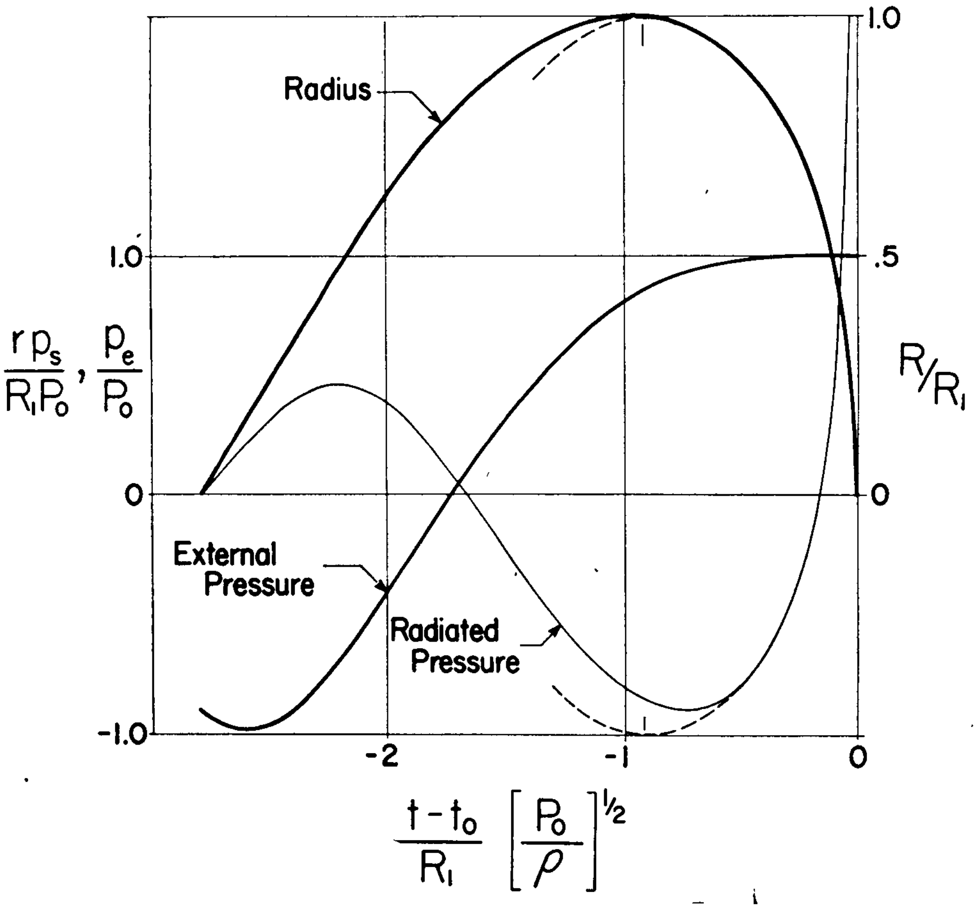


Figure 4. **Computed radial motion and sound pressure for a growing and collapsing vapor cavity.**

encountered by a passing nucleus can be deduced from a knowledge of the velocity and pressure fields in the vicinity of the curved boundary, Eq. (12) can be solved by numerical integration to obtain the radius of the cavity as a function of the time. Plessett [17] has done this and, despite the obvious idealizations introduced, found the computed radial motion to resemble the observed behavior reasonably well. Figure 4 shows the results of a typical, though hypothetical, example of such a computation,

* Note that, to avoid needless repetition throughout the discussion, all pressures are referred to the vapor pressure.

illustrating not only the radial motion of the cavity but also the resulting sound pressure. The abscissa represents time. The curve labelled "External Pressure" indicates a temporal variation in environmental pressure typical of that encountered by a cavity under conditions like those being considered. The radius of the cavity is shown by the heavy solid curve. The constant P_0 is the value of the pressure difference, $(p_e - P)$, at the final instant of collapse, t_0. A characteristic radius, R_1, may be defined so that, in the final stages of collapse, the total energy, kinetic and potential, is $\frac{4}{3}\pi P_0 R_1^3$.

The sound pressure, shown by the light solid line, oscillates once and, at the end of the collapse, rises in an extremely high narrow "spike." The details of this high narrow pulse (which determine the high-frequency portion of the sound spectrum) are not given correctly by the acoustic theory. While most of the energy radiated as sound is associated with the pulse at the end of the collapse, the sound associated with the earlier part of the motion is by no means negligible.

The broken lines indicate the solution for the special case, considered by

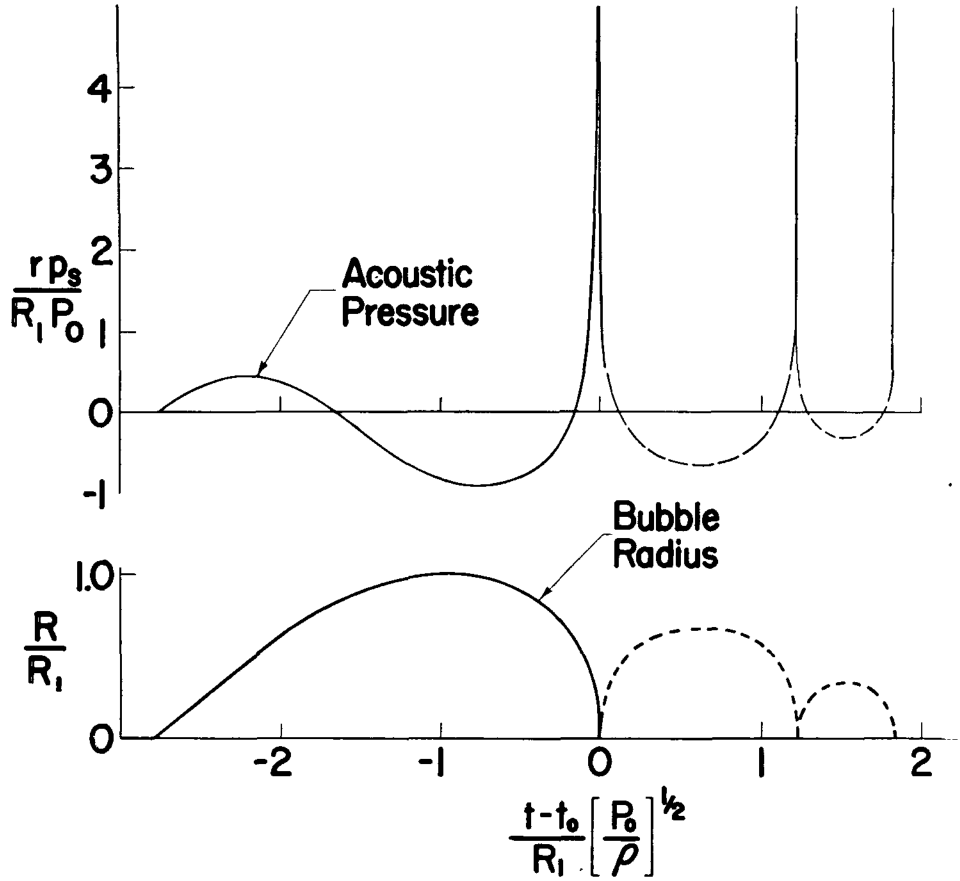


Figure 5. Radial motion and sound pressure for a vapor cavity which rebounds after growing and collapsing.

Rayleigh [18], in which $(p_e - P)$ is constant and equal to P_0. He found, by expressing the constancy of the sum of the kinetic energy, $2\pi\rho R^3\dot{R}^2$, and the potential energy, $\frac{4}{3}\pi R^3 P_0$, that the radius and the wall velocity are related by the equation,

$$(R_1/R)^3 = 1 + (3\rho\dot{R}^2/2P_0). \tag{13}$$

For this case, R_1 is the maximum radius of the cavity.

Some cavities rebound after collapse. The mechanism by which the rebound comes about is not known. Apparently a small quantity of gas contained in the cavity plays an essential role, but details are uncertain. What is known is that the flow velocity and pressures attain such values that the acoustic theory is not applicable to the part of the motion for which the cavity is very small. Figure 5 continues the illustration, however, showing the behavior typical of those cases in which rebound occurs. The assumption of an empty cavity and an incompressible liquid have been retained. The cavity is assumed to collapse to an indefinitely small radius and to rebound with an arbitrarily postulated loss of energy. The whole sequence is clearly only an illustration; actual cavities show a wide range of behavior. Knapp and Hollander [13] photographed cavities which rebounded as many as five times. Benjamin [19] has also observed repeated rebounds under different conditions.

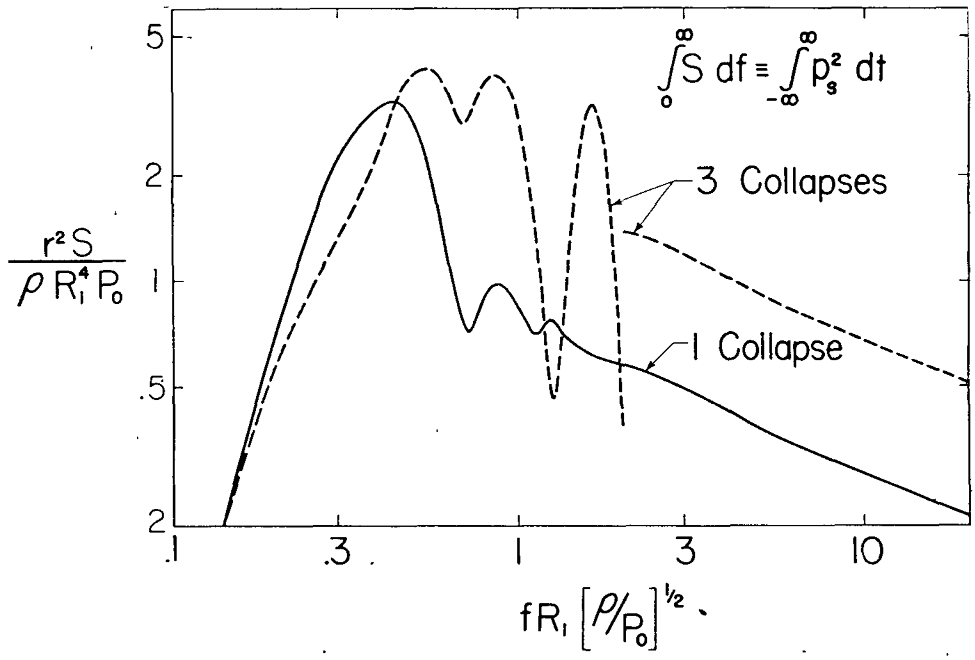


Figure 6. Spectral distribution of the sound, for motions shown in Figure 5, as computed for an incompressible liquid.

The spectrum of the sound: acoustic theory.—From the sound pressure, the spectral distribution of the radiated energy may be computed. Figure 6 shows the spectra corresponding to the growth and single collapse and to the growth and multiple collapse postulated in the example. The spectra exhibit maxima at frequencies of the order of the reciprocal of the time required for growth and collapse. At lower fre-

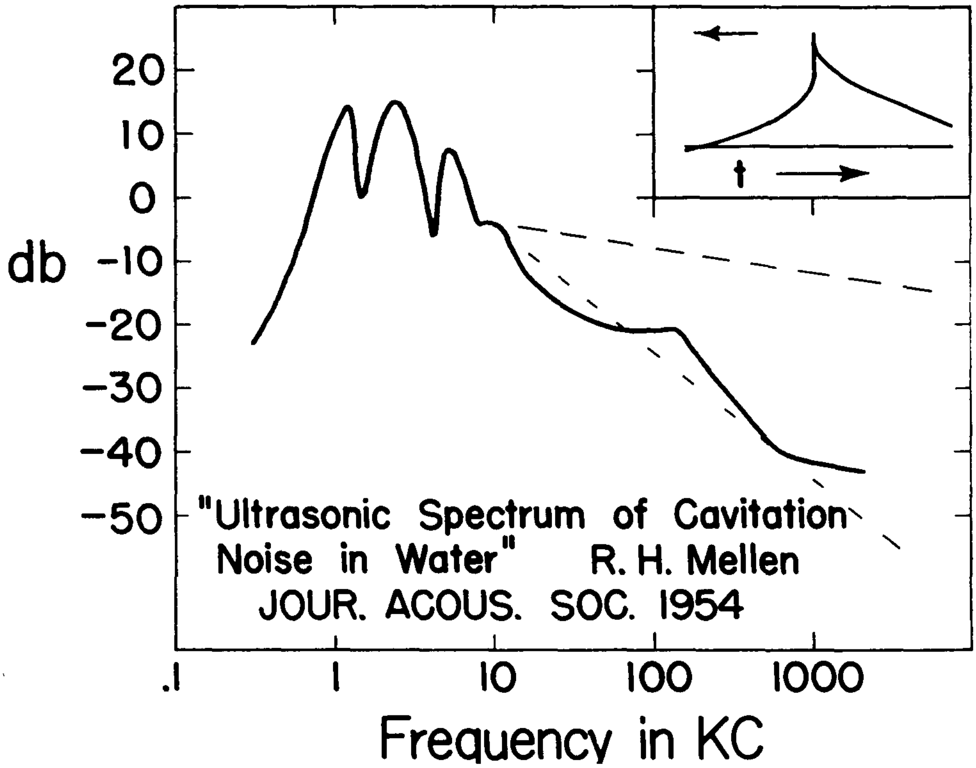


Figure 7. A measured spectrum of cavitation noise. The solid curve represents the spectrum of the steady noise observed at a distance of 1 meter from a metal rod, 2 inches long and 1/16 inch in diameter, rotated at 4300 rpm about a transverse axis through its center. The reference level for the decibel scale corresponds to sound pressure of 1 dyne/cm^2 in a 1 cy/sec band of frequency. The cavities observed were about 1 mm in diameter. The pressure was approximately 1 atmos.

quencies, the spectral density necessarily varies as the fourth power of the frequency.* The oscillations in the spectrum of the sound generated by the multiple collapse do not disappear from the computation at the higher frequencies. However, the oscillations will not appear in the spectrum of the sound generated by a succession of cavities which do not all grow, collapse, and rebound in exactly the same way. Accordingly, at the higher frequencies the "smoothed out" spectrum is shown. The asymptotes

* This follows from the fact that the volume of the cavity is necessarily positive during its lifetime and essentially zero before and after. The magnitude of the Fourier transform of the volume (the latter considered as a function of time) is then, by well-known properties of the Fourier transformation, to the first order, independent of the frequency at values of the latter which are small in comparison with the reciprocal of the total lifetime of the cavity. Since the sound pressure is proportional to the second time derivative of the volume of the cavity (Eq. (4), Sect. 2), the exponent relating the spectral density and the frequency at small values of the latter is thereby determined. The conclusion applies to the spectrum of the sound generated by a single cavity or by a random succession of cavities, but must be modified if referred to a sequence of cavities in which the behavior of successive cavities is in any way correlated. The presence of a reflecting boundary, especially a free surface will, of course, alter the frequency dependence also.

indicating that the spectral density at high frequencies varies as the reciprocal of the two-fifths power of the frequency cannot, of course, correctly describe the spectra of real cavitation noise, if only because of the physical requirement of finite energy. The departure is simply a manifestation of the fact that the high-frequency portion of the spectrum is determined by details of the "spike" of sound pressure which are not given correctly by the incompressive and acoustic theories.

The spectrum of the sound: experimental.—Figures 7 and 8 show examples of spectra of cavitation noise obtained experimentally by Mellen [20] and Jorgensen [21] respectively. Three features characterize the spectrum: the location of the peak, and the two exponents corresponding to the asymptotic behavior at low and at high frequency. At high frequencies, the spectral density is observed to vary roughly as the reciprocal of the square of the frequency (—6db/octave). This feature of the spectrum suggests that the sound pressure undergoes a sharp rise of the nature of a shock. Direct evidence of the sudden rise, such as an oscillograph record showing it, is difficult to obtain because of the extremely short time interval which must be resolved (perhaps 10^{-7} second or smaller). However, such records showing indications of a shock have been obtained through the use of tiny barium titanate transducers [22]. Shock waves emitted by collapsing cavities have also been shown by Schlieren photography [23, 24].

Compressive flow in the collapse of a cavity.—The explanation of the outgoing

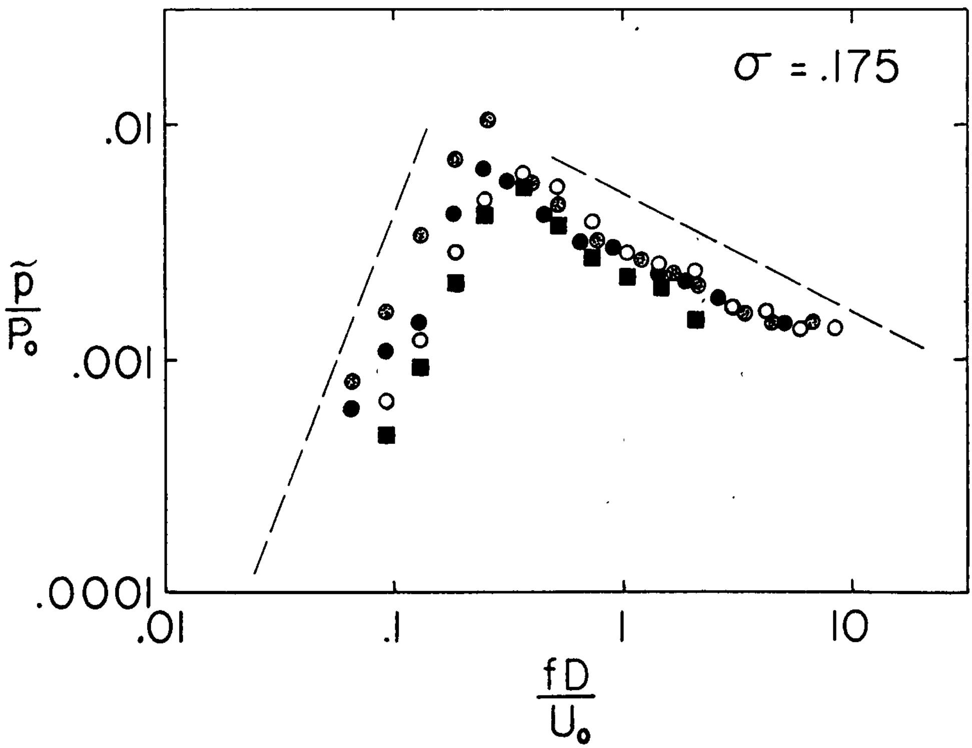


Figure 8. **The measured spectrum of noise produced by a cavitating jet. The symbol $\tilde{p}$ represents the rms pressure in a half-octave band of frequency, at a point four diameters off the axis and four diameters downstream from the orifice. The different spots represent different combinations of jet diameter, D (3/8, 3/4, and 1 1/2 inch), and ambient pressure, P_0 (1/2, 1, and 2 atmos). U_0 is the efflux velocity; σ is the cavitation index, $2P_0/\rho U_0^2$. (From Jorgensen [21])**

shock developed at the collapse of a vapor cavity must begin with a consideration of the effect of the compressibility of the liquid upon the motion. Various extensions of acoustic theory in which the wave equation in the velocity potential has been retained have provided relations which can be said to give "first order" corrections for the effect of compressibility of the liquid [25]. These results are useful in the consideration of the collapse of the gas globe formed by an underwater explosion, where the flow velocities do not exceed one-tenth of the velocity of sound and the pressures do not exceed about six percent of the compressibility modulus of the liquid. In the case of a collapsing vapor cavity, however, the significant effects of compressibility are manifest while the cavity contains essentially only condensing vapor: the small amount of gas inside is not sufficient to arrest the collapse before the inward flow velocities exceed the velocity of sound [26]. The problem can be put then, in idealized form, as the calculation, first, of the motion and, then, of the surrounding pressure and velocity fields, in the case of the collapse of an empty spherical cavity in a compressible liquid [27].

The equations of compressive flow do not, in general, admit of explicit solutions. Gilmore [28, 29] however, has derived an equation relating the radius of the cavity explicitly to the velocity and acceleration of the wall of the cavity. In the special case of the empty cavity, Gilmore's differential equation is

$$R\ddot{R}(1 - \dot{R}/c_0) + \tfrac{3}{2}\dot{R}^2(1 - \dot{R}/3c_0) = - P_0/\rho_0, \qquad (14)$$

which reduces to the corresponding form of (12) for vanishingly small values of $\dot{R}/c_0$. For the initial conditions considered by Rayleigh in the incompressive case, Gilmore gives

$$\left(\frac{R_1}{R}\right)^3 = \left(1 - \frac{\dot{R}}{3c_0}\right)^4 \cdot \left(1 + \frac{3\rho_0\dot{R}^2}{2P_0}\right), \qquad (15)$$

which also reduces to the corresponding incompressive solution, Eq. (13), as $\dot{R}/c_0 \rightarrow 0$. Gilmore's results are derived from a hypothesis of Kirkwood and Bethe which states that in the spherically symmetric flow about the cavity the quantity $r(h + \tfrac{1}{2}u^2)$, which in isentropic flow is exactly equal to $r\dot{\phi}$, is propagated outward with variable velocity $(c + u)$. Here r is the radial coordinate; c is the local value of the velocity of sound and $\dot{\phi}$ is the time derivative of the velocity potential. The enthalpy h is defined in Fig. 9. Where the pressure in the fluid differs from the ambient value by only a few atmospheres, h is simply $(p - P_0)/\rho_0$, but where the pressures are not negligible in comparison with the modulus of compressibility, ρc^2, of the liquid (about 21,000 atmos for water), the value of h depends also on the relation between the pressure and the density of the liquid. (In the present discussion, c and ρ must be considered variable; their values at ordinary pressures (near zero) will be denoted by c_0 and ρ_0). Here, p and u are the pressure and the radial flow velocity.

The sound pressure, p_s, at some large value of the radial coordinate is, according to the theory, determined as follows: The value of the propagated quantity $r(h + \tfrac{1}{2}u^2)$ is easily evaluated at the wall of the cavity as $R\dot{R}^2/2$. The "outgoing path" of each of its successive values may be traced through the pressure and velocity field surrounding the collapsing cavity according to the rule given by the Kirkwood-Bethe hypothesis. The assumed behavior is illustrated qualitatively in the diagram, Figure 9. Several outgoing paths or "characteristics" are shown, each corresponding to a different value of the propagated quantity, the particular value being the value of $R\dot{R}^2/2$ at the instant at which the path "left" the wall of the cavity. The slope of the line representing each such outgoing characteristic represents, at each point in

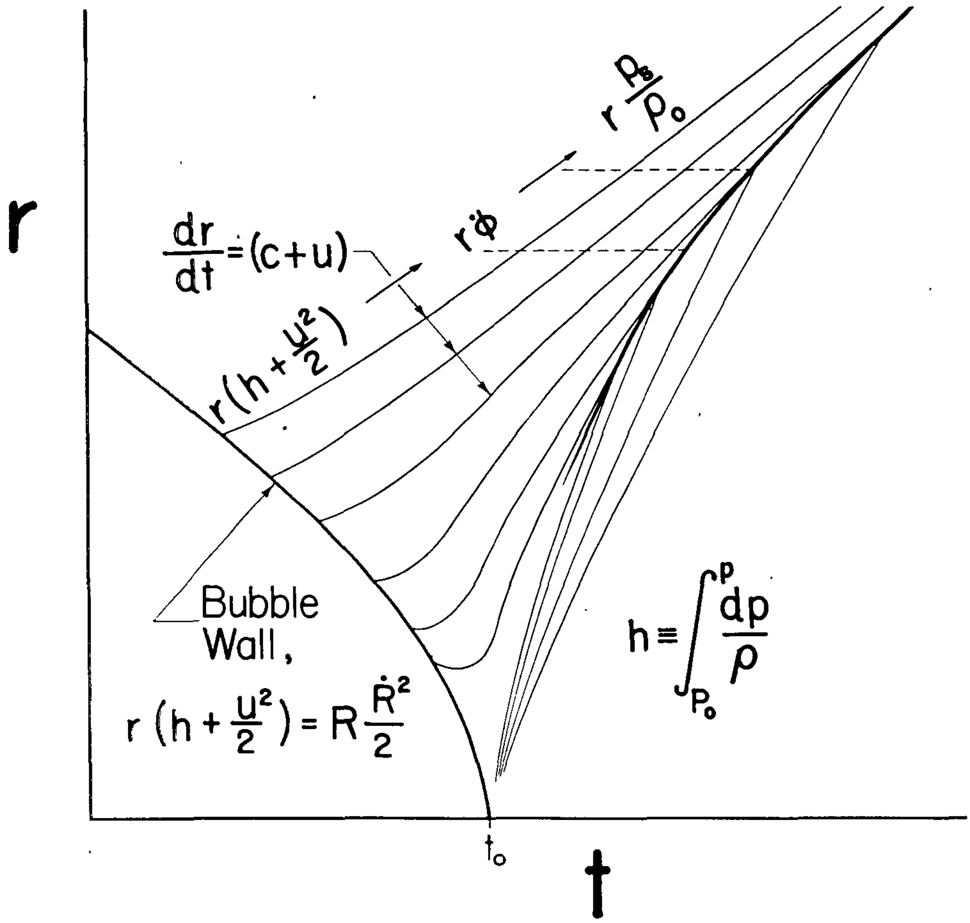


Figure 9. **Sketch illustrating the method of calculating the sound pressure radiated by a collapsing vapor cavity according to theory of Gilmore.**

the diagram, the value of $(c + u)$ at the corresponding radial coordinate and instant of time. Outside the immediate vicinity of the cavity, the theory is nearly exact: the value of the propagated quantity gives the value of $r\dot{\phi}$ and hence, at large values of the radial coordinate r, the acoustic pressure p_s (since $p_s = \rho\dot{\phi}$). In order to construct the outgoing paths, it is necessary to determine the value of $(c + u)$ at each radial coordinate and instant of time, information itself partly dependent upon the result of the calculation, so that it is necessary to employ either iterative methods or an adequate analytic approximation to the required velocity field. The calculation is further complicated by the development of a shock, whose path of propagation must be computed by separate considerations. The development of the shock also is indicated qualitatively in the diagram, which shows how larger values of the propagated quantity "overtake" smaller values which "left" the wall of the cavity at an earlier instant. The entire diagram is only crudely qualitative; it is impossible to detail the actual situation in undistorted form suitable for ready visualization.

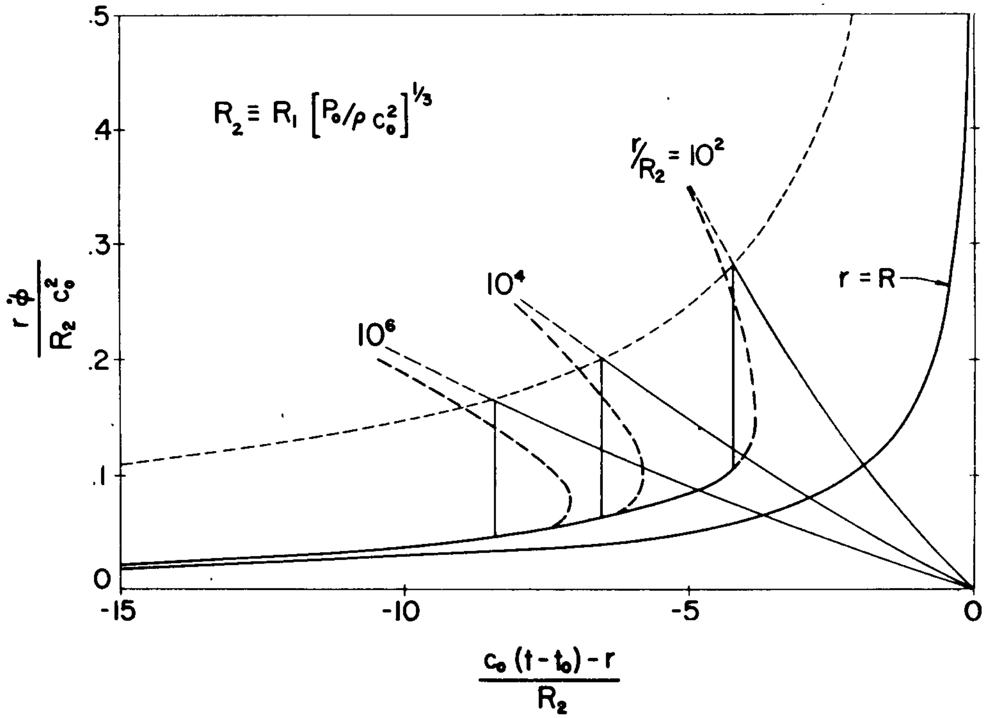


Figure 10. **Pressures and shocks radiated by a vapor cavity collapsing in a compressible liquid. (Values according to Mellen [30].)**

Mellen [30] has made the indicated calculations, taking approximate values for $(c + u)$, with the result shown in Figure 10. Each solid line exhibiting a vertical rise shows the value of $r\dot{\phi}$ (very nearly equal to rp_s/ρ) as a function of time for one selected value of a parameter involving the radial coordinate. In the case of the collapse of a cavity in water with external pressure P_0 equal to one atmosphere, the three radial distances indicated would be approximately 3.6, 360, and 36000 R_1. The peak pressures at the "shock" front are, for the same case, 59, 0.42, and 0.0035 atmospheres. Mellen also obtained estimates of the peak pressures experimentally and found good agreement with the calculations.

From the pressures shown in Figure 10 as functions of the time, the corresponding frequency spectra might be computed. The spectrum shown in Figure 6 could be corrected, at high frequencies, for the compressive effects neglected in its original derivation. In view of the somewhat limited accuracy of the values shown and of a number of idealizations made tacitly in the brief treatment presented above, it is sufficient merely to indicate the high-frequency asymptote corresponding to the discontinuity in the sound pressure. The resulting computed spectra (Figure 11) do show a fair resemblance to observed spectra of cavitation noise. It appears, however, that the computed magnitude of the sudden rise in pressure at the shock front, relative to the parts of the wave which determine the low-frequency part of the spectrum, is higher than is really the case. This is not surprising. The computations assume no loss of energy in the propagated wave except that inherent in the Hugoniot conditions, whereas, in fact, other losses do occur in the propagation of the high-frequency components of sound waves. It is possible, also, that small amounts of

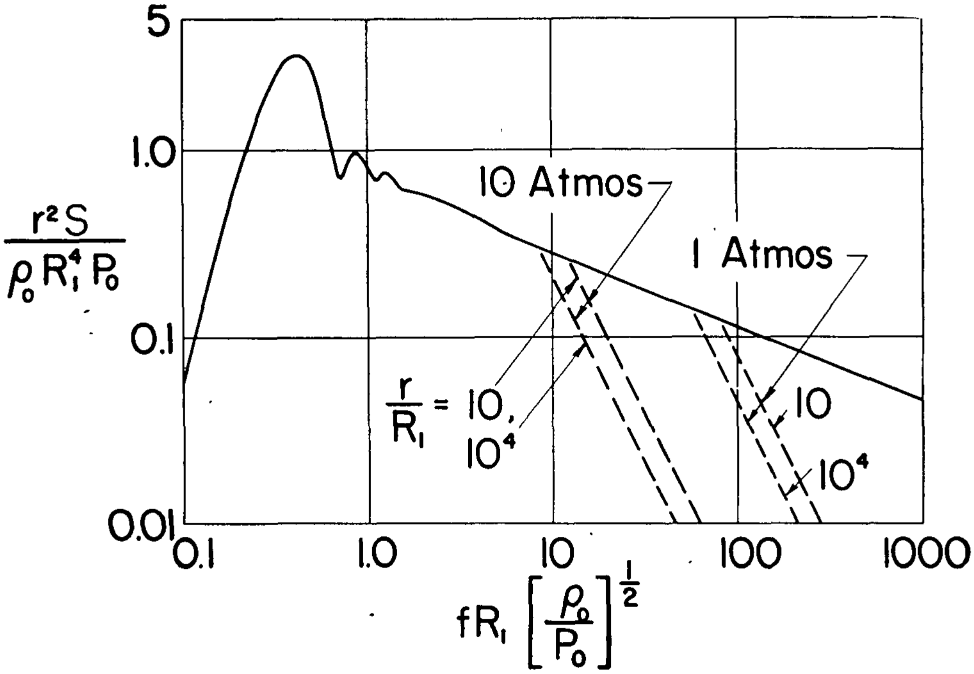


Figure 11. **Spectrum of sound radiated by a growing and collapsing vapor cavity in a compressible liquid. The dash lines show the asymptotes corresponding to the shocks indicated in Fig. 10.**

gas in the cavity affect the amplitude of the shock. A calculation would be necessary to ascertain the effect in a specific case.* A more important reason for the discrepancy, however, is the fact that real cavities may not collapse as spheres. Ample evidence, both theoretical and experimental, indicates that collapsing cavities undergo radical distortions in shape and may even break up into clouds of smaller bubbles. The cavities which produce the noise represented in Figure 8 resemble those shown in Figure 12 which is taken from Ref. [31]. A quantitative description of the motion and, especially, of the development of the shock for any case other than that of spherical symmetry would be very difficult, however. All that can be said is that, despite the uncertainties concerning details, the main features of the sound generated by cavitation seem to be accounted for.

IV. Surface Disturbances

The airborne sound which accompanies the splash made by a droplet or a solid object falling onto the surface of water is well known. It is rather to be expected that concomitant sounds exist in the water below the surface and, indeed, such is the case. This is not to say that there is any resemblance or necessary relation between the two sound fields—only that disturbances of the surface which make sounds in air generally produce other sounds in the water.

---

* Dr. T. Brooke Benjamin has indicated that theoretical work on a related question, the development of the pressures about a collapsing cavity containing a significant amount of gas, is underway at King's College, Cambridge. Results indicate that a shock will develop in the vicinity of a gas-filled collapsing cavity if the pressure at maximum compression exceeds a value of about 2000 atmospheres. This pressure is slightly greater than that reached at the first collapse of the gas bubble formed by an underwater explosion.

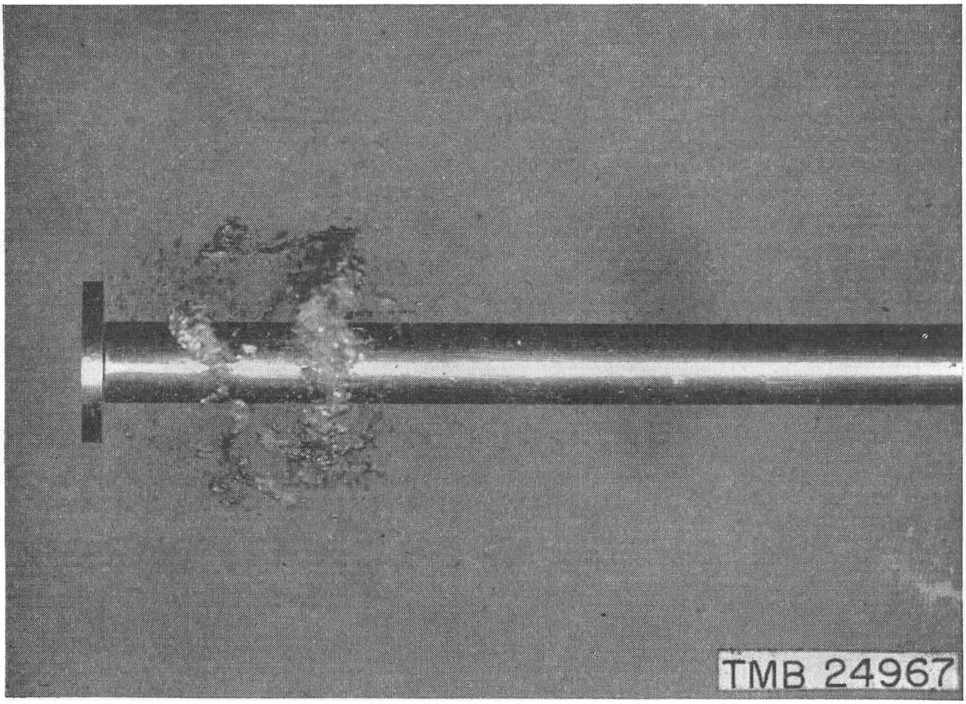


Figure 12. **Transient cavities in the vortex sheet behind a sharp-edged disk. The water flows from left to right. (From Ref. 31.)**

Many other kinds of surface disturbances, natural and artificial, result in underwater sounds. In this section, we shall devote our attention principally to sounds made by impinging solid or liquid objects. One reason for this limitation is that scientific results available to us are concerned principally with such disturbances. Actually, however, the main features of sound production by various kinds of surface disturbances are the same. For example, it will be shown that the underwater sound of sprays and of rainfall is explained simply in terms of the sound produced by the impingement of a solitary droplet. Similarly the underwater sounds of breaking waves can be at least partially understood in the same light.

The study of splashes, cavities, and other visible aspects of water-entry phenomena has occupied the attention of a great many researchers, of whom a number have elucidated the object of their studies by some remarkable photographs. Special mention is due to the study made more than fifty years ago by Worthington [32]. He produced extensive series of photographs showing consecutive stages in the motion of the surface of the liquid during splashes produced by the impingement of liquid droplets and spherical pellets. In 1918, Mallock [33] studied similar splashes and adverted to the existence of cavities and to the relation of the size and shape of the latter to the characteristic pitch of the sounds heard in air. In recent years, a number of researchers have studied various aspects of water-entry phenomena in connection with the behavior of missiles, usually without reference to the sound. The photographs, Figure 13, were obtained by May [34] and show the cavity and the splash produced by the vertical entry of a steel sphere. Recently, Richardson [35] has called attention

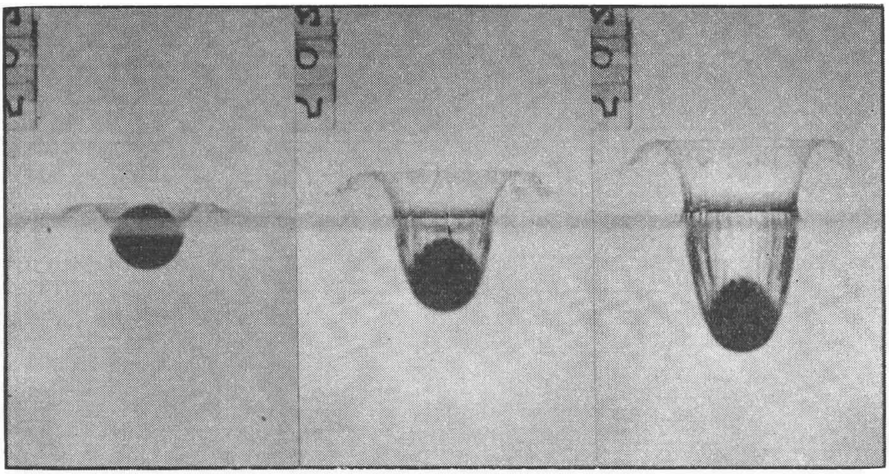


Figure 13. **High-speed motion pictures of a 1-inch steel sphere entering the water at a velocity of 35 ft/sec. The time interval between frames is 1.6 millisec. (From Ref. 34.)**

to the waterborne sounds produced by the impact, the splash, and the volume pulsations of air trapped below the surface. The only systematic attempt to relate the sounds quantitatively to the size and velocity of the impinging object seems to be that of Franz [36], some of whose results will be presented toward the end of this section.

In principle, the understanding of the generation of sounds by surface disturbances requires no theoretical considerations other than those which enter into the question of the origin of all sounds. Thus those relations which determine solutions of the wave equation in terms of the motions and pressures occurring at the bounding surface of the fluid are valid and, in principle, applicable to the determination of the sound field in water. Here, the "bounding surface" of the fluid, i.e. the water, includes the entire air-water interface as well as the boundary at which the water is in contact with any penetrating object. But the determination of the boundary conditions necessary to specify the sound field may, in a given practical situation, present a hydrodynamical problem whose solution is unattainable. It is therefore expedient to seek a more direct way of expressing the relation between the data characterizing the cause of the disturbance and the sound field which results.

The dominant boundary condition on the underwater sound field of sources at or near the free surface is, of course, the requirement that fluctuations in pressure be zero at the free surface. For sound sources which disturb the free surface only acoustically, the modification of the field resulting from the presence of the surface is easily described in terms of out-of-phase reflections at the surface. Where a radical disturbance in the surface itself constitutes the sound source, the situation is more complicated. For simplicity, we consider only disturbances which are symmetrical about a vertical axis. The sketch, Figure 14, indicates the vertical entry of an axially symmetric body of some specified density and shape. We can further specify the body and its motion in terms of a characteristic linear dimension, L, and the velocity, U, with which it strikes the surface (at time $t = 0$). The figure indicates the significance of the coordinates, r and θ. We are interested in the sound pressure, p_s, which, at sufficiently great distances from the disturbance, obeys the wave equation,

$$c^2\nabla^2 p_s - \partial^2 p_s/\partial t^2 = 0. \tag{16}$$

The solution can be expressed in terms of spherical harmonics (Lamb [6], § 292). For our purpose, however, it is more useful to consider the sound field as that

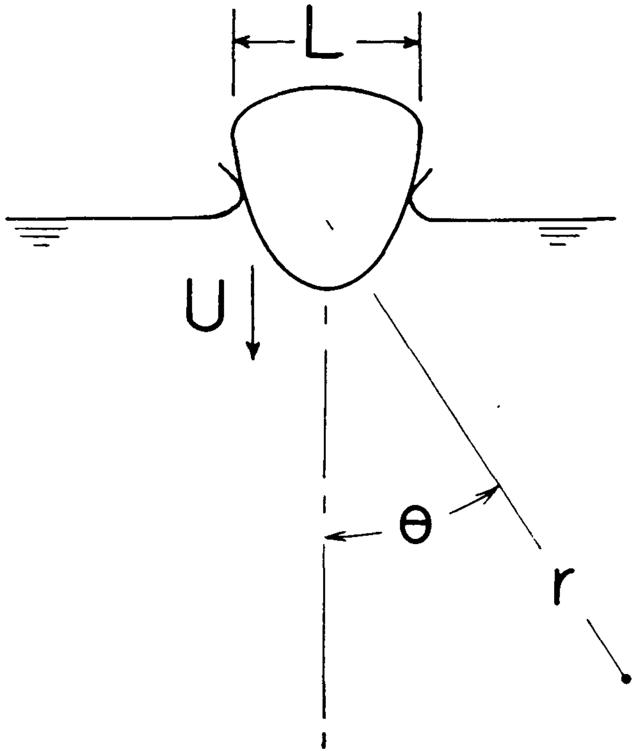


Figure 14. **Sketch illustrating surface disturbance caused by an entering body.**

due to a system of axially symmetric multipole sources located at the origin of the disturbance.* The time-varying strengths of the multipole sources are such as to describe the actual flow velocities outside the immediate vicinity of the disturbance. At some specified stage of the entry, the strength of any such multipole, say of order m, must, from consideration of dimensions, be proportional to UL^{m+2}. But the sound pressure associated with a multipole source of order m is proportional to the $(m + 1)$th time derivative of the strength of the source. These, with appropriate further dimensional considerations, allow the sound pressure to be expressed in the form

$$p_s(r,\theta,t) = \frac{\rho U^2 L}{r} \sum_{m=0}^{\infty} \left(\frac{U}{c}\right)^m A_m\left[\theta, \frac{U}{L}\left(t - \frac{r}{c}\right)\right] \tag{17}$$

The free-surface condition requires that A_0 (corresponding to a simple source) be zero, so that for small values of the Mach number (U/c) the dipole term $(m = 1)$ can be expected to predominate. The dependence of the functions A_m upon the polar angle θ can be expressed very simply: In particular,

$$A_1(\theta,\tau) = Z(\tau) \cos\theta, \tag{18}$$

* The condition for which such a procedure is valid may be stated as follows: (1) all flow velocities involved are very small in comparison with the velocity of sound; and (2) the wavelength of the highest sound frequency of interest is large in comparison with the linear dimensions of the disturbance. Ordinarily this means simply: $(U/c) \ll 1$; $(fL/c) \ll 1$.

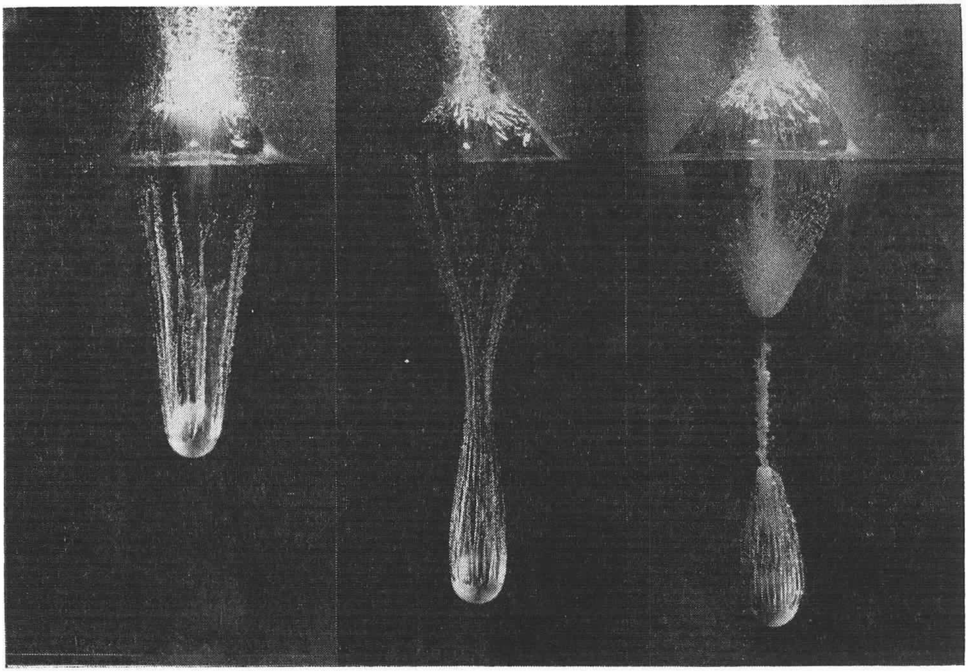


Figure 15. **Splash, cavities, and bubbles accompanying entry of aluminum sphere. The sphere, 3.9 inches in diameter, struck the surface with a velocity of 29 ft/sec.**

so that the dependence of the sound pressure upon ρ, c, U, L, r, θ, and t, can be expressed in the form

$$p_s(r,\theta,t) = \frac{\rho U^3 L \cos\theta}{rc} Z\left(\frac{U}{L}(t - r/c)\right) \qquad (19)$$

It will be recognized that gravity and also various properties of the liquid and gaseous media will to some extent influence the course of the disturbance. Fig. 15 shows some further details of the complicated sequence of events accompanying the vertical entry of a solid sphere. Besides the primary flow implicit in the displacement of the water by the entering object, we can discern a number of more-or-less distinct features such as the spray, the entrance cavity, the cavity formed by the air trapped below the surface after closure of the entrance cavity, and the smaller air bubbles. The manner in which the last two phenomena radiate sound has been discussed in Sect. 2. But even without adverting to details, we recognize various physical parameters not considered in the derivation of [19] as relevant: the acceleration of gravity, g; surface tension, T; the viscosity of the liquid, μ; and the pressure , P_0, adiabatic compressibility modulus, γP_0, and thermal diffusivity, D', of the overlying atmosphere. Their influence may be indicated, in a formal way, simple by the inclusion of the appropriate dimensionless parameters as additional arguments in the function Z in Eq. (19), the essential results being unaffected:*

$$p_s = \frac{\rho U^3 L \cos\theta}{rc} Z\left(\frac{U}{L}(t - r/c), \frac{U^2}{gL}, \frac{\rho UL}{\mu}, \frac{\rho U^2 L}{T}, \frac{\rho U^2}{P_0}, \gamma, \frac{UL}{D'}\right). \qquad (20)$$

* Sounds associated with mechanical vibrations of the entering object are outside the scope of the present discussion. They may be very significant, however.

But even this seemingly extensive list really includes only a few of the more obvious parameters. Eq. (20) is presented, therefore, not as a complete statement of the form of the relation involved but merely as an indication of its complexity.

An experimental investigation of the sound produced by the vertical entry of objects of just one shape and density (say, steel spheres) into water at ordinary conditions would require the empirical determination of a function involving at least three independent arguments. Moreover, even the most complete determinations might describe the actual sound only stochastically, since the details of the motions which result in sound are not necessarily reproducible. It is not surprising, therefore, that such data do not exist.

However, Franz [36] has shown experimentally that an important part of the sound produced by the impingement of water droplets and small solid objects is produced by the impact, so that only the density and compressibility of the water are involved. The term "impact" here refers not merely to the sudden contact of the surfaces but rather to the entire initial regime during the entry of a blunt or pointed object in which inertial reactions predominate. Fig. 16 shows his experimentally determined spectral distribution of the underwater sound energy radiated at the vertical impingement of water droplets. The size and velocity of the droplets were

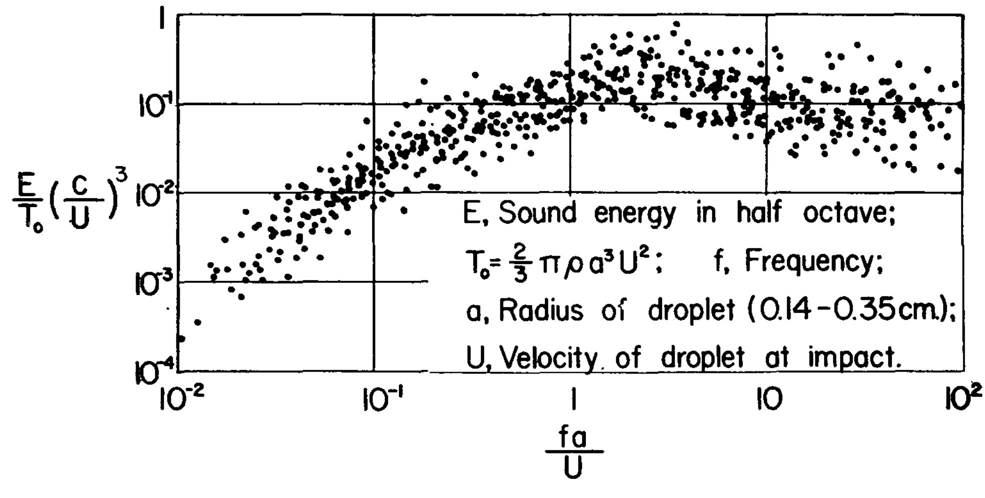


Figure 16. **Spectral distribution of underwater sound energy produced by the vertical impact of water droplets. (Figs. 16, 17, and 18 from Franz [36].)**

varied over wide ranges. Nevertheless, the data define a single function when reduced according to the analysis leading to eq. (19). It is not feasible to distinguish each combination of droplet size and velocity in the plot. but the data show very little systematic dependence upon either variable. Figure 17, traced from an oscillograph record, shows the universal function describing the sound pressure. It will be observed that the major part of the sound energy is generated immediately after the initial contact and during an interval of time smaller than that required for the droplet to travel a distance equal to its own radius.

In addition to the sound of impact, droplets of certain combinations of size and velocity occasionally produced a damped sinusoidal pulse of sound pressure similar to that produced by the formation of an air bubble at a nozzle. The sound, when it occurs, is caused by a tiny air bubble trapped beneath the surface by the splash. Such a bubble is visible in the sequence of photographs in Fig. 18. This sequence is

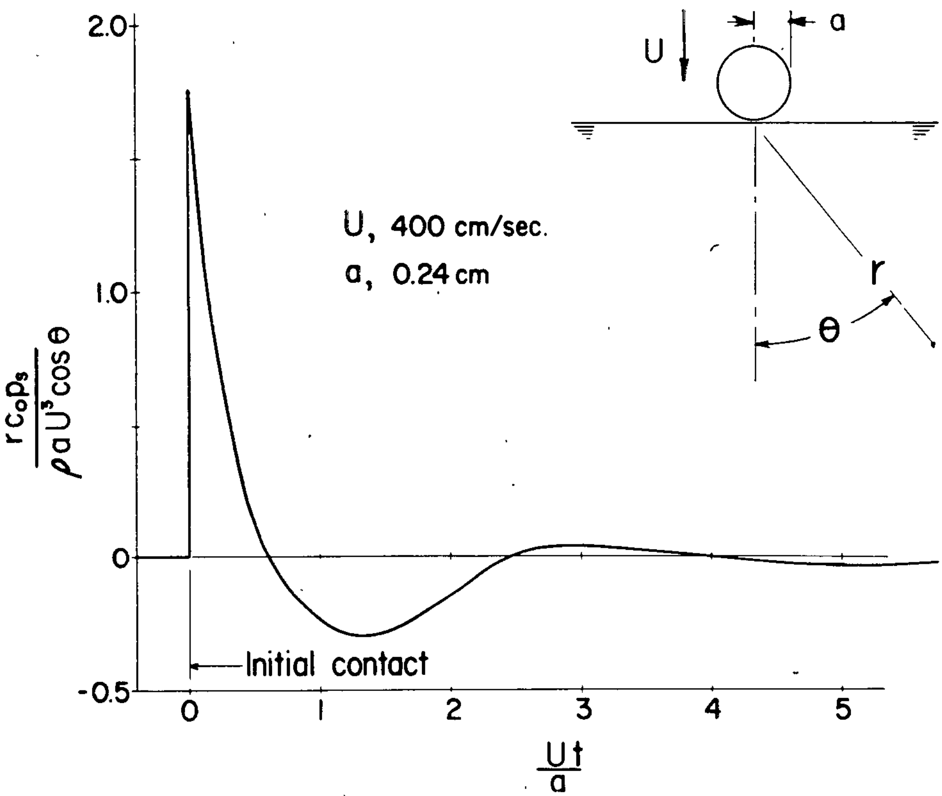


Figure 17. **Pressure pulse produced by the impact of a water droplet.**

part of a high-speed motion picture which Franz made prior to his investigation; the appended oscillographic record does not correspond to the particular splash appearing in the photographs but shows, nevertheless, the considerable time interval between the impact sound and the bubble sound. Attention is directed to the fact that no measurable sound is produced during the greater part of the interval in which the fascinatingly varied undulations of the surface occur.

Franz also measured the underwater noise generated by steady showers of water droplets falling on an extended area. The results agreed with the measurements of the sound made by solitary droplets and, so far as available meteorological data permit comparison, with underwater noise levels measured during rainfall. (The sounds from a continuous succession of randomly spaced impacts add incoherently so that the total energy in each band of frequencies is conserved).

Similar, though less extensive, data were obtained for the entry of solid objects of various shapes. An intriguing theoretical problem is that of relating the time-dependent doublet strength, and hence the sound field, to the shape and motion of the entering body, or to its shape, mass, and entering velocity. It does not appear likely that the answer will be obtained in any simple form. A special case is the entry of a massive solid body having a conical nose. Here, considerations of similarity indicate that the sound pressure pulse must begin as a "ramp function", i.e. must begin as zero and increase linearly with the time.

The preceding discussion has emphasized the gaps in available concrete information concerning sound produced by surface disturbances. Perhaps, however, its pres-

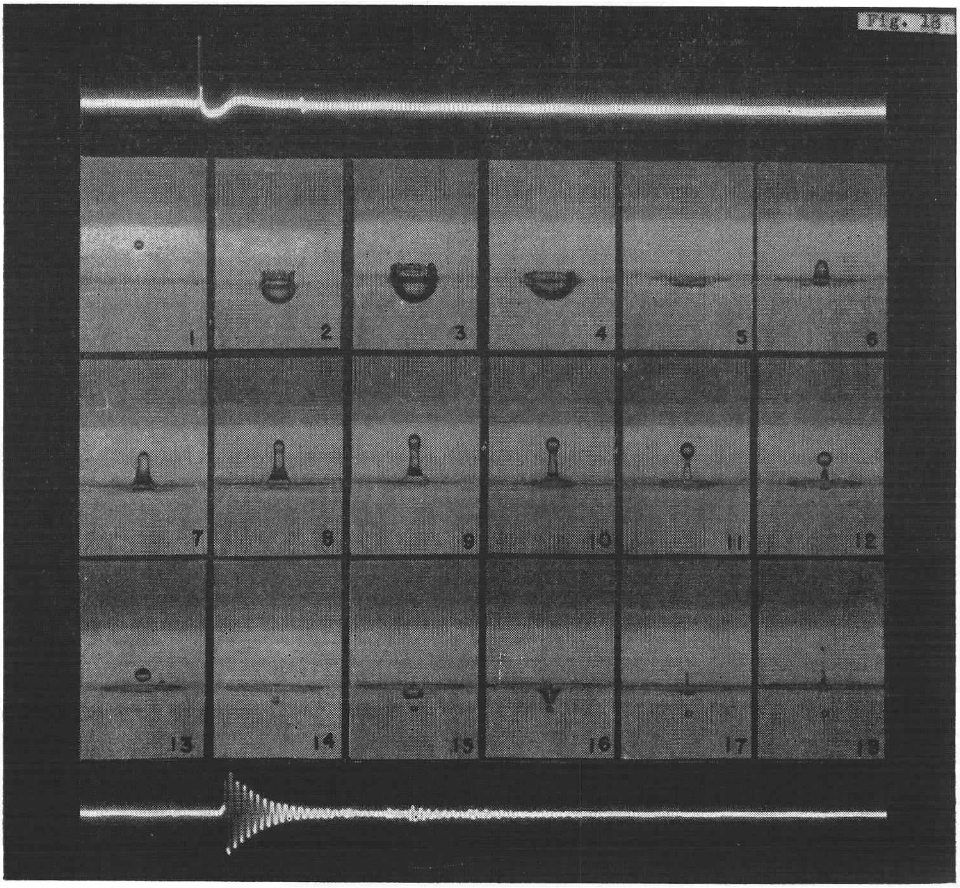


Figure 18. **Oscillogram of sound pressure and photographs of splash produced by impinging water droplet. The interval between frames is 13 millisec. The impact sound occurs between Frames 1 and 2; bubble sound begins between Frames 13 and 14. The bubble can be seen in Frames 14 to 18. The mass of the droplet is 56 mg; its velocity at impact, 350 cm/sec.**

entation will indicate something of the complexity of the subject and the vast opportunity for further research.

V. Unsteady Flow

The generation of sound by an unsteady flow differs in an essential way from the mechanisms discussed in the previous sections. Whereas bubbles and splashes inherently require the presence of both liquid and gaseous states of matter, an unsteady flow can generate sound in a homogenous fluid, either liquid or gas alone.

The recognition of the contribution to the noise from high-speed aircraft made by turbulence and other forms of unsteadiness has led to many investigations of this form of noise. Comprehensive reviews of these investigations have been published recently by Fowell and Korbacher [37] and by Mawardi [38]. The terms "aerodynamic noise" and "flow noise" have been applied to the phenomenon; but we prefer the term "unsteady-flow noise" in order to point up the essential feature causing the noise and distinguishing it from other kinds of fluid-dynamic noise.

Results obtained in the study of noise from air flows are applicable to the analogous situation in water, since the mechanisms involved are fundamentally the same. However, because of the greater density and lower speeds encountered in water, the relative importance of various phenomena can be different. The present brief review will be limited to those aspects of the problem of greatest interest in connection with flowing water.

Recognition of the association of certain sounds with unsteady flow goes back to the nineteenth century. Rayleigh, for example, attributed the "aeolian tones" made by wind streaming past a wire to the vortex wake behind the wire; and the "jet-edge tones" of whistles were also believed to result from fluctuations in the jet. Richardson [39] has described the early investigations of these sounds. These investigations were concerned primarily with the frequency of the sound. No serious attempt seems to have been made to understand the factors controlling the intensity of the sound.

A theoretical basis for investigating the magnitude of the sound pressure associated with unsteady motion was provided by Lighthill [40] in 1950. He discarded the classical form of the acoustic wave equation, which is admittedly only a small-amplitude approximation, and rederived an exact wave equation from the exact equations of fluid motion. This exact wave equation is inhomogeneous and contains terms which represent sources of sound associated with fluctuating velocities. For a liquid, the essential terms in Lighthill's equation are

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = -\rho\frac{\partial^2(u_i u_j)}{\partial x_i \partial x_j}, \tag{21}$$

where p is the pressure, c the velocity of sound, u_i the i-th component of particle velocity, x_i the i-th space coordinate, $\nabla^2 = \partial^2/\partial x_i \partial x_i$ and a repeated index indicates summation in accordance with tensor notation. If the velocity components are sufficiently small, i.e., if $\rho u_i u_i \ll p_s$, then the right side of the equation is negligible and the classical homogeneous wave equation results. If, however, the velocities are not negligible, the right side represents sources of sound.

The fluctuating pressure in an unbounded fluid can be related to the fluctuating velocities by a volume integral over all space of the right side of Eq. (21). Thus

$$p_s(x,t) = -(1/4\pi)\int (1/r)q(x',t')\,dV, \tag{22}$$

where $p_s(x, t)$ is the instantaneous sound pressure at time t and position x in space (x without subscript indicates the three coordinates x_1, x_2, x_3). In the integral, r is the distance between the point x and the volume element dV at position x', and $q(x', t')$ is the value of the right side of Eq. (21) at position x' and earlier time $t' = t\text{-}r/c$. In the application of this solution, the fluctuating velocities constituting $q(x, t)$ are considered to be independent variables whose values are given as part of the description of the flow.

In the usual unsteady flow, the fluctuating component of the velocity differs significantly from zero in only a limited portion of the unbounded space. The associated pressure fluctuations are then very much larger directly within the unsteady region than outside it.

At a point within the unsteady region, the fluctuating pressure depends primarily on the values of q at points in the immediate vicinity, corresponding to small values of r. In this vicinity, the difference r/c between the time t and earlier time t' is negligible. Since the sound velocity c does not appear in Eq. (22) in any other way, the pressure fluctuations are independent of the sound velocity. This implies that the fluctuations inside the unsteady region do not depend on the compressibility of the medium.

Far from the unsteady region, at distances large compared with the dimensions of the region, the situation is different. If the compressibility of the fluid is neglected in the calculation for these distances also, the magnitude of the pressure fluctuations is found to decrease very rapidly with increasing distance, as r^{-3}. If the difference between

t and t' is taken into account in the evaluation of the integral, however, a term results which is proportional to (ρ/rc^2). The pressure fluctuations associated with this term vary inversely as the distance in the manner of ordinary spherical waves of sound. Indeed, these fluctuations correspond to the sound energy radiated by the flow, and this distant region is accordingly called the "radiation field."

At intermediate distances, neither far away nor directly within the unsteady region, the dependence or independence of the pressure fluctuations on the compressibility of the fluid is determined by the frequency of the fluctuations. At low frequencies, associated with sound of wavelength c/f much larger than the distance, the fluctuations do not depend on the compressibility. When it is desired to distinguish the essentially-incompressive pressure fluctuations existing within a wavelength of the unsteady region from the compressive fluctuations in the radiation field, the close-up distances are called the "near field." In the near field, the magnitude of the pressure fluctuations increases more rapidly with decreasing distance than in the radiation field.

Because the pressure fluctuations within and near the region of unsteady flow do not involve the compressibility of the medium, these fluctuations have been called "pseudosound" by Blokintzev. [41] However, a pressure-sensitive hydrophone responds to the pseudosound just as it responds to any sound pressure; the fact that these fluctuations do not involve propagated sound energy makes no difference.

Fluctuations within the unsteady region.—An estimate of the magnitude of the pressure fluctuations within an unsteady flow was made by Taylor [42] in 1936. He related the pressure to the fluctuating velocity by an expression like Eq. (21) but with the time-derivative term omitted; the omission is equivalent to the assumption that the fluid is incompressible. To obtain a tractable velocity field which nevertheless duplicated some of the characteristics of isotropic turbulence, the velocities were assumed to be distributed in space like standing waves of sound in a box. For this synthetic model of isotropic turbulence, the fluctuating pressure and velocity are related by $\tilde{p} = 1.6\rho\tilde{u}_1{}^2$, where u_1 is one component of the fluctuating velocity and the tilde (~) indicates rms values of the fluctuations.

More recently, calculations of the fluctuating pressure in isotropic turbulence were made independently by Heisenberg, [43] by Obukhov, [44] and by Batchelor. [45] Their calculations all involve equivalent assumptions concerning the statistical characteristics of the distribution function for the velocities. These more fundamental calculations lead to the result $\tilde{p} = 0.6\rho\tilde{u}_1{}^2$, the coefficient being lower than Taylor's original estimate. According to Uberoi, [46] however, the value of the numerical coefficient is quite sensitive to the exact form of the statistical distribution function of the velocities.

Batchelor also obtained a relation between the space correlation function of the fluctuating pressure and the correlation function of the velocity. For the specific case of a velocity correlation of the Heisenberg type, the calculated pressure correlation falls to zero more rapidly than the velocity correlation, and the longitudinal integral scale of the pressure is about half that of the velocity.

Ogura and Miyakoda [47] used Batchelor's relations to calculate the spectral density of the fluctuating pressure from simplified spectral functions for the fluctuating velocity. These calculations indicate that the spectral density of the pressure falls more rapidly with frequency, in the high-frequency region, than does the spectral density of the velocity.

The first attempt to measure the fluctuating pressure in an unsteady flow seems to have been reported by Rouse. [48] Within a turbulent jet of air discharging into free space, the fluctuating pressure and velocity were found to be related by $\tilde{p} = 1.1\rho u_1{}^2$. Strasberg and Cooper [49] attempted some measurements of the fluctuating pressure in the turbulent wake behind a cylinder. At a point 24 cylinder diameters downstream, $\tilde{p} = 1.7\rho\tilde{u}_1{}^2$. They also determined the spectral densities of the fluctuating pressure and velocity. The spectra measured 24 diameters downstream are shown on Fig. 19,

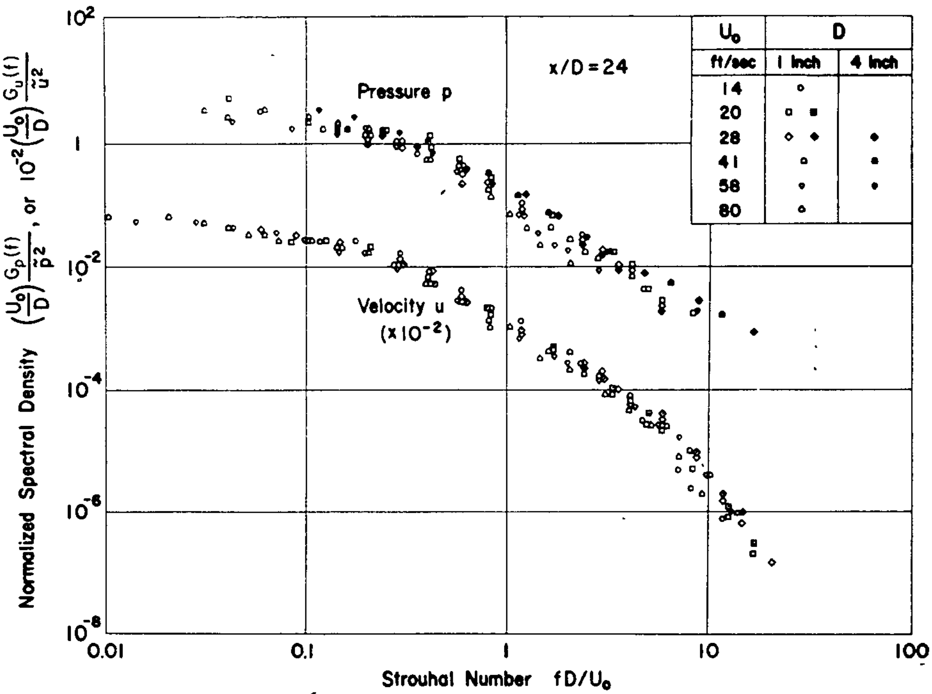


Figure 19. **The spectral densities of the fluctuating pressure and the fluctuating velocity in the wake of a cylinder, at a downstream distance of 24 diameters. The measurements were made in air.**

where the normalized dimensionless spectral densities* of both the pressure and the velocity are plotted against the Strouhal number (fD/U_0), based on cylinder diameter D and free-stream velocity U_0. The two sets of points indicate that the measured spectra of the pressure are essentially the same as those of the velocity. This is an unexpected result, not at all what would be anticipated from the theoretical relations for isotropic turbulence.

However, any comparison between measured values of fluctuating pressure and the theoretical predictions based on isotropic turbulence is subject to two uncertainties:

1. The validity of the measurements is in some question. There is no conclusive evidence that the probes used to measure the pressure do actually indicate the fluctuating pressure that would have existed in the absence of the probe.

2. The measurements were performed in flows which were not isotropic. In this connection, Kraichnan [50] has estimated the pressure fluctuations in several simpli-

---

* The spectral density of a quantity is defined so that the integral of the spectral density on frequency is equal to the mean-square of the fluctuating component of the quantity. In particular, for the spectral density $G_p(f)$ of the fluctuating pressure, $\int_0^\infty G_p(f)df = (\tilde{p}_s)^2 = \langle p_s^2 \rangle_{At}$. Also, if $G_p(f)$ is normalized by division by $(\tilde{p}_s)^2$ and non-dimensionalized by multiplication by (U_0/D), the integral on Strouhal number is then unity.

fied models of anisotropic flow. For the case of a steady mean shear ($\partial U_1/\partial x_2$) with superimposed isotropic turbulent fluctuations of rms value $\tilde{u}$ and correlation scale L, $\tilde{p}_s \propto \rho \tilde{u} L(\partial U_1/\partial x_2)$. This anisotropic model is probably closer than is isotropic turbulence alone to the actual flows in a turbulent jet or in the wake of a cylinder.

Mention should be made of the periodic fluctuations in pressure, of very large amplitude, existing in the wake close to a cylinder shedding vortices. Blokhintzev [41] has calculated the magnitude of these fluctuations, by assuming that the wake is a steady Kármán vortex street. The calculated pressure fluctuation along the centerline of the wake is given by $\tilde{p} = 0.07\rho U_0{}^2$; along this line the fluctuation is primarily at twice the shedding frequency. The calculated value agrees quite well with the value measured by Strasberg and Cooper [49] at this frequency for distances within 3 cylinder diameters of the cylinder. At the shedding frequency, the fluctuations are even larger; close to the cylinder, values were observed as large as $\tilde{p} = 0.3\rho U_0{}^2$. Further downstream, the measured values decrease as the periodic fluctuations degenerate into random turbulence.

The pressure fluctuations in a free turbulent region is a topic worthy of considerable additional attention. The practical importance of these pressure fluctuations can be indicated by an example of their magnitude. In the wake of a cylinder in water at a speed of 5 meters/sec (10 knots), the rms pressure fluctuation 24 cylinder diameters downstream is estimated to be about 5×10^3 dyne/cm^2. This pressure corresponds to a level more than 70 decibels above the ambient noise in the sea at sea state 2.

Pressure in the near field.—The fluctuating pressures outside a turbulent region are much smaller than those mentioned above. In the near field close to a turbulent region, however, their magnitudes can be significant. Jorgensen [21] has measured the pressure fluctuation outside a free turbulent jet of water. The spectral density of the pressure, measured 4 jet diameters off the axis and 4 diameters downstream from the mouth of the jet, is shown in Fig. 20. The bottom and left scales are non-dimensional, but dimensional scales have been put along the top and right side for a jet diameter of 0.6 inch at a speed of 30 knots. For these specific conditions, the fluctuations at 500 cy/sec are some 30 db above the ambient noise at sea state 2.

Radiated pressure fluctuations.—In the radiation field far from an unbounded region of unsteady flow, the pressure fluctuations are even smaller than in the near field. The sound radiated by an unsteady flow has received perhaps more attention than any other form of flow noise, because of its importance in connection with high-speed jet aircraft. At the speeds encountered in water, however, the radiated pressure fluctuations are completely negligible.

The negligible values of the sound pressure radiated by turbulence in water are illustrated by the case of the turbulent jet. Fitzpatrick and Lee's [51] measurements with air jets give for the rms sound pressure, in the direction of average intensity,

$$\tilde{p}_s = 2\times 10^{-3}\rho U^2(U/c)^2(D/r), \tag{23}$$

where D is the diameter of the orifice, and U the mean efflux velocity. At a speed of 15 meters/sec (30 knots) and a distance of 100 jet diameters, the rms sound pressure is only about 0.005 dyne/cm^2; at least 40 decibels below the ambient noise in sea state 2.

The influence of boundaries.—The discussion to this point has been concerned with flows in an unbounded space. If boundaries are present, the sound field is of course modified by the boundaries. If the boundaries are within the unsteady region, sound radiation can be associated directly with the boundaries, and this boundary radiation can be much stronger than the radiation from the unsteady motion itself.

The complete relations describing the sound field generated by an unsteady

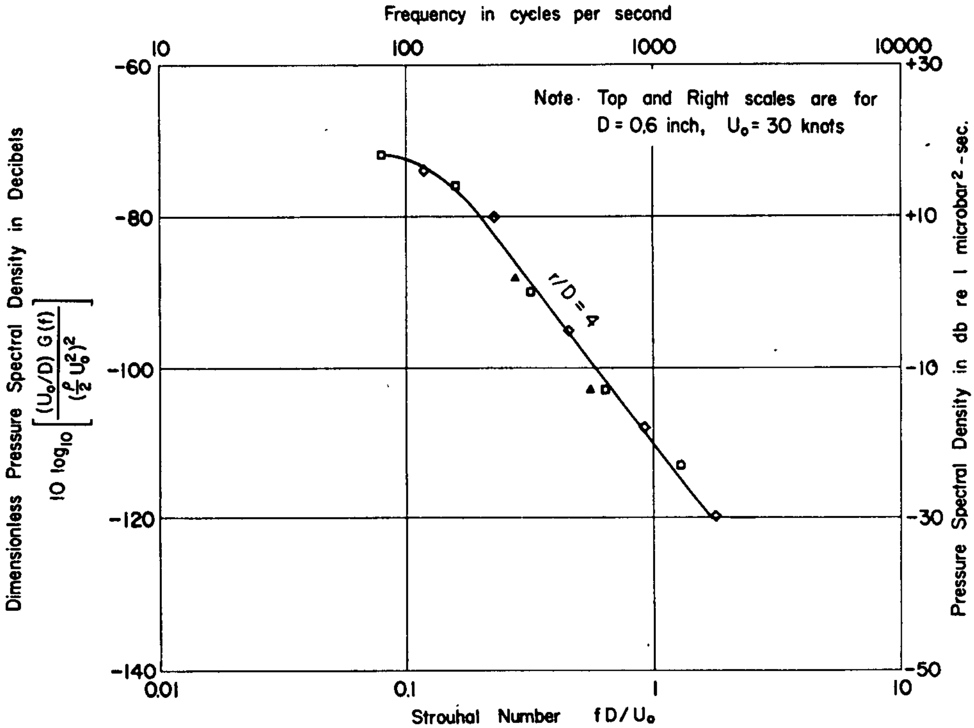


Figure 20. The spectral density of the fluctuating pressure in the near-field of a free jet of water. (From Jorgensen [21].)

flow in the presence of boundaries were developed by Curle [52] in 1955. If the boundaries are directly within the unsteady region, their principle contribution to the radiated sound pressure is given by a surface integral over the boundaries, of the pressure and the boundary acceleration, viz.,

$$p_s(x,t) = -\frac{\rho}{4\pi}\int \frac{\dot{u}_j'}{r}\cos\theta_j\,dS + \frac{1}{4\pi}\frac{\partial}{\partial x_j}\int \frac{p'}{r}\cos\theta_j\,dS. \tag{24}$$

Here r is the distance between the point x and the surface element dS; p' is the pressure and $\dot{u}_i'$ the component of acceleration in the x_i direction, at the surface element dS at earlier time t-r/c; and θ_i is the angle between the x_i direction and the surface normal out of the fluid. It is of interest that the equation is of the same form as the classical solution of the homogeneous wave equation in terms of the pressure and acceleration at the boundary (cf. Lamb [6] §290); the same terms appear in Curle's result even when the motion near the boundaries is so large that the homogeneous wave equation is no longer applicable.

If the boundary is contained within a region small compared with the wavelength of the sound at the frequency of interest, the integrals in Eq. (24) may be evaluated without reference to the shape of the boundary. In the radiation field far from the boundary, the sound pressure is then given by the simple relation

$$p_s(t) = \frac{1}{4\pi r}\left[\rho\ddot{V} + \frac{\rho V\ddot{U}}{c}\cos\theta_u + \frac{\dot{F}}{c}\cos\theta_F\right], \tag{25}$$

where V is the instantaneous volume enclosed by the boundary; U the velocity of the centroid of the enclosed volume; F the total force exerted on the fluid by the boundary; and θ_u and θ_F the angles between the radius vector toward x, and the directions of $\ddot{U}$ and $\dot{F}$ respectively. The dots indicate time derivatives. In this equation, the terms on the right have their values at earlier time $(t-r/c)$. The $\ddot{V}$ term in the equation corresponds to the ordinary simple source associated with volume pulsation (cf. Eq. (4) of Section 2). The other two terms have a dipole-like directionality and are associated with translational oscillation of the boundary and with oscillating forces on the fluid.*

Even if the boundary is rigid and does not vibrate, sound can nevertheless result from oscillating forces acting on the fluid. Thus, the aeolian tones radiated by a cylinder have been explained in terms of the oscillating force, between the fluid and the cylinder, associated with periodic shedding of vortices. To calculate the magnitude of the sound, Etkin, Korbacher and Keefe [53] assume that the transverse oscillating force on the fluid is given by

$$F = 0.90\ (\tfrac{1}{2}\rho U_o{}^2)LD \sin(2\pi f_1 t), \tag{26}$$

where U_o is the free-stream velocity, D and L are here the diameter and length of the cylinder, and f_1 is the frequency of vortex shedding given by $f_1 = 0.2U_0/D$. Substituting Eq. (26) into (25), and assuming that the cylinder is rigid, the sound pressure is calculated as

$$p_s(t) = 0.18(\tfrac{1}{2}\rho U_0{}^2)(U_0/c)(L/r)(\cos 2\pi f_1 t) \cos\theta, \tag{27}$$

θ being the angle between the radius vector and the flow direction. The equation indicates that the sound pressure is proportional to the cube of the velocity.** This relation was verified by their experimental data, some of which are shown in Fig. 21.

Similar calculations and measurements are reported by Phillips. [54] He calculates the fluctuating force from measured values of the fluctuating velocity in the wake of the cylinder and obtains a coefficient 0.76 instead of the 0.9 in Eq. (26). The measured sound pressures of both Etkin et al and Phillips are lower than the value given by Eq. (27) by constant factors. The difference is explained by the fact that the vortex shedding is not correlated along the entire length of the cylinder, so that the phase of the oscillating force varies axially. If the correlation length is assumed to equal the cylinder diameter times a factor b, then the quantity $(bDL)^{1/2}$ should be substituted for L in Eq. (27). Phillips estimates that b is about 17 for the range of Reynolds number U_0D/ν from 80 to 160, and about 3 for Reynolds numbers above 300; whereas Etkins estimates b as about 8.

The intensity of aeolian sounds has also been measured by Gerrard [55] over a wide range of Reynolds numbers. His data can be interpreted as indicating that the sound pressure varies with the cube of velocity, as required by Eq. (27), but that the numerical constant is larger by a factor of about 4 for Reynolds numbers below 300. This higher value may be due to the fact that, at low Reynolds numbers, the vortex shedding is correlated all along the cylinder axis.

Both Phillips and Etkin et al assume that the vibration of the cylinder does not radiate any significant sound. The latter, in fact, report measurements showing that the sound is independent of the elastic properties of the cylinder. However, it is well known that the intensity of the aeolian tones increases when the cylinder vibrates in resonance at the shedding frequency; this phenomenon was observed by Strouhal himself. Phillips' explanation is that the vibration of the cylinder "locks-in" or correlates

* High-order terms, with higher negative powers of c, have been omitted from Eq.(25); such terms are associated with higher moments of the boundary vibration and pressure distribution.

** Eq. (27), without the numerical coefficient, was predicted by Blokhintzev, ibid.

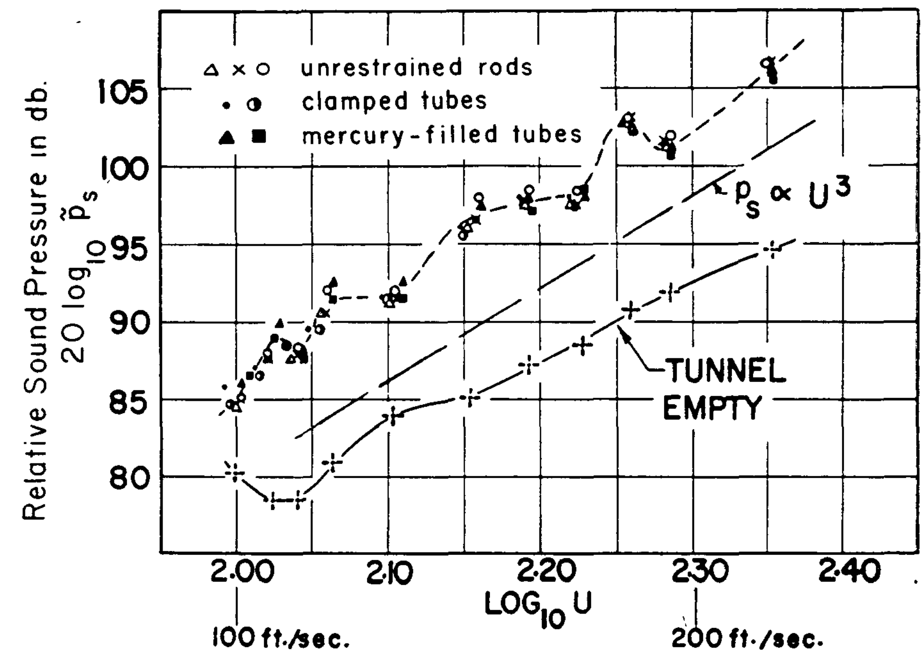


Figure 21. The sound pressure radiated by a cylinder shedding verticces in air, as a function of speed. (From Etkin et al [53].)

the vortex shedding along its entire length, causing the sound pressure to increase by a factor $(L/bD)^{1/2}$.

Although the sound generated by the vibration itself is neglected by both Etkin and Phillips, it should be noted that in a relatively dense liquid such as water, the amplitude of the sound may be increased by the vibration even in the absence of any modification of the pattern of vortex shedding. To indicate the magnitude of the sound that may result from vibration, consider a rigid cylinder held by compliant supports so that the cylinder vibrates transversely, when immersed in water, in free oscillation at a frequency f_r with logarithmic decrement $\pi\delta$. Assume, as a first approximation, that the amplitude of vibration is so small that the gross motion of the fluid is not at all affected. The cylinder vibration will result from a reaction to the force exerted by the cylinder on the fluid. In terms of the force exerted by a fixed cylinder, e.g., the force F given by Eq. (26), the reaction force acting on the cylinder is $-(F + M_a\dot{U})$, with M_a being the added mass due to the water. If the force and vibration are sinusoidal with frequency f_1 equal to the shedding frequency, so that $F = F^0 e^{2\pi i f_1 t}$ and $\dot{U} = \dot{U}^0 e^{2\pi i f_1 t}$, the amplitudes F^0 and $\dot{U}^0$ are related by

$$\dot{U}^0 = [F^0/(M_a + M_c)] \cdot [(f_r/f_1)^2 - 1 + i\delta]^{-1}, \tag{28}$$

M_c being the mass of the cylinder. The sound pressure amplitude p_o is then given by Eq. (25), with $\dot{U}$ and $\dot{F}$ replaced by $2\pi i f_1 \dot{U}^0$ and $2\pi i f_1 (F^0 + M_a \dot{U}^0)$, respectively, and $p_s(t)$ by p_o.

At resonance, $f_1 = f_r$; the sound pressure amplitude is then given by

$$p_o = \frac{f_1 F^0}{2rc}\left[i + \frac{M_a + \rho V}{\delta(M_a + M_c)} \right]. \tag{29}$$

The right term inside the brackets represents the direct contribution of the vibration to the radiated sound; if the displaced mass ρV is of the same order as the cylinder mass, the contribution can be considerable, especially since δ is often smaller than 0.01 for ordinary mechanical systems. The vibration can thus cause a large increase in the sound pressure in liquids, independently of any influence on the vortex shedding or other aspects of the gross flow. The vibration will, in fact, modify the gross flow and the associated fluctuating force. This interaction has not been studied in any detail.

These aeolian sounds are associated with fluctuations at a predominant frequency. However, the fluctuations usually contain a random component. In the flow past a cylinder, the fluctuations become almost completely turbulent at high Reynolds number, above about 10^5. The radiated sound is then also random. The spectral distributions of the random force and vibration are related by an expression similar to Eq. (28); if the vibration contributes to the sound, the sound spectrum will be peaked around the resonant frequency of the system.

The foregoing discussion indicates that there is a satisfactory understanding of the sound generated by flow about a cylinder. However, this case is only the simplest example of the acoustic interaction of a surface with an unsteady flow. A more complicated situation involves the sound associated with a turbulent boundary layer. This is of practical interest in connection with sound generated at the skin of an aircraft or at the hull of a ship.

Noise from a boundary layer.—The sound generated by the fluctuations in the boundary layer on a rigid bounding surface have been investigated theoretically by Phillips. [56] His results indicate that the sound from a plane boundary layer is small, except perhaps near the transition region. The radiated sound pressure is proportional to $\rho U_0^2(U_0/c)$ if the flow maintains similarity. Accordingly, it is likely that only negligible sound will be radiated at the speeds encountered in water.

If the boundary surface is relatively flexible, however, flexural vibration of the surface can result in significant sound. This surface vibration is excited by the local pressure fluctuations within the boundary layer; the resulting sound pressure can be much larger than that radiated by the pressure fluctuations themselves. The vibrating boundary acts like a sounding board and substitutes a simple source with no (U_0/c) dependence for the dipole-like source of a rigid boundary.

A calculation of the radiated sound can be made with the following sequence of steps:

1. The pressure fluctuations at the boundary are estimated from a knowledge of the flow.
2. The flexural vibration of the boundary in response to the pressure fluctuations is determined.
3. The radiated sound pressure associated with the vibrating surface is calculated.

This procedure assumes that the motion resulting from the vibration is too small to modify the grosser flow, so that the pressure fluctuations at the boundary can be treated as an independent variable.

Once the boundary pressure is known, steps (2) and (3) involve the application of well-known acoustical equations which give the sound pressure radiated by a membrane or a plate in terms of an arbitrary distribution of pressure on its surface. Because the fluctuations in the boundary layer are random in both time and space, the calculations are relatively complicated, but no essential difficulty is introduced by the randomness. It is necessary to know the statistics of the fluctuations, viz., the special density and the space correlation of the pressure fluctuations. The evaluation of these quantities, however, which is essentially the first step listed above, has not been accomplished in a completely satisfactory manner as of this writing. A theoretical estimate of the characteristics of the pressure fluctuations can be attempted by following the procedure outlined by Batchelor and already discussed in connection with the local fluctuations within a turbulent region. However, too little is known about the fluctuations within the boundary layer to permit the calculation to be carried out with any rigour.

Some preliminary measurements of the fluctuating pressure in a turbulent boundary layer at the bottom of an open channel have been described by Einstein and Li. [57] Willmarth [58] has reported some measurements of the fluctuating pressure at the boundary of a pipe. He estimates that $\bar{p} = 3.5 \times 10^{-3}(\frac{1}{2}\rho U_0^2)$, where U_0 is the free-stream velocity, but the measurements may be in error because the fluctuations may not be correlated over the entire area of the pressure-sensing device. Certainly, additional measurements are required to make possible an estimate of the space correlation and the spectral density of the pressure fluctuations, as required by step (1) above.

Theoretical calculations of the boundary-layer noise have been attempted by Ribner, [59] and more recently by Corcos and Liepmann, [60] and by Kraichnan, [61] all using certain assumed characteristics of the pressure fluctuations in the layer. The first two of these papers deal primarily with a boundary surface which is "floating" i.e., unconstrained. Kraichnan, on the other hand, treats a square plate constrained at its periphery; he also goes into greater detail concerning the character of the fluctuations in the boundary layer. Which of these calculations is closer to the actual situation cannot be determined until some measurements have been made of this form of noise.

Other unsteady flows.—Before this section is concluded, mention should be made of several other types of sound associated with unsteady flow which have received some attention in recent years:

(1) "Jet-edge tones," which are generated when a jet impinges on a thin plate or wedge. The sound contains a predominant frequency determined by a periodic undulation of the jet, the frequency depending primarily on the velocity of the jet and the separation between the mouth of the jet and the edge. Richardson [39, 62] has described his own and several other investigations of the phenomenon. Other discussions have been published by Curle, [63] by Powell, [64] and by Bouyoucos and Nyborg. [65] Two alternative mechanisms have been suggested. Richardson and Curle believe the undulations are caused by instability of the flow itself, whereas the others believe that the sound reacts on the jet and "triggers" the instability. In either case, the sound is presumably generated by a fluctuating lift force on the edge.

(2) "Orifice-pipe tones," generated when a fluid discharges through a sharp-edged orifice at the end of a tube. The frequency of the sound has been determined for a wide range of conditions by Anderson. [66] The sound, in this case, is probably a simple source associated with periodic fluctuation in the rate of efflux from the orifice.

(3) Resonant cavities excited by external flow past the mouth of the cavity. The frequency of these sounds has been investigated by Blokhintzev [67] and by Harrington. [68] There seems to be some interaction between the cavity and the unsteady flow past the mouth, because the predominant frequency of the sound may differ somewhat from the frequency at which the cavity resonates in the absence of flow. The sound can be considered to be a simple source associated with alternating flow in the mouth of the cavity.

The investigations of jet-edge tones, orifice tones, and resonant cavities have been concerned primarily with the frequency of the sounds, without any attempt to determine their amplitudes. Accordingly, it is not possible to say whether any of them are significant sources of underwater noise. However, the uncertainty results from the lack of adequate descriptions of these flows. Once suitable descriptions of the flows become available, the associated sound pressure can be calculated by the methods described in this section.

REFERENCES

1. R. G. Folsom, E. D. Howe, and M. P. O'Brien, "Hydraulic Noise," Univ. of Calif., OSRD 949, June 1942.

2. J. Bouyoucos, "Self Excited Hydrodynamic Oscillators", Acoustics Research Laboratory, Harvard Univ., Tech. Memo. 36, July 31, 1955.
3. M. Minnaert, "On Musical Air Bubbles and the Sounds of Running Water," Phil. Mag. *16*, 235 (1933).
4. E. Meyer and K. Tamm, "Natural Vibration and Damping of Gas Bubbles in Liquids", Taylor Model Basin Trans, 109, April 1943. Orig. in Akust. Z., *4*, 1945 (1939).
5. H. Pfriem, "Zur Thermischen Dampfung in Kugelsymmetrisch Schwingenden Gasblasen," Akust. Z. *5*, 202 (1940).
6. H. Lamb, "Hydrodynamics," (Dover Publications, New York, (1945)).
7. M. Strasberg, "Gas Bubbles as Sources of Sound in Liquids," J. Acoust. Soc. Am. *28*, 20 (1956).
8. E. H. Kennard, "Radial Motion of Water Surrounding a Sphere of Gas in Relation to Pressure Waves," Taylor Model Basin Report 517, Sept. 1943.
9. B. E. Noltingk and E. A. Neppiras, "Cavitation Produced by Ultrasonics," Proc. Phys. Soc. London, B*63*, 674 (1950); B*64*, 1032 (1951).
10. O. Reynolds, "Experiments Showing the Boiling of Water in an Open Tube at Ordinary Temperatures," (Exhibited before Section A, Brit. Assoc. Adv. Sci., 1894, at Oxford) Papers on Mechanical and Physical Subjects (reprinted) Cambridge Univ. Press (1901).
11. A. B. Wood, "A Textbook of Sound," (Macmillan, New York, 1930) p. 215.
12. H. Mueller, "Über den gegenwärtigen Stand der Kavitationsforschung," Die Naturwissenschaften, 16 Jahrg., Heft 22 (1928).
13. R. T. Knapp and A. Hollander, "Laboratory Investigations of the Mechanism of Cavitation," Trans. A.S.M.E., *70*, 419, (1948).
14. Mark Harrison, "An Experimental Study of Single-Bubble Cavitation Noise," Taylor Model Basin Report 815, June 1952.
15. B. Parkin, "Scale Effects in Cavitating Flow," Calif. Inst. Tech. HL-CIT Report 21-7, Dec. 1951.
16. P. Eisenberg, "Modern Developments in the Mechanics of Cavitation," App. Mech. Rev. *10*, 85, (1957).
17. M. S. Plessett, "Dynamics of Cavitation Bubbles," App. Mech., *16*, 277, (1949).
18. Lord Rayleigh, "On the Pressure Developed in a Liquid During the Collapse of a Spherical Cavity," Phil. Mag. *34*, 94, (1917).
19. T. Brooke Benjamin, "Cavitation in Liquids," doctoral dissertation, Cambridge Univ., Cambridge. (As quoted in Reference 26.)
20. R. H. Mellen, "Ultrasonic Spectrum of Cavitation Noise in Water," J. Acoust. Soc. Am., *26*, 356, (1954).
21. D. W. Jorgensen, "Measurements of Noise from Cavitating Submerged Water Jets," Taylor Model Basin Report 1126, in preparation.
22. R. H. Mellen, "An Experimental Study of the Collapse of a Spherical Cavity in Water," J. Acoust. Soc. Am. *28*, 447, (1956). (In an unpublished communication, Dr. T. Brooke Benjamin indicates that evidence of a shock wave has been observed at King's College, Cambridge, by means of tiny transducers made of chips of barium titanate).
23. E. Meyer, "Some New Measurements on Sonically Induced Cavitation," J. Acoust. Soc. Am. *29*, 4, (1957).
24. W. Güth, "Zur Entstehung Der Stosswellen Bei Der Kavitation," Acustica, *6*, 526, (1956).
25. J. B. Keller and I. Kolodner, "Damping of Underwater Explosion Bubble Oscillations," J. App. Phy. *27*, 1152 (1956).
26. H. G. Flynn, "The Collapse of a Transient Cavity in a Compressible Liquid," doctoral dissertation, Harvard Univ. 1956.
27. A. J. R. Schneider, "Some Compressible Effects in Cavitation Bubble Dynamics," Ph. D. Thesis, Calif. Inst. Tech. 1949.
28. F. R. Gilmore, "The Growth and Collapse of a Spherical Bubble in a Viscous Compressible Liquid," Hydro. Lab. Calif. Inst. Tech. Report 26-4, April 1952.
29. F. R. Gilmore, "Collapse of a Spherical Bubble in a Compressible Liquid," Presented at

meeting of the Division of Fluid Mechanics, American Physical Society, Pasadena, Calif., March 1956.
30. R. H. Mellen, "Spherical Pressure Waves of Finite Amplitude from Collapsing Cavities," U. S. Navy Underwater Sound Laboratory Research Report 326, Sept. 1956.
31. P. Eisenberg, "On the Mechanism and Prevention of Cavitation," Taylor Model Basin Report 712, July 1950.
32. A. M. Worthington, "A Study of Splashes," (Longmans, Greene, and Co., 1908).
33. A. Mallock, "Sounds Produced by Drops Falling on Water," Proc. Roy. Soc. A*95*, 138, (1919).
34. A. May, "On the Entry of Missiles into Water," U. S. Naval Ordnance Laboratory, NAVORD Report 1809, Aug. 1951.
35. E. G. Richardson, "The Sounds of Impact of a Solid on a Liquid Surface," Proc. Phy. Soc. London, B*68*, 541, (1955).
36. G. J. Franz, "The Underwater Sound from the Splashes of Water Droplets," Paper D2, 48th meeting Acous. Soc. Am., Austin, Tex., 1954. (In preparation for publication).
37. L. R. Fowell and G. K. Korbacher, "A Review of Aerodynamic Noise," Inst. Aerophysics, Univ. of Toronto, Review 8, July 1955.
38. O. K. Mawardi, "Aerothermoacoustics," Reports on Progress in Physics, Vol. 19 (The Physical Society, London, 1956).
39. E. G. Richardson, "Sound," (Edward Arnold and Co., London, 1940).
40. M. J. Lighthill, "On Sound Generated Aerodynamically," Proc. Roy. Soc. London: A*211*, 564, (1952); A*222*, 1, (1954). (Orig. ARC 13,298, F.M. 1467, Aug. 1950).
41. D. Blokhintzev, "The Acoustics of an Inhomogeneous Moving Medium," (translation by Res. and Anal. Group, Brown Univ; also as NACA TM 1399).
42. G. I. Taylor, "The Mean Value of the Fluctuations in Pressure and Pressure Gradient in a Turbulent Fluid," Proc. Camb. Phil. Soc., *32*, 380, (1936).
43. W. Heisenberg, Z. Phys. *124*, 624 (1948).
44. A. M. Obukhov, C. R. Acad. Sci. U.R.S.S. *66*, 17 (1949) (Translation by Brit. Ministry of Supply, TPA3/T1B Trans. T3757).
45. G. K. Batchelor, "Pressure Fluctuations in Isotropic Turbulence," Proc. Camb. Phil. Soc. *47*, 359 (1951).
46. M. S. Uberoi, "Correlations Involving Pressure Fluctuations in Homogeneous Turbulence," NACA TN 3116, Jan. 1954.
47. Y. Ogura and K. Miyakoda, "Note on the Pressure Fluctuations in Isotropic Turbulence," Jour. Met. Soc. Japan, *32*, 42, (1954).
48. H. Rouse, "Cavitation in the Mixing Zone of a Submerged Jet," La Houille Blanche, *8*, 9 (1953).
49. M. Strasberg and R. D. Cooper, "Measurements of the Fluctuating Pressure and Velocity in the Wake behind a Cylinder," Paper I.101, Ninth International Congress of Applied Mechanics, Brussels, Sept. 1956.
50. R. Kraichnan, "Pressure Field within Homogeneous Anisotropic Turbulence," J. Acoust. Soc. Am., *28*, 64 (1956).
51. H. Fitzpatrick and R. Lee, "Measurements of Noise Radiated by Subsonic Air Jets," Taylor Model Basin Report 835, Nov. 1952.
52. N. Curle, "The Influence of Solid Boundaries upon Aerodynamic Sound," Proc. Roy. Soc. A, *231*, 505 (1955).
53. B. Etkin, G. K. Korbacher, and R. T. Keefe, "Acoustic Radiation from a Stationary Cylinder in a Fluid Stream," Inst. Aerophysics, Univ. Toronto, UTIA Report 39, May 1956.
54. O. M. Phillips, "The Intensity of Aeolian Tones," J. Fluid Mech., *1*, 607 (1956).
55. J. H. Gerrard, "Measurements of the Sound from Circular Cylinders in An Air Stream," Proc. Phys. Soc. B, *68*, 453 (1955).
56. O. M. Phillips, "On the Aerodynamic Surface Sound from a Plane Turbulent Boundary Layer," Proc. Roy. Soc. A, *234*, (1956).

57. H. A. Einstein, and Huon Li, "The Viscous Sublayer along a Smooth Boundary," J. Eng. Mech. Div., Proc. ASCE, *82*, (1956).
58. W. W. Willmarth, "Wall Pressure Fluctuations in a Turbulent Boundary Layer," J. Acoust. Soc. *28*, 1048 (1956).
59. H. S. Ribner, "Boundary-Layer-Induced Noise in the Interior of Aircraft," Inst. Aerophysics, Univ. Toronto, UTIA Report 37, Apr. 1956.
60. G. M. Corcos and H. W. Liepmann, "On the Contribution of Turbulent Boundary Layers to the Noise Inside a Fuselage," NACA TM 1420, Dec. 1956.
61. R. H. Kraichman, "Noise Transmission from Boundary-Layer Pressure Fluctuations," J. Acoust. Soc. *29*, 65 (1957).
62. E. G. Richardson, "Acoustics in Relation to Aerodynamics," J. Aero. Sci. *22*, 775 (1955).
63. N. Curle, "The Mechanics of Edge-Tones," Proc. Roy. Soc. A, *216*, 412 (1953).
64. A. Powell, "On Edge Tones and Associated Phenomena," Acustica, *3*, 233 (1953).
65. J. V. Bouyoucos and W. L. Nyborg, "Oscillations of the Jet in a Jet-Edge System," J. Acoust. Soc. *26*, 174, 511 (1954).
66. A. B. C. Anderson, "A Jet-Tone Orifice Number for Orifices of Small Thickness—Diameter Ratio," J. Acoust. Soc. *26*, 21 (1954).
67. D. I. Blokhintsev, "Excitation of Resonance by Air Flow," ZhTF, *15*, 63 (1945) (Taylor Model Basin Trans. 270).
68. M. C. Harrington, "Excitation of Cavity Resonance by Air Flow," Paper J4, 52nd meeting Acoust. Soc. Am., Los Angeles, Nov. 1956

DISCUSSION

S. Byard

I would just like to describe in a very few words the results of some simple laboratory experiments which have been carried out by Mr. Hey at the Admiralty Research Laboratory, and which seem to illustrate some of the characteristics of propeller cavitation noise.

These experiments have been made with quite small hydrofoils, the smallest being less than one inch by one inch, either mounted in a miniature water tunnel with transparent sides, or mounted at the periphery of a disc which is rotated under water. By suitably orienting the foil, a tip vortex cavity can be formed at the trailing edge, or alternatively blade cavitation induced at the leading edge, and the noise spectra of the two forms of cavitation are markedly different.

In these model tests the noise spectrum associated with the tip vortex cavity is localized around 2 to 3 kilocycles/second, whereas the spectrum of the noise due to blade cavitation is more generally distributed, extending to the higher frequencies.

In the case of a ship's propeller, where cavitation is well established, both forms of cavitation are normally present and give rise to a continuous noise spectrum with a marked peak.

Some simple scaling experiments in which the effect of doubling the size of the hydrofoil was observed, showed an approximate doubling of the diameter of the tip vortex, and a lowering of the frequency of the peak in the noise spectrum. It was also possible to show experimentally that the source of the noise associated with the tip vortices is located close to the hydrofoil, and does not extend along the vortices.

It is thought that experiments along these lines, which can be regarded as supplementing and extending the work on the single collapsing cavity which has been carried out by Kendrick some time ago at the Admiralty Research Laboratory, can help us understand the mechanism of propeller cavitation noise.

G. K. Batchelor

I should like to ask a question of Mr. Fitzpatrick. He remarked that, in his experiments, no noise generated by the turbulence in jets could be detected. Such noise would presumably vary as the 8th power of Mach number, and I think a few rough calculations suggest that the level would be so small that one couldn't expect to be able to detect it. I should like to ask if he has done any experiments in which there was a lifting surface somewhere in the flow. There is a connection between the fluctuations in the lift on a solid body, and the amount of surface noise generated. The intensity of the surface noise varies as the 6th power of the Mach number, and might therefore be greater than that of the volume noise at low Mach numbers.

I suppose the most favorable circumstances for generation of surface noise might be some kind of flat plate edge-on to a stream.

H. M. Fitzpatrick

I know of no such experiments involving a jet and lifting surface except, of course, the "jet-edge" studies mentioned.

M. Strasberg

When you try to do such an experiment you run into a complication. If you put a plate in a jet—you are, I presume, thinking of a rigid plate which would not oscillate in the jet, so that the radiation would be the kind which is associated with the lift forces on the plate. What happens in practice when you try to do that is that the plate itself cannot be made infinitely rigid, and begins to oscillate and respond to local pressures the jet exerts. As a result of the oscillation of the plate, sound will be radiated, which very much complicates any analysis of the experimental results.

That sound which is radiated by a vibrating plate, excited into vibration by the fluctuations of pressure on it, is a very important type of sound. A theoretical analysis has been given by Dr. Kraichnan, who is, I see, a couple of rows behind you there.

M. J. Lighthill

I could make comments on this point. But I would like to say first how much, how very much I enjoyed Dr. Fitzpatrick's lecture. He tried to say some things I would disagree with, but he wasn't able to. I find myself in complete agreement with everything he said. I think it was extremely desirable that a survey of that kind should have been made.

On the question raised by Dr. Batchelor, whether hydrodynamic sound of a one-phase character (involving no bubbles of any kind) could be detectable, I suppose that the dipole radiation due to moving a circular cylinder about in a fluid is one of the most promising kinds. A lot of information about the sound radiated in this case is available. Phillips (J. Fluid Mech. 1 (1956), p. 607) has recently correlated extensive measurements by Holle and Gerrard in the range of Reynolds numbers from 400 to 40000, by means of a formula which gives

$$P = 0.006 \frac{\rho U^3 l d}{a^3}$$

where P is the total acoustic power radiated, ρ the density, U the velocity of the cylinder, l and d its length and diameter, and a the velocity of sound; and he gives reasons why the constant should be independent of Reynolds number in the range in which the wake is irregular but the boundary layer is laminar at separation.

In water, if $U = 30$ knots, $l = 10$ ft. and $d = 1/2$ in., then the Reynolds number is in this range (it is about 10^5), and the formula gives $P = 10^{-3}$ watt.

C. A. Gongwer

I concur, of course, the aeolian tone will be generated, but it only occurs in the fairly narrow range of Reynolds number associated with the Karman vortex street.

I noticed Dr. Fitzpatrick's curve of the sound from the underwater jets, which were made so carefully, that the sound peaks at the Strouhal number, which is usually associated with the vortex street. This suggests some kind of regular pattern of vortices associated with submerged jets, the value .18 or .2, suggests the vortex street.

The purely hydrodynamic noise is, of course, hard to find, unless you go to a submerged hydrodynamic oscillator in which there may be no free interfaces whatsoever. In this case one can generate pure hydrodynamic noise by exciting a resonant system in relatively incompressible liquid.

There are old cases of controversies of this type, about whether you should streamline hydrophones, where people have said that there is no hydrodynamic noise of significance, but experimental tests have shown some anomalies. For example, pointing directional hydrophones at submerged bumps on objects, one will get a noise coming from this bump, under conditions in which cavitation really could not exist at all. There is a real mystery there, and if one can throw some light on this subject, it would be well worth while.

There may be the question of compound interferences from roughness, and so forth, putting the interface into the system somewhere, or tiny bubbles, but in the absence of the interface, it is really hard to explain some of these noises.

M. Strasberg

Dr. Gongwer's comment about one of our spectrum curves indicates that the curve requires some clarification. I believe he stated that our curve of the spectrum of the radiated sound from a jet had a peak at a Strouhal number of about 0.2, suggesting that some regular vortex pattern might be associated with the jet. However, on the curve referred to we did not use the Strouhal number, but rather a dimensionless frequency based on the sound velocity.

We use two types of dimensionless frequency. For the sound close to the jet, where the behavior is the same as in an incompressible fluid, we do use the Strouhal number, frequency times diameter divided by flow velocity, as the dimensionless frequency parameter. On the other hand, for the sound radiated far from the jet, we think it is more appropriate to use a dimensionless frequency based on the sound velocity, that is, frequency times diameter divided by sound velocity.

There is some question whether either frequency parameter by itself is adequate. Actually, both parameters may influence the radiated sound. But we think that the radiated sound depends more on the parameter fD/c than on fD/U_0.

So it was just an accident that the peak occurred at a value of 0.2, the same as the Strouhal number for vortex shedding.

T. B. Benjamin

I wish only to emphasize the importance of one aspect of underwater noise touched upon in Strasberg and Fitzpatrick's excellent survey. This is the formation of real shock waves, akin to underwater blast waves, by collapsing cavitation bubbles. To call to mind this effect, it may be remembered that the pressure pulse produced at a distance by a cavitation collapse can have an effective duration of about 10 microseconds; but under some circumstances the pulse may develop a shock front whose transit past a fixed station may occupy considerably less than one microsecond, that is, very much less than that of the whole pressure wave.

There appear to be two respects in which the presence of shocks may be specially important, the first being their role in cavitation damage. In writings on this subject the term "shock wave" is used extensively, and the importance of the brief duration of cavitation pressures is fully appreciated; however, the distinction between very short yet continuous pulses, such as would always occur if water were strictly incompressible, and real shock waves is not always recognized. Nevertheless, one is naturally led to give weight to this distinction, since it is known that the stresses developed in a solid boundary by the incidence of shock waves are much greater than those due to continuous pressure pulses of the same amplitude. Some experiments done at

Cambridge have revealed great difficulties in trying to detect whether or not shock waves are present in cavitation pressure fluctuations, but left no doubt that they do arise at least sometimes. It does not seem untimely to express a hope that in the hands of able experimenters such as Professor Knapp and Dr. Ellis, this aspect of cavitation damage will soon be fully elucidated.

The second reason justifying attention to the matter of shock waves concerns the cavitation noise spectrum, as is found by making frequency analyses of hydrophone outputs. It has been pointed out before now that if a finite pulse possesses a discontinuity of the sort represented by the mathematical property $f(t^+) - f(t^-) \neq 0$ (i.e., a finite "jump" as at a shock front), its energy spectrum is asymptotically proportional to the inverse square of frequency; in other words, it decreases at a rate of 6 decibels per octave. Moreover, no other kind of pulse has a spectrum with this asymptotic property. This fact may possibly have a bearing on some experimental measurements of noise spectra; but a probably more useful consideration is as follows. If a pulse having the property described above is applied to a resonant system, the frequency response decreases asymptotically (i.e., well above the resonance frequency) as the inverse fourth power of frequency. Again, no other kind of pulse produces this response. Thus the response at very high frequencies from a practical hydrophone, which is bound to be affected by self-resonances at a number of frequencies, should be sustained at a slope of -12 decibels per octave if shocks are present.

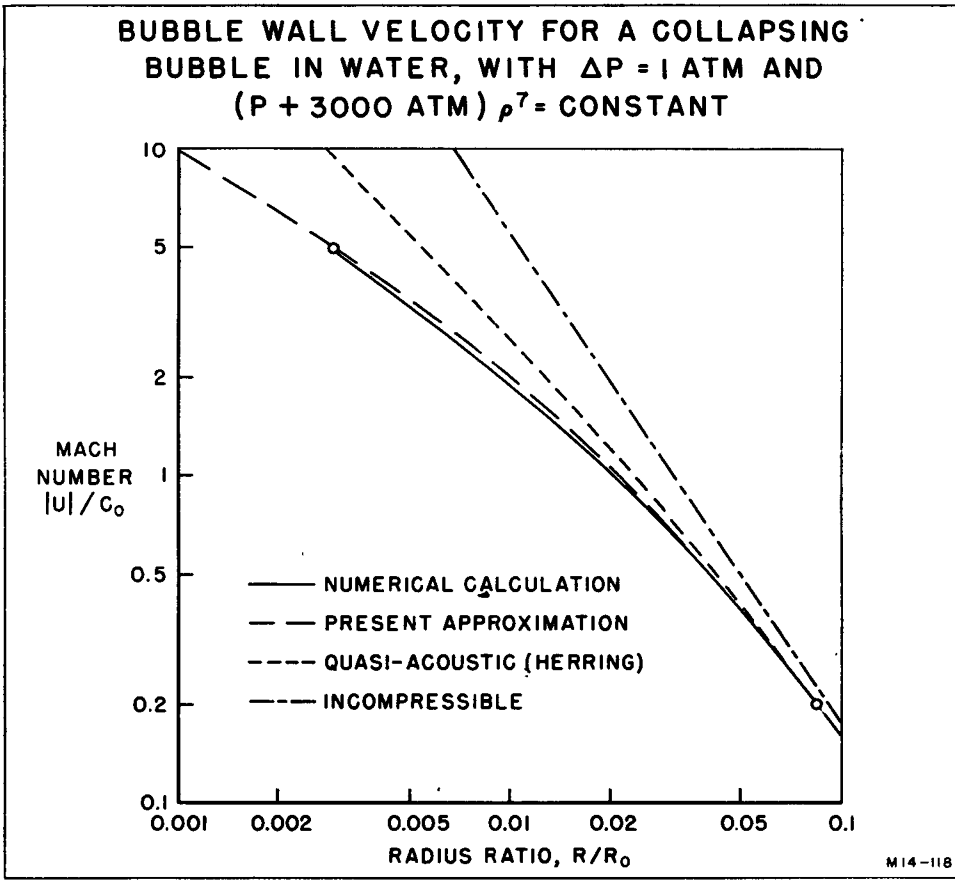
The theoretical problem of shock formation by collapsing bubbles is clearly a very difficult one. The allied but evidently simpler problem where a spherical shock arises as the result of conditions *prescribed* over a spherical boundary, such as Sir Geoffrey Taylor, Dr. Whitham and many others have treated, seems formidable enough; but the cavitation problem presents the great additional difficulty that the motion of the inner boundary (the bubble surface) is not initially known. Dr. Gilmore, for instance, has made an important contribution towards the understanding of bubble motion in compressible fluids, and his method of treatment was the first to give useful results for cases where the fluid velocities are of the order of the sound velocity; but apparently much remains to be done to clear up the question of strong cavitation where a shock wave probably forms very close to the bubble surface and has a profound effect on the bubble motion.

I understand that Mr. Fitzpatrick has made some calculations, not mentioned explicitly during his talk, which enabled him to estimate the least severe conditions of cavitation which should give rise to shock waves, and independently I have also attempted this. My own work stopped short after considering a weak shock formed far from the cavity centre, but I hope eventually to hear that Mr. Fitzpatrick has proceeded closer to the "heart of the matter."

These remarks are intended only to reaffirm the desirability of regarding water as a compressible fluid as far as cavitation collapse is concerned, and so to commend compressibility effects in cavitation as an interesting and useful field of study. Recognition of the theoretical difficulties need not reflect a pessimistic view of progress in this subject, for the signs are that a great deal will be achieved towards a complete solution within the next few years.

F. R. Gilmore

The authors have given a very interesting and well organized review of the problem of hydrodynamic noise. I wish to comment only on the section dealing with the noise produced by a collapsing cavity. According to the theoretical work of Lord Rayleigh, as a spherical bubble in an incompressible liquid collapses to infinitesimal size, velocities and pressures in the neighborhood of the bubble approach infinity, provided that the pressure of any vapor or gas in the bubble either remains constant or at least does not rise rapidly enough to prevent complete collapse. Recent theoretical work yields similar results even when the compressibility of the liquid is



taken into account (see my comments on M. S. Plesset's paper). For actual cavitation bubbles, however, there must be a point in the collapse after which these simple theories no longer apply, either because the bubble is no longer spherical, or because the pressure of the vapor inside starts to increase rapidly as the collapse becomes too rapid to permit vapor pressure equilibrium, or for any of a number of other possible reasons. Such complicating factors, which are very difficult to treat theoretically, provide an effective "cut-off" to the infinite pressure peak given by theory. However, if one is interested in the pressure pulse propagated to some finite distance from the bubble, there is another cut-off which may be more amenable to theoretical treatment. This arises from the well-known tendency of finite compression waves to become steeper as they propagate. In a compression wave having a sharp peak the peak will move faster than the rest of the wave, until a vertical front (shock wave) is formed. Thereafter, the peak will gradually advance into the shock wave and be effectively "lost." The height of the pressure peak is thus significantly reduced as it propagates (in addition to the geometric reduction in the spherical situation), and the pulse at some distance from the bubble may be independent of the very last stages of the bubble collapse. This possibility deserves careful theoretical investigation, using perhaps the methods developed for underwater explosion shocks during World War II. Since I am presently occupied with Air Force instead of Naval problems, I would particularly like to encourage someone else with the appropriate theoretical background to undertake such an analysis.