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HYDRODYNAMICS OF UNDERWATER EXPLOSIONS

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ABSTRACT

A survey is given of the various hydrodynamic aspects of underwater explosions. The topics selected deal with shockwave phenomena, the behavior of the bubble, surface phenomena and hydrodynamic problems of damage to structures.

In particular, the following items will be discussed:

a. The shockwave formation by a detonating charge. The long distance propagation of shockwaves. The anomalous surface reflection.

b. The change of shape of migrating bubbles. The energy dissipation at the bubble minimum. The energy partition of non-migrating and of migrating bubbles. The theoretical treatment of pulsating bubbles. Migration caused by rigid or free boundaries.

c. The spray dome. The gas break-through of shallow explosions. The surface phenomena of venting and non-venting underwater explosions. The plume formation. The rise of the bubble from deep explosions.

d. The reloading of an air-backed plate subjected to an underwater explosion. The response of cylindrical shells to a shockwave. The whipping of ships.

INTRODUCTION

When an explosive charge is fired under water, a sequence of complex events is started. Out of the great number of phenomena which occur, those have been selected which seem to be of greatest interest from the hydrodynamic point of view. Many of the problems which will be discussed are not completely solved today and more work is necessary until a satisfactory understanding is obtained.

SUMMARY OF THE BASIC UNDERWATER EXPLOSION PHENOMENA

The detonation of the charge converts the solid explosive material into gaseous reaction products which have an exceedingly high pressure. This pressure is transmitted to the surrounding water and propagates as a shockwave in all directions.

Fig. 1 illustrates the pressure-time history which is observed in the water at a fixed distance from the point of explosion. Upon arrival of the shockwave the pressure rises practically instantaneously to the peak value. Subsequently the pressure decreases steadily but at a very fast rate. It takes only a few milliseconds until the pressure has decreased to 1/e or 36.8% of its maximum value. The shockwave peak pressure and the decay constant depend on the charge weight and in the distance of the point of observation. Empirical equations for these as well as other shockwave parameters can be found in the literature [1].

Fig. 1 shows that subsequent to the shockwave other pressure pulses occur. These pulses arise from a much slower phenomenon, namely the pulsating of the gas bubble which contains the gaseous products of the explosion. The high pressure of the gas causes an initially rapid expansion of the bubble and the inertia of the outward

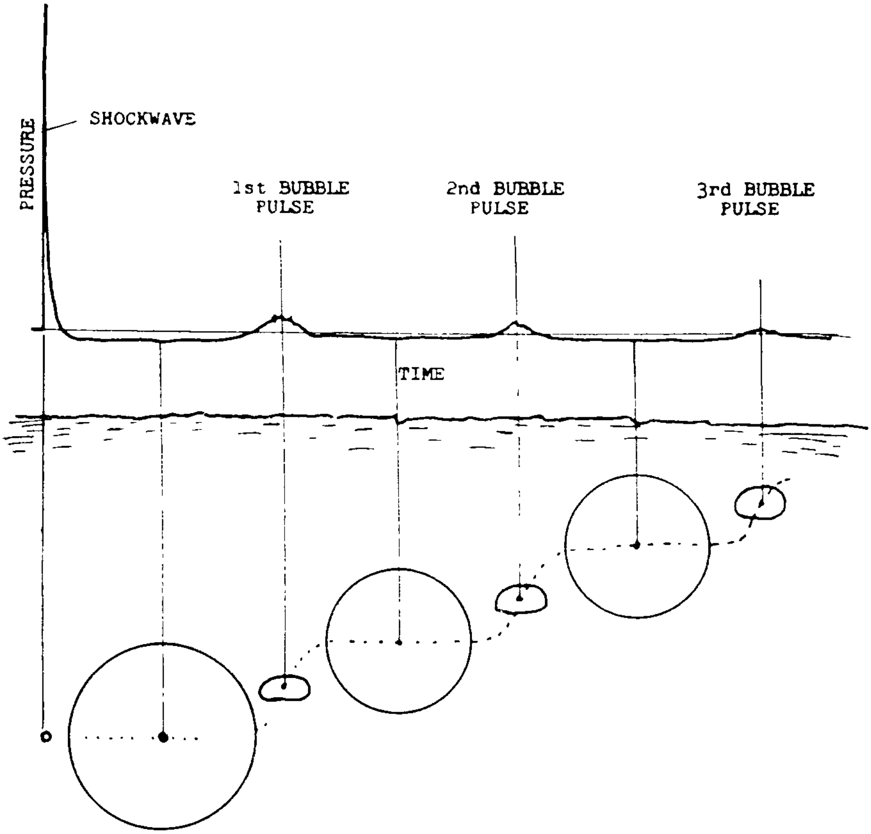

Figure 1. **Pressure waves and bubble phenomena of underwater explosions. The upper part shows a pressure-time plot, the lower, the position and size of the bubble for specific moments which correspond to the curve above as indicated by the vertical lines.**

moving water carries it far beyond the point of pressure equilibrum. The outward motion stops only after the gas pressure has fallen substantially below the ambient pressure. Now the higher surrounding pressure reverses the motion. Again the flow overshoots the equilibrium and when the bubble reaches its minimum size the gas is recompressed to a pressure up to several hundred atmospheres. At this point we have effectively a second explosion and the whole process is repeated. The bubble oscillates in this way several times.

In Fig. 1, the position and the size of the bubble is shown for a few specific moments which correspond to the pressure-time curve as indicated above. The pressure-time history reflects the low gas pressure during the phases where the bubble is large and it shows the pressure pulses which are emitted from the bubble near its minimum.

The period of the first pulsation [1] is more than one half second for common charges. This is a long time when compared with the extremely fast processes occurring with explosions. In particular, this duration is long enough for gravity to become effective. Such a bubble has great buoyancy and, therefore, migrates upward. However, it does not float up like a balloon, but shoots up in jumps.

In Fig. 1, the dotted curve represents the position of the bubble center as a function of time. This curve shows that the rate of rise is largest when the bubble is near its minimum, but is almost zero when the bubble is large. (Note that Fig. 1 is a time plot of bubble position and size. It must not be interpreted that the bubble moves sidewards. Actually, the bubble rises vertically upward.)

The interaction of the pressure waves and the pulsating bubble with the water surface, the bottom of the sea and the target structures produces a great variety of interesting phenomena and effects. Some of these will be discussed in this paper.

SHOCKWAVE PHENOMENA

Shockwave Formation

Fig. 2 illustrates the formation of the shockwave by a detonating charge. The graph shows the pressure-distance distribution at two moments t_1 and t_2. We assume a spherical charge which is ignited at the center. A spherical detonation wave proceeds outward causing the reaction of the explosive in concentric shells one after the other. At the moment $t = t_1$ this detonation wave has reached the charge surface and the explosion is completed. The graph shows the pressure distribution inside of the gas globe for this instant, curve labeled t_1. The maximum pressure occurs at the surface of the gas globe. The pressure-distance curve decreases rapidly—in fact, discontinuously—at this point when going toward the center. Proceeding further in this direction, the pressure levels off and becomes constant at a distance of about one-half of the radius from the center of the gas sphere. The infinitely sharp pressure spike at the surface of the gas sphere is a consequence of the spherical propagation of detonation waves (G. I. Taylor, Doering [2]).

The pressure transmission from such a high pressure gas ball to the ambient medium has been calculated by Wecken [3] as well as Holt and Berry [3]. They have studied the problem of a detonating charge in air. Fig. 2 shows an adaptation of their results to an explosion in water, curve labeled t_2. The gas globe has expanded as indicated by the dashed circle. Ahead of the bubble the shockwave has moved into the water and a rarefaction wave runs back into the gas. There is a small second shock which propagates toward the center. At a later moment this shock will be reflected at the center and will be finally transmitted into the water. Experimentally observed shockwaves show a small but reproducible irregularity in their tails. It is believed that this caused by the "second shock."

When the maximum shockwave pressure is traced back to the gas globe, one finds that the initial shockwave pressure in water is about one-half of the detonation pressure. This means the gas expands only partially when the shockwave is formed and only a fraction of the total energy of detonation is transmitted into the shockwave; the remainder produces the expansion of the gas globe and the bubble pulsations. It turns out [4] that about 50% of the total energy is radiated in the shockwave and that the other 50% constitutes the bubble energy.

Although the problem of the shockwave formation by a detonating charge is basically understood, no numerical calculations applicable to underwater explosions have been made so far. The same holds for the subsequent propagation of the high-amplitude shockwave through the water. It might be mentioned that there are several approximate treatments (Kirkwood-Bethe, Kirkwood-Brinkley, Snay-Mathias [5]) which describe these phenomena accurately enough for many practical purposes. However,

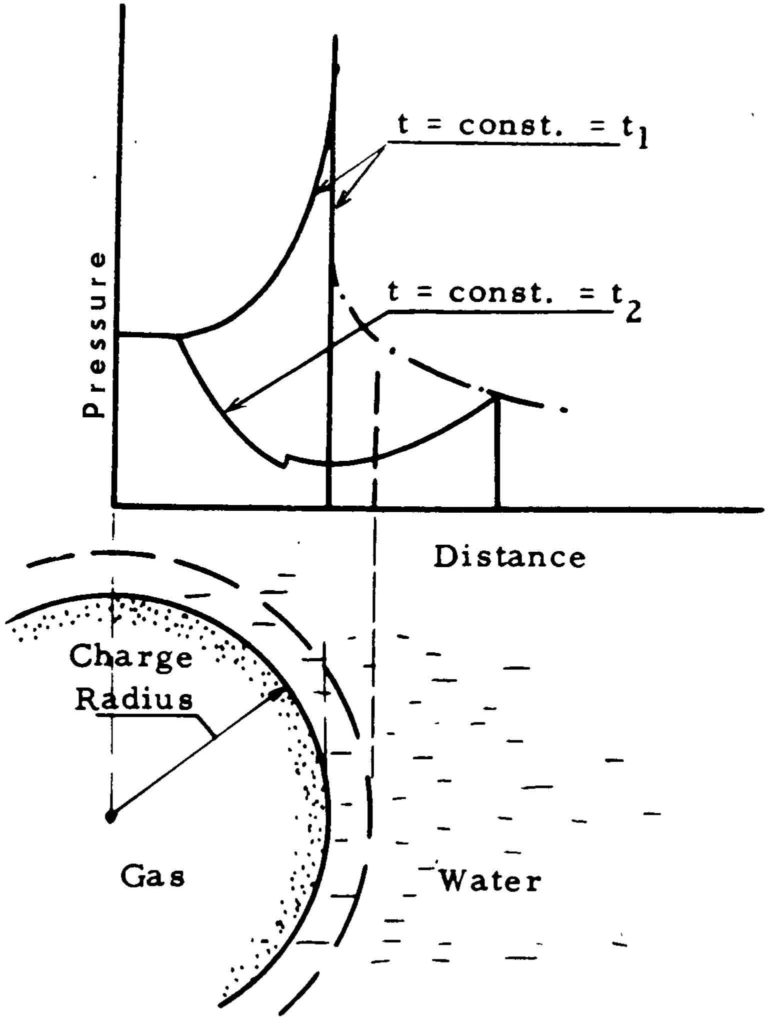

Figure 2. Shockwave formation by a detonating spherical charge. The vertical dashed line in the pressure plot designates the position of the gas-water interface at $t = t_2$.

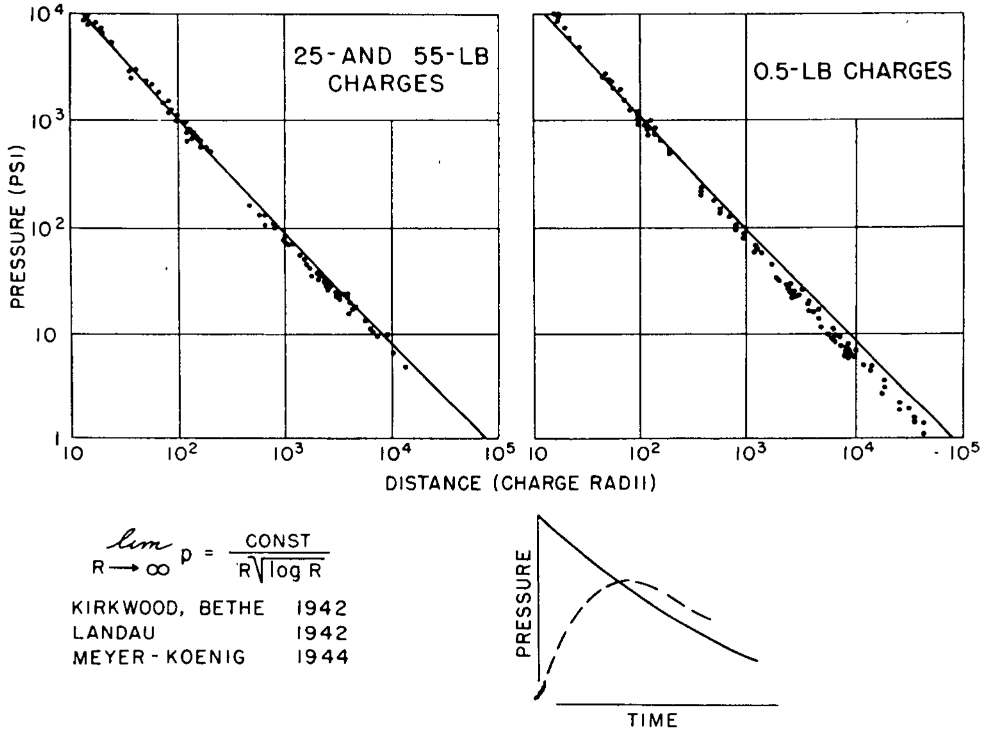

Figure 3. **Comparison of the asymptotic pressure decay relation with experiments. The curve which represents the equation noted is adjusted to a good fit with experimental points near the 10^3 psi pressure level. Experimental data are due to ARONS, reference 7.**

a rigorous treatment of the shockwave phenomena produced by a detonating charge has not been made. It would require the use of an electronic computer.*

Long Distance Propagation

Since the shockwave pressure decreases when it propagates through the water, one might assume that there will be a point where the amplitude has decreased so far that the wave has lost its high-amplitude character and can be treated as a low amplitude acoustic wave. Several workers in this field [6] have stated independently and almost at the same time that a shockwave never becomes exactly an acoustic wave. The asymptotic relation is noted on Fig. 3. A spherical acoustic wave decreases inversely proportional with distance, but a weak shockwave decreases somewhat more rapidly due to the factor $\sqrt{\log R}$ in the denominator.

If we compare this asymptotic law with experimental evidence, Fig. 3, we find

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* Such a calculation has been recently reported by BRODE.

poor agreement for tests carried out with one half pound charges, but somewhat better agreement for the tests with 25 lb. and 50 lb. charges.

The reason for this disagreement may be either inaccurate experimental data or an unrealistic assumption for the derivation of this formula.

The asymptotic law was obtained from the Rankine-Hugoniot conditions for the shockfront and the equations of *inviscid* fluid motion behind the shockfront. This means that outside the discontinuity all dissipative effects are ignored and that the asymptotic equation is based on a wave profile as illustrated in Fig. 3 by the solidly drawn curve. Actually, weak shockwaves have a profile as sketched in Fig. 3 by the dotted curve: The front shows a steep but steady rise and the peak is rounded off by the effect of viscosity. The question has been raised [7, 8] as to whether the equation shown in Fig. 3 would still hold for such conditions. Arons [7] has attacked this problem by means of an approximate calculation and found a decay stronger than that given by this equation. In this calculation an acoustic treatment is used to account for the viscous effects and afterwards a correction is applied for the cumulative effects of the non-acoustic contributions. Although the results are probably qualitatively correct, it is not clear whether the accuracy of the calculation is sufficient to demonstrate a valid deviation from the asymptotic law.* On the other hand Aron's calculations agree very well with the experimental points shown in Fig. 3. This brings us to the question of the accuracy of the experimental results. To measure such low pressures with electronic equipment, very sensitive and therefore very large gages are necessary. The presence of such a large gage in the water distorts the incident wave so that a true pressure measurement is in principal not possible.

The data of the graphs in Fig. 3 are corrected for finite gage size assuming that the response-time of the gage is equal to the transit-time of the wave; hence, the hydrodynamic distortion of the pressure field is neglected. It is not known today how large this effect is. Unfinished and unconfirmed studies of Slifko at NOL have resulted in response times which are about twice the transit time. The use of such a large response time would move the experimental points in Fig. 3—in particular those for the ½ lb. charges-upward. This would improve the agreement with the asymptotic relation.

Surface Reflection

When the shockwave emitted from an explosion (Fig. 4), hits the free water surface, the pressure in the water at this interface must be equal to that of the air. It is well known that this leads to the formation of a rarefaction wave, which—if we use the acoustic approximation—is seemingly emitted from the image of the point of the explosion. This yields the dashed pressure-time-curve for the point P. Considering the high pressures of shockwaves, it appears that the negative phase shown will not occur in actuality. Only highly purified water can sustain a substantial tension without beginning to cavitate. Sea water contains numerous cavitation nuclei and cavitates as soon as the pressure drops below a certain level usually somewhat above the vapor pressure [7]. This results in the solidly drawn pressure curve:—the tail of the wave is cut off. At the pressure scale of such a record no distinction between cavitation pressure and the pressure of the undisturbed medium can be made.

This cavitation of an elastic liquid is called bulk cavitation. Kennard [9] has given the basic laws of this phenomenon, but very little more has been done. Kennard [10] and Arons [7] have calculated the extent of the cavitated region. It is a surprisingly thin layer which extends far to the side, Fig. 4. Nobody has so far attempted to

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* In a recent publication [23] LIGHTHILL has derived the asymptotic formula, Fig. 3, from the "Burgers" equation which includes dissipation at all points of the wave profile. This theory will go far towards dispersing the above mentioned objections against the asymptotic equation.

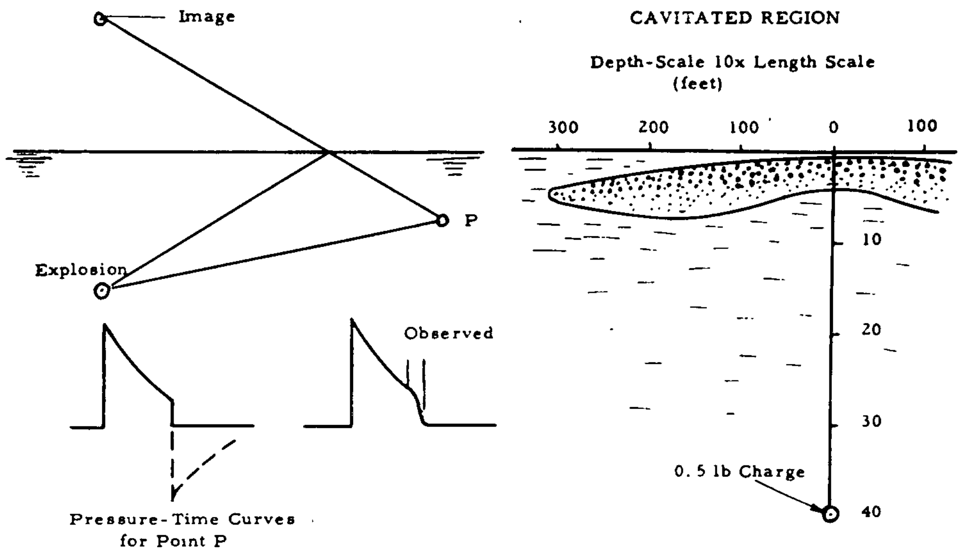

Figure 4. **Reflection of a shockwave from the water surface.**

calculate the fluid motion within such a cavitated region or to understand when and how the closure of this cavitation occurs.

The acoustic approximation seems to describe the surface effect reasonably well. But, if we give the problem a closer look, we find again that we are dealing with high amplitude waves and that the acoustic approximation can be misleading. The first evidence is seen in Fig. 4 and it is inconspicuous enough to be overlooked. The surface signal arrives earlier than calculated by the acoustic theory, also there is not a clean cut-off,—but a gradual decrease of pressure. Obviously this faster-than-acoustic-propagation is a finite amplitude effect. This fore-running of the surface signal becomes more and more pronounced the shallower the point of observation is, Fig. 5. Finally, the surface signal arrives so early that it reaches the shockfront itself. This point lies on the boundary of the anamolous surface reflection (Penny and Keil [11]). Beyond this point the rarefaction from the surface engulfs the whole wave and changes drastically the profile of the wave. Rosenbaum and Snay [12] have treated this problem theoretically by using the approach of the pseudo-stationary fluid motion. The agreement of these calculations with experimental evidence is good. However, we do not have a precise mathematical description of this phenomenon yet.

BUBBLE PHENOMENA

The Migrating Gas Bubble

Fig. 6 shows a migrating pulsating bubble. The pictures are from a model test where the air pressure above the water surface was reduced. By this way it is possible to scale the effects of gravity and to simulate realistically the bubble migration. By adjusting the air pressure to a certain value, it is possible to scale any desired charge weight. For instance, this test corresponds to a 354 lb. TNT charge detonated in a depth of 80.8 feet.

The change of the shape of the bubble is interesting. At the moment of the maximum expansion the bubble is an almost perfect sphere. But, when the bubble con-

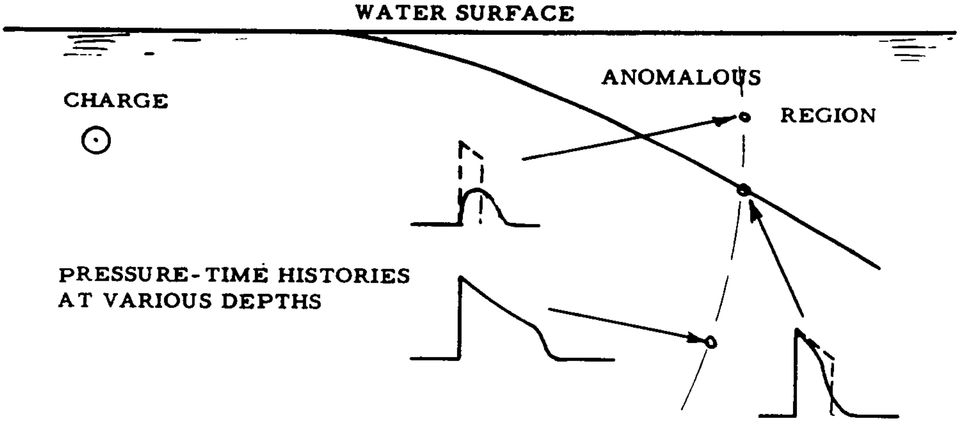

Figure 5. **Anomalous surface reflection. The pressure-time curves refer to the points indicated by the arrows. The dashed curves show the shape of the wave from acoustic treatment.**

tracts, the sphere is distorted. The straight bottom seen on the second frame is due to the fact that the bubble surface is inverted into the inside. The actual lower bubble surface is faintly visible in the interior of the bubble.

This change of shape is caused by the difference of the hydrostatic pressure between bubble top and bottom. Therefore, upon contracting, the bottom of the bubble is pushed inward faster than the top and the kidney-like shape seen in the second frame of Fig. 6 is formed. A short instant later the two interfaces impinge on each other and the bubble becomes a torus (third frame). The gas is near the periphery and there is water all the way through the bubble near the center line. As in a vortex ring there is a strong upward velocity near the centerline and this carries the bubble upward when it re-expands.

The last frame shows the moment of the second bubble maximum. The bubble is almost spherical again. The black region below the bubble is water which is darkened by the solid products of the explosion.

The inversion of the bubble is apparently the mechanism by which such a migrating bubble dissipates its translational energy or in other words it is the mechanism by which the drag of such a moving bubble is originated. The analogy to the re-entrant jet of fast moving bodies under water is apparent. This impinging of the two interfaces produces a water-hammer effect and dissipates energy. It also introduces vorticity into the sofar irrotational fluid motion.

Kolodner and Keller [13] and, earlier, Penny and Price [1] have attacked the problem of the bubble contraction in a gravitational field and carried the calculations to a point shortly before the collision of the interfaces. These calculations describe the change of shape very well. No attempt has been made so far to follow up the later phases, for instance to treat the bubble as an expanding vortex ring.

The Bubble Pulse

Fig. 7 shows the pressure-time record of a bubble pulse. This record has been obtained from a 1600 lb. TNT charge exploded at a depth of 100 ft. In this case,

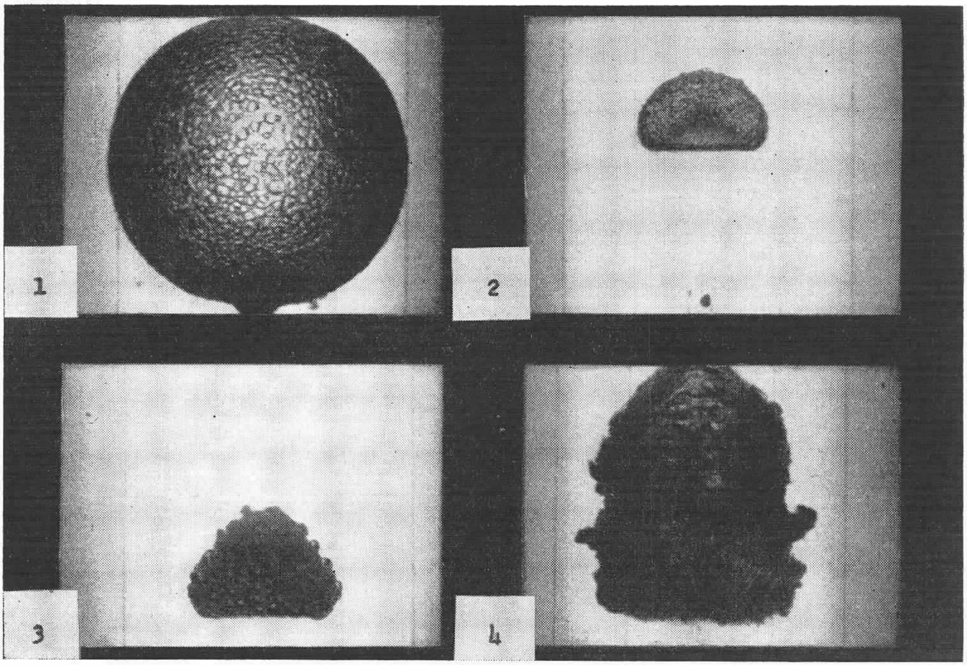

Figure 6. **Gas Bubble of an Underwater Explosion.**

Frame 1: Bubble maximum

Frame 2: About 6msec before bubble minimum

Frame 3: About 6msec after bubble minimum

Frame 4: Second bubble maximum

(times quoted refer to the condition scaled in this model test: 354 lbs TNT in 80.8 ft. depth)

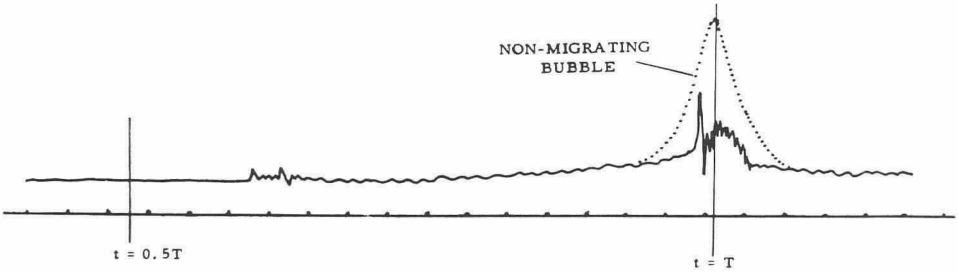

Figure 7. **Shape of the bubble pulse. The record shows a later portion of the pressure-time history. The shockwave is omitted. T means the duration of the first bubble oscillation.**

there is a strong bubble migration and therefore also an inversion of the bubble. The spike or pressure jump is probably caused by the water hammer effect of the colliding bubble surfaces. (The oscillations which suddenly appear shortly after $t = 0.5T$ are caused by the reflection of the shockwave from the bottom of the sea.)

If we explode a small charge in a very great depth (say 1 lb. in 500 ft.) the period of bubble oscillation is so small that the gravity or buoyancy cannot produce any significant bubble migration. The shape of the bubble pulse from such a non-migrating bubble is indicated by the dotted line in Fig. 7. It is smooth, without any irregularities and shows a somewhat higher pressure. Apparently no inversion of the bubble takes place here and the bubble contracts to a smaller volume. Underwater photographs have shown indeed that a non-migràting bubble does not change its shape but remains spherical over many pulsations until it finally dissolves into many little gas bubbles.

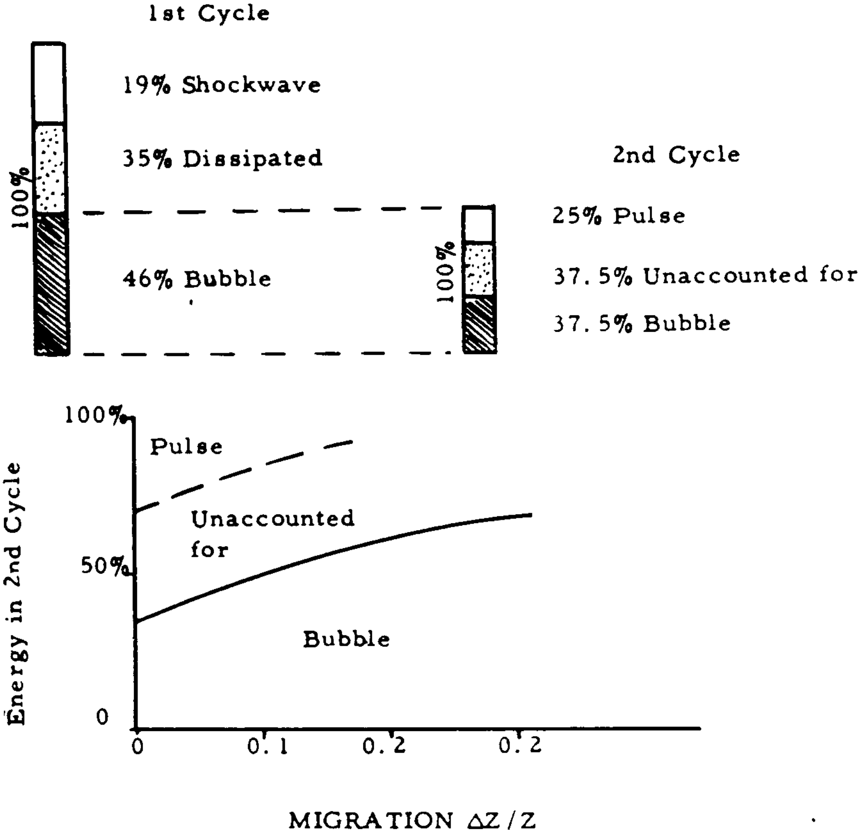

Figure 8. **Energy partition of an underwater explosion. The upper portion refers to a non-migrating bubble. The energy distribution of the 2nd cycle is that found after the first bubble minimum. This distribution is affected by the bubble migration as shown in the graph. ΔZ is the migration between the first and second bubble maximum, Z, the total hydrostatic head at the depth of explosion.**

Energy Partition

An analysis of the oscillations of a non-migrating bubble by Arons and Yennie [4] has shown that despite the fact that the bubble remains spherical a strong energy dissipation takes place near the bubble minima. For instance, at the first bubble minimum 25 per cent of the original bubble energy is acoustically radiated by the bubble pulse. Another 37.5 per cent is dissipated by a mechanism which is not clearly understood at the present time. This leaves only 37.5 per cent of the original bubble energy for the next pulsation. This is illustrated in Fig. 8.

This energy dissipation of a non-migrating bubble is somewhat mysterious. It is not caused by a collision of the bubble surfaces as we have seen it for the migrating bubble. It may have something to do with the stability of the gas-water interface. According to G. I. Taylor's [16] criterion, this interface is highly unstable near the bubble minimum, but is stable for the major portion of the oscillation. This unstability may produce a spray of water which is projected into the compressed gas and reduces the internal energy of the gas by cooling.

For migrating bubbles [14] the energy radiated by the bubble pulse decreases with increasing migration Fig. 8. The amount of the dissipated energy (called "unaccounted for" in the figure) remains about the same, although the mechanism probably changes. Therefore, the bubble energy (of the second cycle) increases with increasing bubble migration as shown in Fig. 8.

Theoretical Treatment for Non-Migrating Bubbles

There is a great number of publications on the theory of the oscillating gas bubble [1, 15]. Most of these assume that the bubble pulsates in an incompressible medium. Since the bubble pressure is low for a great portion of the oscillation, such approximate theories describe the movements of the bubble very well, except for short moments near the bubble minimum. Of course, these theories cannot describe the early bubble expansion shortly after the detonation and the energy losses at the minima. An acoustic approximation or improved version of it can account for the energy radiation by the bubble pulse, but not for the additional energy dissipation at the minimum. Therefore, all these calculations are applicable only to one cycle of the oscillation. For each of these cycles, the parameters of the theory must be redetermined on the basis of the empirical evidence on the energy redistribution at the bubble minimum. It is possible to adjust the parameters of the acoustic theory in such a way that the observed radius-time curve is reproduced over several cycles. Such calculations would yield too large amplitudes for the bubble pulse and, therefore, are in poor agreement with pressure measurements.

Migration Caused by Bounding Surfaces

Besides gravity other factors may produce a migration of the bubble. For instance, a rigid surface attracts the pulsating bubble and a free surface such as the water surface repells it. This phenomenon is analogous to the classical Bjerknes spheres. Bjerknes demonstrated long ago through the use of air-filled rubber bags in water that bubbles attract each other when their oscillation is in phase and that they repel each other when pulsating out of phase. A bubble that oscillates near a rigid surface can be represented by the bubble and its image pulsating in phase. Therefore, the bubble and its image attract each other. At a free surface, the image of a sink is a source and vice versa. Hence, the image of the bubble pulsates out of phase and the bubble is repelled from the free water surface.

For an explosion in shallow water these two effects oppose the upward migra-

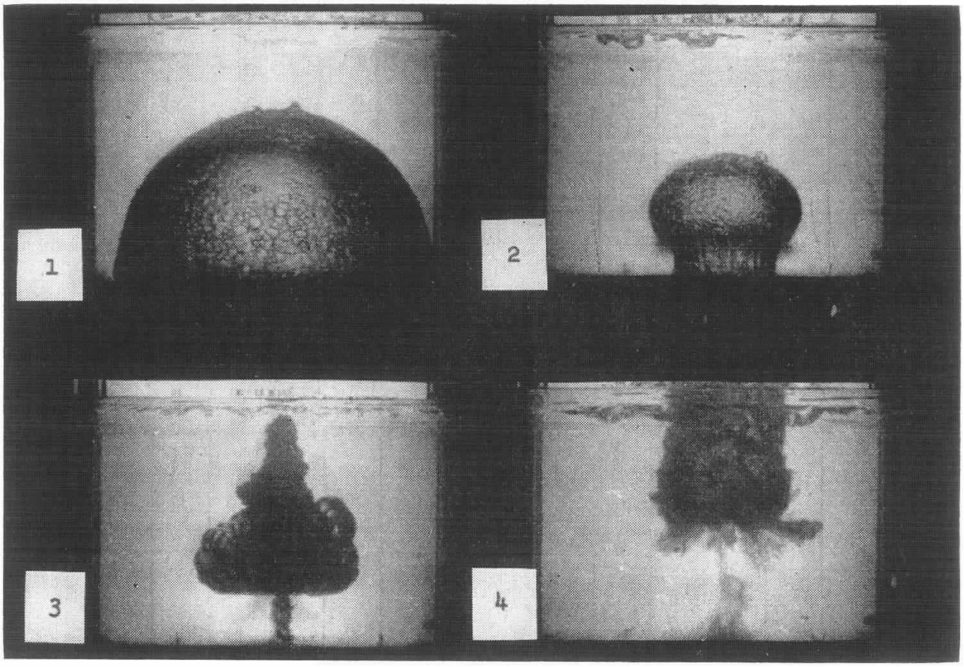

Figure 9. **Bubble Behavior of an Explosion on a Rigid Surface. The charge was fired on a steel plate. The water surface is visible near the top of each frame.**

tion due to gravity. Fig. 9 shows an explosion on the bottom. The attractive action of the bottom is clearly visible. The bubble seems to cling to the bottom, but the effect of gravity is stronger and finally tears the bubble away. The repulsive effect of the surface is not noticeable here, because of the strong upward momentum of the bubble.

In the test shown in Fig. 10 a rigid cylinder has been mounted under water. Frame 1 shows the bubble maximum. In frames 2 and 3 the attraction force of the cylinder becomes apparent. Frame 4 shows that the bubble has not migrated straight upward but toward the solid body in the water.

SURFACE PHENOMENA

The Spray Dome

When the shockwave strikes the water surface, the air-water interface becomes highly unstable. It breaks up into many spikes or needles which quickly disintegrate into droplets. According to the mathematically treatment of G. I. Taylor [16] the original irregularities grow exponentially with time. This is nicely seen in Fig. 11. (Test made by Young and Goertner, NOL.) The jets spring up predominantly from the crests of the ripples which were there before the explosion. This phenomenon is of particular interest, since one can determine the peak pressure of the shockwave photographically by measuring the initial velocity of the spray dome. However, the spray which is emitted from the peak of an irregularity moves faster than the average and this may

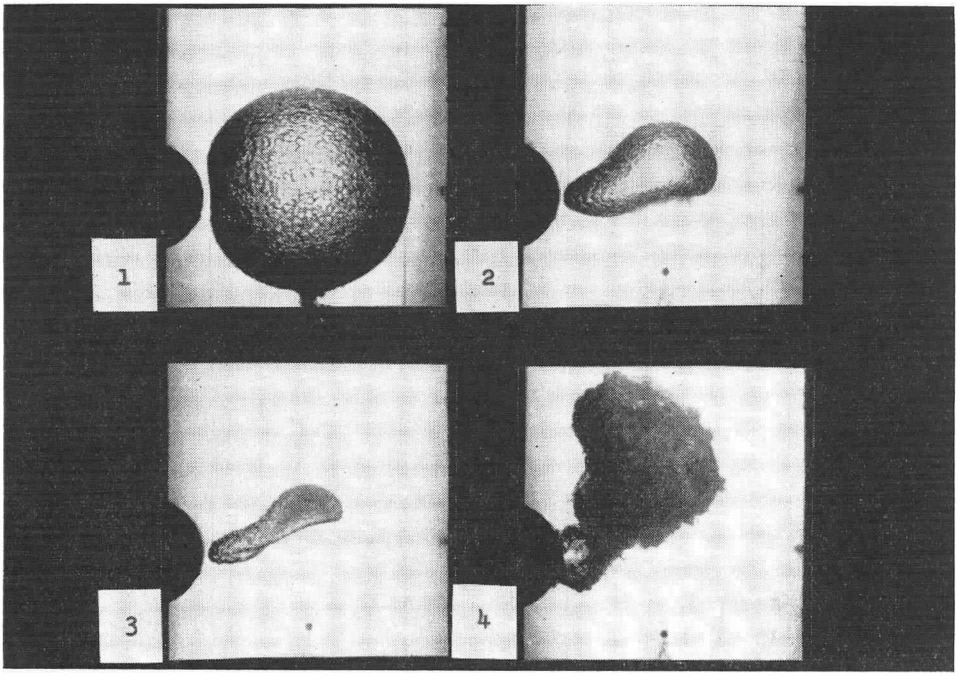

Figure 10. Bubble Pulsation Near a Rigid Cylinder.

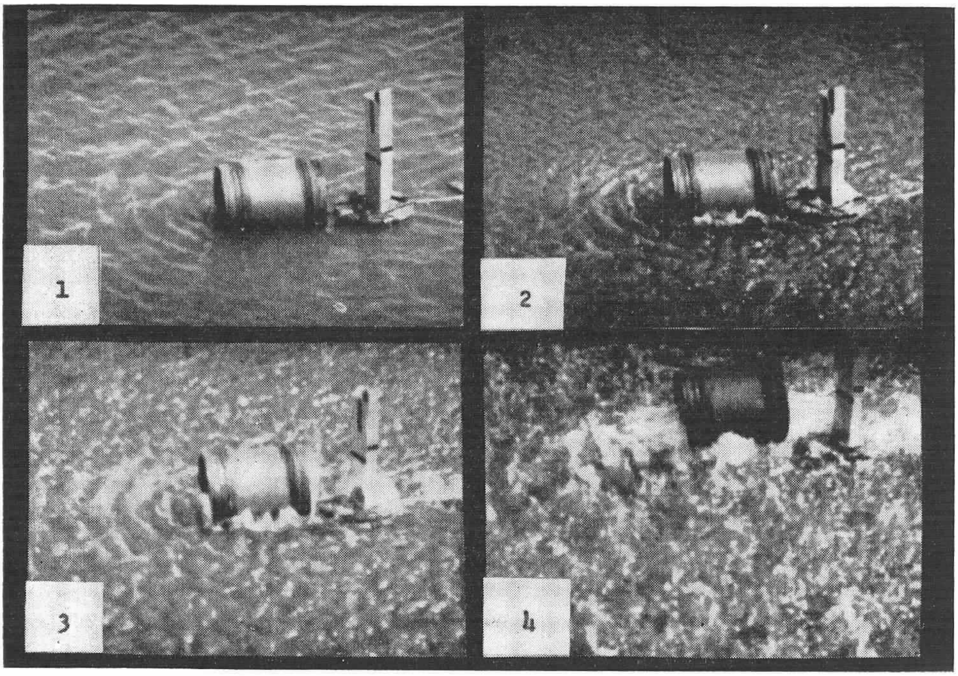

Figure 11. Formation of the Spray Dome from a Rippled Water Surface. The jets emerge from the crests of the ripples.

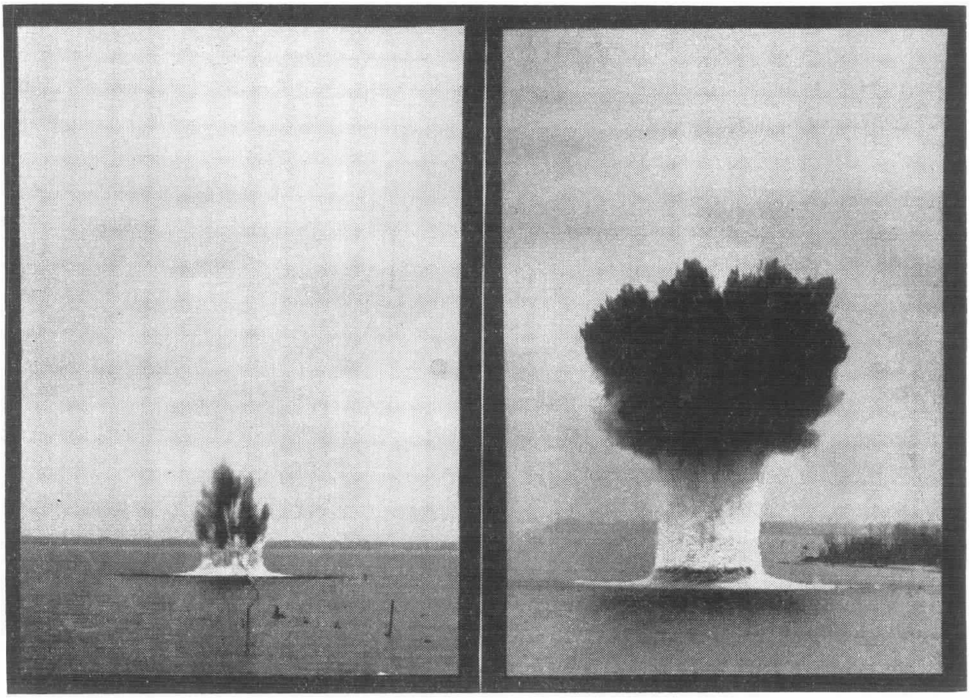

Figure 12. **Shallow Explosion with Gas Blow-Out. Charge: 4200 lbs TNT. Depth: 2.1 charge radii.**

be one reason why this method did not always give consistent results. There is a theory by Keller and Kolodner [17] on this spray formation. A quantitative correlation of their calculations with experimental evidence has not been made.

The Water Column

Fig. 12 illustrates the development of the water column from a very shallow explosion. (Test made by Young and Willey, NOL.) The gaseous reaction products of the explosion blow out into the atmosphere and form a dark cloud of smoke and water. The column below is practically cylindrical and it is hollow inside.

Rosenbaum and Snay [18] have theoretically studied the growth of such a water column. They calculated the fluid motion which is produced by an expanding cylindrical piston. Of course, this analysis cannot explain why this water column has a cylindrical shape. This has been studied by Hudson (see his discussion at the end of the paper).

Fig. 13 shows a somewhat deeper[*] explosion. There is very little or no gas

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[*] The tests discussed here were not carried out with the same charge weight. Therefore, the terms "shallow depth" or "great depth" must be understood as relative measures that account not only for the actual depth of the explosions, but also for the weight of the charge. A suitable way of doing this is to measure depth either in charge radii or in maximum bubble radii. This parameter assures geometric scaling and the qualitative appearance of the surface phenomena depends largely on it. Some quantitative effects, in particular the height of the column, depend on a further parameter which is essentially the absolute weight of the charge. This also holds for phenomena induced by the gravity migration of the bubble, for instance the formation of the plumes.

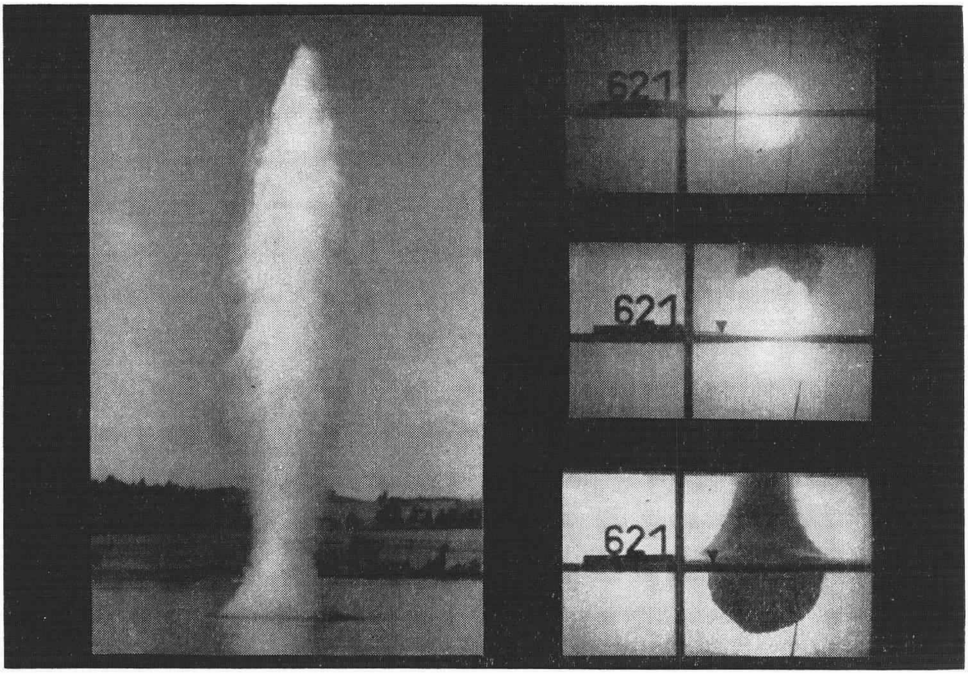

Figure 13. **Water Column of an Explosion with Venting Bubble. Charge: 2.75 lbs TNT and 3 grams $KClO_4/Al$ 50/50. Depth: 0.4 maximum bubble radii.**

blow-out in this case. The small scale test shown on the right hand side concerns the same conditions as shown by the larger explosion on the left. The water surface is at the little triangle. The black lines are steel bars which support the windows of the tank. In this model test an explosive is used which yields highly luminous products. This makes it possible to study the motion of the gas. No blow-out is visible here. On the contrary, there are indications that air is sucked into the bubble, because, after reaching its maximum, the bubble does not contract again. Hence the pressure inside the bubble must have been in equilibrium with the ambient pressure. This venting of the bubble occurs through the column and probably causes the contraction near the base.

The Case of the Non-Venting Bubble

If the depth of explosion is increased further, the appearance of the water column is radically changed, Fig. 14. A thin water jet shoots upward which is surrounded at the base by a lower and broader spray formation. The bubble does not vent in this case. The first frame of the model test which scales the same test geometry shows the bubble shortly after the maximum in the stage of contraction. There is clear water above the bubble which proves that there was no communication between the bubble and the atmosphere.

The model test in Fig. 14 scales a small charge weight (1 gram). Consequently, the effect of gravity is small and the above mentioned repulsion of the pulsating bubble from the water surface becomes dominant: Frames 2 and 3 show that the bubble migrates downward.

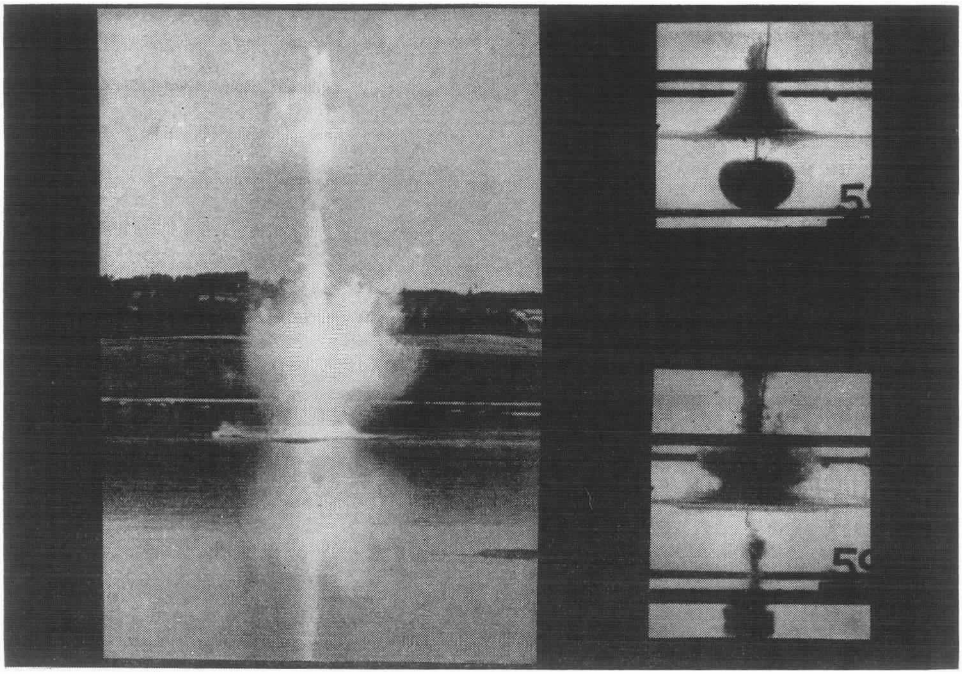

Figure 14. **Surface Phenomena for the Case of a Non-Venting Bubble. Charge: 2.75 lbs TNT and 1 grame KC10$_4$/A1 50/50. Depth: 0.9 maximum bubble radii.**

The Plumes

Placing the charge still deeper, the so-called plumes occur, Fig. 15. The first frame shows the spray dome which is caused by the shockwave. Somewhat later (second frame), a jet suddenly appears above the spray dome and bursts upward with great violence. Such a jet is commonly called a "plume". If a distance-time plot is made of this plume and if it is extrapolated back to zero distance from the water surface, it is found that the plume started to move at the moment of the bubble pulse. Thus, it appears that this plume is formed by the impact of the bubble pulse. Later (fourth frame) more plumes spring up when the gas bubble breaks through the water surface. Very little is known about the mechanism which leads to the formation of the plumes. It seems, that a strongly accelerated water surface breaks up either into a spray if the surface was originally smooth, or into plumes if the surface was already grossly disturbed, for instance by the prior impact of the shockwave. Small charges (below say ten pounds) do not produce such conspicuous plumes as are shown in Fig. 15.

Fig. 16 shows the water column of the nuclear underwater explosion at Bikini. Here, also, plumes are formed, but in an entirely different situation from that shown in Fig. 15. They emerge horizontally from the water column and later fall downward.

Deep Explosion

In Fig. 17, we see an explosion of a small charge in great depth. The spray dome is minute and disappears quickly. After a while the gaseous reaction products appear in form of a compact ball on the water surface. In the second frame, the

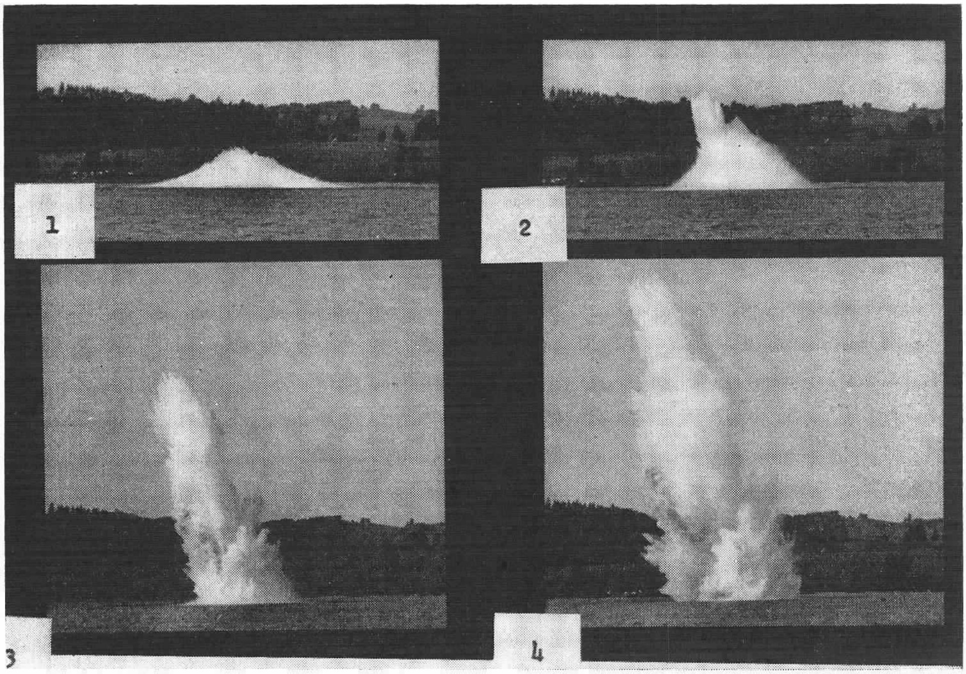

Figure 15. Plume Formation. Charge: 275 lbs TNT. Depth: 1.8 maximum bubble radii.

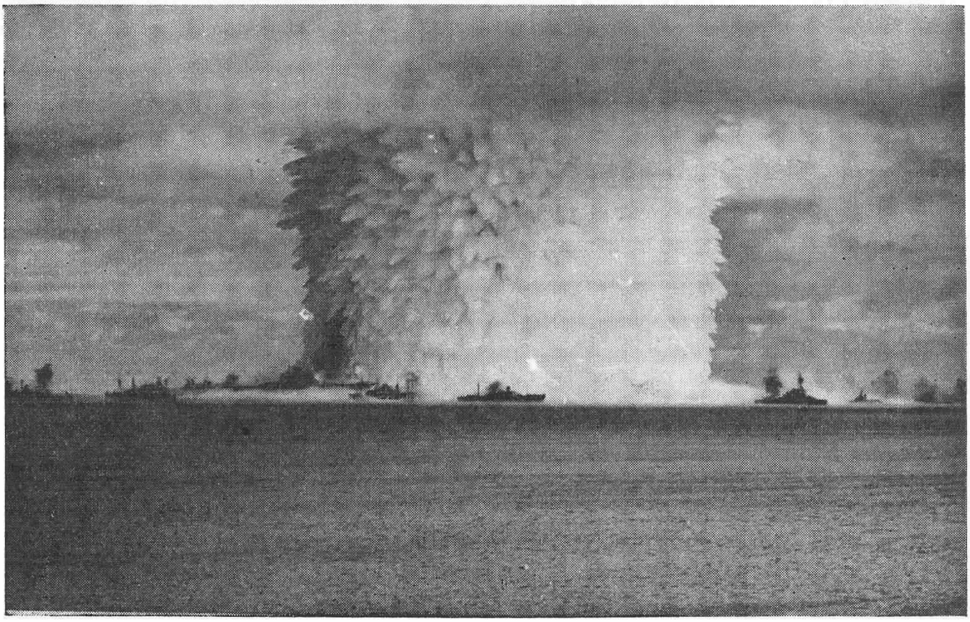

Figure 16. Plumes on the Bikini Column.

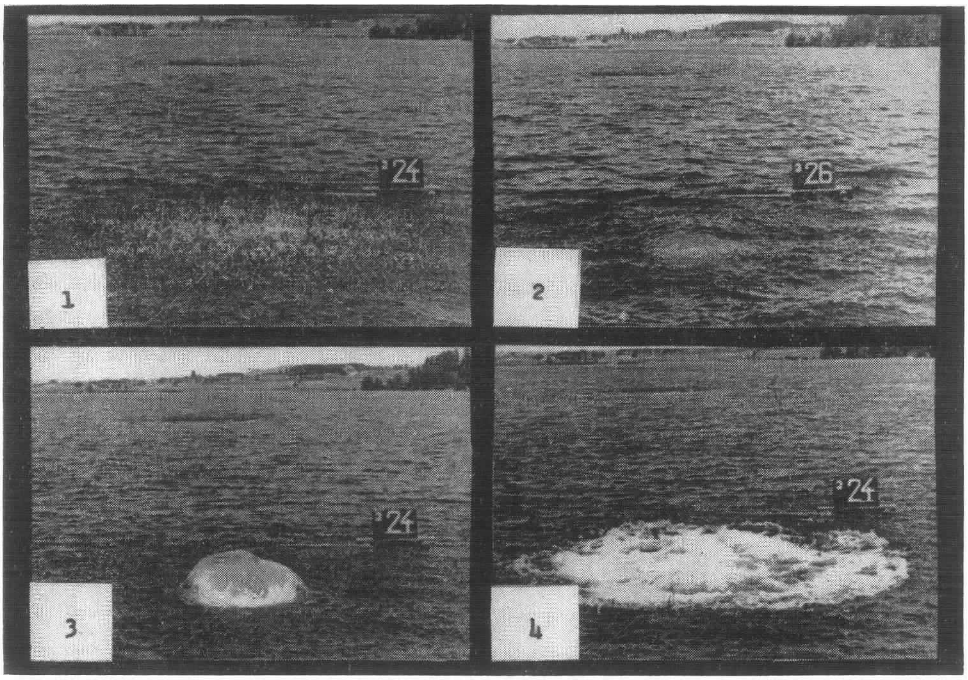

Figure 17. Rise of the Bubble from a Deep Explosion. Charge: 1.1 lbs TNT. Depth: 4–8 maximum bubble radii.

bubble can already be seen through the water. At this time the bubble has virtually ceased to oscillate and had probably the shape of a mushroom head. It is a relatively rare incident that the bubble stays together in this manner. Usually, it dissolves into many tiny bubbles and a sort of soda water appears on the surface. We have here again an interesting hydrodynamic problem, namely the motion and the stability of a large gas-volume under water. (See contribution of Lane and Green in reference [23].)

SOME HYDRODYNAMIC PROBLEMS OF THE DAMAGE TO STRUCTURES

We proceed now to the problem of damage done by underwater explosions to idealized structures. We will not discuss military applications of underwater explosions against actual ships or submarines, but will restrict ourselves to hydrodynamic aspects.

The Reloading of an Air-Backed Plate

It was mentioned that about 50 per cent of the total chemical energy of the charge is found in the energy of the bubble. Therefore, the question arises as to whether this large energy term contributes to the damage, for instance to the deflection or rupture of a plate. Schauer [19] has treated this question and he pointed out that, according to experiments, the initial deflection of an air backed plate takes place in two steps, as illustrated in Fig. 18. The second blow is neither caused by the bubble pulse which comes much later and produces a further deflection, nor by the shockwave, because at this time the shockwave has long disappeared. Also, the total deformation energy of the plate is larger than the energy flux of the shockwave impinging on the plate. It turns out that this second blow is caused by the expanding bubble by means of the following mechanism:

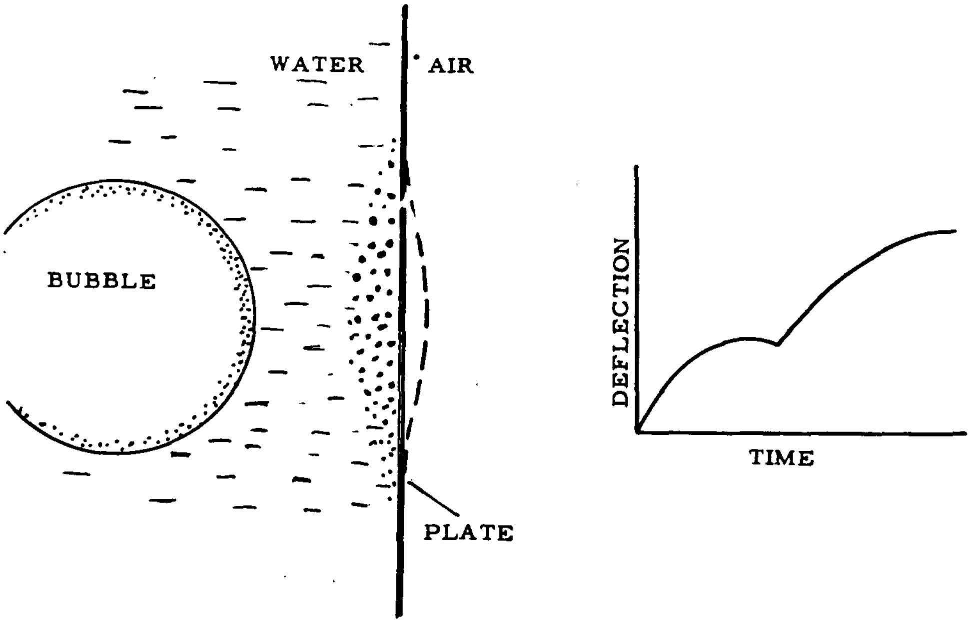

Figure 18. **Reloading of an airbacked plate.**

The impact of the shockwave accelerates the plate to a high velocity. This takes place within a very short time, i.e., near the origin of the plot in Fig. 18. The pressure in the water subsequently decreases rapidly with time and cavitation sets in near the plate. At this moment, the plate has a higher velocity than the water particles which follow;—each cavitated particle being slower than that ahead of it.

When the plate is retarded by the elastic forces of the material, the cavitated water particles catch up. They transmit their momentum to the plate and form a layer of uncavitated water which moves with the plate. This so called spray reloading occurs during the first phase of the deflection. The energy of this reloading stems from the shockwave.

When the plate has lost its velocity (first maximum of the deflection curve in Fig. 18) that portion of water which moved with the plate has also come to rest. But the water farther away from the plate is still moving and is pushed ahead by the expanding bubble. These two masses of water are separated by gaps, namely the cavitation bubbles. These are quickly closed and the moving mass impinges on the resting one. This produces a closure shock and the second reloading. Its energy stems from the bubble energy. By this means the bubble may transmit up to twice as much energy to the plate as the shockwave.

This type of reloading occurs only with thin plates. It is not expected to occur with more rigid structures, such as a submarine hull, because no or only weak cavitation [10, 20] will occur there, and cavitation is necessary to liberate the bubble energy for the reloading.

The Response of Cylindrical Shells to a Shockwave

The response of an elastic cylindrical shell to a pressure wave has been treated by Mindlin, Bleich [21] and others, Fig. 19. Here the acoustic approximation and the neglect of bubble effects is quite realistic. Hydrodynamically seen this is a straight

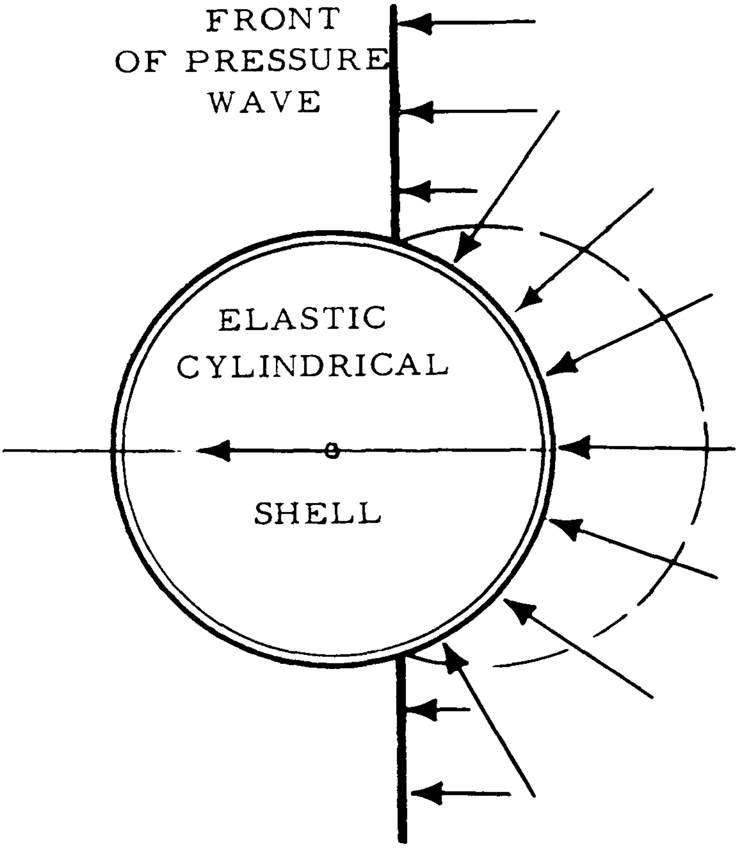

Figure 19. The response of a cylindrical shell to a pressure wave. The arrows indicate velocities. On the surface of the shell, the velocity of the plating must be equal to that of the water particles. The dashed curve indicates the wave reflected from the shell. The translation of the cylinder originates another pressure wave which is not shown in this sketch.

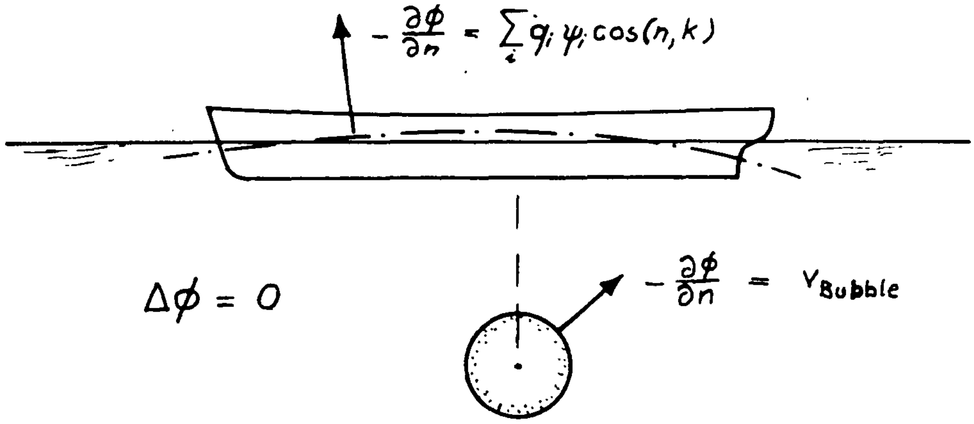

Figure 20. The whipping of a ship.

forward acoustic problem. The boundary conditions for the wave equation must be satisfied at the surface of the shell taking into account the translation, dilatation and the higher order flexural vibrations of the shell. Although the hydrodynamic concept seems simple, the analysis as well as the questions regarding the behavior of the material and the failure are not simple by any means and offer many challenging problems.

The Whipping of Ships

When an underwater explosion takes place under the keel of a ship, violent and dangerous transverse vibrations of the ship as a whole are excited, as is indicated by the bent line in Fig. 20. The girder strength of many ships is not great enough to withstand such a loading and often ships have been broken in two by such an explosive attack.

Chertock [22] has published an analysis of this phenomena. He assumed incompressible fluid motion, namely that the velocity potential has to satisfy the Laplace-Equation and the boundary conditions on the surface of the gas bubble and on the elastic beam which represents the ship.

The assumption of incompressible fluid motion seems at first glance very crude, since the shockwave and the bubble pulses which are compression waves are certainly important factors in producing these transverse vibrations. But such a variable velocity potential produces pressures in an incompressible fluid similar to those of the shockwave and bubble pulses. Of course the pressure-distance relation is somewhat different; one difference being that there is no energy dissipation in an incompressible fluid such as occurs at the shockfront of high amplitude waves. However, all that must be done to make this analysis quite realistic, is to adjust the velocity potential in such a way that it produces the same pressures near the target which the shockwave and the bubble pulses would have produced at this place.

Then the only discrepancy is the time lag which a compression wave has due to its finite propagation velocity, whereas in an incompressible medium the "wave" arrives at all points at the same time. Therefore, it becomes plausible that this seemingly crude approximation is not bad, as long as the period of the oscillations of the beam is large in comparison with the transit time of the wave. In most practical cases this condition is satisfied.

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DISCUSSION

G. Chertock

I think that Dr. Snay has given us a very interesting survey of the hydrodynamics of underwater explosions. I am particularly impressed by how well he managed to avoid all the confidential and classified material. That is quite a difficult job.

I noticed a couple of points that were new to me, but were very interesting. One was the reason why the low shock wave pressures do not match the same curves as the high shock wave pressures.

This is something that we have known for some time, but I was interested in his suggestion that this may be an instrument effect, and I think we will look into it ourselves.

I was also interested in the photograph that we saw of the bubble welling up and not forming plumes—it just seemed to come up as a lazy ball, with water spilling around it. As Dr. Snay remarked, this is probably due to a bubble rising without pulsating. This is important to our work, because the pressures that are due to such a bubble are largely due to the pulsations. A nonpulsating bubble would have a very different pressure field, and a very different damage potential.

At the Model Basin most of our work has been in the interaction of the pressure wave with structures. The most difficult problems have not been the hydrodynamics but rather techniques for handling the structural reactions.

For example, Dr. Snay mentioned three cases of structural response. One of them is the deformation of a diaphragm. We don't know how to treat the mechanics of the diaphragm too well. The diaphragm material is expanding under some yield stress which is a function of the strain rates, and yielding is often delayed because of the delay time we know so little about. And the whole problem is nonlinear. In fact, there are so many parameters that you are often free to pick and choose, and make it fit any theory you want.

The second thing he discussed was the reaction of the infinite cylinder to a shock wave. You may have noticed the fact that the shock wave he took was a plane shock wave. In practice, of course, it almost always results from the point charge and we have a spherically divergent wave. We have difficulty treating that problem with any precision.

The third problem he mentioned is the whipping problem. There we made quite a few measurements on highly idealized models and we have had very good success in comparing the measurements with theoretical predictions, but it was models where we have knowledge of the structural characteristics of the model.

S. G. Reed

I hope it will not be an undue distortion to expand a bit on one or two points out of the wealth of interesting material presented by Dr. Snay.

Dr. Snay mentioned briefly the Bjerknes fields when talking about the attraction or repulsion of the explosion bubble near surfaces. The early discussion of Bjerknes is very suggestive . . . at least to one interested in the physics of the phenomena. I would like to give an example.

There is an analogy to electrostatics which has been used by Dr. Kennard and others in connection with the attraction or repulsion: the pulsating bubble far from surfaces can be replaced in first order by a "charge" Q proportional to the rate of change of its volume:

$$Q = \frac{1}{4\pi}\frac{dv}{dt}$$

The analogy is that the forces between pulsating volumes are proportional, to first order, to the product of their respective charges, but with sign reversed to the electrostatic case. Thus in a free surface the induced charge is of opposite sign, so that there is repulsion; and in a rigid surface the induced charge is of the same sign, and there is attraction.

The Bjerknes' developed the analogy and brought in the notion of polarization, and this suggests a way of interpolating between free and rigid boundaries . . . within the realm of inviscid, incompressible hydrodynamics. Recently Birkhoff has used the idea of polarization and stressed many other analogies in an article in the von Mises memorial volume. The interpolation can be made in analogy to the behaviour of a homogeneous dielectric. What corresponds to the dielectric constant ε is the inverse of the density: ρ^{-1}. As an example, consider the model of an explosion bubble as a charge Q in water above a "bottom" taken to be a fluid of density ρ. The fluid boundary is assumed to remain plane. The square of velocity is neglected, otherwise there is no straightforward electrostatic analogy. The boundary conditions are continuity of velocity, and pressure continuity. These can be met by using images just as in the electrostatic problem of a point charge and a plane dielectric: in the water the potential is (r from bubble; r' from image)

$$\varphi_1 = Q/r + \left(\frac{\rho - 1}{\rho + 1}\right) Q/r'$$

and in the bottom fluid

$$\varphi_2 = \frac{2Q}{\rho + 1} \cdot \frac{1}{r}$$

In the appropriate limits this agrees with the well known cases. This may be useful to improve calculation of bottom effects on explosion bubbles well away from the bottom.

One can make a somewhat more realistic model of the bottom, imagining it to be a mixture of two fluids of mean density ρ, and then proceed in analogy with the microscopic physics of dielectrics.

The other remark has to do with the behaviour of a rising bubble. Calculations on Herring's theory give consistently larger rates of rise than are observed. Herring's theory constrains the bubble to remain spherical. Work by Kolodner and Keller, in which deformations of the bubble are taken into account, improves the agreement with experiment. The trend is evident from a simple consideration: that the bubble flattens owing to underpressure at its sides, once moving, and as it flattens the virtual mass increases. For a torus, for example, the virtual mass is roughly twice that of a sphere of equal volume. One would then expect the bubble to slow down appreciably compared with a sphere.

G. E. Hudson

Dr. Snay has commented on the studies of water columns arising from shallow explosions, and he showed us a picture of the Bikini test Baker water column as an illustration. Now this column was vertical, or nearly so, and I should like to point out that this is by no means always the case. In fact Dr. E. Swift and Mr. George Young of the Naval Ordnance Laboratory made a series of experiments in which the ambient atmospheric pressure was varied above a shallow layer of water in which a small charge was detonated at mid-depth. I have spent some time in analyzing the data from these experiments.

The most striking effect noticeable in high speed motion pictures of the resulting water columns is the variation in slope of the column generators as the pressure changes. The accompanying Figures I A, B, C, D show sequences of frames of the motion pictures which illustrate this. The depth d of the water layer was also varied, being 0.39 inches in Figure I A, 0.78 inches in Figure I B, 1.17 inches in Figure I C and 1.56 inches in Figure I D.

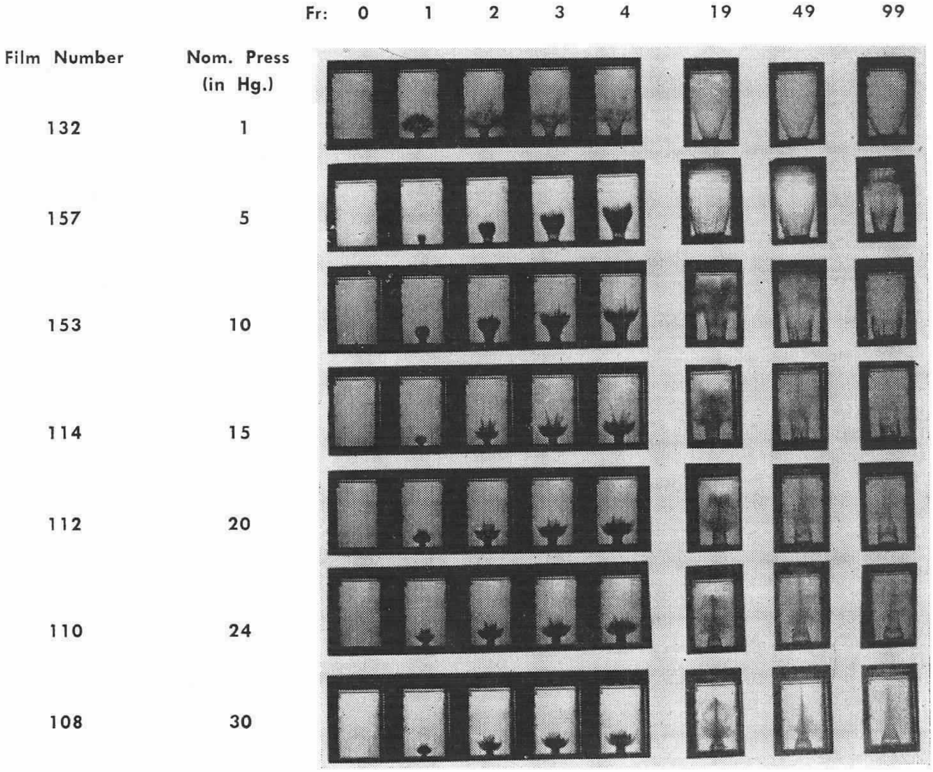

Figure 1A. Water Depth: 0.39 inches.

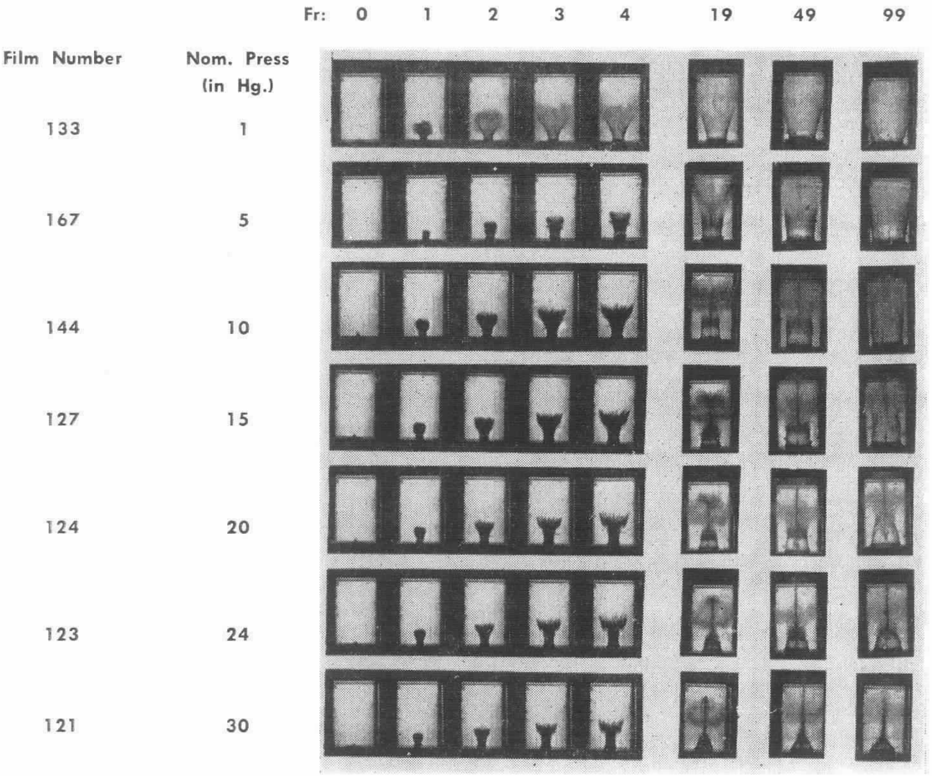

Figure 1B. Water Depth: 0.78 inches.

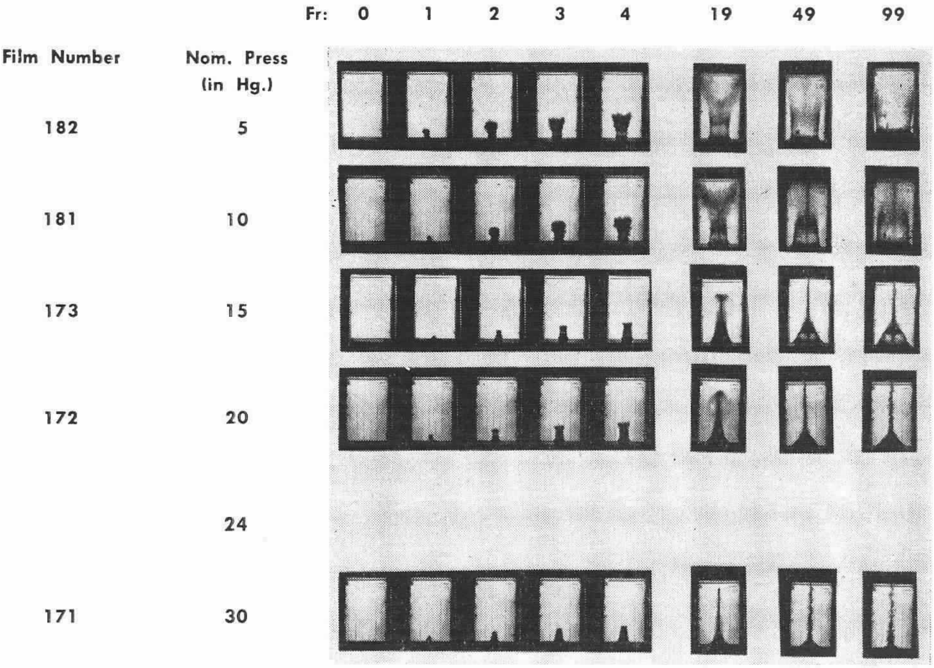

Figure 1C. Water Depth: 1.17 inches.

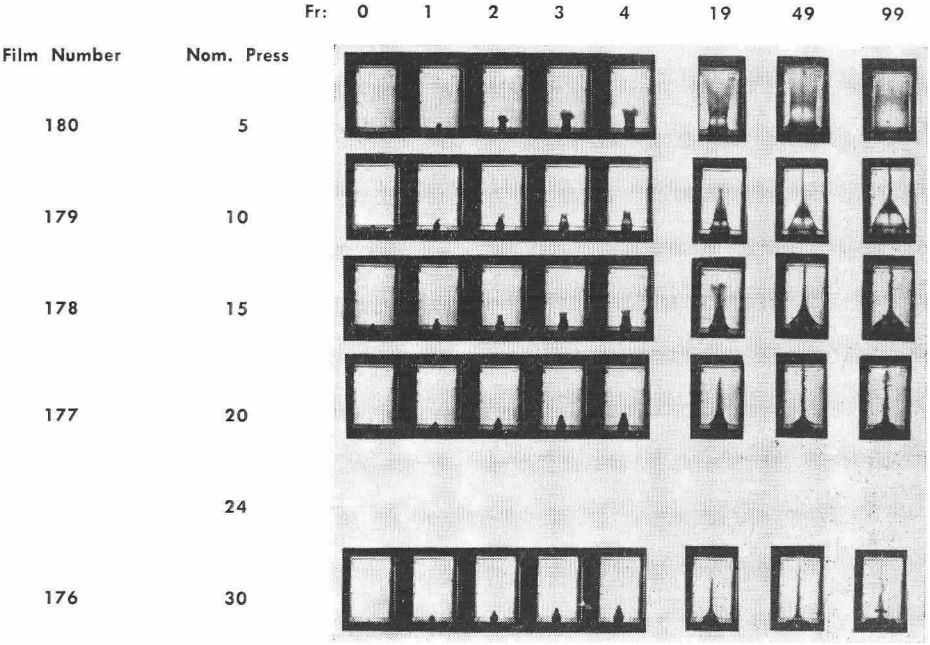

Figure 1D. Water Depth: 1.56 inches.

The generators of the water columns tend to become straight in many cases with time; their ultimate inclinations α from the vertical turn out to be a linear function of the pressure. In fact we may write this empirical relation in the form

$$\alpha = mp - \alpha_0 \tag{1}$$

where α_0 is nearly independent of depth and pressure and m is a function of depth only. For these experiments, we had

$$\alpha_0 = 20^\circ \qquad \text{(average)}$$

and

d (inches)	m (degrees/in. Hg)
0.39	1.51
0.78	1.24
1.17	3.11
1.56	4.60

Another empirical rule which seems to hold roughly over a large part of the column motion is that the ratio of the diameters w_1 and w_2 of a column at two different heights, z_1 and z_2, above the initial water surface is a constant in time. Hence the motion is in a kind of normal mode, at least for large enough times. For a wide variety of cases the profile motion may be expressed by

$$w(z,t) = w_0 \ln \frac{t}{t_0}, \tag{2}$$

in which w_0 is a slowly varying function of z, and in fact may often be treated as constant. Not much is known empirically concerning t_0 however. Since

$$\frac{\partial w_0}{\partial z} \approx 0, \tag{3}$$

we can infer that the angular slope of a column generator is approximately

$$\alpha = -\tan^{-1}\frac{\partial w}{\partial z} = \tan^{-1} w_o \frac{\partial \ln t_0}{\partial z}. \tag{4}$$

In this way we may relate $w_0 \ln t_0$ to z, $m(d)$, and p.