A METHOD FOR A MORE PRECISE COMPUTATION OF HEAVING AND PITCHING MOTIONS BOTH IN SMOOTH WATER AND IN WAVES

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BACKGROUND

In the Symposium "On the Behaviour of Ships in a Seaway," in Wageningen, 1957, a paper "Durch Wellen an einem Schiffskörper erregte Kräfte" was read by the author, which will now be continued. In part II of that paper ideas were deduced which may be considered an improvement of the well-known strip method. At that time some simple numerical results were given on the basis of this improved strip method although the method was not entirely completed.

In the meantime, some progress has been made and, in addition, numerical work could be done to a much greater extent since an electronic computor is now available. This paper gives the results obtained starting from a complete summary of the method which is deemed useful since the representation given in 1957 is no longer considered satisfactory.

The methods which have been applied to the theoretical treatment of the problems of the ship waves may be crudely subdivided into two groups, viz., (a) methods of singularities and (b) strip methods. The methods mentioned under (a) have as a basis a representation of the ship body by periodical singularities [3,4,5]. They enable us in an elegant manner to consider certain important parameters within an expression for the velocity potential and then to examine the influence of these parameters. The weak point of these methods consists in the loose connection between the motion and the shape of the ship on the one hand and the distribution of the singularities on the other hand. As a consequence the condition on the surface of the body is hardly sufficiently satisfied. Therefore, these methods can only be expected to give qualitative information as to the influence of the main parameters and not quantitative results as to the influence of the shape of the ship.

The methods mentioned under (b), viz., strip methods [6], assume solutions to be known for corresponding problems on two-dimensional bodies of which the sections coincide with the sections of the ship body. The results for the three-dimensional body will then be obtained by adding those obtained from the two-dimensional bodies. It must be considered an advantage of these methods that one can start from relatively accurate results and that the influence of the shape of the sections can be allowed for. The well-known strip method, however, has several disadvantages, e.g., that the influence of the three-dimensional flow can scarcely be taken into account and that the influence of the speed of the ship cannot be established exactly.

In the method proposed by the author the advantages of the two methods shall be combined and their disadvantages shall, if possible, be avoided. A strip method will be applied for the distribution of the singularities; i.e., for each section of the ship the singularity will first be chosen to satisfy the condition on the surface of a two-dimensional body of the section in question. The distribution of singularities so obtained is not yet accurate, however, even if an accurate representation of the two-dimensional cases is taken as a basis, because the representation of the three-dimensional ship body requires a somewhat different distribution of singularities. An improved distribution will be obtained with the help of an integral equation.

The method will be described here for the speed $V = 0$ only. For $V \neq 0$ it is necessary to complete the formulae for the flow potential and, in addition, to allow for the speed in the boundary condition. These extensions have been worked out; since, however, numerical results are not yet available, this extension of the method will not be discussed here.

The following problems will be treated: the heaving motion, the pitching motion, and the forces generated in a vertical direction by waves. An ideal fluid free from friction will be assumed and, further, the problem will be linearized. It should be mentioned that the boundary condition at the body cannot be satisfied exactly. These approximations as well as additional simplications will be discussed in the course of the paper.

I. TWO-DIMENSIONAL PROBLEMS

Introduction

Since the treatment of the complete three-dimensional problem requires solutions of two-dimensional problems to be known, it is necessary to briefly discuss the latter. The method used is the same, in principle, as deduced by the author in 1953 [10] on the basis of which later published results were computed [12]. As now an electronic computor is available it was possible to carry out the computations both more precisely and for a sufficient large number of transverse section contours.

Computations have been made for three problems, viz.,

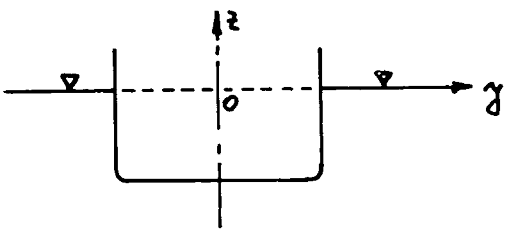
(a) for the periodical heaving motion of the body in smooth water,

(b) for the vertical force generated on the restrained body by a transverse surface wave, and

(c) for the hypothetical case of a two-dimensional body in a "longitudinal wave" which cannot be realized physically. For this case which is important relative to the method described under II results have been given by Abels [9].

Description of the Method

The system of coordinates is fixed in space. Its origin lies in the plane of the waterline when at rest and in the midst of the contour, the y-axis being horizontal, the z-axis being vertical. The computations have been carried out for transverse profiles which can be represented by the transformation formula of Lewis. On the basis of this formula the coordinates of the profile can be expressed by means of the parameters a and b and of the coordinate θ running from 0 to $-\pi$.



The formula for the boundary of the contour is expressed in nondimensional form as follows:

$$(y+iz) = e^{i\theta} + a e^{-i\theta} + b e^{-i3\theta}. \tag{1}$$

The following formulae for the potential and for the stream function caused by a heaving motion are applied:

$$\Phi = U \left[A_o \lim_{\mu\to 0} \int_0^\infty \frac{e^{Kz} \cos(Ky)}{K - \nu + i\mu} dK + \sum_{n=1}^{\infty} A_n \int_0^\infty K^{2(n-1)} (K+\nu) e^{Kz} \cos(Ky) dK \right]$$

$$\psi = U \left[A_o \lim_{\mu\to 0} \int_0^\infty \frac{e^{Kz} \sin(Ky)}{K - \nu + i\mu} dK + \sum_{n=1}^{\infty} A_n \int_0^\infty K^{2(n-1)} (K+\nu) e^{Kz} \sin(Ky) dK \right] \tag{2}$$

The time factor $e^{i\omega t}$ has been omitted. U describes the amplitude of the oscillatory velocity of the body. Both the condition of continuity and the condition on the free surface are satisfied. The problem is now to define the coefficients a such that the condition on the boundary of the contour is also satisfied. This requires a sufficient number of terms in the rows to be considered.

The condition on the boundary of the contour for the heaving motion is

$$\psi = Uy. \tag{3}$$

In case (b) – the restrained body in transverse waves – a surface wave with the orbital velocity 1 is assumed so that

$$\Phi_w = \frac{e^{\nu z}}{\nu} \cos (Ky)$$
$$\psi_w = \frac{e^{\nu z}}{\nu} \sin (\nu y). \tag{4}$$

The whole potential or the stream function, respectively, consists of the potential of the wave (4) and the potential (2) by which the deformation of the wave caused by the body is described. In Eq. (2) the factor U will then be omitted. For instance:

$$\psi = \frac{e^{\nu z}}{\nu} \sin (\nu y) + A_o \lim_{\mu \to 0} \int_0^\infty \frac{e^{Kz} \sin (Ky)}{K - \nu + i\mu} dK$$
$$+ \sum_{n=1}^{\infty} A_n \int_0^\infty K^{2(n-1)} (K + \nu) e^{Kz} \sin (Ky) dK. \tag{5}$$

Of course, the coefficients A have different values in this case as for the heaving motion.

In case (b) the condition at the boundary of the contour is

$$\psi = 0. \tag{6}$$

In the hypothetical case (c) the basis is a representation of a transverse section of the three-dimensional ship in a longitudinal wave. The potential of the nondeformed wave with the orbital velocity 1 is

$$\Phi_w = \frac{e^{\nu z}}{\nu} \cos (\nu x). \tag{7}$$

From this it follows that the velocity of the water particles in a vertical direction amounts to

$$e^{\nu z} \cos (\nu x) \tag{8}$$

and the hydrodynamic pressure to

$$-i\rho\omega \frac{e^{\nu z}}{\nu} \cos (\nu x). \tag{9}$$

The value of cos (νx) may be understood as the phase shift for the following and may be omitted in Eqs. (8) and (9).

The following question may be asked: Which two-dimensional potential $\Phi(y, z)$ describes such a velocity on the contour of the section of a two-dimensional body that the boundary condition will be satisfied by this velocity together with Eq. (8)? This potential, for which Eq. (2) will be applied putting $U = 1$, can be defined as a transverse deformation of the longitudinal wave. At the boundary of the contour the following condition will be used:

$$d\psi = e^{\nu z} \, dy. \tag{10}$$

The boundary conditions (3), (6), and (10) cannot be satisfied exactly (except for $\omega = 0$ or $\omega = \infty$). It has been proved that the following procedure is well converging.

Into the boundary conditions (3), (6), or (10), Eqs. (2), (5), or (2), respectively, will be introduced. Within these equations the coordinates of the boundary of the contour can be replaced by Eq. (1). Then only one coordinate appears, viz., θ. The equations are written as follows:

$$B_o \left(\frac{\pi}{2} + \theta\right) + \sum_{n=1}^{\infty} B_n \sin (2n\,\theta) = 0, \qquad \text{for} \quad -\pi \le \theta \le 0. \tag{11}$$

Of course, the coefficients A are linearly included in the coefficients B.

The boundary condition is satisfied if all coefficients B vanish in Eq. (11). In Eqs. (2) or (5) the series are cut off after N terms so that N unknown coefficients A are included. Then Eq. (11) is also cut off after N terms and N linearized equations are obtained:

$$B_o = 0; \qquad B_n = 0. \tag{12}$$

By Eq. (12) $2N$ equations are represented since the coefficients A or B, respectively, are complex numbers. Solving these equations the potential is found. The boundary condition is nearly satisfied. An error remains which changes sign on the contour several times and this more frequently the larger the N that is chosen.

Having computed the unknown coefficients A the problem is now to determine the hydrodynamic force. To obtain the force in a vertical direction the following integration around the contour is required:

$$\int \Phi \, dy\,. \tag{13}$$

In case (a) this integration yields only the hydrodynamic force. It is possible to add both the hydrostatic and the inertial force of the body and then to deal with the total force which is responsible for the heaving motion.

In case (b) only a hydrodynamic force exists. Therefore the integration yields immediately the force in a vertical direction which is caused from the wave on the restrained body.

In case (c) the force which follows from the pressure in the undisturbed wave is added to the force from (13). This is necessary since (13) yields only the force which arises from the deformation of the wave.

Representation of the Results: Figs. 1 to 24

More convenient than the parameters a and b of the transformation formula are the parameters H and β for the designation of the section contours:

$H = B/2T$ denotes the ratio of the half breadth of the profile in the waterline to the depth of the profile

β = (area of the section)$/BT$ denotes the fullness of the section profile.

Together with the transformation formula (1) the two parameters suffice to exactly define the sections.

The following results of the computations are represented in Figs. 1-24:

For the heaving motion: R, $A_o\pi/B$, C, and $\bar{A}$:

$R = R_r + iR_i$ represents the total force required to produce the heaving motion (real and imaginary part). R is made nondimensional by the amplitude of motion and by δB.

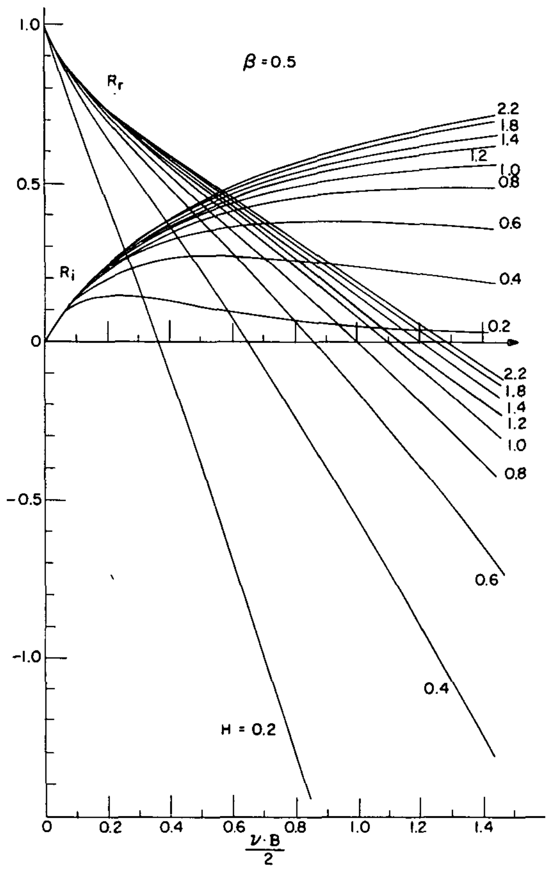


Fig. 1. Heaving motion; plot of force R for $\beta = 0.5$

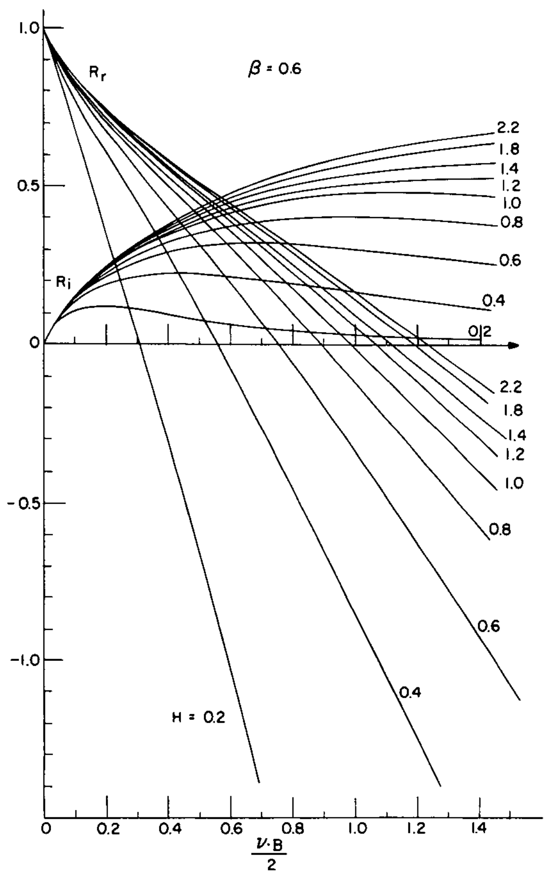


Fig. 2. Heaving motion; plot of force R for $\beta = 0.6$

Therefore, in the statical case, i.e., $\omega = 0$, R equals 1. For a periodical heaving motion, viz.,

$$z_o = \overline{Z}_o \, e^{i\omega t} \tag{14}$$

the force which is required to generate the heaving motion is

$$R \, \delta \, B \, \overline{Z}_o \, e^{i\omega t}. \tag{15}$$

This force is identical with

$$(m + m'') \, (\ddot{z}_o) + (N) \, \dot{z}_o + Bz_o. \tag{16}$$

This form is known to be the one side of the equation of motion. It contains: the hydrodynamic force $m'' \ddot{z}_o + N\dot{z}_o$, the hydrostatic force $\delta B z_o$, and the inertial force $m\ddot{z}_o$.

$A_o \pi / B$ is the coefficient of the first term in (2) in nondimensional representation

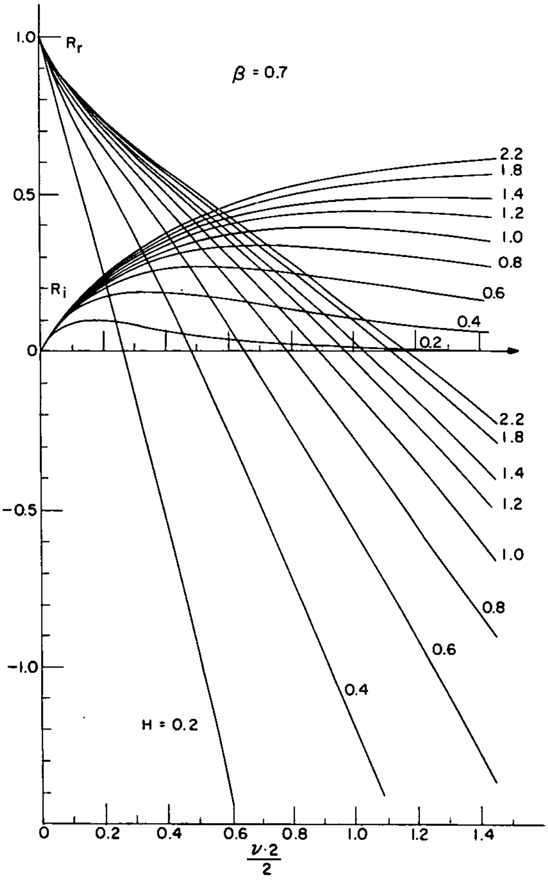


Fig. 3. Heaving motion; plot of force R for $\beta = 0.7$

C represents the hydrodynamic mass in nondimensional form, i.e.,

$$C = \frac{m''}{\rho \frac{\pi}{8} B^2} . \tag{17}$$

$\bar{A}$ represents the ratio of the amplitudes, i.e., the ratio of the amplitude of the surface waves drifting away from the body to the amplitude of the heaving motion. Between $\bar{A}$ and R_i the relation

$$R_i = \frac{g}{\omega^2 B} \overline{A^2} \tag{18}$$

holds because the work done by the force in unit time equals the energy dissipated by the waves.

For cases (b) and (c), viz, the restrained body in transverse or longitudinal waves, respectively, the following results are represented:

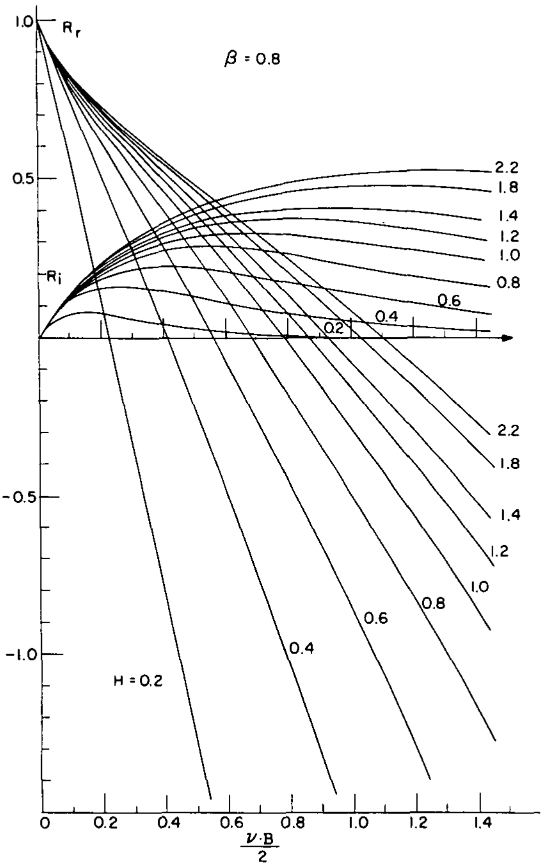


Fig. 4. Heaving motion; plot of force R for $\beta = 0.8$

$E = E_r + E_i$ which is the force in the vertical direction generated on the body by a surface wave of amplitude 1. This force is made nondimensional by δB. In the case of a very long wave ($\omega \to 0$) E necessarily equals 1. For this case the force can be looked at as the hydrostatic buoyancy which corresponds to the additionally displaced volume. This force has first of all a physical meaning only for the case (b).

For case (c) there is also plotted

$A_o\pi/B$ which is the coefficient of the first term in the formula for the potential in a nondimensional representation. (The diagrams for case (c) are not given here since they are published in Ref. 9.)

All results are plotted against a frequency parameter which includes the beam B, viz.,

$$\frac{\nu B}{2} = \frac{\omega^2 B}{2g} \,. \tag{19}$$

In cases (b) and (c) this frequency parameter can also be expressed by the wavelength λ:

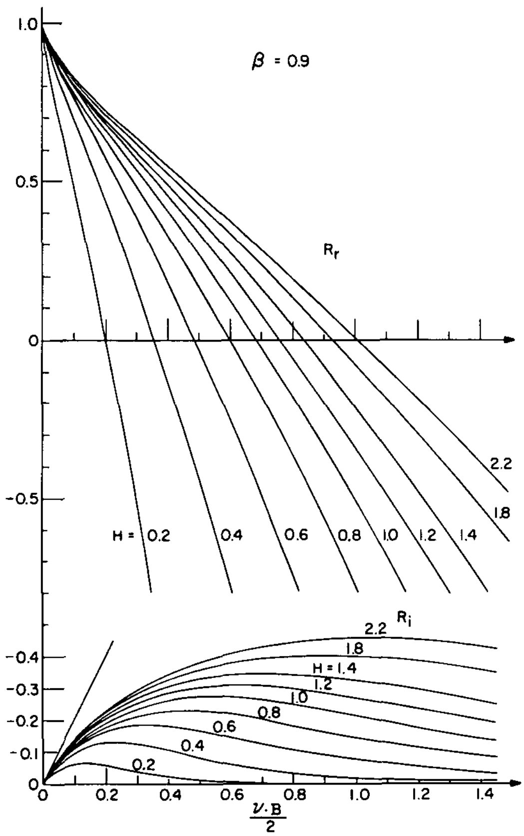


Fig. 5. Heaving motion; plot of force R for $\beta = 0.9$

$$\frac{\omega^2 B}{2g} = \pi \frac{B}{\lambda} . \tag{20}$$

With the help of these results the heaving motion of the free body generated by transverse surface waves may be determined. If the radius of the orbital circle of the surface wave is denoted $\bar{h}$ and the heaving motion produced by the surface wave z_o, the following equation must be satisfied:

$$z_o R = \bar{h} E . \tag{21}$$

From this the heaving motion of the free body can be computed both in magnitude and phase.

Convergence of the Method

To prove the convergence, first four terms in the series for the potential and then five terms were taken into account. The difference of the results is not significant. It, therefore, can be expected that both the convergence and the accuracy of the method are sufficient.

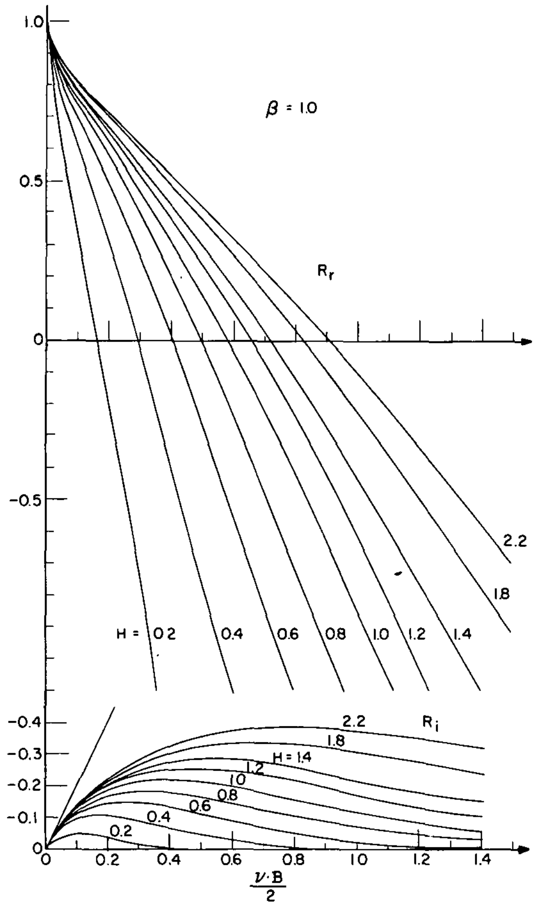


Fig. 6. Heaving motion; plot of force R for $\beta = 1.0$

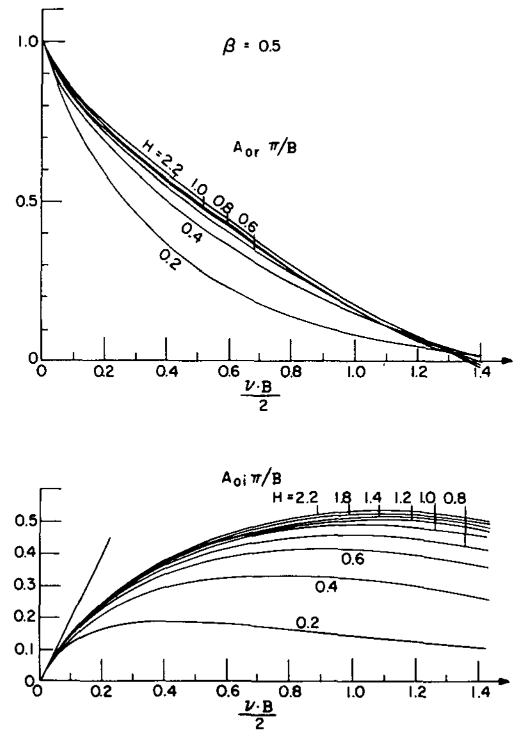


Fig. 7. Heaving motion; plot of singularity distribution $A_o\pi/B$ for $\beta = 0.5$

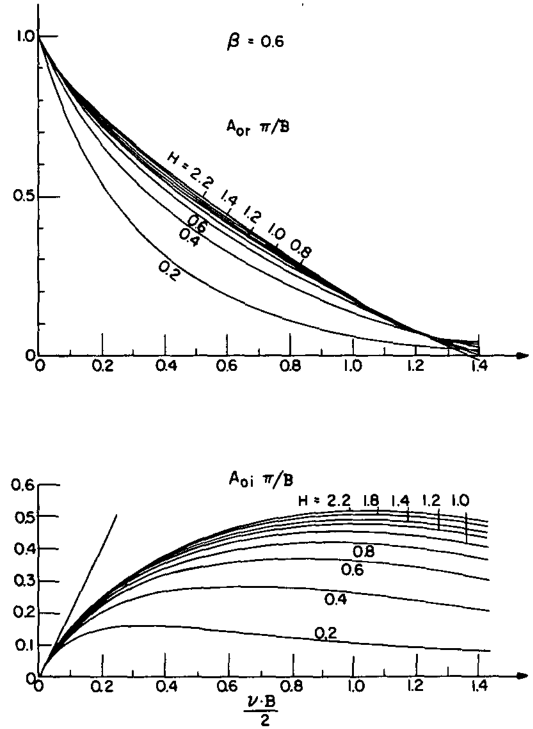


Fig. 8. Heaving motion; plot of singularity distribution $A_o\pi/B$ for $\beta = 0.6$

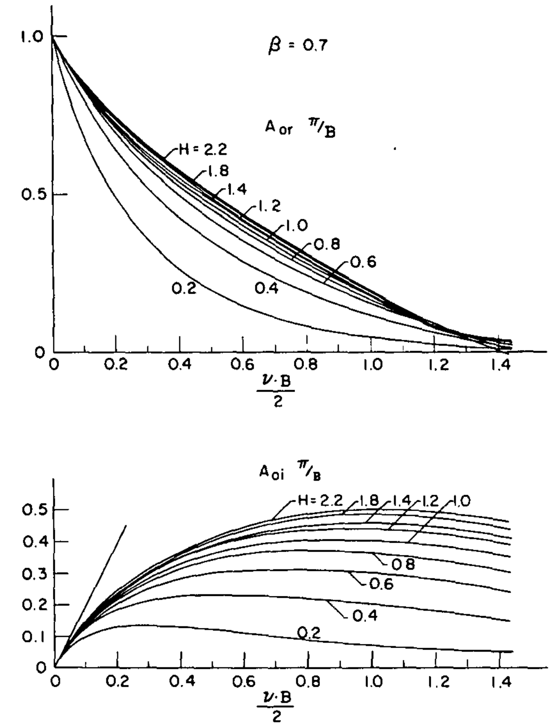


Fig. 9. Heaving motion; plot of singularity distribution $A_o\pi/B$ for $\beta = 0.7$

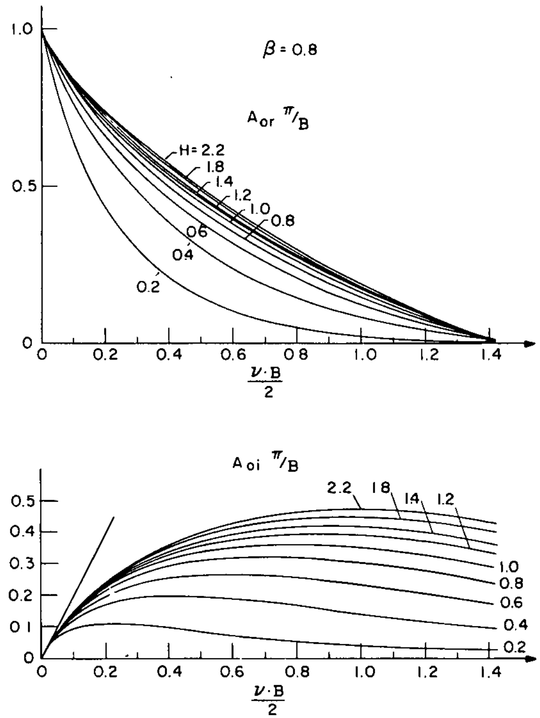


Fig. 10. Heaving motion; plot of singularity distribution $A_o\pi/B$ for $\beta = 0.8$

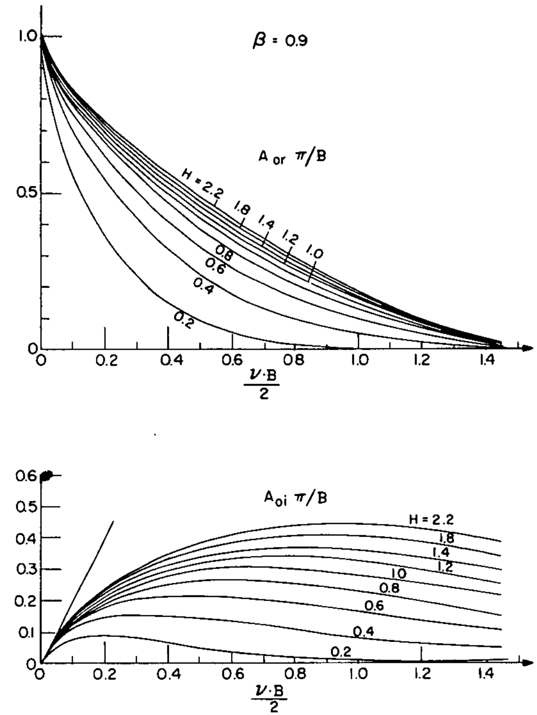


Fig. 11. Heaving motion; plot of singularity distribution $A_o\pi/B$ for $\beta = 0.9$

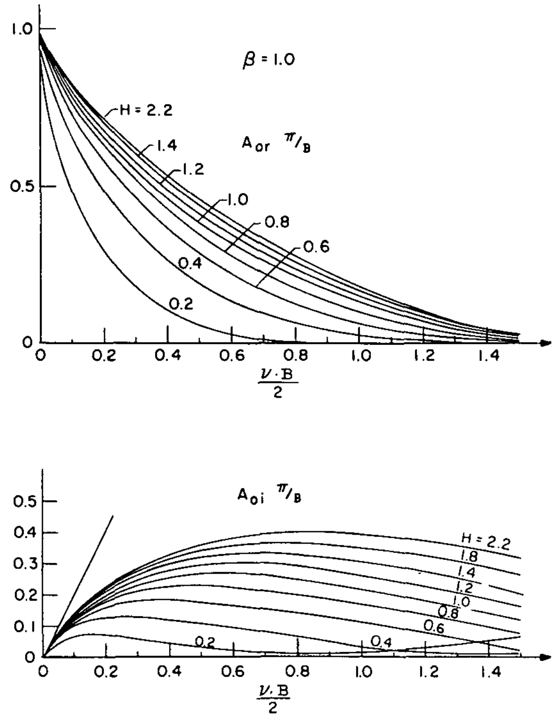


Fig. 12. Heaving motion; plot of singularity distribution $A_o\pi/B$ for $\beta = 1.0$

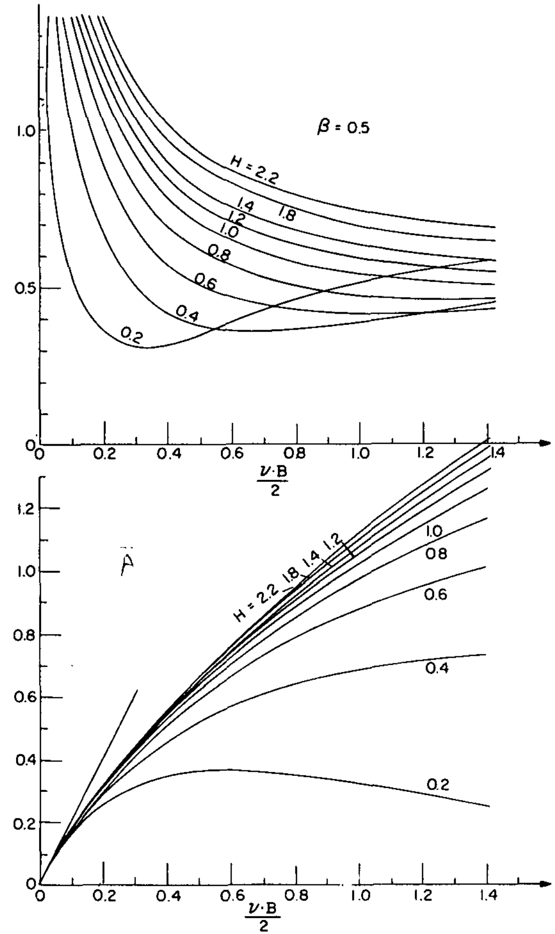


Fig. 13. Heaving motion; plots of C and of $\overline{A}$ for $\beta = 0.5$

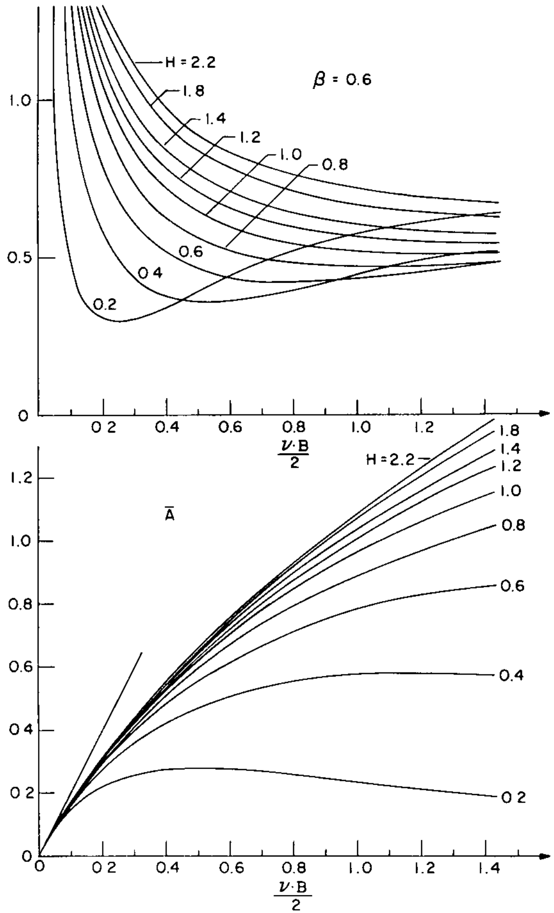


Fig. 14. Heaving motion; plots of C and of $\bar{A}$ for $\beta = 0.6$

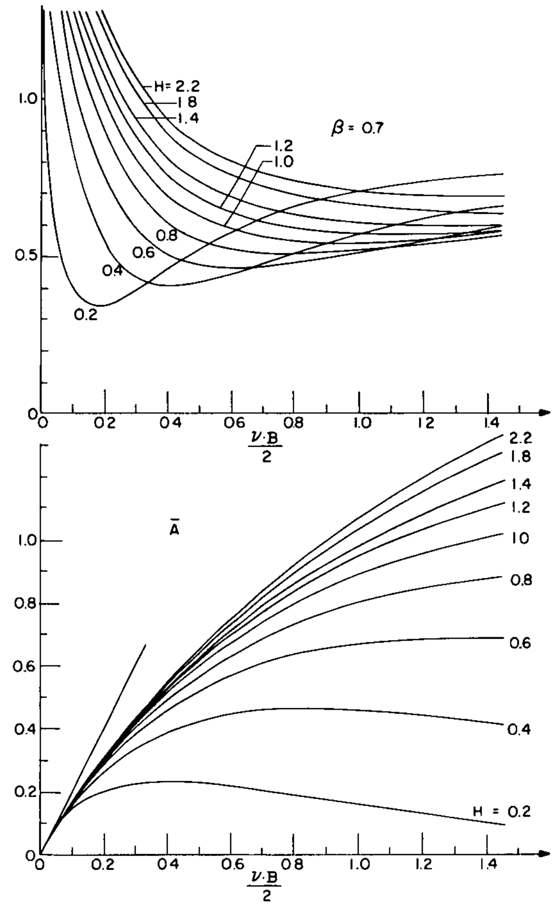


Fig. 15. Heaving motion; plots of C and of $\bar{A}$ for $\beta = 0.7$

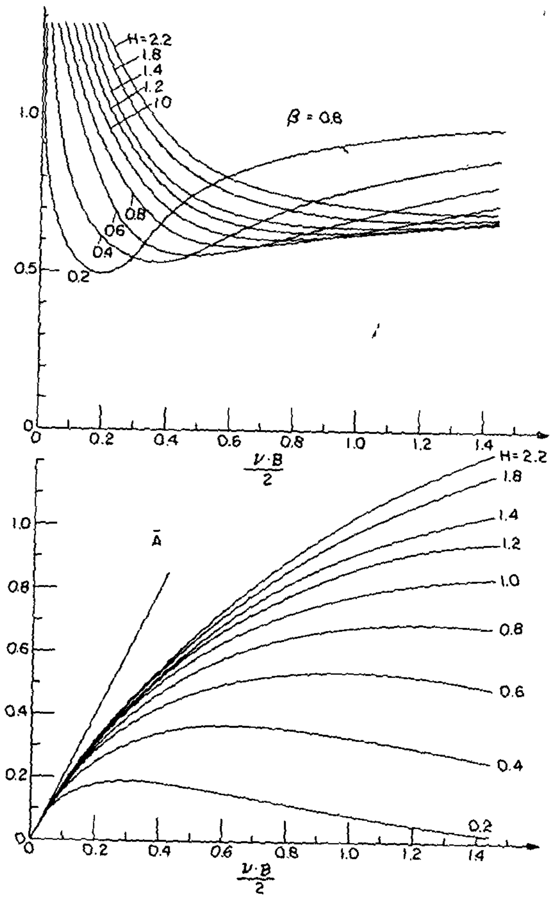


Fig. 16. Heaving motion; plots of C and of $\bar{A}$ for $\beta = 0.8$

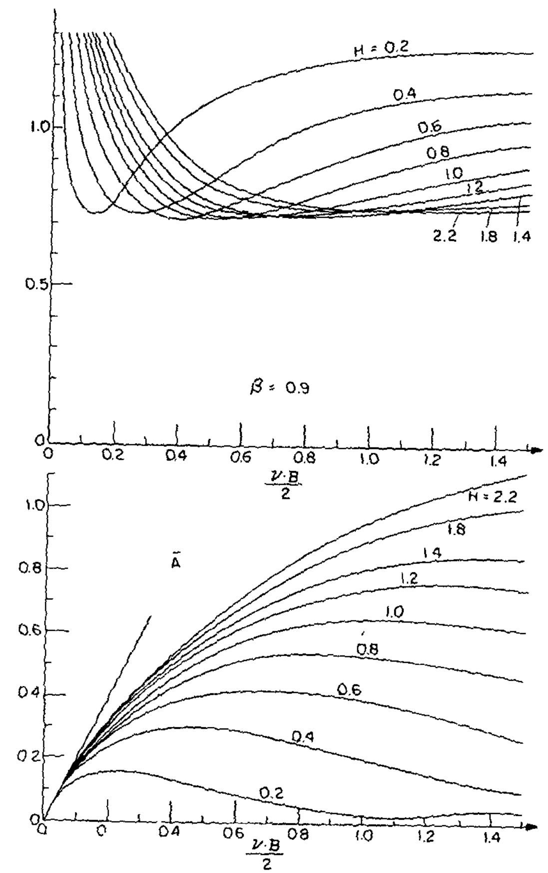


Fig. 17. Heaving motion; plots of C and of $\bar{A}$ for $\beta = 0.9$

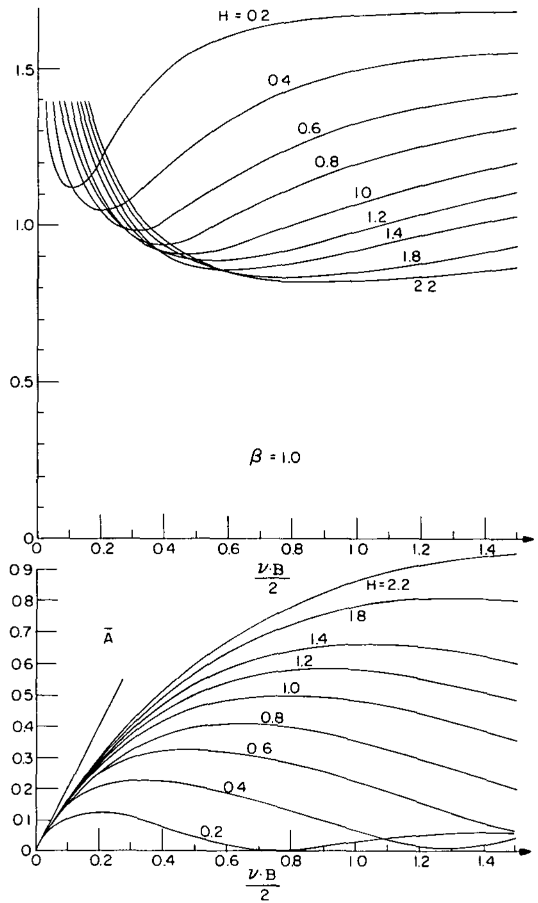


Fig. 18. Heaving motion; plots of C and of $\bar{A}$ for $\beta = 1.0$

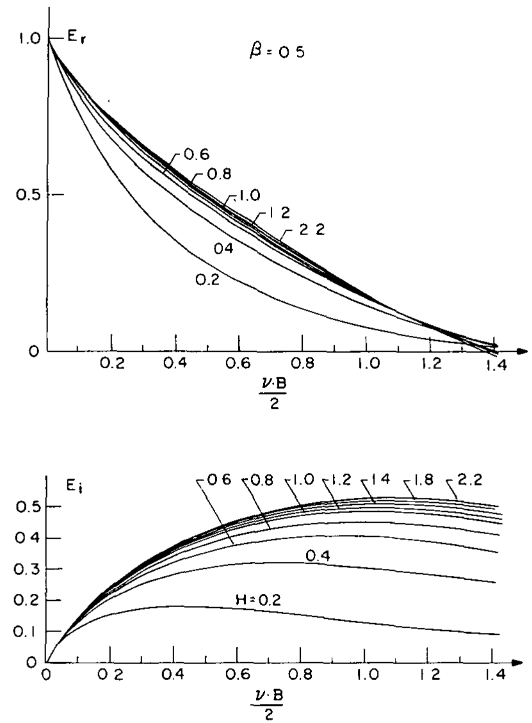


Fig. 19. Transverse wave; plot of force E for $\beta = 0.5$

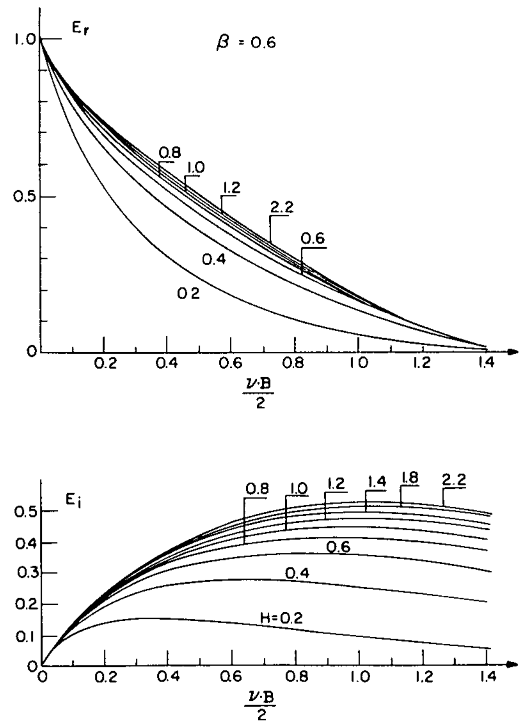


Fig. 20. Transverse wave; plot of force E for $\beta = 0.6$

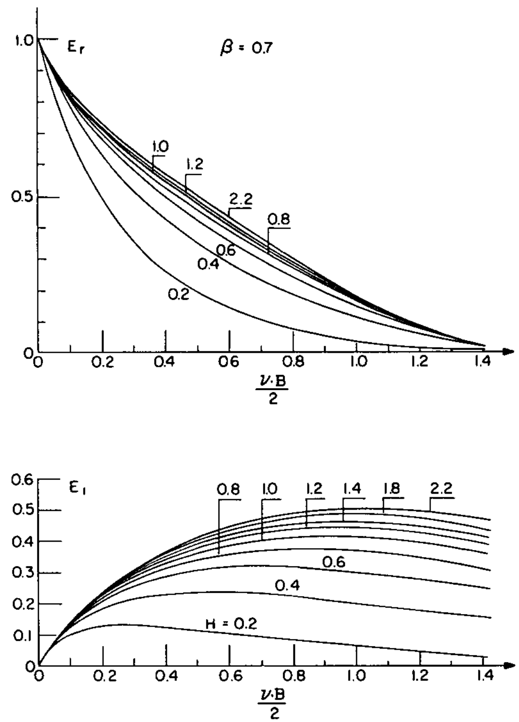


Fig. 21. Transverse wave; plot of force E for $\beta = 0.7$

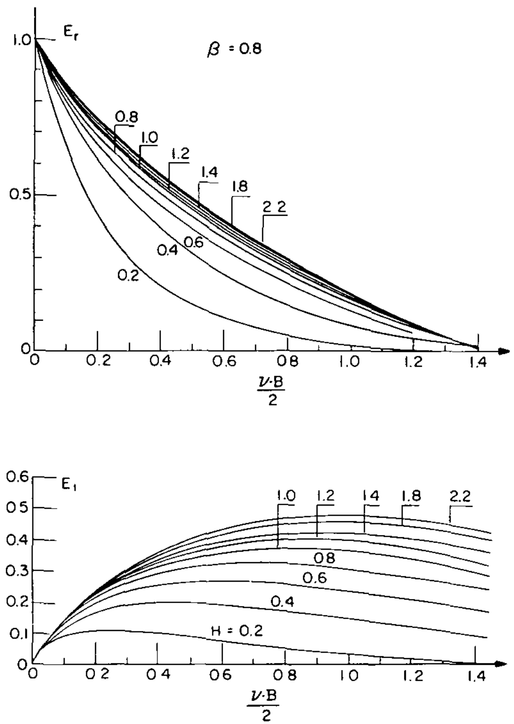


Fig. 22. Transverse wave; plot of force E for $\beta = 0.8$

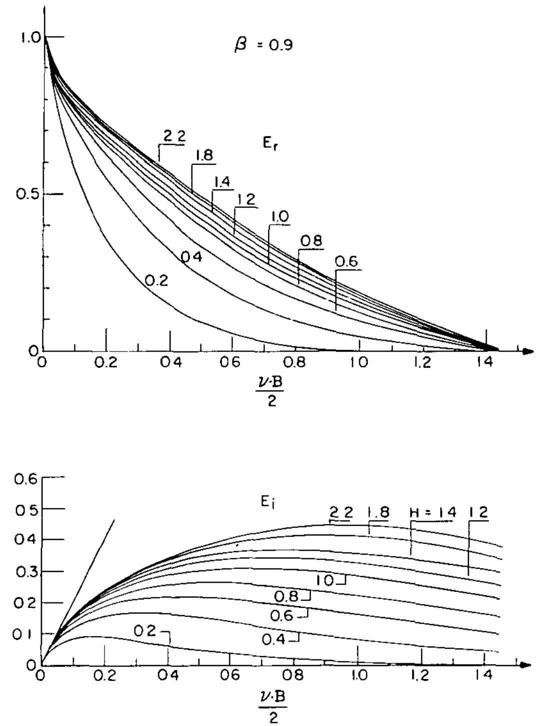


Fig. 23. Transverse wave; plot of force E for $\beta = 0.9$

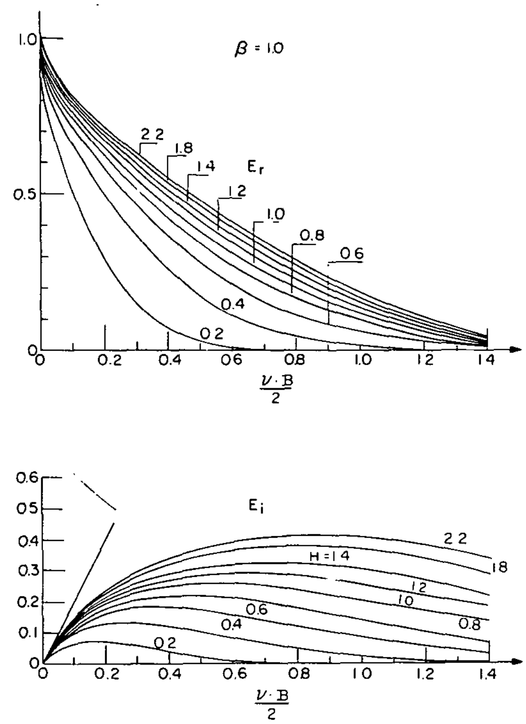


Fig. 24. Transverse wave; plot of force E for $\beta = 1.0$

Summarizing Remarks to Part I

It is certain that sufficiently exact solutions are obtained in the way here described. Recently, a Japanese study by Tasai [7] has been published in which numerous numerical results are presented which agree closely with those given in this paper.

It should be mentioned that the method may be applied in the whole range of frequency from $\omega = 0$ to $\omega = \infty$. It appears necessary, however, that the sections are amenable to conformal mapping. In this respect difficulties may arise for sections of which the contour is not perpendicular to the waterline at their intersection.

The first term in Eq. (2) for the potential represents the potential of a periodical source in $y = 0$, $z = 0$. The coefficient A_0, therefore, represents a measure for the strength of the source. It is now extremely remarkable to learn from Figs. 7-12 that the coefficient A_0 is complex and depends very strongly on both the shape of the section and the frequency. It is therefore a very crude and inaccurate approximation if only the shape of the waterline and the amplitude of the motion are considered for a method of singularities when choosing the singularities.

II. THREE-DIMENSIONAL PROBLEMS

Heaving and Pitching Motion in Smooth Water

The motion of the body is determined by the velocity $U_0(x)$ in a vertical direction. This velocity is constant and independent of x relative to the heaving motion. Relative to the pitching motion this velocity grows linearly when x increases. The time factor $e^{i\omega t}$ is omitted.

In Ref. 11 the formula

$$\Phi(x,y,z) = \frac{1}{2}\sum_{n=0}^{\infty}\int_L U_o(\xi)\, A_n(\xi)\, \varphi_n\left[(x-\xi),y,z\right]\, d\xi \tag{22}$$

is chosen for the flow potential. The functions $\varphi_n[(x-\xi),y,z]$ within the integrals are denoted partial potentials and are expressed as follows:

$$\begin{aligned}
&\varphi_o\left[(x-\xi),y,z\right]\\
&= \frac{2}{\pi}\int_0^{\infty}\cos\left[m(x-\xi)\right]\, dm\int_0^{\infty}\frac{e^{\sqrt{m^2+K^2}\,z}\cos(Ky)}{\sqrt{m^2+K^2}-\nu+i\mu}\, dK \qquad \text{for} \quad \mu\to 0\\
&\varphi_n\left[(x-\xi),y,z\right]\\
&= \frac{d^{2(n-1)}}{dz^{2(n-1)}}\left\{\frac{3z^2}{\left[(x-\xi)^2+y^2+z^2\right]^{5/2}}-\frac{1+\nu z}{\left[(x-\xi)^2+y^2+z^2\right]^{3/2}}\right\}.
\end{aligned} \tag{23}$$

This formula satisfies both the condition of continuity and the condition on the free water surface outside the ship body. When applied to the limit case of an infinitely long two-dimensional body this formula is identical with Eq. (2). The transition to the limit is performed as follows:

1. The distribution functions $A_n(x)$ become constants A_n which are placed before the integrals.

2. These coefficients A_n are chosen such that they are identical with those computed in part I.

3. The integrations are carried out from $-\infty$ to $+\infty$.

The formula furnishes a nearly exact solution in this limit case. Relative to a ship of finite length the distribution functions $A_n(x)$ are chosen identical at each section x with those distributions $A_n(x)$ which hold for the corresponding motion of the two-dimensional body. In Eq. (22) there are, therefore, no unknowns. It may also be expected that the condition on the surface of the body is satisfied to a sufficient degree as long as the ship body is sufficiently slender, i.e., as long as L/B and L/T are great and the angle between the tangent on the waterlines and the plane of symmetry is small.

Relative to the methods of singularities an improvement is obtained because the relation between the singularities, the frequency and the contour parameters is satisfied to a higher degree.

To better recognize the improvement, Eq. (22) is transformed into the identical formula

$$\Phi(x,y,z) = \frac{1}{2}\sum_{n=0}^{\infty} U_o(x)\, A_n(x) \int_{-\infty}^{+\infty} \varphi_n\left[(x-\xi),y,z\right]\, d\xi$$

$$+ \int_{-\infty}^{+\infty} \left\{\left[U_o(\xi)\, A_n(\xi) - U_o(x)\, A_n(x)\right]\; \varphi_n\left[(x-\xi),y,z\right]\; d\xi\right\} . \tag{24}$$

The first term represents the nearly two-dimensional flow around the section x because the integrals are independent of x. Only in consequence of the functions $A_n(x)$ a velocity component in x-direction arises. By this velocity component, however, the boundary condition is not considerably disturbed and it is nearly satisfied if the distribution functions are correspondingly chosen. Only the first term is taken into account for the common strip method. If the hydrodynamic pressure and the force in a vertical direction generated on a section are computed only from this term the same force as in part I is obtained. Applying the representation R used in part I one obtains

$$\frac{\delta}{\omega} \int_L U_o\, BR\; dx \tag{25}$$

for the total force to generate the motion with velocity U_o or for the total moment

$$\frac{\delta}{\omega} \int_L x\, U_o\, BR\; dx\ . \tag{26}$$

Here it is assumed for the computation of the moment that the weight of the body is distributed over the length in the same manner as the displacement. Of course, a different distribution of the weight can be taken into account.

The common strip method will be improved if the second term in Eq. (24) is also applied. This improvement is already included in Ref. 11. An additional improvement is discussed in the following considerations.

The second member in Eq. (24) may be explained physically as a flow generated at x by a distribution $[A_n(\xi) - A_n(x)]$ situated outside x. Since this distribution lies in $y = 0$, $z = 0$ and since the ship body is slender (i.e., the distribution does not change too rapidly with ξ, this additional flow at x will depend only a little on y inside the ship body, i.e., for

$|y| \leqq B/2$. Only a small error arises if in this second member of (24) $\varphi_n[(x-\xi),0,z]$ is written instead of $\varphi_n[(x-\xi),y,z]$. Since it is well-known that circular waves are generated by a periodical singularity of which the dependence on the coordinate z is described by the factor $e^{\nu z}$ (provided that the distance from the origin of the circular waves is not too small) it is suggested that the same dependence on the coordinate z holds for the second member of Eq. (24). For all singularities within this member have the same frequency and there are no singularities at x itself. It, therefore, may be permitted to further simplify the partial potentials in this second term, viz.:

$$\varphi_n\left[(x-\xi),y,z\right] \rightarrow e^{\nu z}\ \varphi_n\left[(x-\xi),0,0\right] \tag{27}$$

In this way the coordinates y and z are removed from the integrals. This term can then be described as a product of a function of x times the exponential function $e^{\nu z}$. This holds within the region of the ship body, i.e., for ordinates y which do not go far beyond $B/2$.

Since the condition on the surface of the body is well satisfied by the first member of Eq. (24) the second member causes a disturbance of this boundary condition. To reduce the error an additional velocity U as function of x will be introduced. U may be explained as the velocity of an additional deformation of the water surface (depending on x and z for $|y| \leqq B/2$) so that a certain section of the ship body has the velocity $(U_o + U)$ relative to the water surface. This deformation is called "additional" because a deformation (depending on y and z) takes place already in the two-dimensional case. The additional deformation diminishes when the depth increases following the function $e^{\nu z}$ because the influence of singularities with circular frequency ω lying far outside of x will be eliminated.

Therefore, the following formula instead of Eq. (22) will be chosen for the total potential:

$$\Phi(x,y,z) = \frac{1}{2}\sum_{n=0}^{\infty}\left\{\int_L \left[U_o A_{na}(\xi) + U A_{nc}(\xi)\right]\ \varphi_n\left[(x-\xi),y,z\right]\ d\xi\right\}. \tag{28}$$

Two different distribution functions A_{na} and A_{nc} are introduced. The second indices of these functions denote the functions belonging to cases (a) and (c) treated in part I. Since the additional deformation U decreases when the depth increases, the distributions corresponding to the two-dimensional case (c) of the body "in longitudinal waves" will be chosen for this part since the motion of the water particles decreases with the same exponential function. This formula has been transformed in the same manner discussed previously into the identical formula:

$$\begin{aligned}\Phi(x,y,z) = \frac{1}{2}\sum_{n=0}^{\infty}\Bigg\{ & U_o(x)\ A_{na}(x)\int_{-\infty}^{+\infty}\varphi_n\left[(x-\xi),y,z\right]\ d\xi \\ & + U(x)\ A_{nc}(x)\int_{-\infty}^{+\infty}\varphi_n\left[(x-\xi),y,z\right]\ d\xi + \int_{-\infty}^{+\infty}\Big[U_o(\xi)\,A_{na}(\xi) \\ & + U(\xi)\ A_{nc}(\xi) - U_o(x)\ A_{na}(x) - U(x)\ A_{nc}(x)\Big]\ \varphi_n\ (x-\xi),y,z\ \ d\xi\Bigg\}.\end{aligned} \tag{29}$$

Again, as mentioned before, the common strip method results if only the first row is taken into account. The second row, again, represents essentially a plane flow around the sections.

To carry out the computations a further simplification, besides the one already mentioned, is applied for the third row. In this row only the member for $n = 0$ of the series over n members will be considered. The reason is that this member causes the greatest long-distance effect since the functions φ_n fade away at a greater distance from the origin of the disturbance more rapidly the greater n is. Besides, the potential of a periodical source is described by φ_o and only by this term circular waves, which transport energy, are described. Therefore, instead of the third row in Eq. (29),

$$\sim \frac{e^{\nu z}}{2} \int_{-\infty}^{+\infty} \left[U_o(\xi)\, A_{oa}(\xi) + U(\xi)\, A_{oc}(\xi) - U_o(x)\, A_{oa}(x) \right.$$

$$\left. - U(x)\, A_{oc}(x)\right] \; \varphi_n \left[(x-\xi),0,0\right] \; d\xi \qquad (30)$$

is written and, the integral being a function of the coordinate x only, this row can be written

$$e^{\nu z}\; F(x) . \qquad (31)$$

The next problem is to determine U such that the condition on the surface of the body is satisfied to a sufficient extent. This means that the influence of the second member in (24) on the boundary condition is approximately eliminated.

A simplification is introduced relative to this boundary condition; viz., the velocity in longitudinal direction is neglected so that

$$\left[\frac{\partial \Phi}{\partial y}\, dz - \frac{\partial \Phi}{\partial z}\, dy\right]_S = U_o \; dy \qquad (32)$$

where S = the surface of the ship body.

The potential (29) is introduced into the left-hand side of this simplified boundary condition. This can be done without difficulty since solutions are known for the corresponding two-dimensional cases.

$$\left[- \frac{\partial \Phi}{\partial y}\, dz + \frac{\partial \Phi}{\partial z}\, dy\right]_S = U_o\, dy + U e^{\nu z}\; dy + \nu e^{\nu z}\; F(x)\; dy\, . \qquad (33)$$

The sum of these members equals $U_o\ dy$ from Eq. (32). The simplified boundary condition leads, therefore, to the following equation for the unknown function U:

$$U(x) + \nu F(x) = 0 . \qquad (34)$$

The application of the hypothetical two-dimensional case (c) is again confirmed by the appearance of the function $e^{\nu z}$ in the second and third row of Eq. (33).

In the equation for U, only the coordinate x appears. This equation is an integral equation.

Having determined U the task of determining the hydrodynamic forces remains.

Having satisfied Eq. (34), one may write for the potential instead of (29)

$$\Phi = \frac{1}{2} U_o \sum_{n=0}^{\infty} A_{na}(x) \int_{-\infty}^{+\infty} \varphi_n \left[(x-\xi), y, z\right] \, d\xi$$

$$+ U \quad - \frac{e^{\nu z}}{\nu} + \frac{1}{2} \sum_{n=0}^{\infty} A_{nc}(x) \int_{-\infty}^{+\infty} \varphi_n \; (x-\xi), y, z \quad d\xi \quad . \tag{35}$$

This expression does not satisfy the continuity condition any longer and is, therefore, not exact. However, the expression may be used for coordinates which correspond to the surface of the ship. For these coordinates this equation certainly represents a sufficient approximation. The computation of the hydrodynamic pressure and the integration of this pressure over the contour of the section in transverse direction to determine the force acting in a vertical direction has already been carried out in part I.

From the first member in Eq. (35) one obtains as in the common strip method:

$$\delta \frac{B}{\omega} U_o R \, . \tag{36}$$

From the second and third member in (35),

$$\delta \frac{B}{\omega} UE_c \tag{37}$$

for, the second member in (35) describes a "longitudinal wave" at x of orbital velocity U and the third member the additional two-dimensional potential computed for (c) in part I. E_c represents the nondimensional parameter of the force which is related to (c) of part I and which is generated by the "longitudinal wave."

The total force in a vertical direction on a section of a ship body amounts then to

$$\delta \frac{B}{\omega} \left[U_o R + UE_c\right] \tag{38}$$

The second member represents the correction to the common strip method. U, R, and E are complex functions.

The force and the moment for the whole ship body are obtained by corresponding integrations of Eq. (38) over x.

The Force Generated by Waves on the Ship Body

To compute the force it is assumed first that the ship body is restrained. The generating surface wave is supposed to run in longitudinal direction from fore to aft. The method can, however, also be applied for all running directions.

As compared to the Froude-Kryloff method an essential progress would be obtained if, instead of the quasi hydrostatic force computed from the nondeformed wave, the force E_c computed for case (c), viz., the ship in a "longitudinal wave" is introduced.

A different method is proposed in Ref. 11. Also this method could be improved.

The formula for the potential of the nondeformed surface wave of orbital velocity 1 is as follows, omitting the time factor $e^{i\omega t}$

$$\Phi_w = \frac{1}{\nu} e^{\nu(z+ix)}. \tag{39}$$

The following formula will be used for the total potential of the wave and deformation caused by the body:

$$\Phi(x,y,z) = \frac{e^{\nu(z+ix)}}{\nu} + \frac{1}{2}\sum_{n=0}^{\infty} \int_L e^{i\nu\xi} G(\xi) A_{nc}(\xi)\, \varphi_n\left[(x-\xi),y,z\right] d\xi. \tag{40}$$

This potential still exactly satisfies both the continuity equation and the condition on the free water surface. $G(x)$ represents a presently unknown complex function in a similar manner as $U(x)$.

Using the same simplifications as before, which are deemed permissible relative to points on the surface of the ship, Eq. (40) is simplified to

$$\begin{aligned}\Phi \sim e^{i\nu x} \Bigg\{ &\frac{e^{\nu z}}{\nu} + \frac{1}{2} G(x) \sum_{n=0}^{\infty} A_{nc}(x) \int_{-\infty}^{+\infty} \varphi_n\left[(x-\xi),y,z\right]\, d\xi \\ &+ \frac{e^{\nu z}}{2} \int_{-\infty}^{+\infty} \Big[e^{i\nu(\xi-x)}\, G(\xi)\, A_{oc}(\xi) \\ &- G(x)\, A_{oc}(x)\Big]\, \varphi_o\left[(x-\xi),0,0\right]\, d\xi \Bigg\}. \end{aligned} \tag{41}$$

The last member in this equation can be written as follows:

$$e^{i\nu x}\, e^{\nu z}\, H(x) \tag{42}$$

since the integral in this row is a function of x only.

The following simplified condition on the surface of the ship body S will be used:

$$\left[- \frac{\partial \Phi}{\partial y} dz + \frac{\partial \Phi}{\partial z} dy\right]_S = 0. \tag{43}$$

The right-hand side of this boundary condition is zero since the body is assumed to be restrained and since the whole potential will be introduced. The required computations have already been carried out or can easily be carried out. The boundary condition furnishes the relation

$$e^{\nu z} dy + G(x)\ e^{\nu z} dy + \nu H(x)\ e^{\nu z} dy = 0 \tag{44}$$

or the definition equation for the unknown function $G(x)$, viz.,

$$1 + G(x) + \nu H(x) = 0. \tag{45}$$

This is an integral equation for the unknown function $G(x)$.

After having determined this function G the next problem is to determine the exciting force.

Into the formula for the potential $H(x)$ will now be introduced

$$\Phi \sim e^{i\nu x}\ G(x) \left\{- \frac{e^{\nu z}}{\nu} + \frac{1}{2} \sum_{n=0}^{\infty} A_{nc}(x) \int_{-\infty}^{+\infty} \varphi_n \left[(x-\xi), y, z\right]\ d\xi\right\}. \tag{46}$$

Although this formula does not yield an exact value for the potential, it gives a very close approximation for coordinates of points at the surface of the ship.

The hydrodynamic force per unit length in vertical direction has, for this expression, already been determined in part I under (c). This force amounts to

$$\delta \frac{B}{\omega} G e^{i\nu x}\ E_c. \tag{47}$$

The result Eq. (46) may be conceived a two-fold deformation of the longitudinal wave caused by the ship body. The first member, viz.,

$$- \frac{G}{\nu}\ e^{\nu(z+ix)} \tag{48}$$

may be considered the potential of a longitudinal wave with variable effective wave amplitude of which the reduction factor relative to the oncoming wave amounts to $-G(x)$. This member represents, therefore, a deformed wave which, in the region of the ship body, is independent of the coordinate y. The second member in (46), however, mainly represents a deformation of the wave which depends on the coordinate y. It may be convenient, although not quite exact, to denote $G(x)$ an effective wave amplitude.

Free Moving Ship Body in Longitudinal Waves

All hydrodynamic problems relative to the free moving ship body are already solved in the foregoing considerations since a linearized treatment is assumed.

Some hints will now be given how the motions are determined. All forces or moments, respectively, may be found by an integration of the forces per unit length over the length of the ship body. The exciting forces are found by an integration of Eq. (47), the restoring forces by an integration of Eq. (38). The results for the nondimensional heaving and pitching motions (made nondimensional by the wave amplitude or by the amplitude of the wave angle, respectively) are as follows:

$$z_o \int_L B(R + U_H E_c)\, dx + \psi \int_L xB(xR + U_P E_c)\, dx = \int_L BG\, e^{i\nu x} E_c\, dx \tag{49}$$

and

$$z_o \int_L xB(R + U_H E_c)\, dx + \psi \int_L xB(xR + U_P E_c)\, dx = \int_L xBG\, e^{i\nu x} E_c\, dx. \tag{50}$$

where E, G, R, and U are complex functions of x. The values of U are nondimensional values of the function $U(x)$ which follow from Eq. (34) for the case $U_{oH} = 1$ or $U_{oP} = x$, respectively. The indices H and P indicate that the equation for U is satisfied either for heaving or for pitching.

Using these values for the amplitudes of the motions two interesting functions* can be determined, viz., the resulting wave deformed in a longitudinal direction:

$$Ge^{i\nu x} - z_o U_H - \psi U_P \tag{51}$$

referred to a space fixed zero-line or

$$Ge^{i\nu x} - z_o U_H - \psi U_P - z_o - \psi x \tag{52}$$

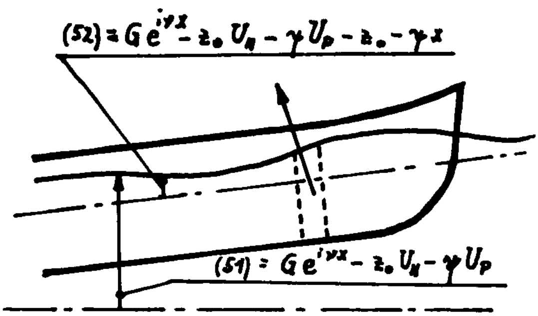
referred to a zero-line moving with the ship.

The function (51) appears for the case of the free moving ship body instead of the function $Ge^{i\nu x}$ for the case of the restrained body. Taking the amplitude of the function (52) this function may be conceived as envelope of the wetted ship surface.

*Of physical significance.

Resistance

Since the deformation of the longitudinal wave has been determined one could try to compute the mean force generated by the wave in direction of the negative x-axis, i.e., the resistance W. However, it is necessary to simplify the distribution of the pressure for this purpose.



For the free moving ship body (see sketch) the contour of the surface of the wave is given by (51) relative to a line fixed in space and by (52) relative to a zero-line moving with the ship. An element of the ship body of length dx, bounded by two sections, displaces the volume:

$$V = \left[BT\beta + B(Ge^{i\nu x} - z_o U_H - \psi U_P - z_o - \psi x)\right] dx. \tag{53}$$

The simplifications now are introduced, when computing the resistance, that the magnitude of the buoyancy force equals δV and that its direction is perpendicular to the contour of the surface of the wave. These assumptions lead to the following formula for the force in the direction of the negative x-axis:

$$P = \delta \int_L \left[BT\beta + B(Ge^{i\nu x} - z_o U_H - \psi U_P - z_o - \psi x)\right] d(Ge^{i\nu x} - z_o U_H - \psi U_P). \tag{54}$$

Only the time mean value of this force is of interest, viz.,

$$W = \frac{\omega}{2\pi} \int_{\omega t = 0}^{\omega t = 2\pi} P\, dt. \tag{55}$$

The first member in (54), viz., βBT does not contribute to this mean value. The remainder may be written as the following sum:

$$\left.\begin{aligned} W &= W_1 + W_2 \\ W_1 &= \left[\delta \int_L B(Ge^{i\nu x} - z_o U_H - \psi U_P)\ d\ Ge^{i\nu x} - z_o U_H - \psi U_P)\right]_M \\ W_2 &= \left[\delta \int_L B(-z_o - \psi x)\ d(Ge^{i\nu x} - z_o U_H - \psi U_P)\right]_M \end{aligned}\right\} \tag{56}$$

The index M indicates that in both cases the mean value over the time is to be taken.

The first part can be looked at as caused by the reflexion of the wave on the ship body and the second part by the phase shift between the motions of the ship and the wave. This representation of the resistance is known [1,8] and it now appears possible to compute both parts of the resistance.

Example

The results for an example which have been computed from the method discussed before will now be given in detail. The ship body chosen is as follows:

$L/B = 6.4$; $B/T = 2.8$; T = constant for <u>all</u> sections; $\beta = 0.9$, constant for all sections;

$$\frac{B_{\text{section}}}{B_{\text{max}}} \text{ (for 10 sections)} = 0.33, 0.70, 0.90, 0.99, 1.0, 1.0, 0.99, 0.90, 0.70, 0.33.$$

This ship body is symmetrical about $x = 0$. The 10 given sections lie in the midst of 10 equally long intervals.

Each integral equation (34) or (45), respectively, has been transformed into a system of linear equations and has then been solved. The 10 given sections were chosen as supporting points for this system of equations. Each system of equations contains, therefore, the 10 complex values U_H or U_P or G, respectively, of these sections as unknowns and, therefore, three times a system of 20 equations for 20 real unknowns had to be solved.

Figures 25-31 show the results in a nondimensional representation.

Figure 25 shows the deformation of the smooth water surface caused by the heaving and pitching motions. This deformation would not exist for an infinitely long body. The velocity of the deformation represented on the diagram is made nondimensional by the velocity U_o of the motion and the velocity of the bow is chosen for the pitching motion, i.e., U_o at $x = L/2$. Further λ/L is chosen a parameter. The ratio λ/L is defined by the relation

$$\frac{L\omega^2}{g} = 2\pi\frac{L}{\lambda}$$

since the wavelength λ has in this case no physical meaning.

Figure 26 gives the deformation at the point $x = 0$ which arises from the heaving motion, plotted against the frequency. This diagram is presented to show the course of U_H/U_o for $\omega \to 0$. From this course it follows that the virtual mass does not go to infinity for $\omega \to 0$, as it does with the common strip method. For the virtual mass, the virtual moment, the damping force, and the damping moment about the same values are obtained for $\omega \to 0$ as given in Ref. 11.

Figure 27 shows for the section $x = 0$ the coefficient C of the virtual mass and the hydrodynamic damping force both for the two-dimensional and for the three-dimensional case. The latter is computed on the basis of the former by means of the additional velocity.

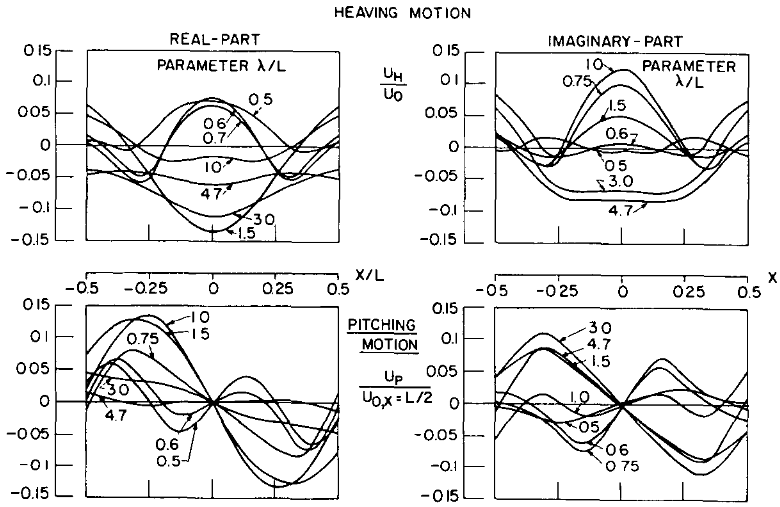
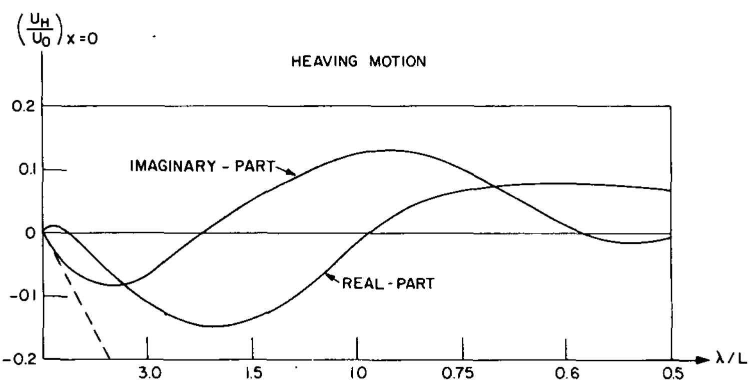


Fig. 25. Heaving motion and pitching motion



$$\left(\frac{U_H}{U_0}\right)_{x=0,\,\omega\to 0} \div \frac{\omega^2}{g} \cdot \frac{B}{\pi} \cdot \left[-\ln\cdot(1.783\cdot\frac{\omega^2}{g}\cdot L) - i\pi\right] =$$
$$= 0.313 \cdot \frac{L}{\lambda} \cdot \left[-\ln\cdot(11.2\cdot\frac{L}{\lambda}) - i\pi\right]$$

Fig. 26. Heaving motion

In Fig. 28 the amplitude of the motion of the sections of the ship body in vertical direction, viz., $z_o + \psi_x$ is plotted, made nondimensional by the wave amplitude $\overline{h}$.

In Fig. 29 the amplitude of the deformed wave is presented, made nondimensional by the wave amplitude. The upper diagram holds for the restrained body and the lower diagram for the free moving body. The amplitudes of the wave deformed in a longitudinal direction are, as is expected, less reduced relative to the undisturbed wave in the case of the free moving ship than of the restrained ship body. These differences are, again as is expected, smaller in the case of small wavelengths than of large ones. The enlargement of the wave amplitude for λ/L = 1.0 to 1.5 in the force part of the ship body in the case of the free moving ship is striking.

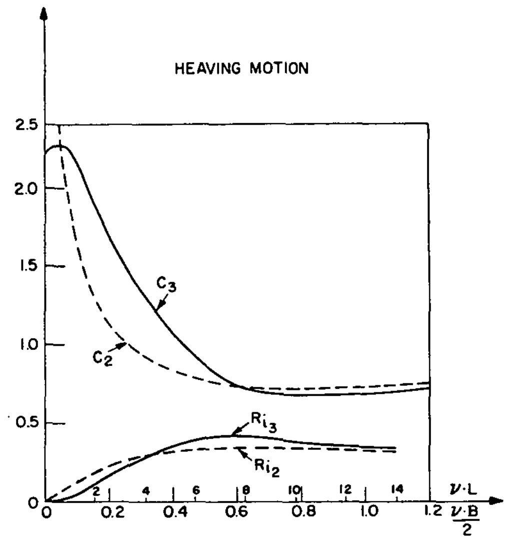


Fig. 27. Heaving motion; coefficient C and force R_i for the section $x=0$, two- and three-dimensional

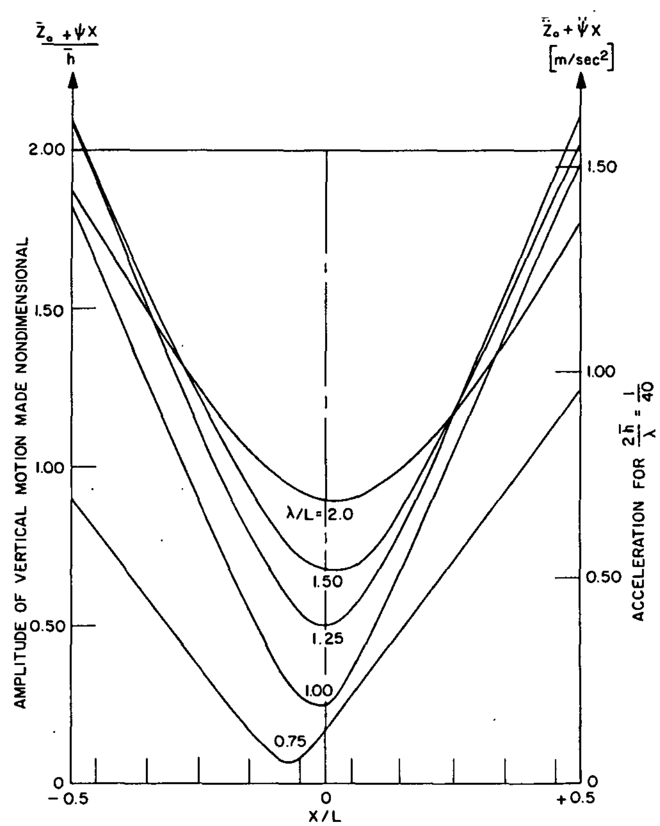


Fig. 28. Vertical motion of the sections of the ship

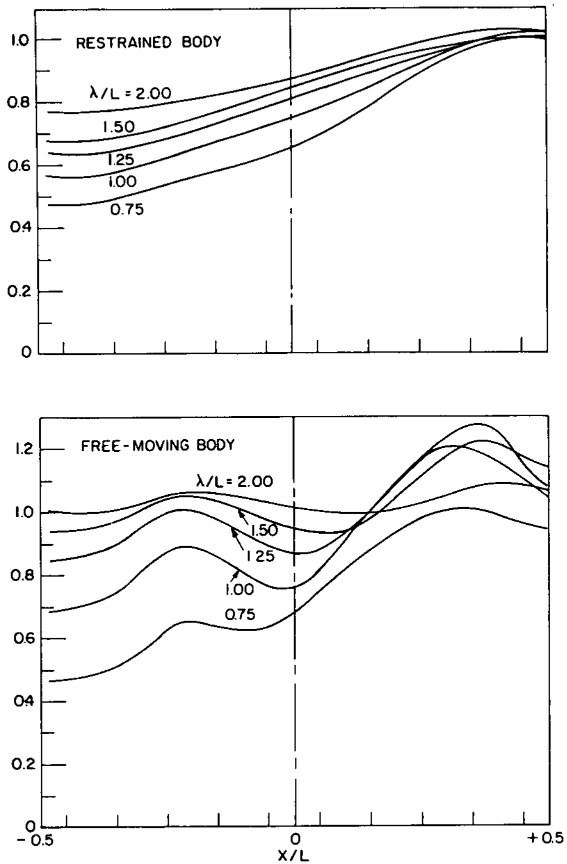


Fig. 29. Amplitude of the deformed wave made nondimensional by the undeformed wave amplitude

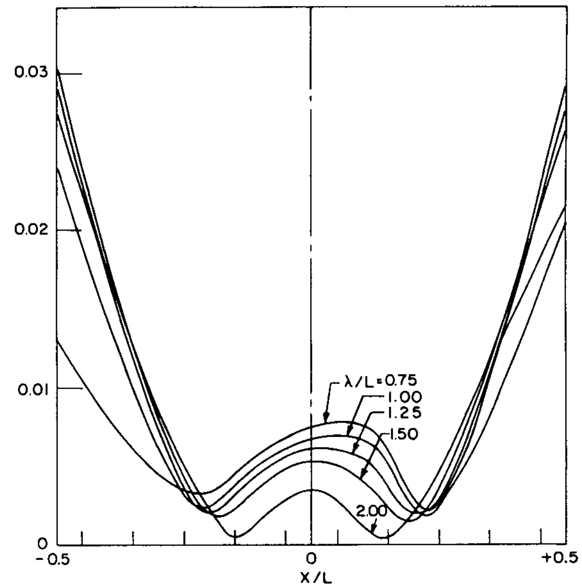


Fig. 30. Amplitude of the relative motion between the body and the wave surface made nondimensional by the length of the ship for waves of $2\bar{h}/\lambda = 1/40$

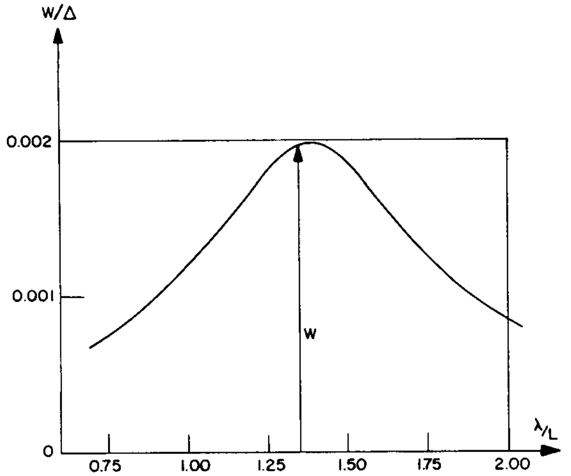


Fig. 31. Resistance arising from reflexion made nondimensional by the weight of the ship for waves of $2h/\lambda = 1/40$

Figure 30 deals with the amplitude of the relative motion between the deformed wave and the ship body. This amplitude is not made nondimensional with the amplitude of the wave but with the length of the ship. The ratio of the wave height to the wavelength is taken 1/40. This representation has been selected because the envelope of the plotted curves shows to which maximal degree the water surface can move relative to the ship body for all waves of the ratio chosen.

Figure 31 shows the ratio of the resistance to the weight of the ship for a constant ratio of wave height to wavelength of 1/40. The first part of the resistance only arising from reflexion is represented.

Calculations for this example were previously carried out for a subdivision of the ship body in 8 sections and applying a quite different program for the computor. The results of both the computations closely coincide for both U and G, i.e., for the deformation of the free water surface in consequence of the heaving and pitching motion and for the deformation of the wave in the case of a restrained ship body. In the case of the heaving and particularly of the pitching motion somewhat greater values have been obtained with a subdivision into 10 sections. These deviations, as checking computations have shown, are mainly caused by the different subdivision of the ship body. It can be stated on the basis of this comparison that the accuracy of the calculation is sufficient.

ACKNOWLEDGMENT

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NOMENCLATURE

x, y, z coordinates

ξ same coordinate as x (used only within integrals)

ω circular frequency

λ wavelength

$\bar{h}$ wave amplitude

$$\nu = \frac{\omega^2}{g} = \frac{2\pi}{\lambda}$$

δ density of water

ρ specific weight of the water

L, B, T length, beam and depth of the ship

B beam of a section (if used within integrals)

$$H = \frac{B}{2T}$$

$\mathfrak{B}$ fullness of section

z_o heaving displacement

ψ angle of pitch

Φ velocity potential

φ_n partial potential

A_n coefficients of singularity distributions in the two-dimensional case and distribution functions in the three-dimensional case

U amplitude of the velocity

U_o velocity of a section in the three-dimensional case

U velocity of the water surface in the three-dimensional case which arises from the finite length of the body

G amplitude of the wave deformed by the restrained body

m mass per unit length

m'' virtual mass per unit length

N coefficient of the hydrodynamic damping force

R the total nondimensional force which is necessary to produce the heaving motion in the two-dimensional case

E nondimensional force excited by the wave in the two-dimensional case

C coefficient of the virtual mass

$\bar{A}$ ratio of amplitudes

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DISCUSSION

John V. Wehausen (University of California)

The following remarks are supplementary to that part of Dr. Grim's paper which deals with the heaving motion of two-dimensional bodies. However, they are also supplementary in an important sense to the whole paper, for they deal with the applicability of theory to experiment. The work described below was carried out by W. R. Porter as part of a doctoral dissertation at the University of California, Berkeley.

The aim of this work coincided in some respects with that of Dr. Grim in that we wished to obtain information for forms of shiplike section which could be used later in computations for three-dimensional bodies by a strip method or some modification. However, there was a further aim. Experimental evidence confirming the applicability of perfect-fluid theory with a linearized free-surface condition to oscillatory motion of a body in a real fluid with a free surface seemed to be very scarce, and we felt that more was needed. In order to make a comparison of theory and experiment, configurations were needed for which the theoretical calculations could be made with controllable accuracy and for which the corresponding experimental measurements could be carried out with available equipment. Two-dimensional shapes had several advantages for the experimental work. Furthermore, it was known how to generate by conformal mapping of a circle families of shiplike sections, the so-called Lewis forms, Landweber forms, Prohaska forms, etc. In addition, Ursell* had already carried through computations of added mass and damping coefficients for a circular cylinder by a method which seemed likely to lend itself to further generalization to include the forms mentioned above. This turned out to be the case. Further modification allowed the inclusion of a horizontal bottom.

In order to have a more sensitive test of the perfect fluid theory, it was decided to compute and measure the pressure distribution around the cylinders as well as the added mass and damping coefficients. The computations were programmed for and carried out on an IBM 704. We believe them to be accurate to within 0.5 percent.

At the time this work was carried out we did not know that Dr. Grim was also engaged in similar computations for infinite depth by the method which he had proposed much earlier in 1953. However, Tasai's paper in J. Zosen Kyokai 105:47 had come to our attention, after this work was well under way, and we recognized that the two approaches were identical.† However, since Tasai had not included the pressure distributions, in which we were especially interested, or finite depth, we decided to continue with the computations. Although Porter's computations show some small discrepancies with Tasai's, they confirm them generally, as well as those of Dr. Grim.

The experimental measurements have at this time been carried through only for a circular cylinder (10-inch radius). However, other shapes, especially U-shaped and bulbous sections, will be tested later. In carrying out the force and pressure measurements a retreat was necessary in one respect. Although, a phase resolver had been designed, constructed, and tested, it was not feasible during the first set of experiments to make use of it because of excessive noise in the signal, a difficulty we are confident of being able to overcome later. As a result, the experimental points shown in Figs. D1 and D2 are for the amplitudes of fluctuation of the total pressure and force. For these the agreement between theory and experiment seems to be very satisfactory. Although, extremely square U-sections may show greater deviations, it seems likely that perfect-fluid theory can be used to give adequately reliable predictions of the motion of stationary oscillating bodies.

A detailed description of this work may be found in the report of W. H. Porter, "Pressure Distributions, Added-Mass, and Damping Coefficients for Cylinders Oscillating in a Free Surface," Inst. of Engrg. Res., Univ. of Calif., Berkeley, Series No. 82, Issue No. 16 (July 1960).

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*Quart. J. Mech. Appl. Math. 2:218 (1949).

†An English version of Tasai's paper (Rep. Res. Inst. Appl. Mech., Kyushu Univ., Fukuoka, Japan, 8(No. 26):131 (1959)) was later brought to our attention, and presumably renders our translation of the Japanese version (Inst. of Engrg. Res., Univ. of Calif., Berkeley, Series No. 82, Issue No. 15 (July 1960)) superfluous.

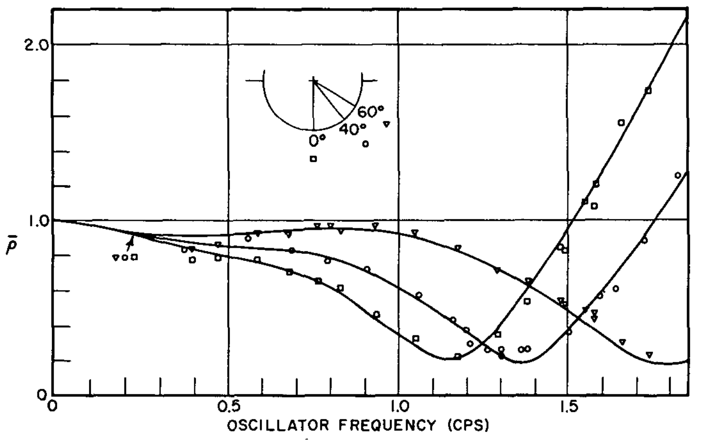

Fig. D1. The calculated and measured amplitude of total pressure fluctuation

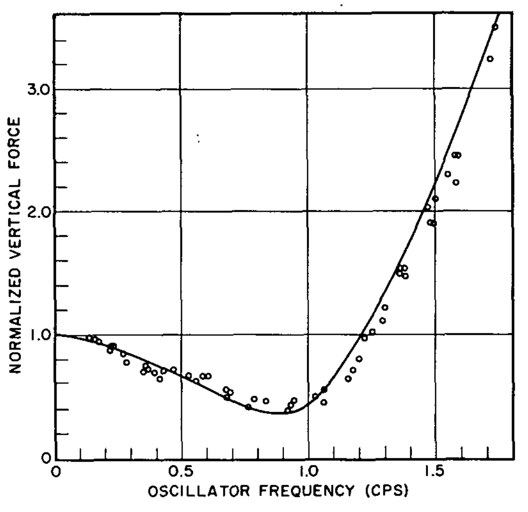

Fig. D2. The calculated and measured total vertical force

O. Grim

Prof. Wehausen's report about the work carried out at the University of California by W. R. Porter must be welcomed. I have seen this work only recently and did not know about it before the symposium. Indeed this work represents a helpful contribution to the problems dealt with in my paper. It is important that the mentioned experiments will be continued because only a few experimental results with respect to the forces on oscillating two-dimensional bodies are known. Such experimental results, however, are needed. Tasai, too, has carried out such experimental work. Since no severe differences seem to exist between the known theoretical and experimental results for the two-dimensional cases, we can be encouraged to continue the theoretical work for the three-dimensional or any other more complicated cases. I am very pleased with this contribution.

P. Kaplan (Technical Research Group, Syosset, N.Y.)

We have received in this rendition by Dr. Grim, quite an outstandi ng piece of work in the sense that it covers many problems that have been considered in the past 10 years from the point of view of ship motion analysis. Dr. Grim, in the past, has provided the largest amou nt of available data for use in the predictions of motions of ships in waves. This latest work is an improvement in the sense that three dimensionality is rather important, since there are not many three-dimensional solutions for determining the coefficients and the resulting motions of ships in waves. One of the main points which I see here, from my point of view, leads me to a question which I would like to place before Dr. Grim. The very fine thing that you have found is a creation of a distortion in the wave pattern, even in the case of a ship which is restrained in waves. This means you have taken account of the influence of the free surface in computing the excitation. Now, this has never been done in all the latest work used for computing the motions of ships and also it has not been taken into account in determining the bending moments which act upon ship forms. Therefore, I would like to know if you can tell me if you have made a comparison of the magnitudes of the exciting forces on a restrained ship with your theory and compared it to the simple slender body theory which just replaces the effect of the interaction by a dipole and does not take into account the influence of the free surface. Secondly, I would also like to make a point here concerning Fig. 27 in the paper. I gather here that what has been found is an expression for the magnitude of the virtual mass coefficient and also the damping coefficient at the particular section. Two dimensionally, one finds the value at a section all the time, but now you have a three-dimensional effect which shows the inclusion of interactions. The point I would like to make is that the appearance of close values for the damping terms in both the two-dimensional and three-dimensional cases is not necessarily to be taken as saying that there appears to be only a small difference between two-dimensional and three-dimensional values when you look at a section. It happens to be so for this particular case. I can report, and will some time in the future, the fact that there are large differences in the local values of damping; that is, the distribution along the hull is quite different, yet the total values are about the same. So I think it important for this one case to see this closeness yet realize local values must be looked at with care. That, of course, is most true for bending analysis, which is of vital importance for the structure of ships.

O. Grim

Dr. Kaplan has put two questions: At first he asks how large the differences are between the normal strip method of computing the forces and the bending moments without any deformation of the wave and the other computations with the deformation of the wave. I have not made such a comparison, but I think the last figure in which the reduced wave amplitude is represented will enable us to make a judgment about this comparison. It may be that the forces and the bending moments will be reduced in about the same magnitude as the wave amplitudes. Dr. Kaplan's second question is whether the added mass and the damping force, mentioned in Fig. 27, are typical. I think that this is so and that the differences between the two-dimensional and the three-dimensional results for heaving and pitching motion are only important for the range where the frequency is near to zero, while in the other range these differences are not so important. More important are the differences for the ship in waves, and the reason for this is the following: For the ship which undergoes a heaving motion with a large frequency, the forces exerted by the ship upon the water are in the same phase along the whole length of the ship. At each section of the ship an oscillatory force acts upon the water. The force at each section excites an elementary wave. If the ship makes a heaving motion with a large frequency, the resulting wave is small due to the phase relation between the elementary waves. However, for the ship in head seas the sum of all the elementary waves, which gives the deformation of the original head wave, is largely due to the phase relation between the elementary waves, which is different from the phase relation for heaving motion. This is the reason why the deformation is large for the ship in head seas and why the deformation of the surface of the water in the longitudinal direction is small for heaving motion with large frequencies.

J. B. Keller (New York University)

My comment is based on the following consideration. Dr. Grim's work begins with the solution of a certain two-dimensional problem and then he shows how to utilize a solution of the two-dimensional problem to solve the three-dimensional problem. I would like to point out another method of using two-dimensional problems to solve a three-dimensional problem. The method I have in mind is especially useful for short waves, i.e., waves that are short compared to the dimensions of a ship. The method is that of geometrical optics. I would like to suppose that Fig. D3 is the waterline of a ship which we are looking at from the top and I would like to consider the forces exerted on this ship by a wave coming in at some oblique direction. Then, according to the principals of geometrical optics, the wave can be thought to travel along rays and each ray will hit the waterline and be reflected according to the law of reflection and give rise to a reflected ray. The calculation of the reflection coefficient, that is the amplitude and phase of the reflected wave, is a local affair and depends only upon the geometry of the ship in the neighborhood of the point of reflection. Furthermore, since waves of short wavelength penetrate a very short distance into the water, it is only the geometry of the ship very near the surface at the point of reflection that determines the reflection coefficient. Since the geometry of a ship at a single point near the waterline can be described in terms of the radius of curvature of the vertical section and the slope of the ship at the waterline, and also, of course, the curvature of the waterline itself, those three quantities – the two radii of curvature and the slope – will determine completely the reflection properties for this particular ray. The radius of curvature of the waterline will be taken into account because neighboring rays get reflected in slightly different directions. The lateral divergence of the reflected rays will account for the curvature of the waterline. In order to compute the reflection coefficient for a given ray, it suffices to solve a two-dimensional problem, namely, reflection from a circular cylinder with the same curvature as the ship has in a vertical section at the waterline. The circular cylinder should cut the water with the same slope the ship has at that place. That is a two-dimensional problem that hasn't been solved, but never mind, I just promised to tell how to relate a two-dimensional problem to a three-dimensional one. If we could solve that two-dimensional problem for each position along the ship, then by such a geometrical construction we could calculate the reflected wave in the three-dimensional problem. The result for the two-dimensional problem might also be

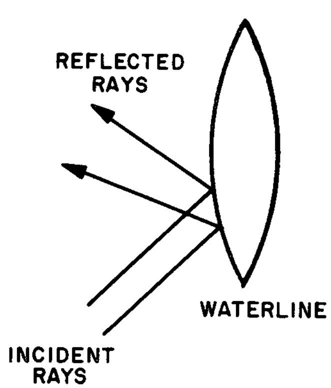

Fig. D3. Geometric optics analogy

determined by measurement or by someone solving it. Thus, it is possible, by solving a relatively simple problem, to get results that should be useful for a relatively complicated one. I would like to conclude by saying that the same ideas can be applied to calculating the waves produced by the motion of the ship in pitching or heaving or other periodic motion. Again, in each case it is necessary to find the corresponding waves produced by the motion of a circular cylinder with the same radius of curvature and slope as the ship has at the point of interest and that problem hasn't been completely solved.

O. Grim

I can imagine that this is a good idea as a way to compute the wave propagation for any large frequency. However, for a prediction of the motions of a ship a more complete solution than only for the wave propagation is needed. It could be helpful to obtain by Prof. Keller's proposed simple method results which confirm those obtained from a more complete solution.

Owen H. Oakley (U.S. Bureau of Ships)

I am sure that if my colleagues, that is, my practical naval architectural-type colleagues, could see me standing here with the temerity to discuss Dr. Grim's paper they would be amazed, and I assure you I am too. I have two points to make, one trivial and one more on the serious side. First of all, in 1957 I was a shipmate of Dr. Grim on an icebreaker in the Sea of Bothnia. We were concerned with vertical movements of the ship, because, installed in the bow of this ship was an eccentric weight device known as the Stampflanage which, when properly tuned at the remote control station in the pilot house, created a pitching motion in the bow of the ship and thus helped to break ice. Dr. Grim was at the controls and occasionally he would turn the knobs the wrong way and a fore and aft surging motion would result which was a bit disturbing to say the least. I wonder whether Dr. Grim has considered the matter of surge in this present analysis; I should think it would be something he would be very much aware of. Now a more serious comment. As a general rule practical naval architects do not have occasion to give serious thought to this sort of theoretical work. However, occasionally one encounters problem where such mathematical assistance would be most welcome. Such an experience was the design of the escort research ship. We would have liked very much to have been able to calculate the response of this ship to waves, but since we could not, we had to build models and go to the trouble of testing them, finding where we were wrong, correcting it, and trying again. The theoretical and empirical development of this subject has been going on for a number of years; many excellent minds have worked on it and produced a prodigious quantity of papers on the various aspects of the problem. I hope that some day all of this work to which Dr. Grim has contributed so greatly will bear fruit and we will be able to sit down with pencil and paper or with computer or what have you and predict quantitatively the amplitudes of response of arbitrary shipforms. I am sure that the contribution which Dr. Grim has told us about today is another significant step in that direction.

O. Grim

I am glad to hear that Mr. Oakley likes to remember the time we spent together on the icebreaker in the Sea of Bothnia and I hope he forgives me for the wrong way in which I twiddled the knobs of the pitching plant of the icebreaker. To the other point of Mr. Oakley's comments I have to say that I also am a naval architect and I too hope that the results of such complicated investigations can, in the future, be used by naval architects.

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