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PHYSICAL EFFECTS IN CAVITATION AND BOILING

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INTRODUCTION

The problem to be considered here is the growth of a vapor bubble in a superheated liquid. When the vapor pressure of the liquid exceeds the ambient pressure, it becomes possible for a vapor bubble to grow from a small "nucleus" in the liquid. This nucleus is a region of nonliquid phase and presumably consists of a gas or vapor phase stabilized on a solid particle. The factors which affect the rate of bubble growth are the surface tension, the liquid inertia, and the difference between the pressure within the bubble and the ambient or external pressure. We shall suppose that the bubble begins its growth near the condition of equilibrium of the forces acting upon it. The initial growth will be slow, but it is accelerated with increase in size because of the reduction in surface tension. When the rate of bubble growth becomes appreciable, however, the temperature, and hence the pressure within the bubble, drops and the rate of growth is decreased. One therefore might expect a maximum in the velocity of the bubble wall. The reduction of the temperature within the bubble is a consequence of the latent heat requirement of the evaporation which takes place at the vapor-liquid interface as the bubble grows.

FORMULATION OF THE PROBLEM

For a quantitative solution of the problem, some simplifying assumptions may be made. These simplifications will be presented for the example of a vapor bubble growing in moderately superheated water. It will be assumed that the bubble is spherical throughout its growth. One can show that the spherical shape is stable for a growing bubble in a spherically symmetric pressure field [1]. We are clearly excluding from consideration the asymmetric buoyant force of gravity which becomes important if the bubble growth is followed for so long a time that a significant translational velocity is acquired. A translational velocity of the bubble as a whole not only causes a deformation of shape but also increases the rate of heat inflow over that used in the present analysis. In water superheated by about 10°C no great error is introduced by the buoyant force provided the bubble growth is not followed beyond a radius of 1 mm or for a time longer than a few milliseconds.

With a superheat of the order of 10°C at one atmosphere pressure, a vapor bubble grows from its initial microscopic size of approximately 10^{-3} mm to a radius of 1 mm in a time of the order of 10 millisec. The corresponding average radial velocity is 10 cm/sec. The maximum radial velocity is about 10 times this value. We see that the velocities of concern in the bubble growth are very small compared with the sound velocity in either the liquid or the vapor. The hydrodynamic equation of motion which contains the effect of liquid inertia is thus rather simple: the liquid motion of concern is incompressible and spherically symmetric. With the limitation to irrotational flow one has the velocity potential

$$\varphi = \frac{\dot{R}R^2}{r}, \tag{1}$$

where $R(t)$ is the liquid interface or bubble wall. Further, the liquid equation of motion gives the Bernoulli integral

$$-\frac{\partial \varphi}{\partial t} + \frac{1}{2}(\nabla \varphi)^2 = -\frac{p(r,t)}{\rho} + C(t). \tag{2}$$

Evaluation of this equation at $r = R$ gives the equation of motion of the bubble wall

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{p(R) - P_0}{\rho} \tag{3}$$

where ρ is the liquid density, $p(R)$ is the pressure in the liquid at the bubble boundary, and P_0 is the external pressure in the liquid or the pressure at infinity. The energy equation for the liquid may be written as

$$\rho c_v \left(\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T \right) = k \triangle T + \dot{q} + \mu \triangle v^2 \tag{4}$$

where μ is the viscosity, k the thermal conductivity of the liquid, and $\dot{q}$ is the heat source per unit volume per unit time. With the velocity field $v = R^2\dot{R}/r^2$, the viscous heat generated is maximum at the bubble wall and has the value

$$(\mu \triangle v^2)_{r=R} = 12\mu \frac{\dot{R}^2}{R^2}. \tag{5}$$

In a typical bubble growth situation (103°*C*, $\dot{R}_{\max} = 32$ *cm/sec*, $R = 3 \times 10^{-3}$ *cm*) this term is of the order of 0.1 *cal/(cm³sec)*. The rate of heat loss by conduction at the bubble wall $\left| \rho c_v \dfrac{dT(R)}{dt} \right|$, is of the order of 10^4 *cal/(cm³sec)*. Clearly the viscous heat generation is negligible.

In considering the conditions within the bubble, one has the simplification that the vapor density is so small compared with the liquid density that inertia effects in the vapor may be neglected. The vapor density is about 10^{-3} of the liquid density. Actually the pressure gradients in the vapor are an additional order of magnitude smaller than 10^{-3} of the liquid pressure gradients. The reason for the additional reduction is that the velocity and variation in velocity in the vapor are at least an order of magnitude smaller than the liquid velocity. Evaporation at the liquid interface fills the bubble as it grows and there is negligible motion of the vapor. We may therefore take the pressure in the bubble as uniform and, since the bubble wall moves at a velocity small compared with the sound velocity in the vapor, we may say further that the pressure within the vapor follows practically instantaneously its value at the bubble wall. We now argue that the uniform pressure within the bubble is the equilibrium vapor pressure of the liquid. When the liquid interface moves with a velocity $\dot{R}$, then the pressure in the vapor, p_{vap}, is related to the equilibrium vapor pressure p_{eq} (the vapor pressure for a stationary interface) by [2]

$$\frac{p_{vap}}{p_{eq}} = \frac{\rho_{vap}}{\rho_{eq}} = \frac{c^*}{c^* + \dot{R}} \tag{6}$$

where c^* is connected with the sound velocity c by

$$c^* = \frac{\alpha c}{\sqrt{2\pi\gamma}}. \tag{7}$$

In Eq. (7), α is the accommodation coefficient and γ is the ratio of specific heats. α is not known at the temperatures of present interest; it has been measured to be 0.04 for water at 0°C. This would give $c^* = 8$ *m/sec.* There is some basis for believing that α is larger than 0.04 at the temperatures of concern here, but in any case the pressure of the vapor is essentially the same as the equilibrium vapor pressure.

We may consider finally the temperature within the bubble. In general the temperature of the vapor would vary with position as well as with time. The thermal diffusivity of the vapor, D', is significantly larger than the thermal diffusivity of the liquid, and the characteristic diffusion length in the vapor, $\sqrt{D't}$, is large compared with the bubble radius. We may therefore make the approximation that the temperature is uniform in the bubble.

In summary, the physical model upon which the analysis is based consists of a spherical vapor bubble which has uniform temperature and pressure, the temperature of the vapor is that of the liquid at the bubble wall, and the pressure is the equilibrium vapor pressure for that temperature. In addition, the effects of viscosity and compressibility may be neglected.

THE EQUATION OF MOTION AND THE BOUNDARY CONDITIONS

We have seen that the viscous effect is negligible in the energy equation and we shall show that it is also negligible in our boundary condition. In the equation of motion for the bubble (Eq. (3)), one needs the pressure $p(R)$ in the liquid at the bubble wall; this pressure is related to the pressure in the vapor p_v by

$$p(R) = p_v(T) - \frac{2\sigma}{R} - 4\mu\frac{\dot{R}}{R}. \tag{8}$$

The effect of viscosity is to increase the surface tension by $2\mu\dot{R}$, and this increase is about 0.5 *dyne/cm* and is therefore negligible. We shall therefore write

$$p(R) = p_v(T) - \frac{2\sigma}{R}. \tag{9}$$

The boundary condition for the heat equation

$$\rho c_l \frac{\partial T}{\partial t} + \vec{v}\cdot\nabla T = k\Delta T + \dot{q} \tag{10}$$

may be simply deduced in the following way. The heat $\dot{Q}$ which must be supplied to the bubble per unit time is

$$\dot{Q} = \frac{4\pi}{3} L \frac{d}{dt}(R^3\rho') \tag{11}$$

where L is the latent heat of evaporation per unit mass and ρ' is the vapor density. This heat is supplied by conduction into the bubble so that

$$\dot{Q} = 4\pi R^2 k \left(\frac{\partial T}{\partial r}\right)_{r=R} \tag{12}$$

where k is the thermal conductivity of the liquid. Hence

$$\left(\frac{\partial T}{\partial r}\right)_R = \frac{L}{3k}\frac{1}{R^2}\frac{d}{dt}(R^3\rho'). \tag{13}$$

The small variations of L and k with temperature are neglected. It will be assumed that the temperature of the liquid is initially uniform

$$T(r,0) = T_0. \tag{14}$$

The bubble growth is supposed to begin at $t = 0$; at a distance from the bubble the temperature will be uniform, undisturbed by the cooling produced by the bubble growth. We have, then,

$$T_\infty = T(\infty,t) = T_0 + \frac{D}{k} q(t). \tag{15}$$

We shall admit, for the present, heat sources which correspond to relatively slow temperature changes. For example, a temperature rise of 1°*C/sec* would correspond to an insignificant change in T_∞ in the time of a few milliseconds over which we follow the bubble growth. The initial superheat temperature T_0 will be essentially the liquid superheat throughout the bubble growth. It is convenient to introduce a radius R_0 defined by the relation

$$\frac{2\sigma}{R_0} = p_v(T_0) - P_0. \tag{16}$$

Physically, R_0 is the effective initial radius of the bubble; it represents an extrapolation of the free spherical bubble down to the equilibrium radius for the given initial conditions. It should be noted that a bubble at rest with radius R_0 is in unstable equilibrium. The actual nucleus from which the bubble grows is not necessarily spherical and its surface energy may be less than $4\pi\sigma R_0^2$ (it may be zero); however, such a nucleus and the free spherical bubble of radius R_0 will have the same growth behavior when their radii are too or three times R_0.

In terms of the parameter R_0, the equation of motion (3) may be written

$$R\ddot{R} + \tfrac{3}{2}\dot{R}^2 = \frac{p_v(T) - p_v(T_0) + (2\sigma/R_0)(1 - R_0/R)}{\rho}; \tag{17}$$

or

$$\frac{1}{2R^2\dot{R}} \frac{d}{dt}(R^3\dot{R}^2) = \frac{p_v(T) - p_v(T_0) + (2\sigma/R_0)(1 - R_0/R)}{\rho}. \tag{18}$$

If the cooling effect of evaporation is disregarded so that $p_v(T) = p_v(T_0)$, Eq. (18) may be integrated to give

$$\dot{R}^2 = \frac{R^3_0}{R^3}\dot{R}_0^2 + \frac{4\sigma}{3\rho R_0}\left(1 - \frac{R_0^3}{R^3}\right) - \frac{2\sigma}{\rho R}\left(1 - \frac{R_0^2}{R^2}\right). \tag{19}$$

This solution will be called the Rayleigh solution. For $R \gg R_0$, Eq. (19) gives

$$\dot{R}^2 \approx \frac{4\sigma}{3\rho R_0} = \frac{2}{3\rho}[p_v(T_0) - P_0]. \tag{20}$$

The actual motion may be expected to deviate markedly from the Rayleigh solution because of the cooling effect.

Let us denote the temperature at which the vapor pressure equals the external pressure P_0 by T_b, which is thus the "boiling temperature,"

$$p_v(T_b) = P_0. \tag{21}$$

Then, for superheats not too far above the boiling temperature, the vapor pressure may be approximated by a linear function of the temperature so that we may write

$$\frac{p_v(T) - P_0}{\rho} = \frac{p_v(T) - p_v(T_b)}{\rho} = A(T - T_b). \tag{22}$$

While a more accurate linear approximation to $p_v(T)$ has been used over the interval T_0-T_b, A is very nearly the slope of the vapor pressure, $p_v(T)$, at T_0 divided by the liquid density. The equation of motion now becomes

$$\frac{1}{2R^2\dot{R}} \frac{d}{dt}(R^3\dot{R}^2) = A(T - T_b) - \frac{2\sigma}{R}. \tag{23}$$

When T is given as a function of R, the solution of the problem is of course determined.

To find this relation, it is necessary to solve the heat transport problem at the moving boundary of the bubble wall. Because of the boundary condition at the moving bubble wall it is convenient to transform the heat flow equation to Lagrangian coordinates. It does not appear possible to solve this heat equation in closed form; however, physical considerations suggest an approximation which gives an analytic solution. The thermal diffusivity of the liquid is so small that the characteristic diffusion length $\sqrt{D\,t}$ is small compared with the bubble radius $R(t)$. In other words, the liquid layer over which the temperature changes from its value at infinity to its value at the bubble wall may be taken to be small. This "thermal boundary layer" assumption leads to an analytic solution in successive approximations [3]. The first and second approximations have been found; the second approximation has been used to demonstrate that the first approximation has acceptable accuracy. This first approximation to the temperature field in the liquid gives for the temperature at the bubble wall

$$T(R) = T_\infty - \frac{L}{3k} \int_0^t \frac{\frac{d}{d\tau}(R^3\rho')\, d\tau}{[\int_\tau^t R^4(\epsilon)\, d\epsilon]^{\frac{1}{2}}}. \tag{24}$$

THE BEHAVIOR OF THE SOLUTION FOR BUBBLE GROWTH

The bubble growth problem is now determined, and we may consider the behavior of the solution for vapor bubbles growing in moderately superheated water. The initial conditions considered are specifically that the bubble is spherical of radius R_0 and at rest for $t = 0$. Such a bubble is unstable. The growth is initiated by the rising temperature of the bulk liquid for which we have

$$T_\infty = T_0 + \frac{D}{k} q(t). \tag{25}$$

The heat source is supposed to give slow changes in this external temperature. As we have already remarked, for ordinary heating rates the bulk temperature of the liquid will remain essentially unchanged during the few milliseconds of bubble growth which will be followed here. The external temperature rise of the liquid is really only a device for initiating the growth. It may be emphasized that the important range of bubble growth is not appreciably affected by the initial conditions. Thus, one might suppose that the growth was initiated by expansion of a permanent gas in the bubble. The permanent gas content of the nucleus will soon become unimportant and the subsequent behavior will be essentially that of the pure vapor bubble described here. Another possibility is that the initial surface tension condition is not that of the free bubble, but again, so long as the surface tension is normal when the bubble is two or three times its initial size, the bubble growth will subsequently be that given by the present analysis.

The bubble growth as found for our initial conditions may be divided into three phases: A delay period in which the growth of the bubble from equilibrium is very slow being initiated by the slow rise in the liquid temperature; there is essentially no temperature drop in the bubble vapor during this "delay" period. When the bubble has reached approximately twice its initial radius, it has acquired a small velocity and the surface tension effect is reduced. For superheats of the order of 10°C the time t_d required for this delay period is about 0.01 millisec; for a superheat of 2°C, t_d is 0.07 millisec. The bubble growth appears to begin abruptly near $t = t_d$. In the second or early phase of growth, the bubble velocity increases rapidly and continues to increase until the cooling effect becomes significant. The growth velocity goes through a maximum and thereafter continues to decrease as the third, or asymptotic, phase of growth is reached. At the beginning of the early or second phase of the growth, the heat source term which initiates the growth is negligible and makes no contribution to the subsequent bubble behavior. From a physical standpoint, this means that the observable bubble behavior is independent of the particular conditions initiating its growth.

It is clear that liquid inertia effects are important in determining the bubble growth in the second phase, during which the bubble experiences its maximum radial acceleration. After this phase, however, the rate of bubble growth is controlled by a balance between the rate of evaporation and the rate of cooling it produces. It is evident that the bubble must continue to grow since a stationary bubble would be at the temperature of the superheated liquid and therefore would have an excess internal pressure. Hence, the temperature at the bubble wall must continue to drop because of evaporation. But the temperature cannot drop below the boiling point and still maintain the pressure difference necessary for growth. It follows that the temperature of the bubble wall must approach the limit T_b as $t \to \infty$, and this fact is sufficient to characterize the asymptotic phase of bubble growth.

We may approximate the leading term in the asymptotic phase by simple physical arguments as follows. At a time when $R \gg R_0$, the difference between the temperature in the liquid at the bubble wall and that in the liquid at a distance is only slightly less than $T_0\text{-}T_b$. This temperature drop takes place principally in a liquid layer around the bubble of approximate thickness given by the diffusion length $\sqrt{Dt}$. The heat flow into the bubble per unit time is therefore given roughly by

$$\dot{Q} \approx \frac{k(T_0 - T_b)}{\sqrt{Dt}} 4\pi R^2. \tag{26}$$

The heat requirement per unit time for evaporation, on the other hand, is

$$\dot{Q} = L\frac{d}{dt}\left(\frac{4\pi}{3}R^3\rho'\right) \approx 4\pi R^2 \dot{R} L\rho'. \tag{27}$$

The approximation indicated in Eq. (27) is based on the fact that the heat flow requirement because of the volume change is considerably greater than that arising from the relatively small change in vapor density. When (27) is equated to (26), there results

$$\dot{R} \approx \frac{k}{L\rho'}\frac{(T_0 - T_b)}{\sqrt{Dt}}. \tag{28}$$

Analytically one finds that the leading term in the asymptotic region is

$$\dot{R} \approx \sqrt{\frac{3}{\pi}}\,\frac{k}{L\rho'}\frac{(T_0 - T_b)}{\sqrt{Dt}}. \tag{29}$$

This leading term in the expansion for $\dot{R}$ is in error by only 10 to 30 per cent in the range of easy experimental measurement.

The results of the analysis for bubble growth in water at 1 atmosphere pressure and 103°C are shown in Figs. 1-4. The radius-time behavior for several superheat temperatures is shown in Fig. 5. Comparisons with experimental observations [4], are shown in Figs. 6-8.

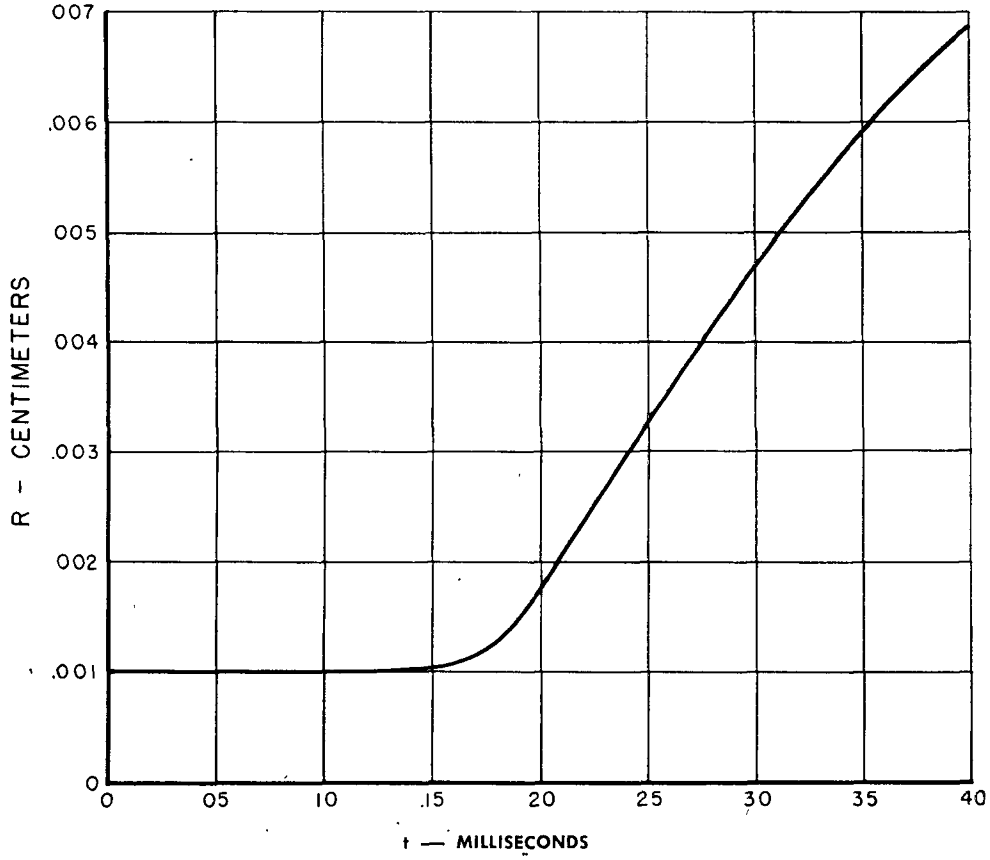

Figure 1. Theoretical radius vs time curve for the growth of a spherical vapor bubble in water at 1 atm external pressure, superheated to 103°C. The bubble is originally (at time t = 0) in unstable equilibrium. The growth is initiated by the introduction of a uniform heat source in the liquid (e.g. by irradiation), such as to produce a temperature rise in the liquid of 1°C in 100 sec. The curve shows the slow initial rate of growth.

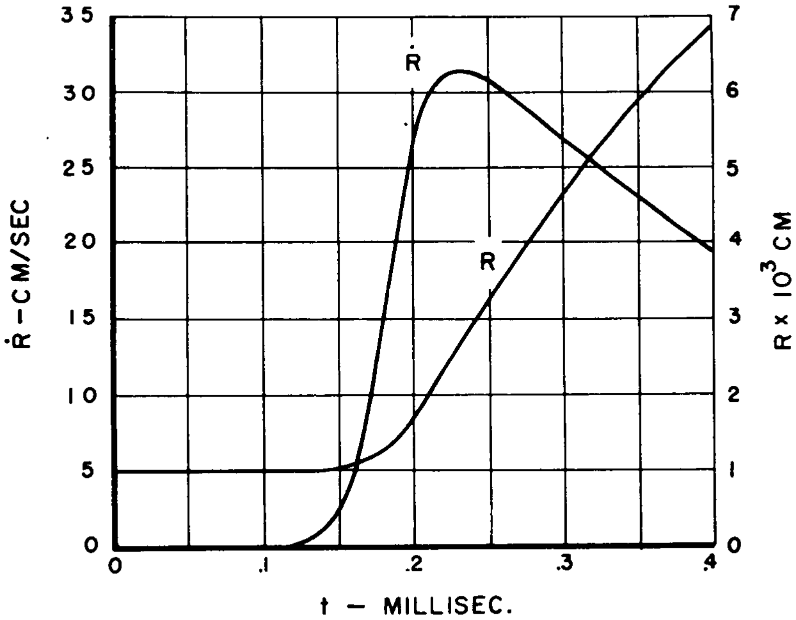

Figure 2. Theoretical radius and radial velocity curves for the growth of a pure vapor bubble in water at 1 atm external pressure, superheated to 103°C.

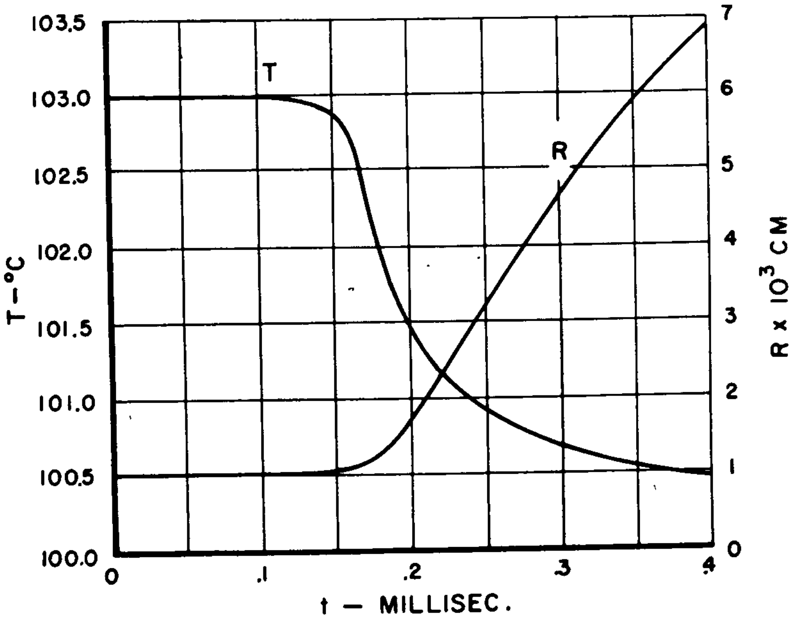

Figure 3. Theoretical radius and bubble wall temperature curves for the 103° vapor bubble of Figure 2.

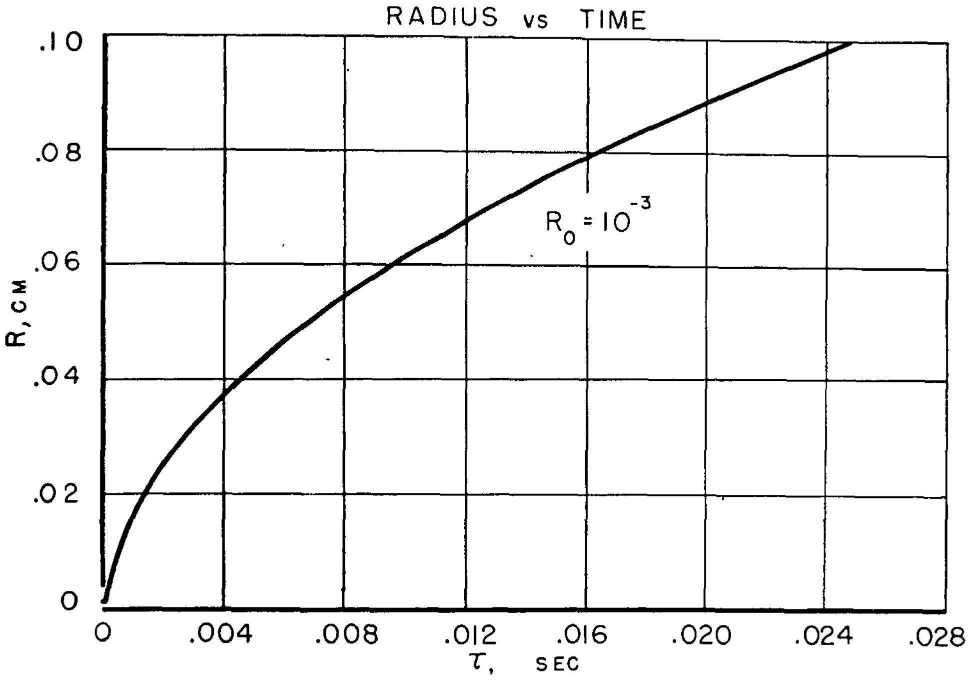

Figure 4. The theoretical radius vs time curve is shown under the same conditions as in Figure 1. The delay time at the beginning of the growth is practically undetectable.

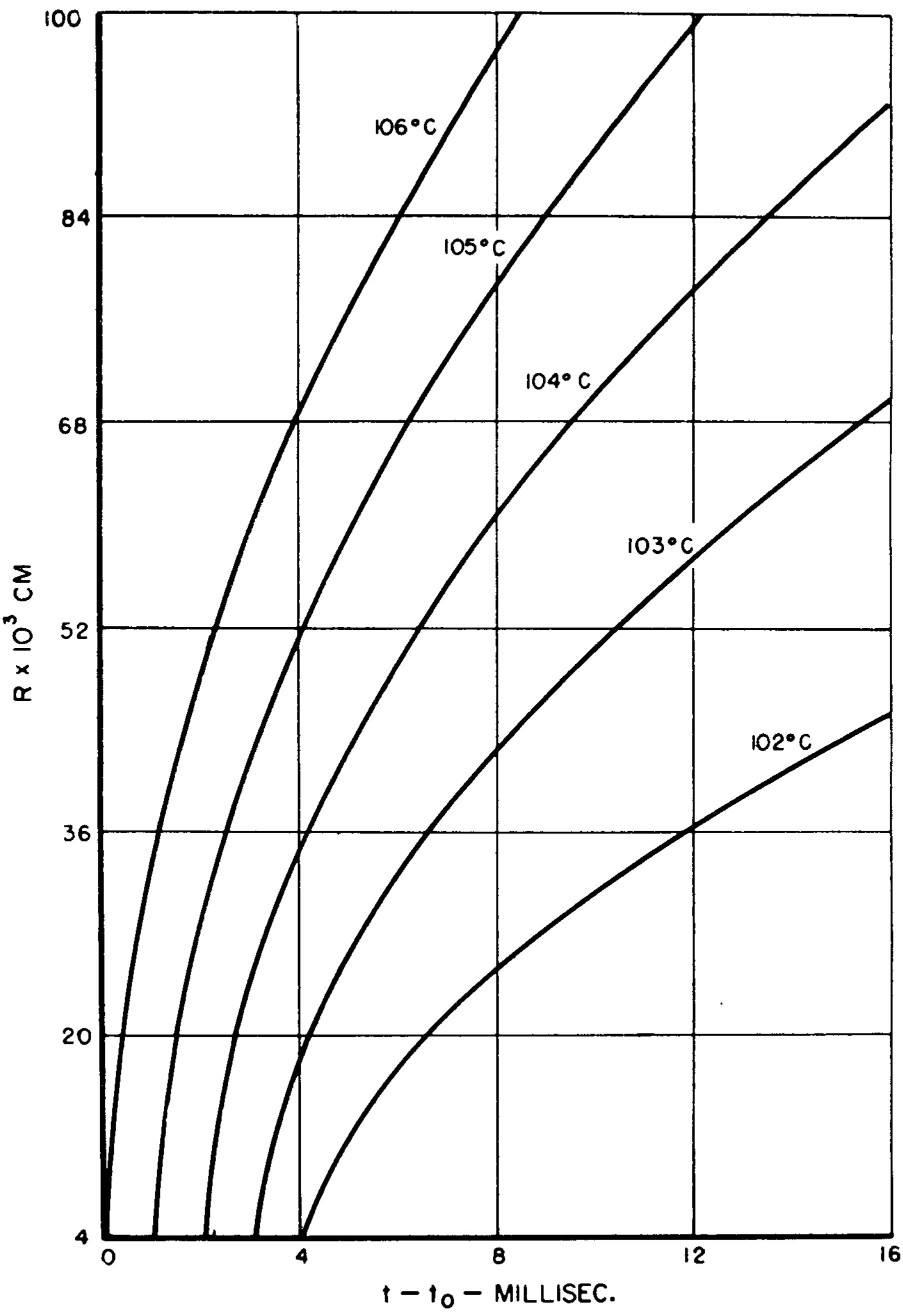

Figure 5. **Asymptotic radius versus time curves calculated for water at 1 atm external pressure and the indicated superheat temperature.**

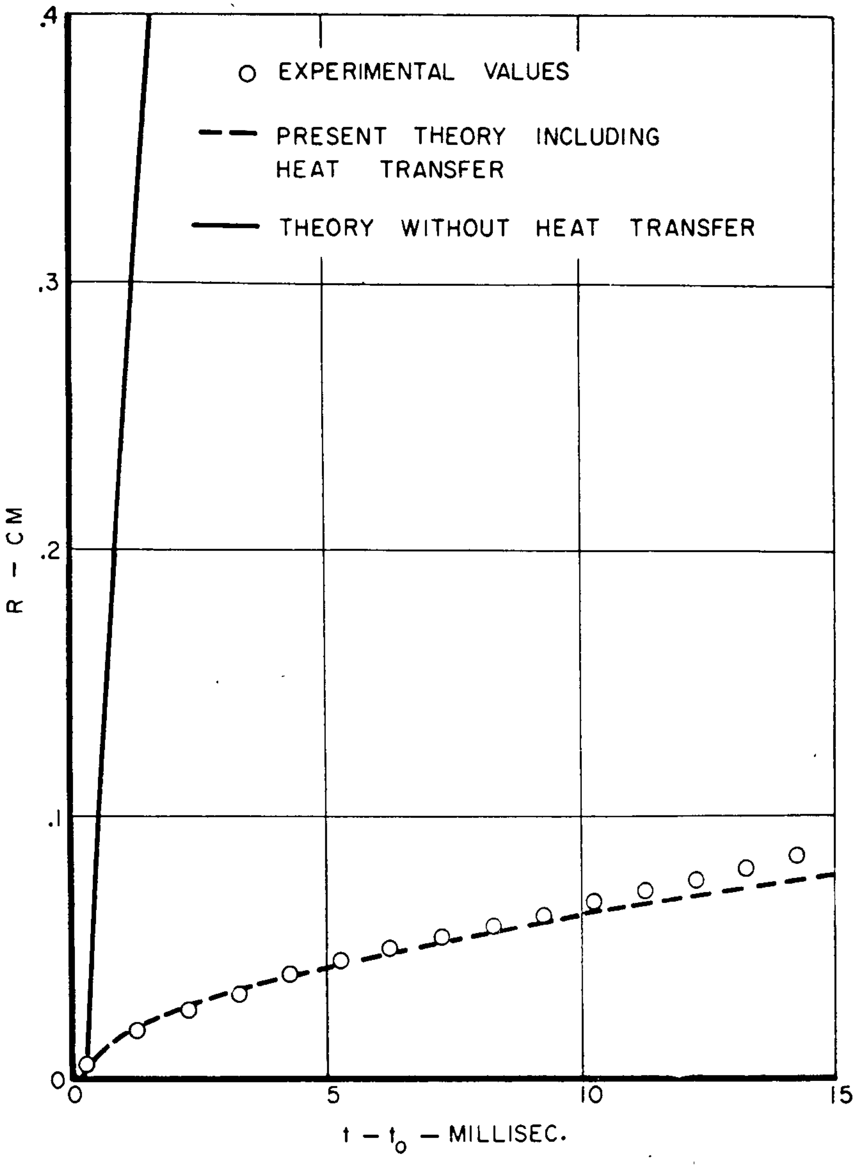

Figure 6. **Comparison of theoretical bubble radius-time values with experimental values for water at 1 atm external pressure, superheated to 103.1 °C. The solid curve is the Rayleigh growth curve, obtained by neglecting heat transfer effects; the dashed curve is that predicted by the asymptotic solution of the text, which takes heat transfer into account.**

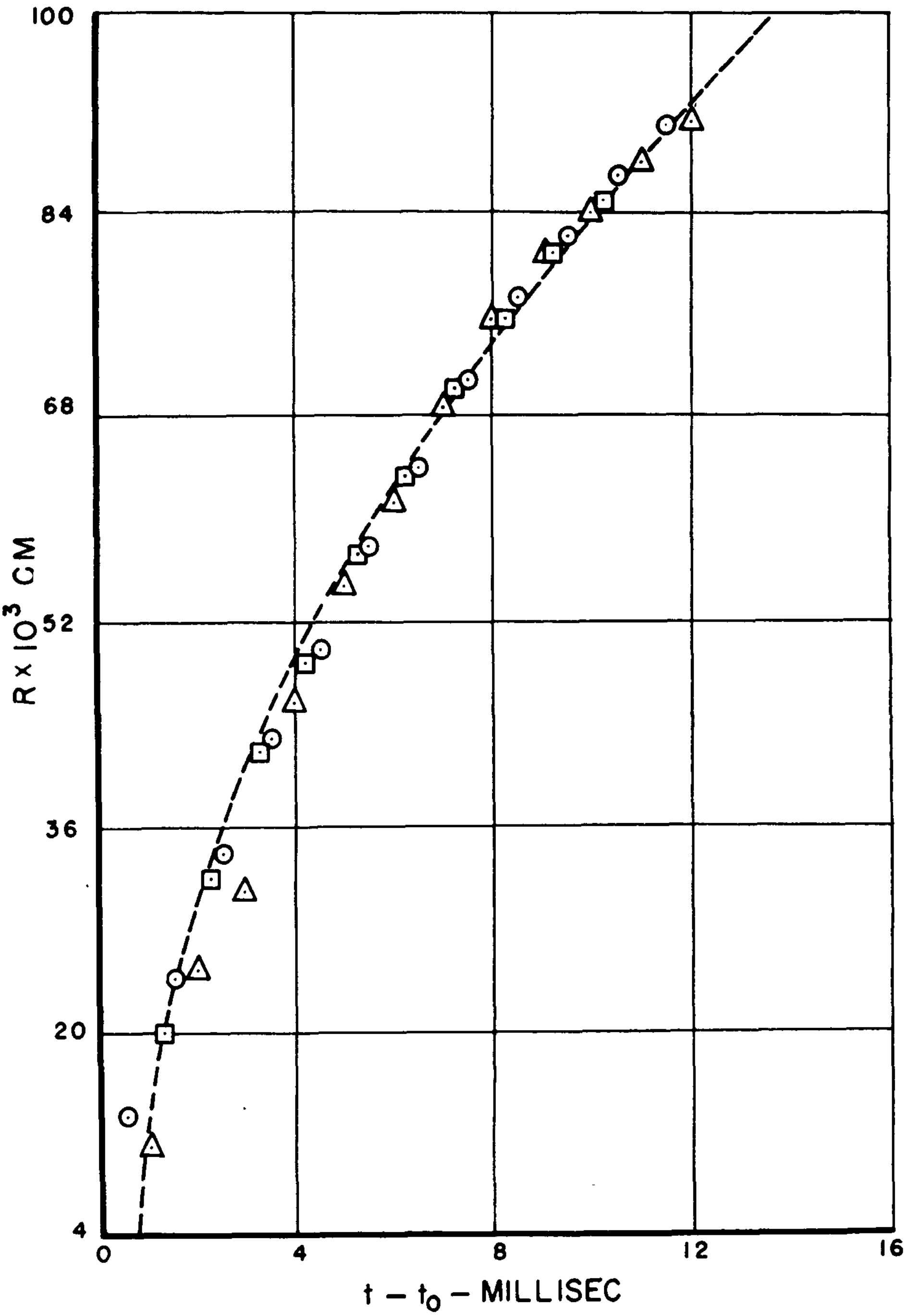

Figure 7. **Comparison of theoretical radius-time values with three sets of experimental values obtained in water at 1 atm external pressure, superheated to 104.5°C.**

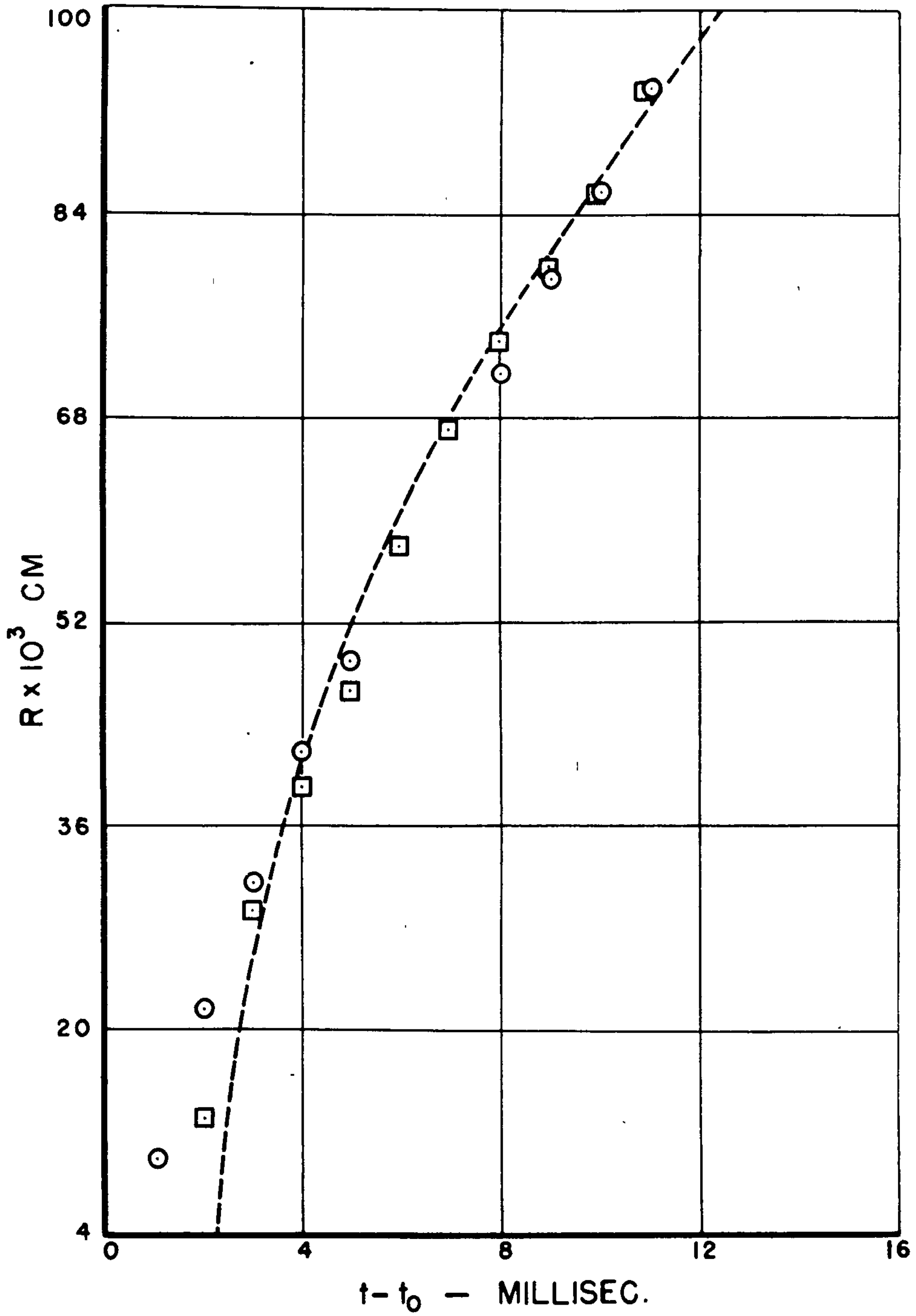

Figure 8. **Comparison of theoretical radius-time values with two sets of experimental values obtained for water at 1 atm external pressure, superheated to 105.3°C.**

EFFECT OF EXTERNAL PRESSURE ON VAPOR BUBBLE GROWTH

Some general remarks may be made concerning the growth of vapor bubbles under various superheat conditions and for different ambient pressures. We are still confining our interest to bulk liquid temperatures which do not change appreciably in times of bubble growth of the order of some milliseconds. A comparison will be made of the bubble growth in superheated water at 1 atmosphere with that in superheated water first under high pressure and secondly under low, or even negative pressures.

If water at 1 atmosphere pressure is superheated, for example, to 106°C, the Rayleigh velocity is 408 cm/sec. Because of the cooling effect, the maximum bubble growth velocity is 160 cm/sec. If we now consider water with a boiling point of 210°C at approximately 19 atmospheres pressure and if we take the same superheat of 6°C, the Rayleigh velocity is 1.39×10^3 cm/sec. The maximum bubble growth velocity is only 16.9 cm/sec. Rayleigh velocities and maximum bubble growth velocities are given in Table I for both 6°C and 3°C superheats for 1 atmosphere and 19 atmospheres-pressure. The striking difference in velocity behavior between the high pressure situation and the 1 atmosphere case is easily explained physically. The heat inflow to the bubble is directly dependent upon the magnitude of the vapor density ρ', and the vapor density

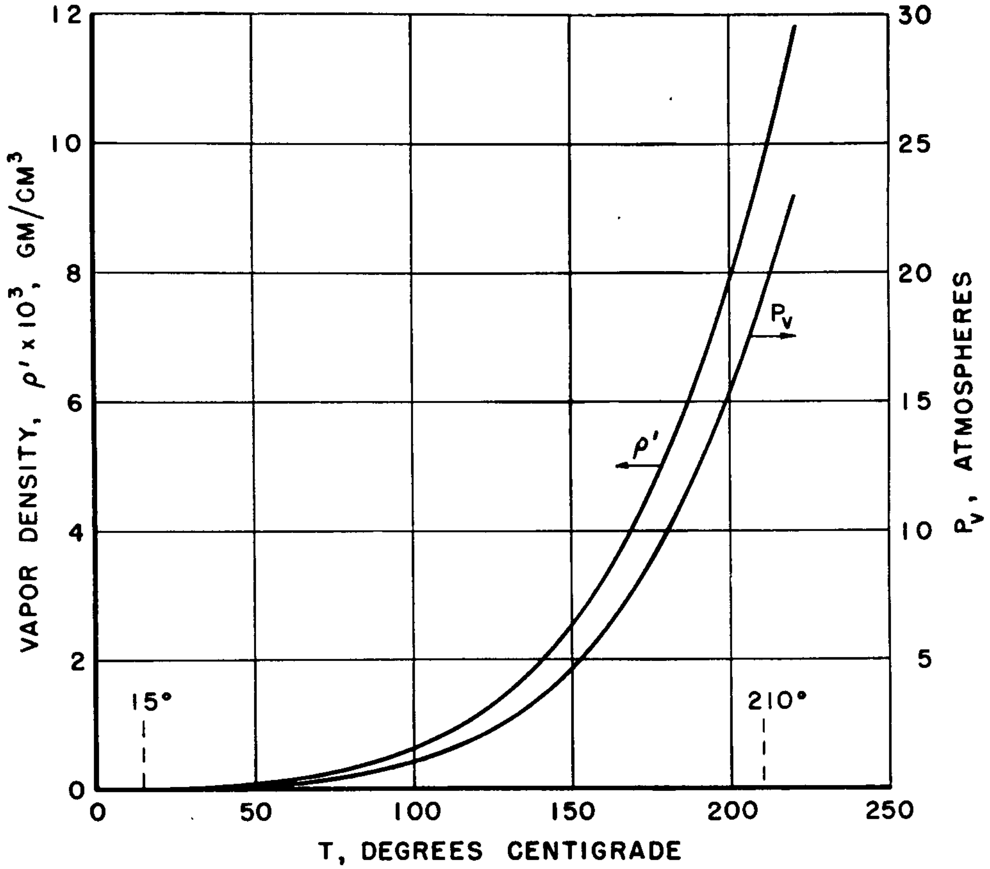

Figure 9. These curves give the variations in the equilibrium water vapor density and water vapor pressure with temperature.

is 16 times greater at 210°C than at 100°C (cf. Fig. 9 and Table II). One must therefore expect a larger cooling effect in the high temperature, high pressure condition. With this larger cooling effect goes also a larger decrease in the driving pressure

$$p_v(T) - P_0 \approx \rho A(T - T_b) \tag{30}$$

since the slope of the vapor pressure curve increases with temperature (cf. Fig. 10). For the specific case considered here, one has

$$\frac{A(210^\circ \text{ C})}{A(100^\circ \text{ C})} \approx 12. \tag{31}$$

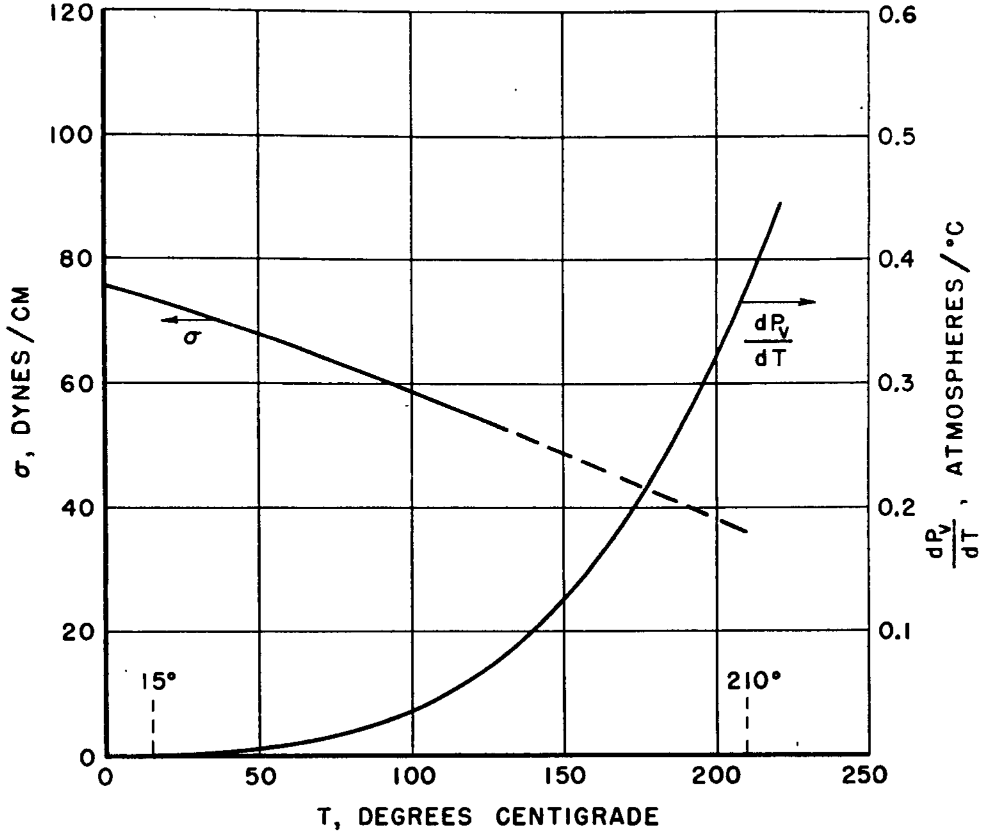

Figure 10. **These curves give the variations of the surface tension of water and of the vapor pressure derivative with temperature.**

We may consider the vapor bubble growth in incipient cavitation with the same arguments. Vapor bubble growth in incipient cavitation at, for example, a water temperature of 15°C is the same phenomenon as that which we have been considering here. A difference to be remarked is that the ambient pressure is often significantly less than zero, but the fact that one is dealing with tensions associated with negative external pressure does not alter the physical description of the bubble growth. It may be observed (Table II) that

$$\frac{\rho'(15^0\,\mathrm{C})}{\rho'(100^0\,\mathrm{C})} \approx 0.2, \tag{32}$$

and further that

$$\frac{A(15^0\,\mathrm{C})}{A(100^0\,\mathrm{C})} \approx 0.03. \tag{33}$$

The consequence of these values is clear: the thermal effects in the growth of cavitation bubbles are unimportant; only inertia effects are of significance. The Rayleigh velocity will be the growth velocity for such bubbles [5].

EFFECT OF RAPID INCREASE OF LIQUID TEMPERATURE

It has already been remarked that for the heating rates ordinarily encountered the temperature rise in the bulk liquid is unimportant when the bubble growth is followed for times of the order of 10 or 100 millisec. The only parameter which is affected by the rate of temperature rise is the delay time and the values which have been given above correspond to a rise in liquid temperature $a = 0.01°C/sec$. Very large rates of liquid temperature rise may be encountered in a power surge in a liquid-cooled or liquid-moderated nuclear reactor. For this reason solutions of the bubble growth equations have been carried through for the temperature increase rates given by $a = 200°C/sec$ and $a = 2{,}000°C/sec$; the former value corresponds roughly to temperature rise rates observed in some reactor safety tests [6].

The results of the calculations at 1 atmosphere pressure are shown in Figs. 11 and 12 for 3°C superheat, and in Figs. 13 and 14 for 6°C superheat. The results at approximately 19 atmospheres pressure are shown in Figs. 15 and 16 for 3°C superheat, and in Figs. 17 and 18 for 6°C superheat. the effect of increasing the rate of temperature rise, *a*, is easily understood. The delay time is decreased noticeably, but the value of $\dot{R}_{\max}$ is essentially unchanged. The maximum growth velocity occurs so early in the bubble history that the bulk liquid temperature is almost unchanged from its initial value. As the bubble growth goes into the asymptotic phase, the radius-time curves are very nearly parallel and the asymptotic behavior is given as before by $R \sim t^{1/2}$. This variation of bubble radius with time cannot persist for too long a time for large values of *a*. It may be shown that the bubble radius changes from a $t^{1/2}$ variation to a $t^{3/2}$ variation at later times. The details of this analysis will be presented elsewhere.

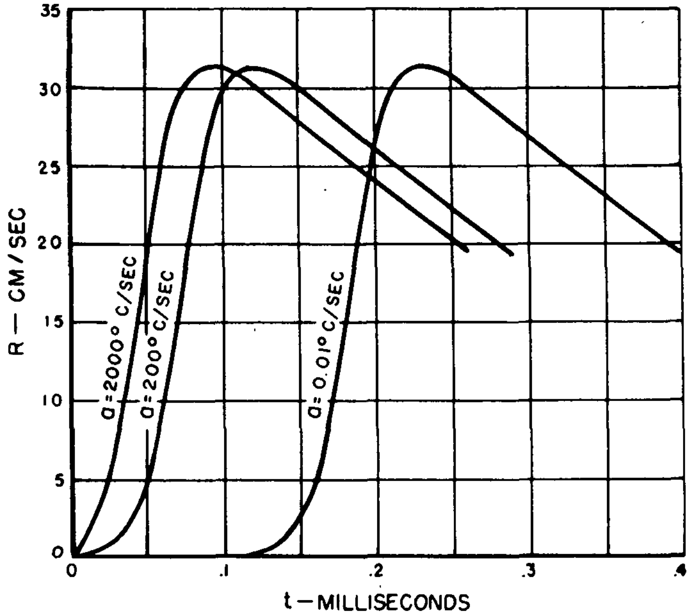

Figure 11. Bubble growth velocities are shown as functions of time for an initial temperature of 103°C; the external pressure is 1 atm. The rate of bulk liquid temperature rise is given by a.

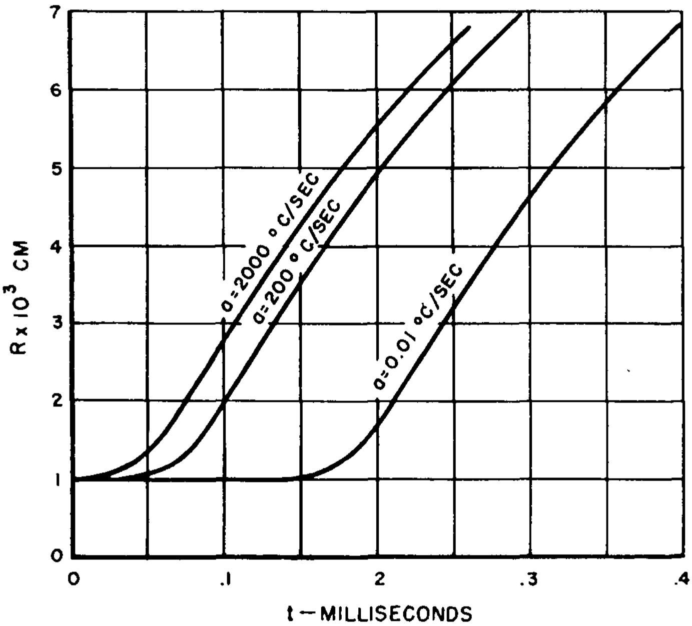

Figure 12. Bubble radius as a function of time is shown for 103°C initial temperature and 1 atm external pressure.

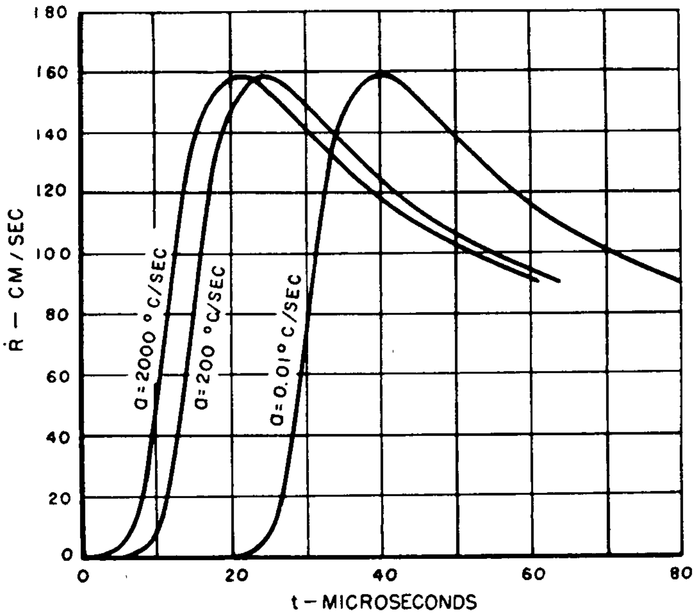

Figure 13. Bubble growth velocities are shown as functions of time for an initial temperature of 106°C; the external pressure is 1atm.

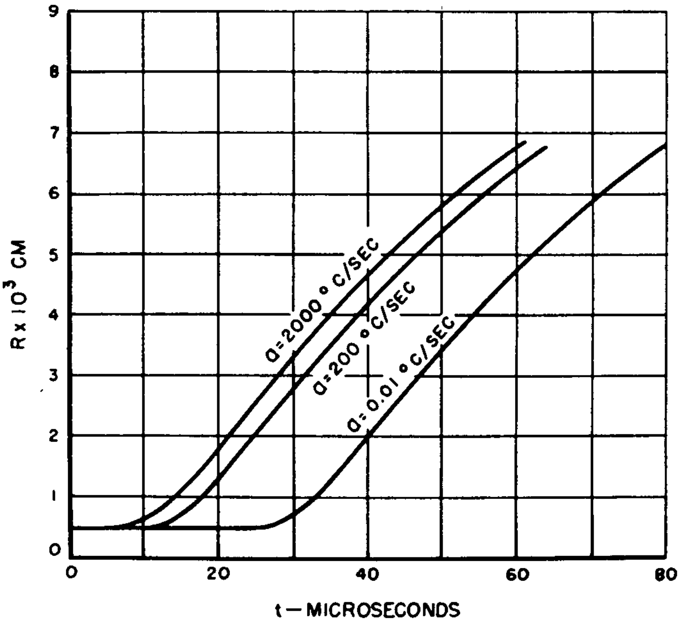

Figure 14. Bubble radius as a function of time is shown for 106°C initial temperature and 1 atm pressure.

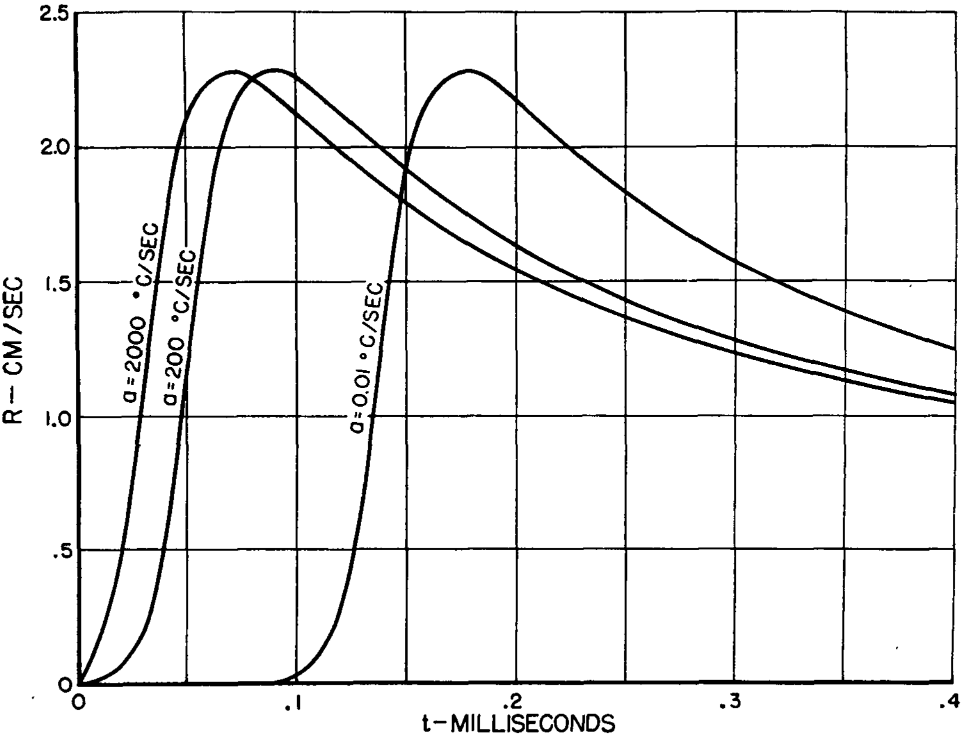

Figure 15. **Bubble growth velocities are shown as functions of time for an initial temperature of 213°C; the external pressure is approximately 19 atmos (boiling point 210°C).**

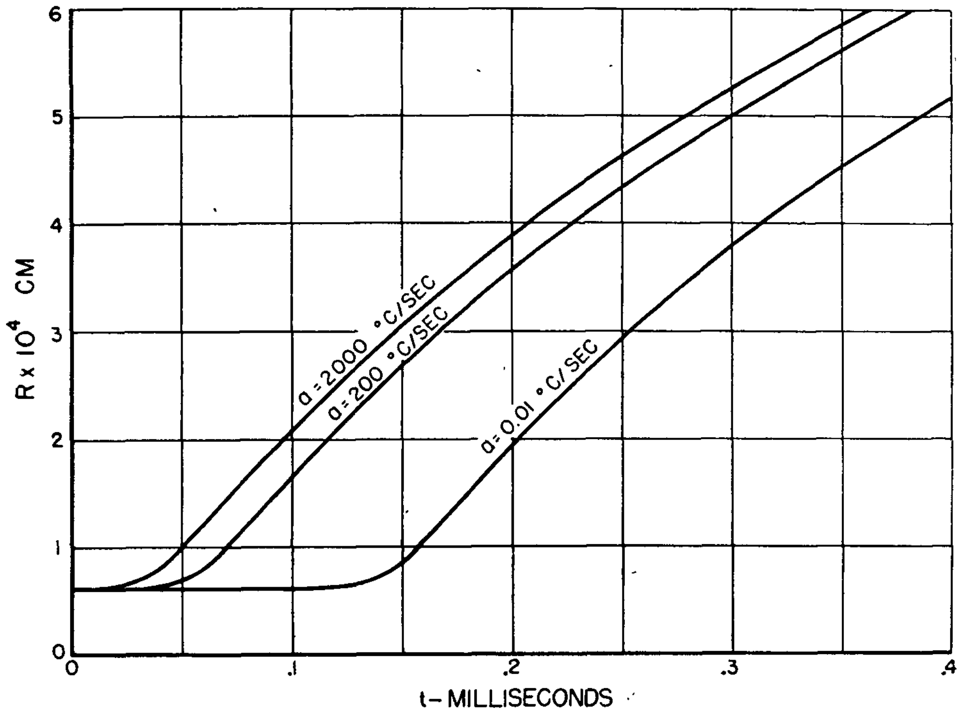

Figure 16. **Bubble radius is shown as a function of time for the conditions of Fig. 15.**

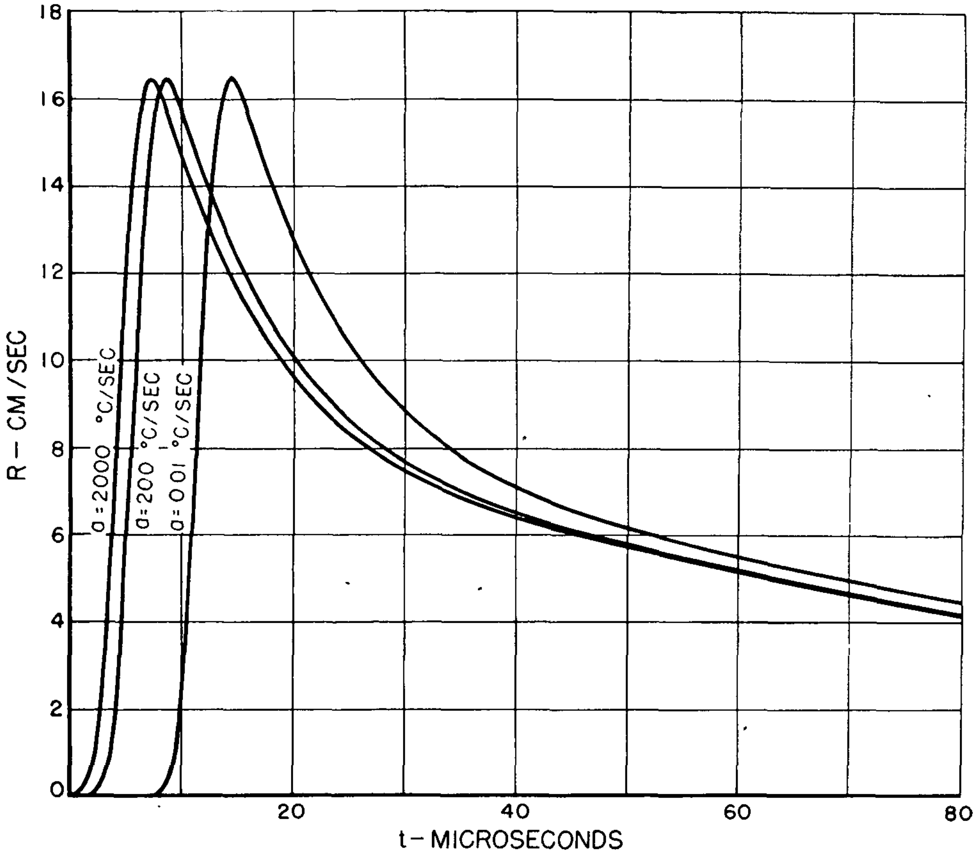

Figure 17. **Bubble growth velocities are shown as functions of time for an initial temperature of 216°C; the external pressure is approximately 19 atmos (boiling point 210°C).**

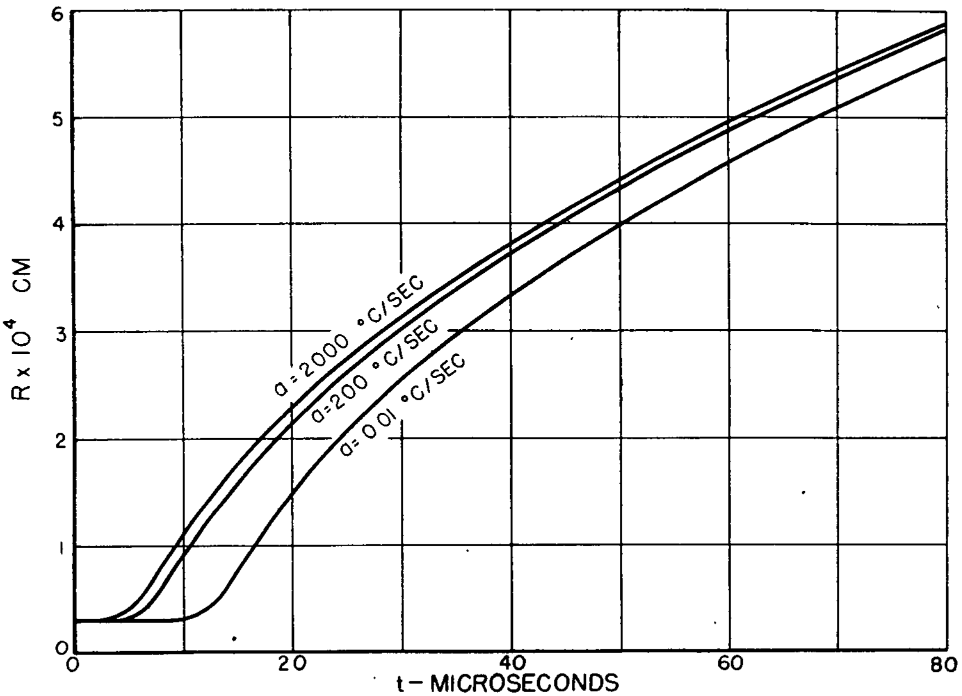

Figure 18. **Bubble radius is shown as a function of time for the conditions of Fig. 17.**

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TABLE I

T_o °C	216	213	106	103
Rayleigh velocity cm/sec	13.9×10^2	9.62×10^2	4.08×10^2	2.82×10^2
$\dot{R}_{max}$ cm/sec	16.9	2.28	159	32.7
$P_o \times 10^{-6}$ dynes/cm^2	19.1	19.1	1.01	1.01
R_o cm	2.96×10^{-5}	6.12×10^{-5}	4.83×10^{-4}	1.02×10^{-3}

TABLE II

PHYSICAL CONSTANTS FOR WATER VAPOR AND WATER

T°C	15	100	210
Water Vapor			
$\mu \times 10^4$ gm/cm • sec	.920	1.26	1.66
$k \times 10^{-3}$ erg/cm • sec • °C		2.4	3.2
$c_v \times 10^{-7}$ erg/gm • °C	1.4	1.4	1.4
$\rho_{eq} \times 10^4$ gm/cm³	.129	5.98	95.9
$D = k/\rho_{eq}\, c_v$ cm²/sec		.29	.024
Gas constant/gm of water vapor:		$B = 4.61 \times 10^6$ erg/gm °C	
Water			
$\mu \times 10^3$ gm/cm •sec	11.4	2.84	1.30
$k \times 10^{-1}$ erg/cm • sec • °C	5.87	6.80	6.59
$c_l \times 10^{-7}$ erg/gm • °C	4.17	3.76	3.29
$L \times 10^{-10}$ erg/gm	2.46	2.26	1.90
ρ gm/cm³	.999	.958	.853
$P_{v_{eq}} \times 10^{-6}$ dynes/cm²	.0170	1.01	19.1
σ dynes/cm	73.5	58.8	~36
$D \times 10^3$ cm²/sec	1.41	1.89	2.35
$A \times 10^{-3} = \dfrac{dP_v/dT}{\rho}\ \dfrac{\text{dyne cm}}{\text{gm °C}}$	1.10	37.7	447.

DISCUSSION

R. W. L. Gawn

I hope you won't think I have gone too far from the strictly physical aspects of Professor Plesset's excellent lecture if I talk about naval architecture and the physical cavitation on ships.

You probably know the physical effects of cavitation as far as a ship is concerned are objectionable in every sense: speed is lost, propeller blades vibrate or sing, noise is emitted, and the material of the propeller is eaten away.

Now despite many years of endeavor and all sorts of research, these defects are still manifest in different degrees depending on the class of the ship and its speed.

The propeller designer is guided in the suppression or limitation of the cavitation by model experiments, but the value of the information thus obtained depends on the extent to which the tests can be relied upon as representative of conditions on the ship.

In the past, models have been tested in axial flow water tunnels at various speeds and pressures, but there is a growing recognition of the necessity of reproducing the velocity field of the ship in magnitude and direction, and facilities are being provided accordingly, one aim being the test of an actual hull-propeller combination. And this we have been doing at Haslar.

This may prove a valuable step forward, although similitude between model and ship conditions will not be completely realized.

Generally, velocity, pressure and density are correlated in the parameter known as the cavitation number. Other physical qualities are involved, such as gas and solids in solution and in suspension, surface tension and viscosity in sea water. I am not sure we know much about the surface tension of sea water. We know a little bit about viscosity.

These may not all be sufficiently significant to call for close matching, but there is at present little information on this, and research is essential for clarification.

Notable researches have been carried out with fresh water and with pure water, but ocean water has received scant attention. Until the relevant physical properties of the sea are known, the ship model experimenter will not be aware of the basic conditions that should be simulated in his propeller cavitation tests.

The malignant disease known as propeller erosion, first recognized about 60 years ago, is still with us despite a good deal of improvement in propellers as regards hydrodynamic design, metallurgy, and the properties of the material.

The root of the propeller is the region most frequently affected, and close attention is directed toward shaping the root sections, root fillets, and propeller boss, accordingly.

Collapse pressure of bubbles and corrosion, separately or in combination, have been adduced as causes and with different emphasis, according to the particular investigator concerned. Perhaps the author would comment on this, and on the recent claims to the effect that the life of the propeller can be prolonged by local cathodic protection.

Most of the bubbles in the boiling water around a ship's propellers are so minute as to provoke the thought that it should be possible to devise means for their elimination. An ad hoc attempt by myself to achieve this by injecting silicones into the water to reduce the surface tensions was not at all encouraging. Perhaps the author would remark on the possibility of such direct attack on the bubbles to remove them as soon as they are formed.

K. F. Herzfeld

I would like to make some comments on the nuclei which are necessary as origin for a small bubble which will then grow, either in boiling or cavitation. The first question is whether bubbles start preferentially at a smooth wall, as compared to the interior of the liquid. While there does not, to my knowledge, exist any systematic investigation of the subject, I have an impression that a smooth wall is not a favorite origin for bubbles. On the other hand, some photographs by Dr. Snay seem to contradict that; here cavitation bubbles form on the smooth lucite wall, either upon reflection of the shock wave or in the negative phase of the bubble pulse. On the other hand, a wad of glass wool is a favorite starting place [1], and it is known that some rough pieces intiate boiling.

Next, what is the nature of the nuclei in the liquid? Fox and I had proposed [2] that they are microscopic gas bubbles, covered by an organic skin, which prevents them from dissolving. This could explain why the previous application of pressure prevents the start of cavitation, since it would crush the skin. However if this were so, there should be a lower limiting pressure—that necessary to crush the skin—below which previously applied pressure would have no effect on cavitation. This disagrees with experiment. According to Strasberg [3] there is no such lower limit, but the tension necessary for cavitation to set in under influence of ultrasonic waves increases smoothly with the amount of pressure previously applied. These and other experiments seem to exclude the assumption of gas bubbles with organic cover as necessary cavitation nuclei.

This leaves as the other alternative solid particles. These must be non-wetted by the liquid and carry undissolved gas, probably in a crack, which acts as the starter of a growing bubble. This assumption has been mainly proposed by E. Meyer at the University of Goettingen and experiments carried out with iron oxide seem to confirm it.

A few theoretical remarks should be added on the origin of cavitation damage. On principle a collapsing bubble produces a very high pressure and a high temperature. If the shock wave produced in the metal were plane I doubt that it would produce pitting; a purely compressional wave does not produce such damage. On the other hand, due to the local concentration, there is probably some shear involved, which might be responsible for damage.

If the mechanism is due to high temperature, it might be threefold: direct, by softening or melting; by sudden liberation of gases previously dissolved in the metal;

by chemical reaction, which is in line with Mr. Gawn's remark, since according to Griffing [4], free radicals are produced in the hot gas of a collapsing bubble.

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F. R. Gilmore

The behavior of expanding vapor bubbles seems to be very well understood, thanks to the good work of Professor Plesset and his students. I would like only to make a few remarks on the related problem of collapsing vapor bubbles. Several years ago I developed a theory which tries to take into account the compressibility of the liquid, a factor which is important when the inward bubble wall velocity approaches or exceeds the velocity of sound in the liquid [1]. This analysis is based on an hypothesis due to Kirkwood and Bethe [2], the accuracy of which is very difficult to assess. To check the validity of this analysis, I have integrated numerically the partial differential equations for a bubble in water, collapsing under a constant pressure difference of one atmosphere, using the "method of characteristics" and an IBM 604 computer. The results are shown in Fig. 1.* The calculated bubble-wall velocity agrees within 6 per cent with that given by the analytic theory, over the calculated range of 0.2 to 4.9 times the velocity of sound in water. The inward velocity thus appears to increase like the radius to the minus one-half power as the radius becomes small, instead of the minus first power given by acoustic theory, or the minus three-halves power given by incompressible theory.

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H. S. Preiser

I would like to amplify the remarks of one of the previous speakers on the possibility of cathodic protection relieving cavitation damage.

Several experiments to this end were carried on in Italy during 1949 by Professor G. Petracchi. Small metal specimens were suspended in the turbulent throat of the venturi section of a water tunnel apparatus. Cavitation bubble collapse was produced on the specimens. A method was incorporated into the experiment whereby the specimens could be made the cathode of a simple electrolytic cell. Pit depth and weight-loss measurements were taken on uncoupled specimens exposed to similar environments. Those specimens under cathodic influence exhibited material resistance to destruction when compared to similar specimens not under cathodic protection.

There is ample opportunity to investigate the influence cathodic protection has on cavitation damage by suitable experimental techniques. In fact, the Bureau of Ships has set up such a program at the Boston Naval Shipyard. A model propeller has been designed specially to produce excessive cavitation on the blades when rotating at appropriate speeds. The cavitation pattern on the flat blades (bevelled edges) of the propeller is controlled by variation of the pitch ratio and rotational speed. The pro-

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* Ed note: This figure appears on page 280, in Gilmore's discussion of the paper by Fitzpatrick and Strasberg.

peller operates in the open sea water of the Boston Harbor from a barge which serves as a supporting platform. The propeller rig is fitted with suitable brush arrangements to permit cathodic protection to be applied to individual blades. A silver-silver chloride reference electrode is built into the plastic hub cap.

The purpose of these Boston experiments is to separate the electro-chemical aspects of cavitation damage from the purely mechanical effect. It is assumed that the total damage in a cavitating environment is the combination of corrosion and impingement attack where the total damage is greater than the sum of each individual component of the damage. It is felt that by arresting the corrosion damage by means of cathodic protection, the severity of mechanical damage can be reduced. So far due to operating deficiencies in the drive mechanism, no data are available. Progress is going forward at present and some information should be reported within the ensuing year.

Another phenomenon which occurs on a metal surface as a result of a cathodic reaction is the occlusion of hydrogen. The hydrogen atoms adhere to the surface of the metal until removed by oxygen depolarization, mechanical washing or overvoltage in which the atomic hydrogen combines to the molecular state and bubbles off as a gas. One theory worthy of investigation is whether the vapor bubble produced by cavitation boiling can combine with the hydrogen bubble when impinging on a cathodic surface to produce a cushioning effect because of its partial compressibility. ONR is sponsoring some exploratory research on the cavitation damage under cathodic protection conditions at the Hydrodynamic Laboratory of the California Institute of Technology.

One other point worthy of mention in a discussion of this nature. Corrosion reactions are comparatively slow and therefore accelerated cavitation testing techniques such as the ultrasonic device used by Professor A. T. Ellis may prevent corrosion reactions from having any influence on the total damage produced. Perhaps exposing specimens under cavitation pulses followed by short intervals of quiescense will reveal the more significant data regarding the role of corrosion in a cavitating environment.

P. Eisenberg

Mr. Preiser has referred to some suggestions that I have made concerning the nature of cathodic protection against cavitation damage. I should like here to elaborate somewhat these ideas and thus give the background for the experiments referred to as being ONR sponsored, i.e., work by Dr. A. T. Ellis at the California Institute of Technology.

As the basis for his experiments on cathodic protection, Petracchi (*La Metallurgia Italiana,* Vol. 41, No. 1, Jan.-Feb., 1949) postulated that cavitation damage is largely attributable to anodic corrosion associated with currents set up between adjacent crystals of the material, such currents resulting from deformation caused by the mechanical stresses produced by collapsing cavities. Actually, similar ideas had been suggested prior to Petracchi's paper—thermal stressing being assumed responsible for such currents. Petracchi applied cathodic protection to a number of specimens in sea water and obtained marked reductions in damage even with very small current densities. On the other hand, experiments can be cited in which very large currents were needed to show any protection. Furthermore, there is the question of what happens in non-conducting materials which are nevertheless susceptible to cavitation damage. Thus, it is necessary to determine the mechanism of cathodic protection in detail—the practical importance is evident.

The idea of protection produced by a blanket of hydrogen evolved at the cathode (in much the same way as large quantities of air introduced into cavitating turbines) suggests itself immediately. However, for such protection it is likely that extremely large current densities are required to produce sufficient quantities of gas to cushion the cavity pressures. In looking for another mechanism in which the current densities would be more commensurate with those of Petracchi's experiments, one can postulate a highly efficient protection system if the hydrogen is introduced directly into the cavities to form a cushion. This may actually occur if hydrogen bubbles (even though microscopic in size) are introduced into the low pressure region since cavities will tend to form preferentially about such bubbles acting as the required nuclei.

Determination of the mechanism of such cathodic protection is complicated by the protection provided against electrochemical corrosion in a corrosive medium, and Petracchi's experiments are not definitive in this respect. It seems clear that mechanical damage will be reduced if attack occurs only on non-corroded material as it would be in a cathodically-protected system.

To attempt to separate the effects of cathodic protection for mechanical vs. electrochemical corrosion and to determine the validity of the mechanism suggested in the foregoing is the goal of work at the California Institute of Technology.

G. K. Batchelor

I have a question, sir. I am not sure whether it should be addressed to the old cock or the the young cock. Perhaps I can address it to the two of them collectively.

The question is this. Can they give us an authoritative statement on the importance of radiation damping during the collapse of a vapour-filled cavity? I refer to the conversion of some of the kinetic energy of the water into sound waves, which are lost from the neighborhood of the cavity.

As a layman to the literature, I have got the impression it is believed to be a small effect. As against that, some calculations by Proudman at Cambridge concerning the collapse of a cavity containing some permanent gas have shown that acoustic radiation might account for as much as 50 per cent of the total kinetic energy in one cycle, under conditions which would be typical for cavities occurring in sea water in the neighborhood of propellers.

M. S. Plesset

I should like to refer Professor Batchelor's question to Dr. Gilmore. He has given this problem some attention.

Concerning the problem of cavitation damage which has been discussed by Mr. Gawn, this is indeed a large subject as Mr. Gawn has mentioned. I appreciate his comments as well as those of Mr. Eisenberg and Mr. Preiser.

I shall now defer to Dr. Gilmore for a discussion of the question posed by Professor Batchelor.

F. R. Gilmore

I really do not know what fraction of the bubble's energy is radiated away acoustically during this period, but I suspect it is not a large fraction. There are, however, basic difficulties in defining the acoustic energy radiated during such a period, because most of the flow energy is still in the neighborhood of the bubble, where amplitudes are so large that the acoustic relations do not apply.

Whether this flow energy is ultimately radiated away acoustically or remains to re-expand the bubble may well depend upon processes occurring near the very end of collapse. For example, experimental evidence indicates that a bubble containing no permanent gases may collapse completely and not rebound at all. In such a case, one might say that at the time of complete collapse all of the bubble energy has been radiated away acoustically and/or dissipated viscously (for example, by a shock wave), unless one prefers to say that the subsequent production of a pressure pulse by the liquid which is still moving inward at the collapse time represents acoustic energy produced *after* the bubble has vanished. On the other hand, evidence indicates that presence of only a small amount of air in a bubble can cause the bubble to rebound to a considerable fraction of its original size. Moreover, only a part of this loss in energy may be due to acoustic radiation, since dissipation in the compressed gas and in the liquid near the wall (which becomes very irregular and nonspherical during the rebound) may be important.

M. Strasberg

I believe there is a paper by Dr. Kennard in which a statement is made of the amount of acoustic energy radiated from a pulsating explosive bubble.

I don't remember what you arrived at, Dr. Kennard.

E. H. Kennard

I don't either, but if anybody writes me I shall send a copy of the paper. The statement about radiated energy was incidental.