

PRESSURE FLUCTUATIONS ON THE WALL ADJACENT TO A TURBULENT BOUNDARY LAYER

Mark Harrison
David Taylor Model Basin

* * * * *

A fully developed turbulent boundary layer with a zero pressure gradient has been studied in a subsonic wind tunnel. The pressure fluctuations on the wall of the wind tunnel were measured by small flush-mounted microphones. Data for the dimensionless spectral density of the pressure fluctuations, $P(f)/\rho^2 U_0^3 \delta^*$, as a function of the frequency parameter $f \delta^*/U_0$ are presented; $P(f)$, ρ , U_0 , δ^* , and f are respectively the spectral density, fluid density, free stream velocity, boundary layer displacement thickness, and frequency. Data for the coefficient

$$\overline{(p^2)}^{1/2} / 1/2 \rho U_0^2$$

are also presented. The transverse cross correlation for the pressure fluctuations was studied by using two flush-mounted microphones. Rather than the longitudinal cross correlation, the longitudinal cross-spectral density was studied, yielding a measure of the coherence of the pressure fluctuations at two points as a function of fx/U_c , where x is the longitudinal spacing of the two points and U_c is the effective convection velocity of the spatial pattern.

* * * * *

INTRODUCTION

Pressure fluctuations necessarily coexist with velocity fluctuations in a turbulent flow. For incompressible flow, the pressure and velocity are related by the equation

$$\nabla^2 p = -\rho \frac{\partial^2 u_i u_j}{\partial x_i \partial x_j} \quad (1)$$

which can be obtained by combining the continuity and the Navier-Stokes equations. The solution to this equation is given in terms of an integration over all space and shows that for the case of boundary-layer flow, the vanishing of the velocity fluctuation at the wall does not imply the vanishing of the pressure fluctuations at the wall.

It is these pressure fluctuations on the wall adjacent to the turbulent boundary layer that are the focus of interest in this study. Some measurements of the characteristics of these pressure fluctuations are reported and the meaning of the measurements are interpreted.

There are several reasons for being interested in the characteristics of the pressure fluctuations. A direct and immediate need exists for data in the field of noise due to unsteady flow. When a turbulent boundary layer develops on a flexible wall, the motion of the wall which results from the pressure fluctuations generates a sound field. Some of the noise in aircraft has been attributed to this mechanism. Kraichnan (1,2) has given a theoretical discussion of this problem which begins with the interpretation of Eq. (1) for the case of boundary-layer flow.

There are also fundamental reasons for being interested in the pressure fluctuations associated with a turbulent boundary layer. There is considerable uncertainty as to how energy is produced, convected, diffused, and dissipated in the turbulent boundary layer. In particular, the role of pressure fluctuations is but vaguely understood. Although this work does not measure the pressure fluctuations inside the boundary layer, it does constitute a start on this very challenging problem and it is believed that a further extension of this work will prove valuable.

To obtain the data for this report, the boundary layer on the wall of a subsonic wind tunnel was studied. Pressure transducers with a very small active area were mounted flush with the wall. Using conventional sound analysis equipment consisting of a narrow tunable filter and an rms meter, it was possible to obtain the spectral density* of the pressure fluctuations at a point on the wall.

By measuring the pressure fluctuations in the total frequency range, it was possible to obtain the mean-square value for the pressure fluctuations. This value was also checked by integrating over the spectral density.

By using two pressure transducers and studying the correlation and the cross-spectral density* between the pressure fluctuations at the two points, it was possible to study the spatial pattern of the pressure fluctuations and how rapidly it evolves as it is convected downstream. By measuring the time delay necessary to maximize the correlation between the two points, it was possible to measure the velocity at which the pattern was convected.

EXPERIMENTAL APPARATUS

The Wind Tunnel

The wind tunnel was a closed-circuit subsonic tunnel. The velocity range of the tunnel was from 50 to 200 ft/sec. The transverse dimensions of the working section were 20 by 15 inches. The measurements were performed on the wall of the working section, 5 feet downstream from the entrance nozzle. At this point the displacement thickness of the boundary layer was 0.105 inch at a velocity of 100 ft/sec. By comparing the mean velocity profiles with those given by Klebanoff and Diehl (3) it was judged that the boundary layer was nearly well enough developed to show similarity. Unfortunately, the working section was not long enough to permit working further downstream so as to prove that the boundary layer was exhibiting similarity.

The Pressure Transducer

The pressure transducer was an adaptation of the Altec 21-BR-180 open-faced microphone (4). This microphone was modified as shown in Fig. 1 to produce the transducer used for the spectral density measurements. A different modification was

*This term will be defined in the section giving the results of the measurements.

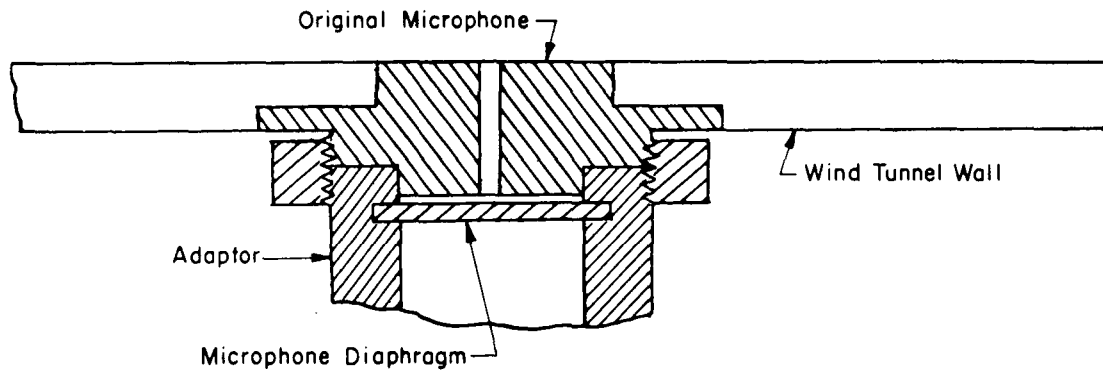


Fig. 1 - Schematic of adaptation of the Altex 21-BR-180 microphone for spectral density measurements

required for the correlation measurements. In both cases the upper frequency limit is determined by a resonance between the mass of the air in the orifice and the stiffness of the air in the chamber adjacent to the diaphragm. Well above this resonance frequency the sensitivity is inversely proportional to the frequency range. The resonant response was damped by a cotton plug in the orifice so that some of this frequency range was salvaged.

For the correlation measurements it was necessary to measure at two points separated by a small distance. This necessitated the use of the transducer in the form shown in Fig. 2. Unfortunately, this arrangement resulted in a much lower resonant frequency due to increased mass in the tube. Using this transducer, arrangements were made to have a series of holes spaced at 0.125 inch apart in the transverse and longitudinal directions which permitted correlation measurements to be made up to 1.5 inches in 0.125-inch steps.

The sensitivity of the transducer was -61 db below 1 volt for 1 dyne/cm². For the spectral density measurements the usable frequency range was up to 8500 cps and for the correlation measurements the usable frequency range was up to 2000 cps. The lower limit on the frequency range was imposed by wind-tunnel noise and not the transducer.

In order to prove that the active area of the transducer was sufficiently small to insure that the pressure was coherent over its entire face, various size orifices were used. It was found that orifices up to 0.125 inch in diameter were satisfactory for obtaining the data on spectral densities. Smaller orifices were used for the correlation measurements in order to improve the accuracy in the measurement of the separation distance between the transducers.

Electrical Equipment

The spectral density of the pressure fluctuations was obtained by the use of the Muirhead-Pametrada wave analyzer. A frequency band of a nominal 10 percent was used in the analysis. The levels were read on the Ballantine True-rms voltmeter. In the section giving the results of the measurements, the details of how to obtain the spectral density from the observed data are given.

The correlation between the pressure fluctuations at two points was measured using rather simple apparatus. The basis for the method used is contained in the algebraic identity

$$\frac{\overline{e_1 e_2}}{(\overline{e_1^2 e_2^2})^{1/2}} = \frac{(\overline{(e_1 + e_2)^2} - \overline{(e_1 - e_2)^2})}{4(\overline{e_1^2 e_2^2})^{1/2}}$$

where the term on the left is the correlation between e_1 and e_2 . The bars indicate time averages. e_1 and e_2 are here regarded as voltages that are proportioned to the fluctuating pressures $p_1(t)$ and $p_2(t)$, respectively, where the subscripts denote the two points at which the measurements are performed. With reference to Fig. 3, the addition and subtraction of the voltages were accomplished by the transformer. The time averages were accomplished by the Ballantine True-rms voltmeter, whose time constant had been increased to 2 seconds. This produced satisfactory smoothing of the fluctuations. Measuring the correlations is then a matter of making four meter readings, $[(\overline{e_1 + e_2})^2]^{1/2}$, $[(\overline{e_1 - e_2})^2]^{1/2}$, $(\overline{e_1^2})^{1/2}$, and $(\overline{e_2^2})^{1/2}$, and doing the computations indicated by the above algebraic identity.

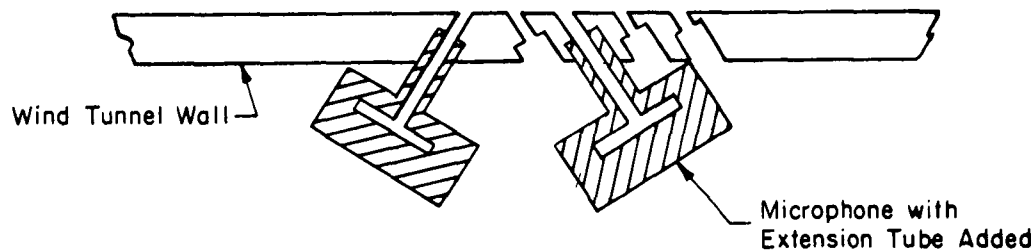


Fig. 2 - Schematic of microphone adapted for cross-spectral and correlation measurements

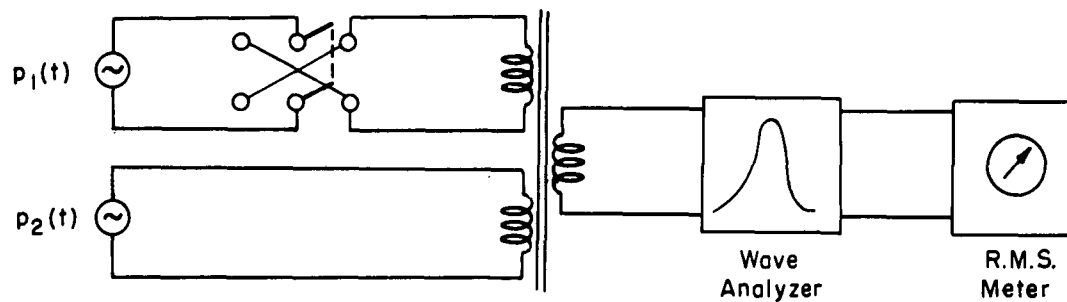


Fig. 3 - Schematic diagram of the equipment used in the measurement of the cross-spectral density

The wave analyzer shown in Fig. 3 serves the function of measuring the correlation in a restricted frequency band. As explained later, if observations are made using a sufficiently narrow frequency band, it is possible to use these measurements to obtain the cross-spectral density between the pressure fluctuations at two points.

The convection velocity for the partially frozen pattern of the pressure fluctuations was determined by examining the correlation and cross-spectral density with time delays. These were produced by a delay line consisting of a multistage low-pass filter. This delay line gave time delays in 40-microsecond steps up to 6 milliseconds. The delay line had a usable frequency range from zero to a little above 5000 cps.

EXPERIMENTAL RESULTS

Spectral Density of the Pressure Fluctuations

The spectral density of the pressure fluctuations $P(f)$ is defined (5) as

$$P(f) = \lim_{T \rightarrow \infty} \frac{1}{T} |A_T(f)|^2$$

where

$$A_T(f) = \int_{-T}^T p(t) e^{-i\omega t} dt$$

is the Fourier transform of the pressure fluctuations over the time $2T$.

The basis for the measurement procedure used in obtaining the spectral density begins with the fact that the spectral density and the autocorrelation function of $p(t)$ are Fourier transforms (5).

$$R(\tau) = \overline{p(t)p(t+\tau)} = \int_0^\infty P(f) \cos \omega\tau df.$$

It then follows that

$$R(0) = \overline{p^2} = \int_0^\infty P(f) df.$$

According to this equation one can experimentally obtain the spectral density by measuring the mean-square output p_0^2 of a narrow tunable filter of band with Δf and unity gain.

That is to say,

$$P(f) = \lim_{\Delta f \rightarrow 0} \frac{\overline{p_0^2}}{\Delta f}.$$

This was the procedure used in the work reported here.

Pressure Fluctuations in Turbulent Boundary Layer

It is desirable to present the measured data in dimensionless form. Since the spectral density of the pressure fluctuations has the dimensions of (pressure)² × time, the dimensionless spectral density was chosen to be $P(f)/\rho^2 U_0^3 \delta^*$ where ρ , U_0 , and δ^* are the fluid density, free stream velocity, and boundary layer displacement thickness, respectively. The frequency f was nondimensionalized as $f \delta^*/U_0$.

The dimensionless spectral density curves are shown in Fig. 4. The data exhibit some lack of similarity over the velocity range studied. As mentioned earlier, the work section of the tunnel was not long enough to permit observations further downstream so as to prove that the boundary layer was exhibiting similarity.

The coefficient

$$K = \frac{\overline{(p^2)}^{1/2}}{\frac{1}{2} \rho U_0^2}$$

can be obtained either by integrating over the spectral density or by direct measurement of $\overline{p^2}$. In view of the lack of similarity exhibited in Fig. 4, a definite value cannot be obtained. The values obtained for K have an average value of about 9.5×10^{-3} which is in excess of the value obtained by Willmarth (6).

A surprising feature of the spectral density is that it does not decrease appreciably at the lowest frequencies measured. For these frequencies, which correspond to wave numbers much smaller than the reciprocal of the characteristic lengths of the boundary layer, Kraichnan (1) has argued that the spectral density should be proportional to the square of the frequency. There is some evidence that intermittency of the boundary layer has a strong influence on the pressure fluctuations at the lower frequency. Qualitative observations at the lower frequencies revealed an intermittency in the amplitude of the spectral density which could not be accounted for by the

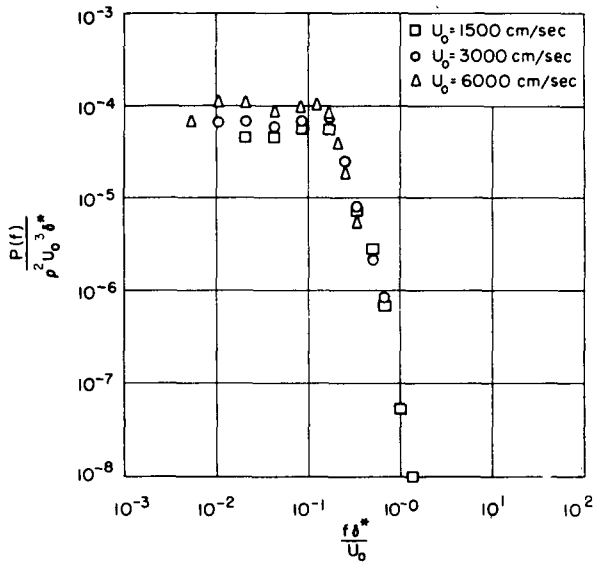


Fig. 4 - Spectral density of the pressure fluctuations

randomness of the process. This intermittency gave pressure versus time records not unlike those observed by Klebanoff (7) for the velocity fluctuations in the boundary layer. This type of behavior diminished with increasing frequency and was not noticeable for values of $f\delta^*/U_0$ exceeding 4×10^{-2} . Further support for the possibility that intermittency plays a role at the lower frequencies can be found in the fact that at these frequencies, the measurements of the convection velocity yielded a value of about $0.8 U_0$. That part of the boundary layer that moves with a mean velocity of $0.8 U_0$ is strongly intermittent.

The validity of the above conjectures could be established by repeating this study using fully developed pipe flow.

In view of the preceding, it would have been desirable to have procured data at lower frequencies. Unfortunately, this was not possible due to the tunnel noise which limited the lowest usable frequencies to between 100 and 200 cps. The signal-to-noise ratio above this range was always in excess of 20 db. This figure was obtained by placing the pressure pickup in the middle of the working section of the tunnel where the flow was essentially laminar. Actually, some noise was undoubtedly induced by the flow around the microphone, so that a better signal-to-noise ratio might actually have existed at the wall.

Vibration pickup by the transducer was not a problem. By simply plugging up the hole in the transducer, the vibration excitation could be measured, which in this case was too low to even consider.

Longitudinal Cross-Spectral Density

In this work it was decided to study the longitudinal cross-spectral density rather than the longitudinal correlation. Since this is a novel approach, a few words of explanation are needed in order to explain the measurement procedure.

The cross-spectral density of the pressure fluctuations is defined as

$$P_{12}(f) = \lim_{T \rightarrow \infty} \frac{1}{T} A_T^*(f) B_T(f)$$

where the asterisk denotes the complex conjugate, and where

$$A_T(f) = \int_{-T}^T p_1(t) e^{-i\omega t} dt$$

$$B_T(f) = \int_{-T}^T p_2(t) e^{-i\omega t} dt$$

with the subscripts 1 and 2 denoting the two observation points.

Using the arguments of Ref. 5 which show that the spectral density and the auto-correlation are Fourier transforms, it can be shown that the cross correlation and the cross-spectral density are also Fourier transforms:

$$R_{12}(\tau) = \overline{p_1(t)p_2(t+\tau)} = \frac{1}{2} \int_{-\infty}^{\infty} P_{12}(f) e^{i\omega\tau} df.$$

Since $P_{12}(f)$ is complex, we can write

$$P_{12}(f) = U_{12}(f) + iV_{12}(f)$$

where it follows from the definition of $P_{12}(f)$ that the real functions $U_{12}(f)$ and $V_{12}(f)$ are even and odd, respectively. Accordingly, we can write

$$R_{12}(\tau) = \int_0^{\infty} U_{12}(f) \cos \omega \tau \, df - \int_0^{\infty} V_{12}(f) \sin \omega \tau \, df .$$

This last equation provides the basis for an experimental method of measuring the real and imaginary parts of the cross-spectral density. If the cross correlation is measured with zero time delay, then

$$R_{12}(0) = \int_0^{\infty} U_{12}(f) \, df .$$

If the cross correlation is measured in a narrow band of frequencies Δf as shown in Fig. 3, then

$$U_{12}(f) = \frac{{}_f R_{12}(0)}{\Delta f}$$

where ${}_f R_{12}(0)$ denotes the cross correlation measured in the narrow frequency band. The real part of the normalized cross-spectral density $u_{12}(f)$ is then

$$u_{12}(f) = \frac{U_{12}(f)}{[P_1(f)P_2(f)]^{1/2}} .$$

Similarly, $v_{12}(f)$ is obtained by using a time delay

$$\tau_0 = \frac{1}{4f_0}$$

where f_0 is here the midband frequency of the narrow filter. The imaginary part of the normalized cross-spectral density $v_{12}(f)$ is

$$v_{12}(f) = \frac{V_{12}(f)}{[P_1(f)P_2(f)]^{1/2}} .$$

It is noted that the measurement of $u_{12}(f)$ and $v_{12}(f)$ yields more information than the more commonly measured normalized cross correlation since the normalized cross correlation may be obtained by integration.

For this work $P_1(f) = P_2(f)$, since the two measurement points are sufficiently close together that the boundary layer has not changed in character.

The real and imaginary parts of the cross-spectral density as a function of $f_0 x / 0.8 U_0$ are shown in Fig. 5. The distance between the two measurement points has been designated as x , and the midband frequency of the narrow filter has been designated as f_0 . The factor of 0.8 will now be accounted for.

By measuring the cross correlation in a narrow band of frequencies, and by determining the time delay τ that maximized this correlation, an effective convection

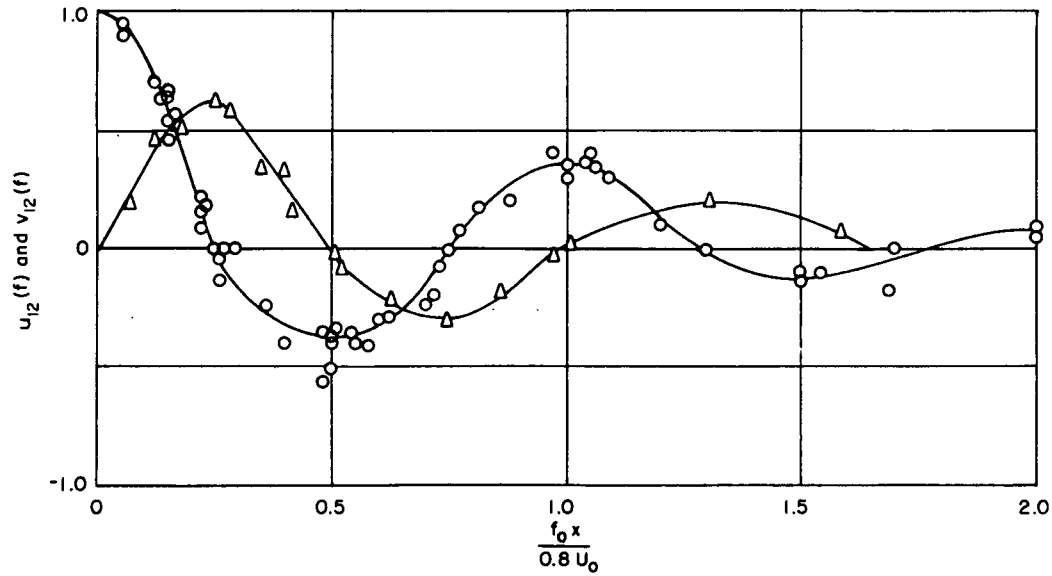


Fig. 5 - Real and imaginary parts of the longitudinal cross-spectral density

Pressure Fluctuations in Turbulent Boundary Layer

velocity $U_c = x/T$ of the spectral pattern was determined. For all frequencies up to 2000 cps (the upper limit of the transducer for this circumstance), it was found that the convection velocity was about $0.8 U_0$. It was expected that the convection velocity would decrease with increasing values of frequency as a consequence of the quadrupole nature of the right-hand term in Eq. (1). The measurement procedure was not sufficiently precise to reveal such a relationship even though there is reasonable certainty of its existence.

It can be seen that the spatial pattern of the pressure fluctuations is not convected downstream as though it is frozen. If this were so, the real part of the normalized cross-spectral density can be shown by a simple computation, for the case of a filter with a rectangular frequency response, to be

$$u_{12} \left(\frac{f_0 x}{0.8 U_0} \right) = \frac{\sin \left(\frac{2\pi \Delta f x}{0.8 U_0} \right)}{\left(\frac{2\pi \Delta f x}{0.8 U_0} \right)} \cos \left(\frac{2\pi f_0 x}{0.8 U_0} \right)$$

where the first factor on the right is an effect due to the finite bandwidth Δf . Since, in these measurements, a constant 10 percent bandwidth was used, this factor is negligibly different from unity.

The departure, for the real part of the data of Fig. 5, from $\cos 2\pi f_0 x / 0.8 U_0$ is a measure of the development of an uncorrelated component in the pressure fluctuations during its travel from the upstream to the downstream measurement point.

A measure for the uncorrelated component can be formulated mathematically. The downstream pressure fluctuation $p_2(t)$ can be resolved into three components whose cross-spectral densities with the upstream pressure fluctuation are, respectively, real, imaginary, and zero. That is,

$$p_2(t) = \alpha(t) + \beta(t) + \gamma(t)$$

where $P_{1\alpha} = U_{12}$, $P_{1\beta} = iV_{12}$, $P_{1\gamma} = 0$. The vanishing of the spectral density is equivalent to saying that the correlation for $p_1(t)$ and $\gamma(t)$ is zero. This resolution can be effected using the definition for cross-spectral density. The spectral densities for $\alpha(t)$, $\beta(t)$, and $\gamma(t)$ are

$$P_\alpha = U_{12}^2 / P_1$$

$$P_\beta = V_{12}^2 / P_1$$

$$P_\gamma = P_2 - |P_{12}|^2 / P_1$$

so that

$$\overline{\gamma^2(t)} = \int_0^\infty P_\gamma(f) df$$

and can be computed in terms of the measured data $P_1(f)$, $P_2(f)$, and $|P_{12}|^2 = U_{12}^2 + V_{12}^2$. Perhaps of greater interest is the quantity

$$\frac{|P_{12}|^2}{P_1 P_2} = 1 - \frac{P_\gamma}{P_2}$$

which can be taken as a measure of the coherence of $p_2(t)$ with $p_1(t)$. This quantity is plotted in Fig. 6.

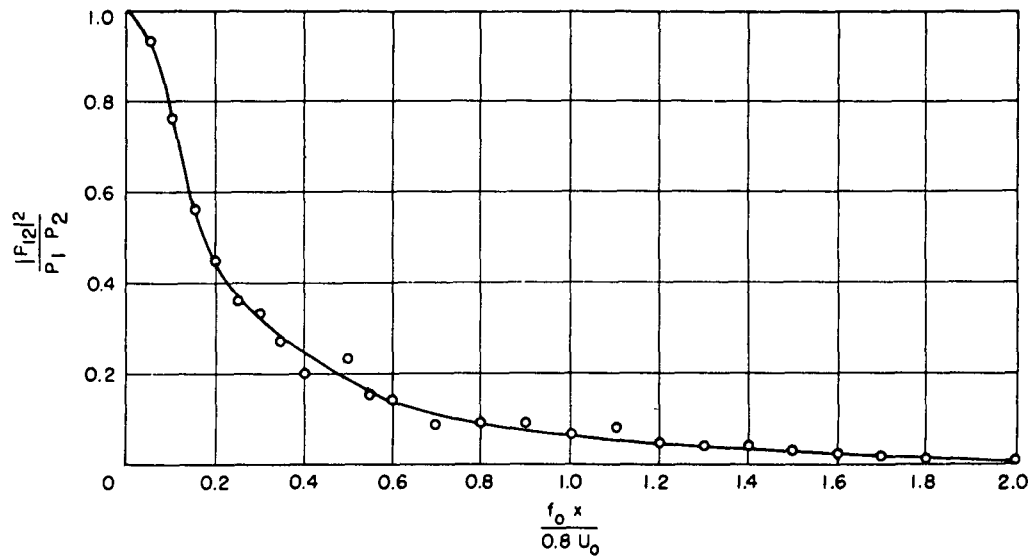


Fig. 6 - Longitudinal coherence of the pressure fluctuations

Pressure Fluctuations in Turbulent Boundary Layer

There are several important limitations in the data presented in Fig. 5. One trend in the data that is desirable to investigate is the effect of the parameter $f\delta^*/U_0$. Unfortunately, the transducer used for the cross-spectral density measurements had a usable frequency range up to only 2000 cps. Consequently, only the flat part of the spectral density, as shown in Fig. 4, could be investigated. The upper limit of 2000 cps set an upper limit on $f\delta^*/U_0$ between 0.083 and 0.33. As can be seen in Fig. 4, this does not permit investigation of the cross-spectral density of the pressure fluctuations in the range where the spectral density is rapidly decreasing with frequency. With reference to the velocity fluctuations in the boundary layer, the region investigated corresponded to the region where the velocity spectral density is described by the $-5/3$ law where inertia forces dominate. It would be interesting to procure data for the cross-spectral density in the range where viscous forces dominate.

THE TRANSVERSE CORRELATION

The transverse correlation was measured using the equipment of Fig. 3 with the wave analyzer omitted. The data are presented in Fig. 7. Unfortunately, these data

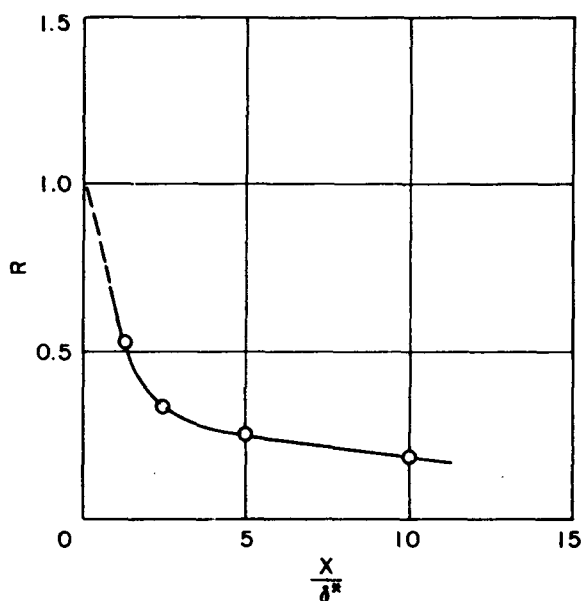


Fig. 7 - Transverse correlation function

have not been checked carefully since they were taken on the last day the wind tunnel was available for this work. Since we are not concerned here with the convection of a nearly frozen pattern, the cross-spectral densities were not investigated. For the purpose of a cross check, however, it would have been valuable to have obtained these data. Another limitation of the data is that their variation of speed was not checked because of limitations on experimental time.

M. Harrison

REFERENCES

1. Kraichnan, R.H., J. Acoust. Soc. Am. 28:378 (1956)
2. Kraichnan, R.H., J. Acoust. Soc. Am. 29:65 (1957)
3. Klebanoff, P.S., and Diehl, Z.W., NACA Report 1110, 1952
4. Hilliard, J.K., and Fiala, W.T., J. Acoust. Soc. Am. 29:254 (1957)
5. James, Nichols, and Phillips, "Theory of Servomechanisms," Chapter 6 New York:McGraw-Hill, 1947
6. Willmarth, W.W., J. Acoust. Soc. Am. 28:1048 (1956)
7. Klebanoff, P.S., NACA Report 1247, 1955

* * * * *

DISCUSSION

W. Willmarth (University of Michigan)

I think Dr. Harrison's work is a valuable contribution to the existing measurements of the wall pressure fluctuations in a turbulent boundary layer. I would like to state that, in general, his experiments agree with those that I did at the California Institute of Technology which will soon be available as an NACA technical note.

However, I would like to point out that Dr. Harrison's value for the root mean square pressure

$$\sqrt{p^2/q}$$

is considerably higher (9.5×10^{-3}) than the value I found (6×10^{-3}) and higher than the value found by others ($\approx 4.5 \times 10^{-3}$) in unpublished NACA technical notes. I don't know why this discrepancy exists.

My measurements of the nondimensional spectra are, of course, lower than Dr. Harrison's, but show qualitatively the same behavior at low values of the Strouhal number ($\omega \delta^*/U_\infty$). I was able to measure the spectra at various stream velocities and boundary layer thicknesses. The data are shown in Fig. D1. It can be seen that the high-frequency response is affected by the size of the transducer. I have attempted to correct these spectra using the method of Uberoi and Kovasznay[†] with the assumptions that the entire pressure pattern is "frozen" and passes by the transducer at $0.83 U_\infty$, and that the statistical properties of the pressure have radial symmetry in the coordinate system moving at $0.83 U_\infty$. The results are shown in Fig. D2.

[†]M. S. Uberoi and L. S. G. Kovasznay, "On Mapping and Measurements of Random Fields," Quart. Appl. Math., Vol. 10, No. 4, January, 1953.

Pressure Fluctuations in Turbulent Boundary Layer

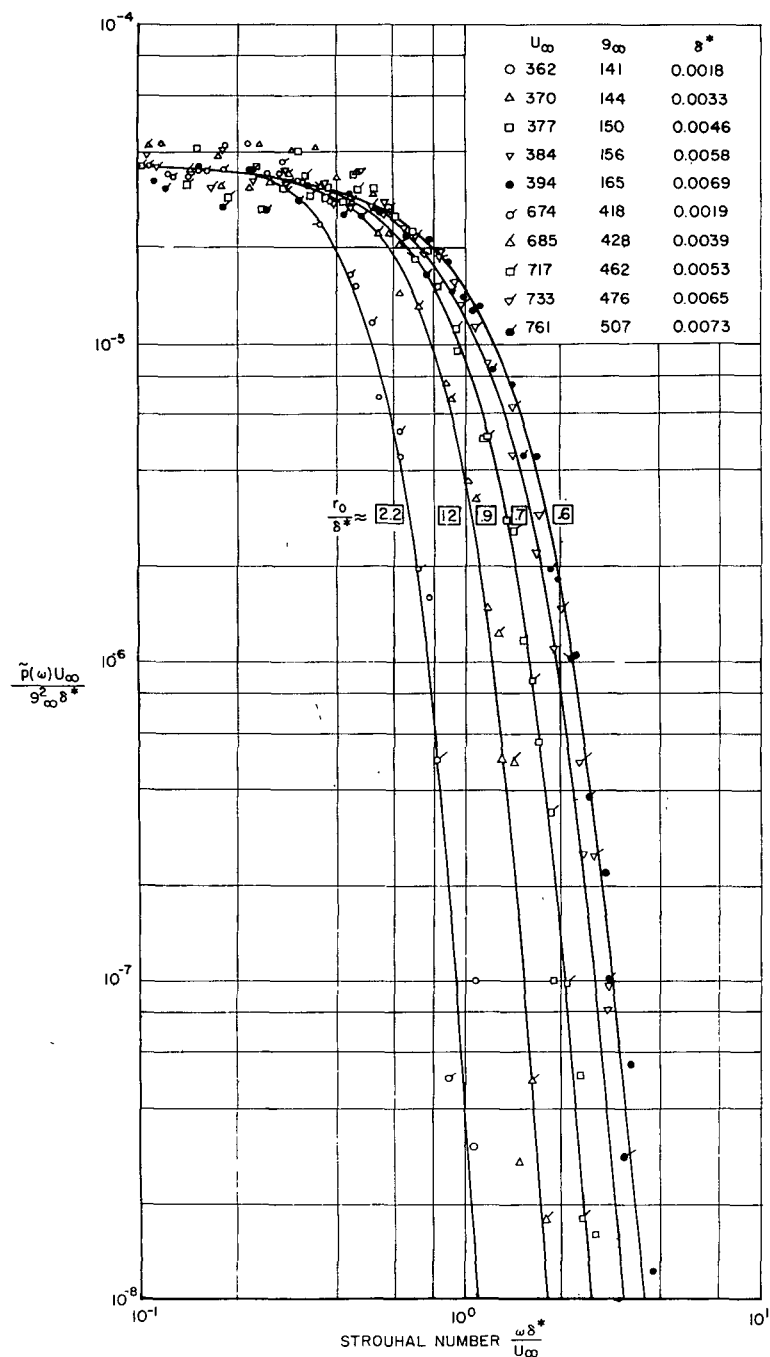


Fig. D1 - Power spectra of the fluctuating wall pressure in a turbulent boundary layer

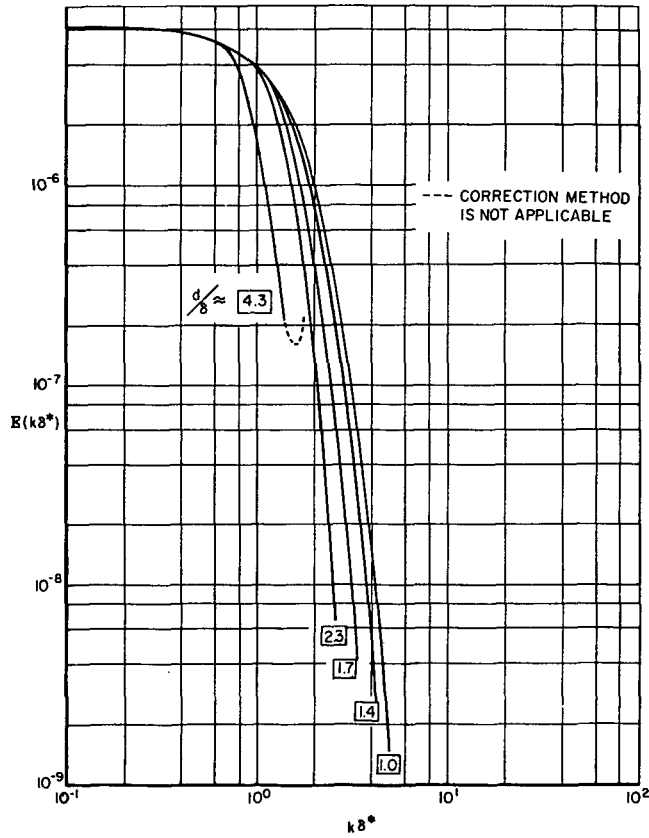


Fig. D2 - Corrected nondimensional spectrum of the wall pressure

The nondimensional spectra and spatial correlation in Fig. 2 are defined by the relations

$$\begin{aligned} \varepsilon(k') &= \frac{\varepsilon(k\delta^*)}{q_\infty^2 \delta^{*2}} = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} R(r') e^{-i(k'_1 x'_1 + k'_2 x'_2)} dx'_1 dx'_2, \\ R(r') &= \frac{R(r/\delta^*)}{q_\infty^2} = \int_{-\infty}^{\infty} \varepsilon(k') e^{i(k'_1 x'_1 + k'_2 x'_2)} dk'_1 dk'_2. \end{aligned} \quad (1)$$

Here ε is the two dimensional spectrum in wave-number space and R is the pressure correlation with respect to distance along the wall. It can be seen in Fig. 2 that the "corrected" spectra are not in agreement for the various ratios of transducer radius to displacement thickness, r_o/δ^* .

I believe this is caused by the lower convection velocity of the small-scale pressure fluctuations. The reasoning is as follows: if everything is moving at $0.83 U_\infty$

Pressure Fluctuations in Turbulent Boundary Layer

then the wave number is given by $k = \omega / 0.83(2\pi)U_\infty$. The correction factor for the spectra, which takes into account the transducer size, turned out to be quite small even for the highest frequencies or wave numbers. The corrected data, however, obviously still depend on the transducer size. Thus, the wave number at high frequency must be much smaller and that means that the convection velocity must be considerably lower than $0.83 U_\infty$ for the high frequencies.

Finally, I think that we have found out something about the turbulent boundary layer when we see that the cross-spectral density of the pressure is a function of the parameter $\omega x / 0.83 U_\infty$. Both Dr. Harrison and I are in agreement on this point. My measurements of the cross correlation of the pressure,[†] see Fig. D3, taken at

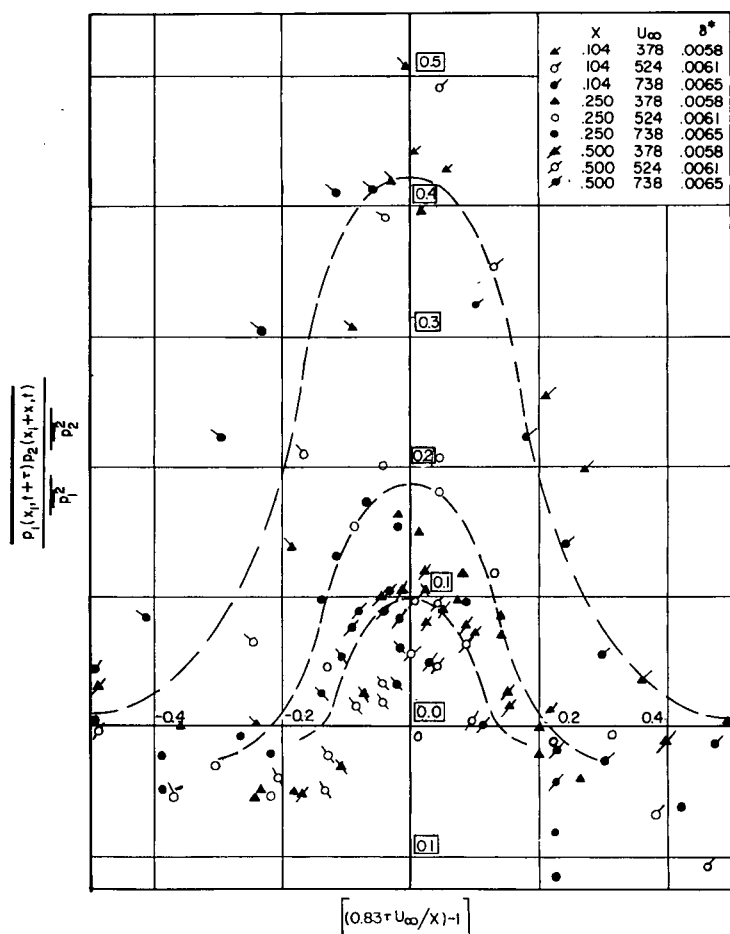


Fig. D3 - Cross correlation of the pressure

[†]W. W. Willmarth, "Space-Time Correlations of the Fluctuating Wall Pressure in a Turbulent Boundary Layer," J. Aeronaut. Sci., Vol. 25, No. 5, May, 1958.

M. Harrison

velocities between three and seven times that of Dr. Harrison's stream velocity, can be written as the function $R(x, 0.83 U_\infty t/x)$, which means, in my case, that the cross-spectral density (i.e., the transform of the cross correlation) can be expressed as the function $(x/0.83 U_\infty) P_{12}(x, \omega x/0.83 U_\infty)$. I do not understand why Dr. Harrison's results do not show this additional dependence on x . I believe this is an important result. It means that the low frequency or larger scale motion dies out much more slowly than the small scale motion, and that the rate of decay of a disturbance of any given scale is related to the distance it travels downstream.

M. Harrison

As Dr. Willmarth pointed out, in the flat part of the spectrum you might expect the convection velocity to travel because of the scale over which the velocity is fluctuating. The very high frequency components on the part of the curve that is falling might be related to a region approaching the inner part of the boundary layer, but not really there, so you might expect the convection velocity for the spatial pattern, which is made up of higher frequency components, to travel at a higher velocity. I tried to measure that but, unfortunately, from the way my pressure components were constructed when I used them for making the wind tunnel measurement, I had a much lower frequency range to work in. So, I may add, all my measurements on the cross-spectral densities were really over the flat part of the spectrum. Consequently I wasn't able to measure the convection velocity very far down, where the spectrum is falling and I got essentially a constant convection velocity with a slight tendency to fall for the higher frequencies, or higher wave numbers. This was rather marginal so I didn't report it because I wasn't sure whether it was real or just an experimental error.

H. S. Ribner (University of Toronto)

Given two microphones at a specified separation, Willmarth measures the product signal as a function of time delay. Harrison, on the other hand, puts the product signal through a tuned filter and measures a spectral density, with and without selected time delays. Harrison is to be commended for developing this cross-spectral density technique. He has given a very nice demonstration that his cross-spectral density and Willmarth's space-time correlation must be Fourier transforms of each other.

Harrison's experimental cross-spectral density functions, as plotted, look like damped cosine and sine functions. The paired curves exhibit a damping factor that varies as the product $f_0 x$. The Fourier transform of a function that simulates Willmarth's measurements, on the other hand, shows a damping depending on f_0 and x separately. Moreover, the inverse Fourier transform of Harrison's curves appears to become infinite at the origin, which is inadmissible. One wonders if Harrison's two curves should not really be two families, with damping depending separately on f_0 and x in each family. More combinations of f_0 and x in the data points are needed.

O. M. Phillips (Johns Hopkins University)

This paper provides a clear verification of the concept of convection velocity of the surface turbulent pressure fluctuations. This is central in Lighthill's theory of

aerodynamic sound and in recent attacks on the problem of wave generation by turbulent wind blowing over a water surface.

There are two remarks that I should like to offer. The first concerns the theoretical prediction of the instantaneous pressure covariance on a fixed surface when one point is held fixed and the other is integrated over the entire plane. This prediction has been made, under conditions of increasing generality, by Phillips,[†] Kraichnan[‡] and Phillips,[§] and expresses the fact, required by the incompressibility and boundary conditions, that the mean-square-linear momentum, per unit area of the boundary layer, normal to the surface vanishes. It requires that the surface pressure covariance should be somewhere negative to balance the positive contributions to the integral. Figure 7 of this paper gives little indication that the transverse correlation will be negative; it is still about 0.2 for $x/\delta^* = 10$. Perhaps the longitudinal instantaneous correlation has negative regions. Unfortunately, Dr. Harrison's data do not appear to bear on this question. The theoretical result appears to be sound, but we would be happier with experimental confirmation (or denial!) of it.

The second remark concerns the spectral densities shown in Fig. 4. The pressure fluctuations are associated with the turbulent velocity fluctuations, whose mean-square-magnitude is proportional to the square of the friction velocity U^* . One would expect, therefore, that this, rather than U_0 , would be the relevant velocity scale to be used in making $P(f)$ dimensionless. If this minor modification is made, it is found that the points for $U_0 = 1500$ cm/sec are raised by about 30 percent relative to those for $U_0 = 6000$ cm/sec, and those for $U_0 = 3000$ cm/sec by about 15 percent, bringing the three sets of results into very close coincidence over the flat, low frequency range and removing the apparent Reynolds number effect indicated by Fig. 4.

[†]O. M. Phillips, Proc. Roy. Soc., A, 234:327-335 (1956).

[‡]R. H. Kraichnan, J. Acoust. Soc. Amer., 28:378-390 (1956).

[§]O. M. Phillips, Talk to Acoustical Society Meeting, Washington, D. C., May, 1958.

* * * * *