IX

RECENT CONTRIBUTIONS TO BASIC HYDROBALLISTICS

R. N. Cox and J. W. Maccoll
Armament Research and Development Establishment

I. Hydroballistics and the Ballistic Sciences

As the word implies, hydroballistics is primarily concerned with the motion of projectiles through water or any other liquid medium. The subject is thus the hydrodynamic counterpart of aeroballistics which concerns the corresponding behavior in air. As it is usual for missiles to have an air flight path before entering the water, water entry is also included as an important section of hydroballistics.

Hydroballistics has however connections with other branches of the ballistic sciences. Thus certain problems of terminal ballistics are closely related to similar problems in hydroballistics: the latter subject can in fact be regarded as the terminal ballistic study of the ideal liquid-like medium. Terminal Ballistics is a field in which the basic scientific background is still largely undeveloped and it may be useful to consider certain terminal ballistic problems from an hydroballistic standpoint. This topic will not be carried further here as the present symposium is concerned with hydroballistics in its relation to naval problems.

In the present paper we deal mainly with some of the basic researches carried out during the last decade: most emphasis will be on work that has appeared during the last five years.

Our subject is nowhere dealt with extensively in the open literature, but Professor Garrett Birkhoff's volume on Hydrodynamics (1950) makes frequent reference to selected topics of a hydroballistic character, and we shall assume some degree of familiarity with the matters touched upon there. Much of the literature on the subject, in both the United States and in the United Kingdom, is in the form of Research Establishment reports. Such reports are not distributed very widely and are not always readily obtainable. Because of this we trust we have not omitted from our review any major contribution to our subject.

II. The several sections of Hydroballistics

The hydroballistic field divides conveniently into a number of phases which can be associated with the typical trajectory. Thus, referring to Fig. 1, we have

a. *Shock phase* which covers an initial, and extremely short, period of shock when the water surface is hit (I).

b. *Water Entry phase* which covers the subsequent cavity forming period (II).

c. *Closure phase* which follows, and which includes either or both surface seal and deep closure (III).

d. *Quasi-Steady Cavity phase* in which the missile moves with an attached cavity (IV).

e. *The Fully-Wetted condition* which will exist at sufficiently great hydrostatic pressures, when a cavity cannot be maintained. The conditions of motion are here similar to those existing in subsonic motion in air at the corresponding Reynolds number: due to the density ratio, the forces in water are about 800 times greater than those in air.

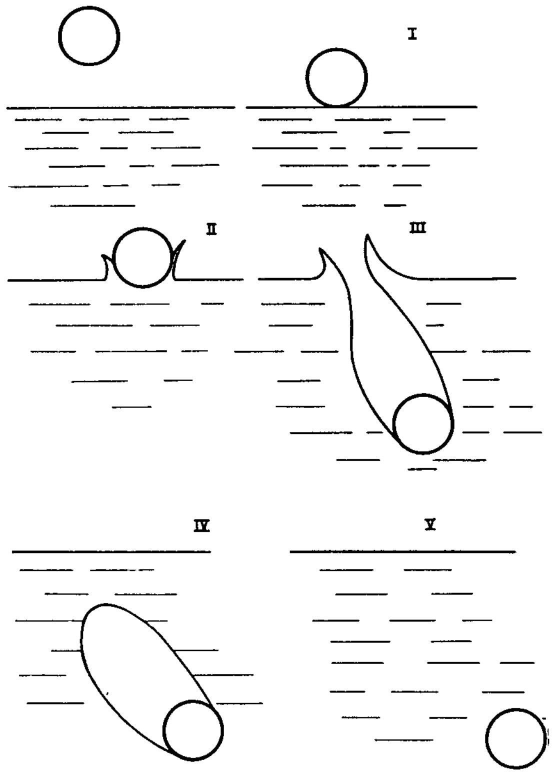

Figure 1. **The phases of water entry.**

Although these divisions of the field are rather artificial in character, and do not embrace conditions such as those which hold for ricochet, they do serve the useful purpose of helping towards a specification of the several regimes which are amenable to mathematical analysis. During the last few years most basic research has been concerned with phase (d), with a more limited amount of activity in (b), and the following review is mainly concerned with these two phases.

III. Shock and Water Entry Phases

When a missile makes contact with a water surface, the water is compressed near the point of contact and a shock wave is formed which radiates at a velocity a little higher than the speed of sound in water. This initial shock phase, in which the compressibility of the water is important, lasts for a few micro-seconds. During this period, the nose of the body experiences a localised peak pressure, the magnitude of which is approximately $\rho c v$ where ρ is the density of water, c the speed of sound in water and v is the vertical component of the velocity of entry. Apart from the possibility of damage to the body, the shock phase is relatively unimportant in the study of underwater ballistics because of its very short duration.

The first stage of importance is the inertial, or flow forming, phase during which the water, which is originally at rest, is accelerated around the nose of the missile. The rapidly changing flow can involve large accelerations and the production of high transient pressures. The determination of the high transient pressure distributions over the surface of the body is of practical importance since the forces produced determine the initial conditions for the subsequent underwater motion of the missile. The most important of these conditions is the change in angular velocity, or "whip," experienced by a missile on water entry; design engineers have already at their disposal a variety of headshape devices which provide pitching moments of varying magnitude and sign. These devices, most of which have been arrived at by empirical methods, give the designer considerable control over the subsequent trajectory.

a. *Normal Entry.* A theoretical study of the flow forming stage is difficult, even with normal entry, as it involves three independent variables: two space coordinates and time. An additional difficulty is that the exact boundary conditions on the free surface are non-linear. For these reasons no exact mathematical solution for a water entry problem, even with the simplest headshape, can be expected. The best we can hope for are either approximate solutions or solutions of the exact equations obtained by numerical methods.

The problem is simplified for wedges and cones entering at a constant speed since the flow pattern produced by such bodies may be pseudo-stationary; under these conditions the pattern of the disturbed flow increases in size uniformly with the depth of penetration. The vertical impact of a wedge on a water surface was investigated many years ago by Wagner (1933); this problem was further developed by Pierson (1950) who calculated in detail the conditions on wedges of several angles. Likewise, Schiffman and Spencer (1951) considered the similar problem of the vertical entry of a cone and dealt in detail with the case of a cone of 60 degs. semi-angle. Solutions of problems of this type can be obtained in a number of ways; for example Schiffman

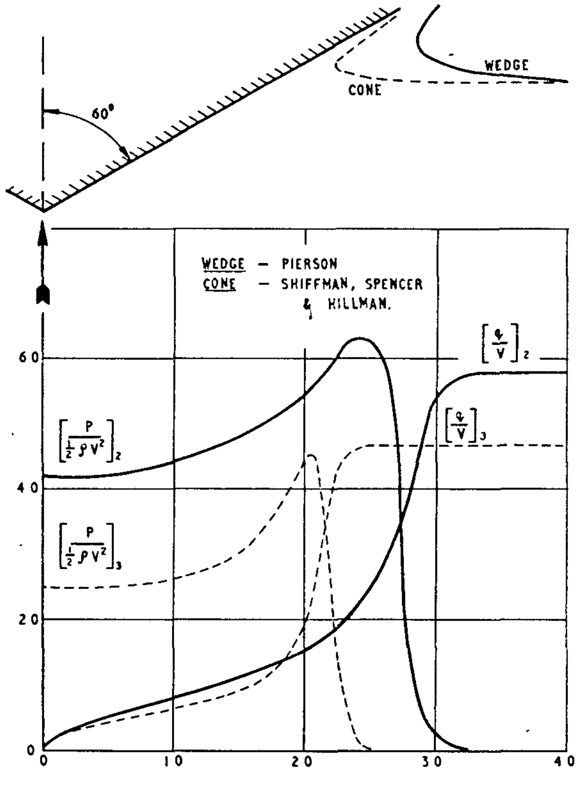

Figure 2. Pressure and velocity distributions on 60° semi-angle wedge and cone during water entry.

and Spencer applied Green's theorem in an iterative process which alternated between conditions on the free surface and those of the solid body. Fig. 2 shows the calculated distributions of velocity and pressure over the surfaces of these wedges and cones. It is interesting to note that the peak pressure on the body is attained close to the origin of the spray jet.

An alternative approach to the normal entry problem is to construct a model flow which reproduces the main conditions of the problem. Von Karman (1929), for instance, used a technique in which he approximated to the conditions of impact of seaplane floats striking the water by the flow produced by the impulsive vertical motion of a plate expanding in the plane of the water surface. For similar horizontal water surface areas, the pressure distribution produced by this model flow approximates to that over the actual submersed body. In his original paper von Karman made no allowance for the rise of the free surface but methods can usually be developed to introduce this refinement, if required. Wagner's (1933) paper did this for the von Karman problem.

Most of the theoretical investigations of water entry carried out during the last decade have been concerned with extensions of the methods originally developed by von Karman and Wagner. Thus problems have been formulated to take account of the actual penetration of the body into the water. This is usually done by considering the flow about an expanding model consisting of the submerged portion of the actual body together with its image in the plane of the undisturbed free surface. An example of this type of investigation is the war-time work of Schiffman and Spencer, who considered the entry of a sphere by means of an expanding lens-shaped model.

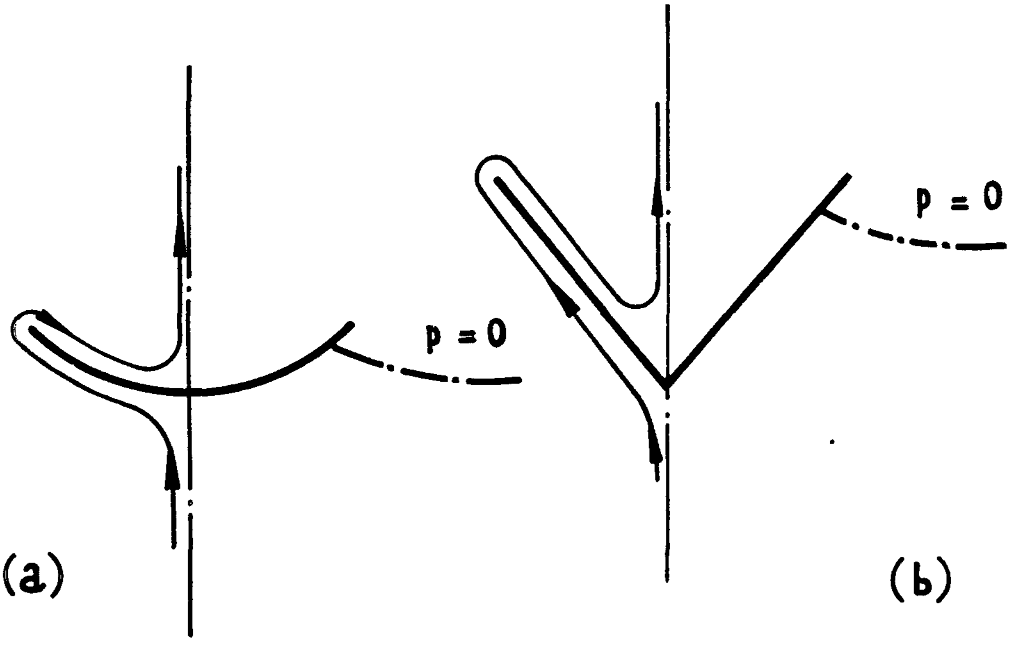

Figure 3. **Expanding hollow body models for water entry flow patterns.**

Likewise Monaghan (1949) extended the work of Wagner by approximating to the entry of a wedge by the flow past an expanding prism.

In addition to the above, a number of model flows have been investigated by comparatively easy analytical methods and are usually found to be in fairly good agreement with experimental data. In this connection mention may be made of some war-time work at the California Institute of Technology on the potential flow about a spherical bowl as the basic pattern for the vertical entry of a sphere (Fig. 3(a)). In this model the $p = 0$ contour is taken to be the free surface. Although the condition of continuity is not satisfied in this model, most of the physical features of the actual problem appear to be reproduced. A similar treatment has been that carried out recently by Coombs at Fort Halstead for an expanding hollow wedge (Fig. 3(b)). Although infinite velocities are present at the rims of both these models, these singularities are outside the field of practical interest.

b. *Oblique Entry*. With oblique entry the problem is further complicated by the presence of an additional independent coordinate and for this reason little work has been carried out in this field. Trilling (1950) has devised a technique for calculating approximately the forces on a body of arbitrary shape during the early stages of submersion. He estimates the hydrodynamic forces on the body at a number of depths of penetration by considering the impulsive motion of half a general ellipsoid having the same depth of penetration, the same length and the same submerged volumes as the actual body. Trilling applied his method to the case of the water entry of a sphere at an angle of 45 degs. to the water surface and obtained for the vertical and horizontal components of the drag the results shown in Fig. 4. It will be seen

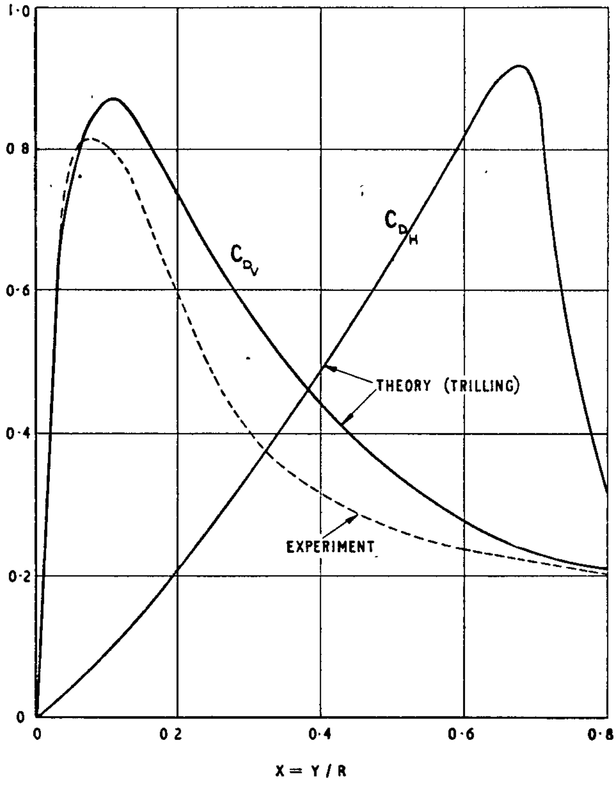

Figure 4. **Variation of drag coefficient with depth for 45′ oblique entry of sphere.**

that the vertical force component is in good agreement with experimental results; it is also of the same type as that obtained by Schiffman and Spencer.

A new method of investigating the oblique entry of a cone has recently been developed by A. Coombs (1956) with promising results. In this method the pressure distribution along a generator is assumed to be the same as that over the corresponding generator of a cone of the same apex angle when entering vertically into a conical water surface. The basic assumptions of this technique are illustrated in Fig. 5; the

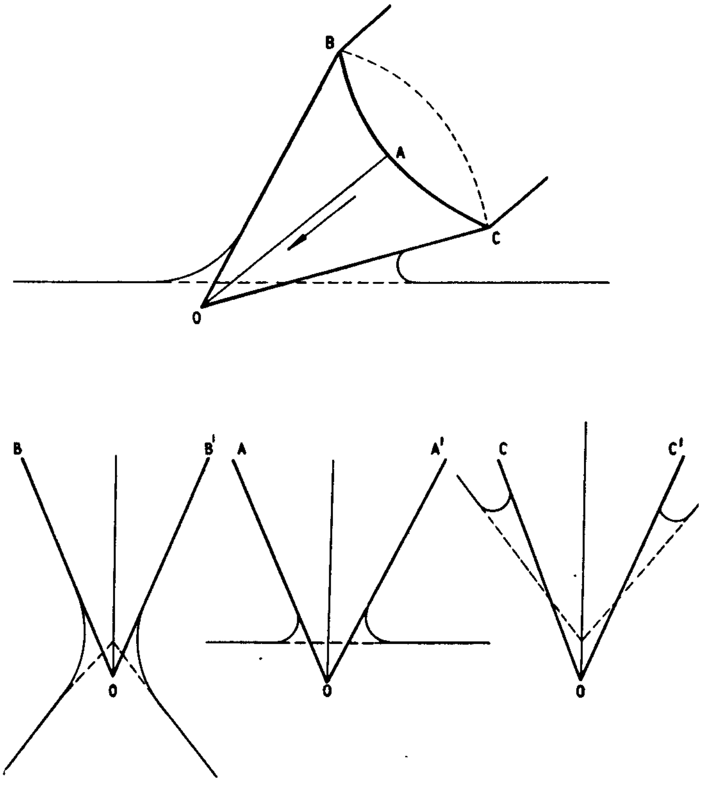

Figure 5. **Strip theory for oblique water entry of cone.**

pressure distribution along the generator OB of the top diagram is assumed to be the same as that over the cone BOB′ shown in the lower left hand diagram. Similarly correspondence is assumed for conditions on generator OA and cone AOA′; also for generator OC and cone COC′.

IV. The Steady Cavity Phase: Axi-symmetric Problems

Hydroballistics is mostly concerned with the motion of bodies of revolution and it is unfortunate that much of the extensive exact theory of two dimensional cavity flow cannot be applied directly to hydroballistic problems. During the past few years, research on the axi-symmetric problem has concentrated mainly on a. the development of analytical techniques capable of solving the exact differential equations and b. methods involving similarity between plane and axi-symmetric flows.

In the present section we propose to go into various aspects of axi-symmetric problems in greater detail.

a. *Exact Theory.* The calculation of the conditions of flow in axi-symmetric problems is much more difficult than the corresponding problem in two dimensions. This is largely because in the axi-symmetric field we have no technique corresponding to the theory of conformal representation of a function of a complex variable. Attempts have been made to overcome this difficulty, with some success in specific problems.

The Rankine method of using sources and sinks in a uniform stream was a first attempt to solve this problem. Although this model gives a superficial representation of the flow, and may be adequate for many engineering problems for certain contours, the source-sink technique often fails to give the finer detail, which is desirable to obtain an accurate representation of conditions near flow singularities, such as at pointed noses and at separation.

Several developments of the last decade have had the object of improving on the Rankine approach. Some of these researches have made use of mathematical singularities of a higher order: thus the simple sources and sinks of Rankine, which were distributed arbitrarily along the axis of symmetry, are replaced by vortex sheets situated along the surface of the body and along the free streamline surfaces. This type of model was proposed by J. W. Fisher in a war time report of 1944 and was later developed in mathematical terms by Vandrey (1951) when working at the A.R.L. in England. If, for example, this method is used to investigate the Riabouchinsky model for the cavitating flow around a head contour (Fig. 6), then the method involves the

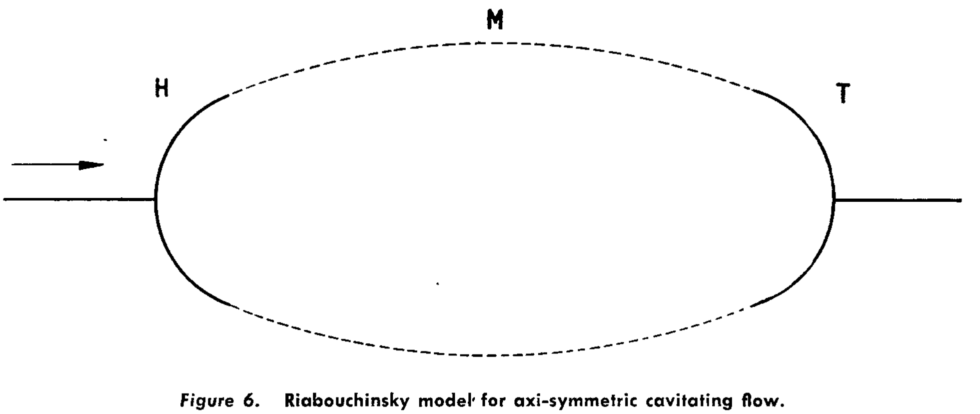

Figure 6. **Riabouchinsky model for axi-symmetric cavitating flow.**

solution of integral equations representing the flow conditions over the head and tail contours H and T and over the free stream surface M. The vortex sheet is, of course, of varying strength over H and T, and of constant strength over M.

Vandrey did not apply his technique to any case of cavity flow but he showed that the method gave satisfactory agreement with experiment in three cases of fully wetted axi-symmetric flow; it is clear from these examples that his iterative method of solving the second order integral equations will, after a few iterations, usually lead to a satisfactorily convergent solution although the computational labour involved may be considerable. This work of Vandrey was followed up shortly afterwards by the investigations of Armstrong and Dunham (1953) who formulated a similar method for obtaining an exact iterative solution of the Riabouchinsky model for axi-symmetric cavitating flow around a flat disc held normal to the main stream. In order to simplify the numerical work associated with their method they made use of certain similarity principles between the two-dimensional and three-dimensional flows. The results of these calculations give cavity dimensions which agree well with the available experimental data; Fig. 7 shows satisfactory agreement between the calculated cavity length

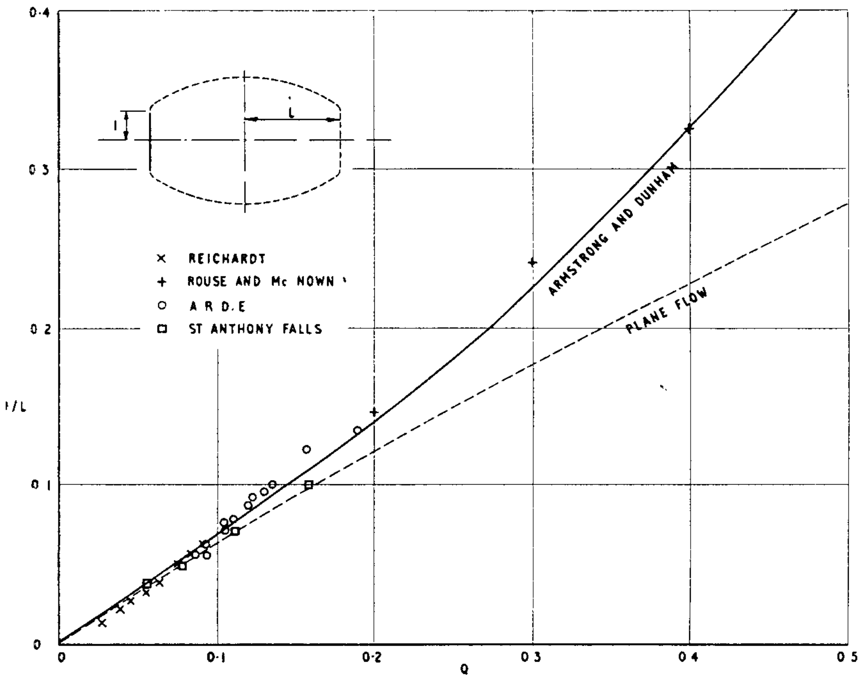

Figure 7. **Variation of length coefficient with cavitation number for flow past disc.**

and the observations of Rouse and McNown (1948) in the Iowa cavitation tunnel. The modified drag coefficient $C_D' = C_D/(1 + Q)$ calculated by Armstrong and Dunham was in good agreement with the Fisher theory; the numerical values, were, however, some 3% greater than those calculated by Plesset and Schaffer using their similitude technique which will be described later (Fig. 8).

During the last year Garabedian (1955) has described further investigations of

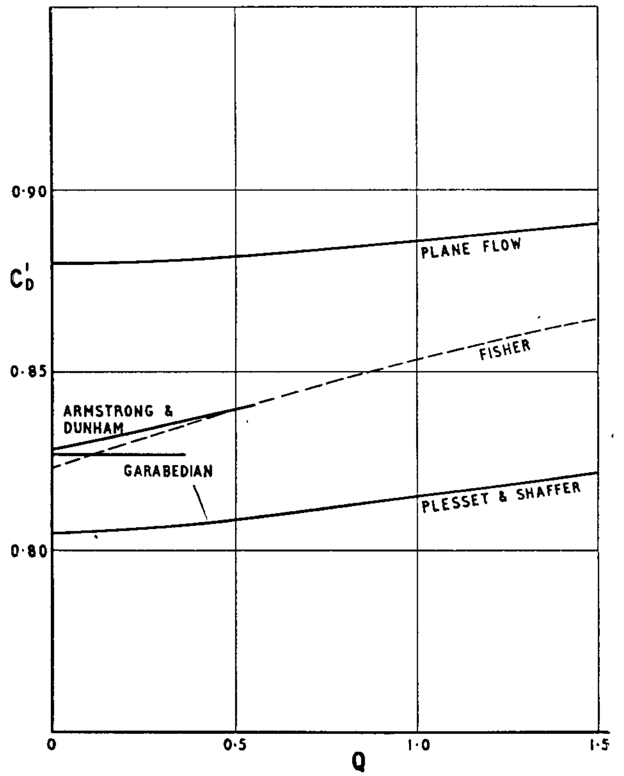

Figure 8. **Variation of modified drag coefficient with cavitation number for flow past disc.**

axi-symmetric problems. His basic technique is to relate the stream functions in plane and axi-symmetrical flow by the partial differential equation

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} - \frac{\epsilon}{y}\frac{\partial\psi}{\partial y} = 0 \tag{1}$$

where $\varepsilon = 0$ corresponds to the plane flow and
$\varepsilon = 1$ corresponds to the flow with axial symmetry.

He considers the flow in the later case to be a perturbation solution of the case of plane flow, i.e. $\varepsilon = 0$. Relationships connecting the boundary conditions on the free streamlines are similarly derived. Garabedian's method offers attractive possibilities: however, like all other techniques in this field, it entails considerable computation in its actual application. It is interesting to note that Garabedian's computation of the modified drag coefficient of a disc is in reasonably good agreement with the value obtained by Armstrong and Dunham.

b. *Approximate methods using Principles of Similitude.* The difficulties involved in applying exact theory have led to attempts to solve certain axi-symmetric problems by the use of similitude considerations linking the axi-symmetric flow with the corresponding problems in two dimensional flow. It can be shown (Armstrong and Tadman, 1954) that for the fully wetted flow past elliptical profiles, an exact similarity exists between plane and axi-symmetric flow for different but specific values of the flow velocity. By assuming that the same similarity law holds for cavitating flows (for which the cavity is approximately ellipsoidal) it is possible to calculate the flow about cavitating ellipsoidal head shapes. Fig. 9 shows the similarity relationship for this case.

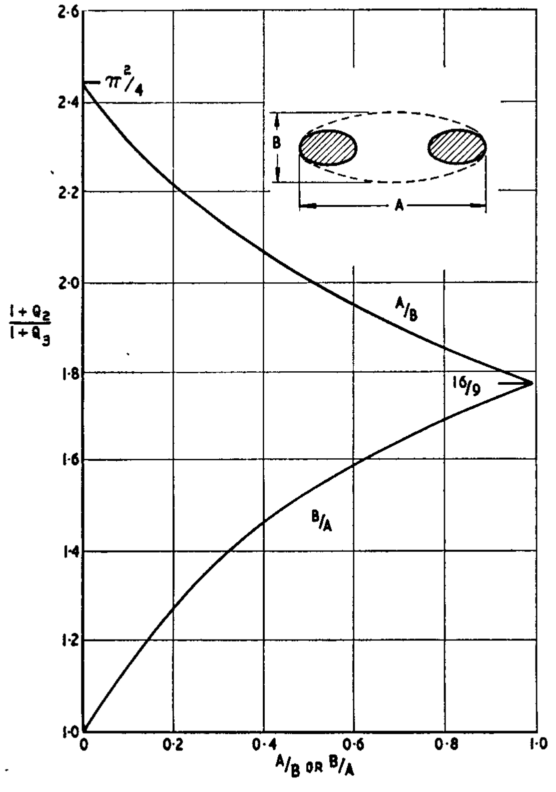

Figure 9. **Similarity relationship between plane and axi-symmetric flow for ellipsoids.**

To obtain the plane flow past curved surfaces it is, however, first necessary to determine the separation position. This type of problem was solved by Brodetsky (1922) and Schmieden (1929) using the Levi-Civita method. The separation positions obtained are in poor agreement with experiment, and it seems possible that the boundary layer plays an important part. The Brodetsky solution for a circular cylinder gives, for instance, separation at 55 degs. whereas experimental observations give values 10-20 degs. greater. A possible explanation of this is afforded by Fig. 10, which shows the calculated distribution of wall shear stress in laminar flow using the Brodetsky velocity distribution: the wall shear stress is high at the calculated separation position, and may inhibit cavitation inception, causing the separation point to move downstream.

There are also some analytical doubts as to the validity of the Brodetsky and Schmieden solutions near the separation point since the calculated position turns out to be a singular point of the head profile for derivatives of the curvature. Complete removal of the infinity is only possible if the free streamline coincides with the body.

Such considerations have to be borne in mind when any attempt is made to obtain solutions of any steady cavity problem by means of high speed digital computers. The application of relaxation or similar techniques is liable to conceal the

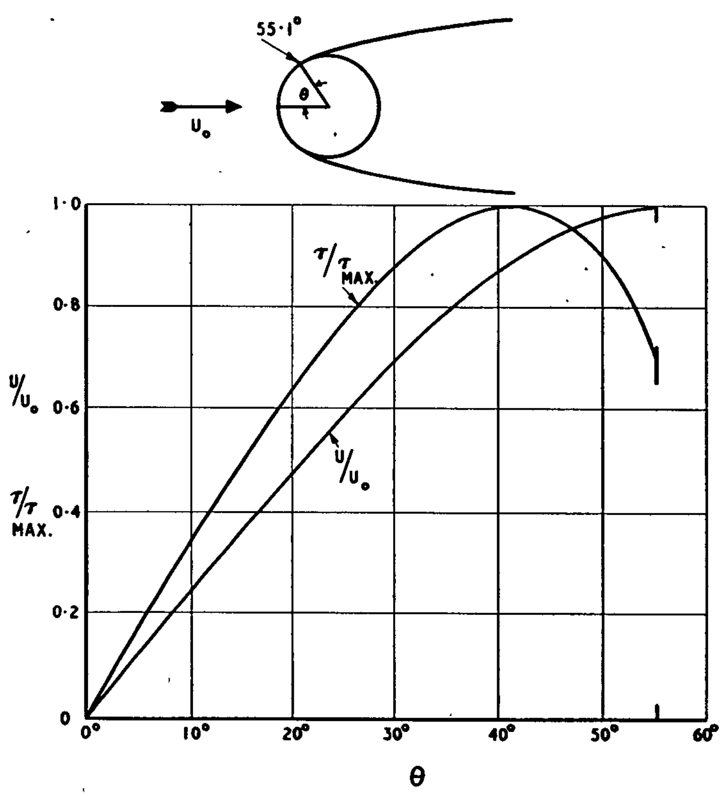

Figure 10. **Velocity and laminar wall shear stress distribution for Brodetsky flow past circular cylinder.**

true conditions of flow near the singularities even using a fine network near these points.

Professor Garrett Birkhoff and his colleagues (1953) have used computer techniques extensively on plane flow problems with curved profiles: they have in fact extended the Brodetsky approach to cavity flow to a much higher degree of refinement. Their investigations have given valuable experience of the formulation of plane flow problems and should prove of considerable value when the physical conditions of flow in the neighbourhood of the separation point are better understood.

These difficulties are no longer present if separation is defined by a corner, as for instance in the flow past wedges and discs. Although the singularity of the flow still exists at the corner in these cases, its behaviour is well understood and can be allowed for. The velocity distribution for abrupt separation at a corner can be seen in Fig. 11 and may be compared with that for smooth separation with a curved profile as shown in Fig. 10.

M. S. Plesset and P. A. Schaffer (1948) calculated the drag coefficients for a series of cones and discs over a range of cavitation numbers using the assumption that the pressure distribution along the generator of the cone or disc was the same as that on a wedge having the same semi-angle at the same cavitation number. The implications of this assumption have been investigated more recently by Armstrong (1953(a)) who, as a result, proposed improved similitude rules for wedges and cones. Armstrong showed that, if the velocity distributions near the apex of a wedge (semi-

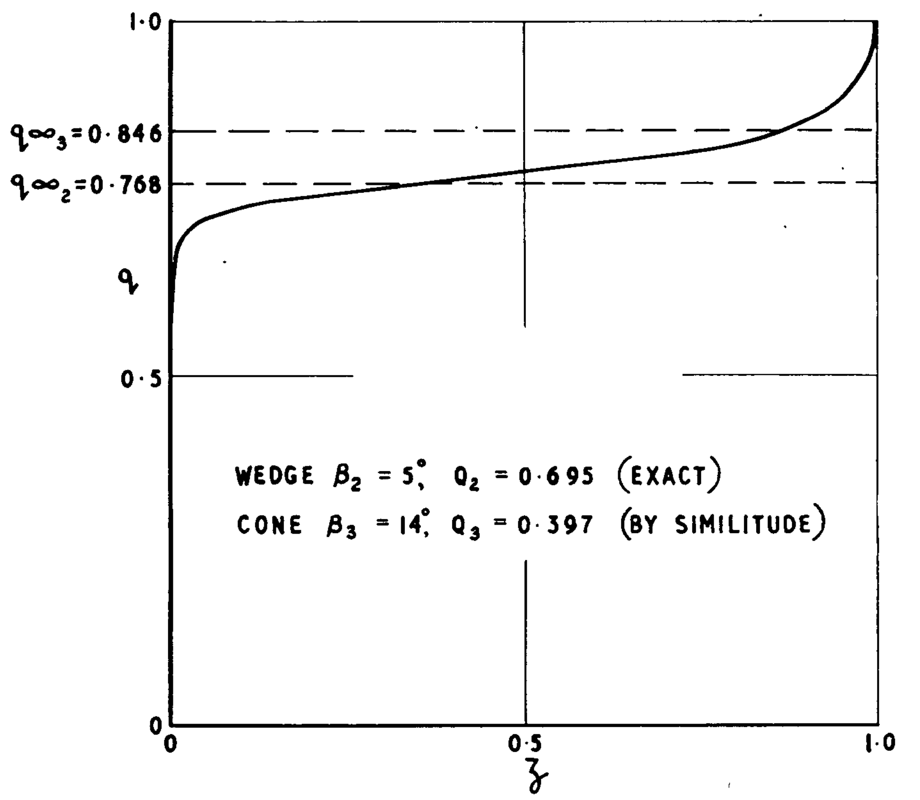

Figure 11. **Comparison of velocity distributions over 5° wedge and 14° cone.**

angle β_2) and a cone (semi-angle β_3) be expanded as power series then, the velocity near the tip of the wedge was

$$\left| \frac{dw}{dz} \right| \propto Z^{n-1} \quad \text{where} \quad n = \pi/(\pi - \beta_2) \tag{2}$$

whereas for a cone the velocity was of the form

$$r^{n-1} P'_n(\cos\theta) \cdot \sin\theta \tag{3}$$

where n must here satisfy the condition $P'_n\ (-\cos\beta_3) = 0$. The exponents of the first terms of the two series for the velocity distributions can be made the same by means of (2) and (3), which define an "angular transformation" and the coefficients of the leading terms become the same if the uniform stream ratio $q_{\infty 2}/q_{\infty 3}$ is defined by

$$\frac{q_{\infty 2}}{q_{\infty 3}} = \left| P_n(-\cos\beta_3) \right| \tag{4}$$

By means (2), (3) and (4) conditions near the apex can be made similar in plane and axi-symmetric flow; Armstrong further shows that the conditions near the rim are the same for the two and three dimensional cases. Fig. 11 illustrates the similarity obtained between a 5 deg. wedge and a 14 deg. cone.

Fig. 12 gives a comparison between drag coefficients of cones calculated by

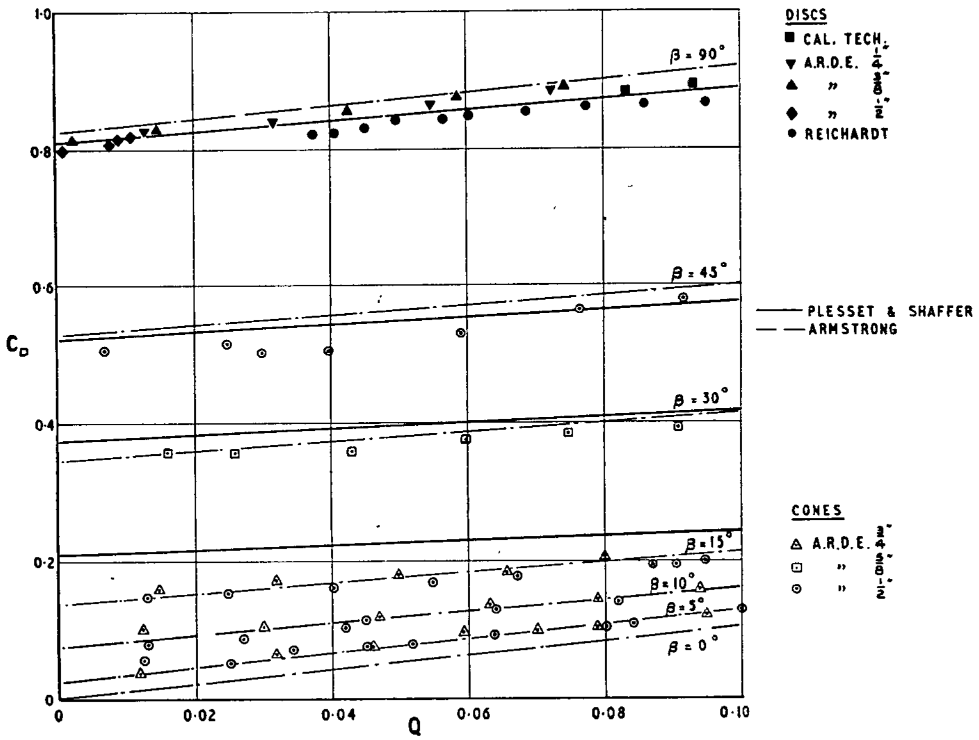

Figure 12. Variation of drag coefficient with cavitation number for cones.

Plesset and Schaffer with the corresponding calculations by Armstrong and with experimental data. It would appear from this diagram that the Plesset and Schaffer results are satisfactory for cones of large apex angle but that Armstrong's results give a marked improvement when the cone is slender. It is interesting to note that this diagram confirms that the drag coefficient can be represented as two components, in the form $C_D = C_{D(Q=0)} + \sigma$, the head drag $C_{D(Q=0)}$ being practically unaffected by the cavitation number for small values of Q.

Fig. 13 gives a general comparison of the two theories with experimental measurements for the limiting case of zero cavitation number. This diagram clearly shows the difference between the Plesset-Schaffer and Armstrong methods near the origin. The former curve approaches the origin with a finite gradient whereas the Armstrong curve approaches the origin with zero slope and infinite curvature.

c. *Bodies with non-zero Incidence.* During the steady running section of an underwater trajectory, most missiles with sufficiently high velocities will have a certain amount of yaw and the tail of the missile will be in contact with the wall of the cavity produced by the head. In these circumstances the moment of forces acting around the centre of gravity will be zero and the total lift force acting on the missile will determine the radius of curvature of its trajectory. The possibility of being able to calculate the forces and moments acting on both the head and tail of a yawing missile in steady cavity flow is therefore of considerable practical importance.

The amount of information concerning actual flow conditions near bodies held at incidence to the flow is still very limited. In this connexion A. D. Cox and W. A.

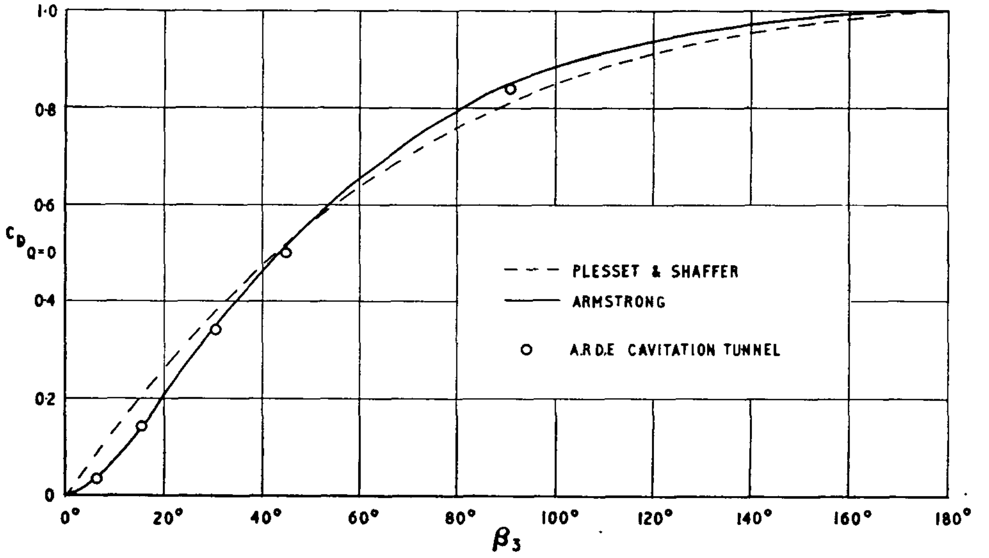

Figure 13. **Variation of drag coefficient with cone angle. (Q = 0)**

Clayden (1956) have recently investigated the two dimensional flow near a wedge of 30 degs. semi-angle at small angles of incidence at zero cavitation number. In this particular example, incidence produced a movement of the stagnation point on to the face of the wedge which was most inclined to the main flow; a subsidiary cavity was also formed on the leeward face at the apex of the wedge. Fig. 14 is a photograph of this wedge at 10 degs. incidence obtained in the A.R.D.E. cavitation tunnel. Fig. 15 is a diagrammatic representation of the mathematical model which was used in a theoretical investigation of the problem when the flow in the neighbourhood of the subsidiary cavity was treated according to the Gilbarg re-entrant jet technique. Analytical expressions were obtained for the lift-slope and moment-slope coefficients for small angles of yaw. Fig. 16 gives a comparison of the theoretical value of $\left[\frac{C_L}{\alpha}\right]_{\alpha \to 0}$ for different wedges with recent measurements made in the A.R.D E. cavitation tunnel.

The above example shows clearly some of the additional complexities which arise when an attempt is made to investigate by exact theory the conditions near a yawed cone, or similar body. In spite of these difficulties it is of interest to explore this problem by means of much simplified models and techniques, such as the application of a. principles of similitude or b. a modified Munk-Jones cross-flow theory. Both these techniques have been under investigation at Fort Halstead.

In applying the similitude technique we use a simplified "Strip Theory" with a two dimensional wedge model having the stagnation point at the nose rather than the model with the subsidiary cavity described above. This is probably justifiable because of the extra degree of freedom in the flow past the cone. Thus we imagine a wedge AOB, of semi-angle β, at a small angle of yaw δ to the stream, we assume that

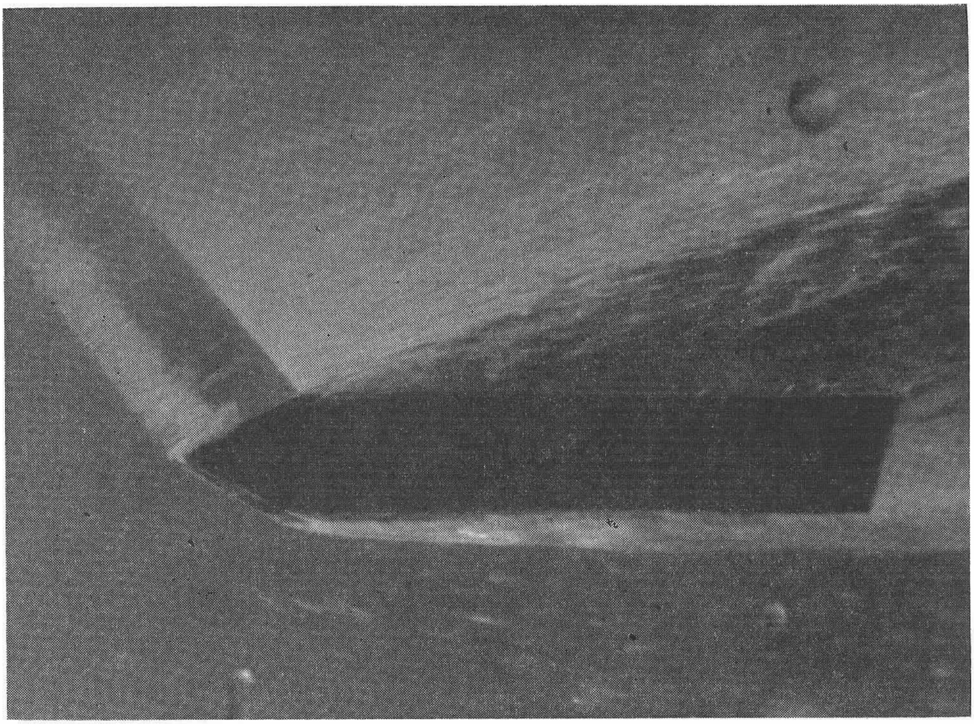

Figure 14. **Photograph of flow past yawed wedge.**

the pressure distribution on the upper face OA is the same as if this face were half a wedge AOA′ of semi-angle $\beta - \delta$, symmetrically placed in the stream (Fig. 17). Similarly the pressure distribution on the face OB is assumed the same as on a wedge BOB′, of semi-angle $\beta + \delta$. If we expand these two distributions by Taylor's theorem we then have, by resolving or taking moments, integrating and then letting tend to zero, expressions for the lift-slope and moment-slope coefficients which are readily calculated by numerical differentiation from the known pressure distributions. Experiments have shown that this approach to the problem, when applied to the cone, at least, gives good agreement with experiment for large cone angles (Fig. 18). This result is in line with the corresponding results obtained by Plesset and Schaffer described above.

An alternative theory, which might be expected to yield good results for small to moderate cone angles, is based on a modification of the well-known Munk-Jones theory for slender bodies. For very slender bodies this theory assumes that the cross-force may be obtained by considering the rate of change of momentum in the two-dimensional flow on a plane perpendicular to the flow direction, and moving with the free-stream velocity V. This yields the value 2/rad. for C_L/α. Fig. 18 shows that as $\beta \rightarrow 0$, $C_L/\alpha \rightarrow 2$ experimentally.

Since for the cavitating flow past moderately slender cones, the surface velocity V_s is constant over much of the surface (V_s is given by $\frac{V_s^2}{V^2} = 1 - C_{Do}$) the condition that the flow must be tangential to the surface of the cone may, over most of the surface, be met by letting the cross flow plane move with a velocity $V_s \cos \beta$ past the body.

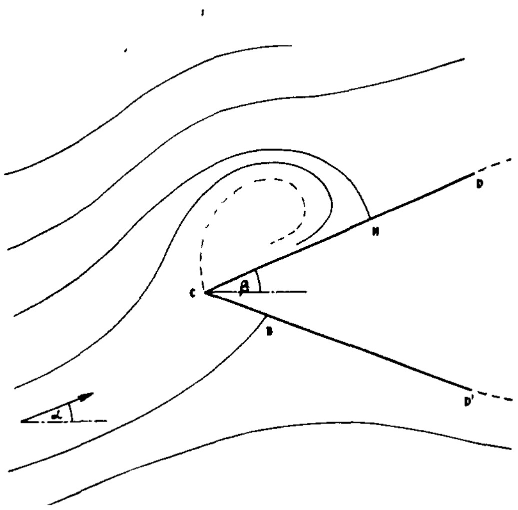

Figure 15. **Mathematical model for flow past yawed wedge.**

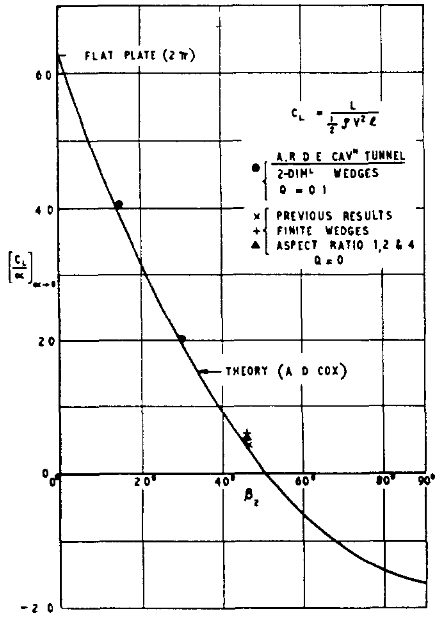

Figure 16. **Variation of lift curve slope with wedge angle (β_2).**

This yields

$$
\begin{aligned}
C_L &= C_N - C_{Do} \\
&= 2\frac{V_s^2}{V^2}\cos^2\beta - C_{Do}/\text{radian} \qquad (5)\\
&= 2(1 - C_{Do})\cos^2\beta - C_{Do}
\end{aligned}
$$

and substituting values of C_{Do}, good agreement with experiment is obtained—not only for moderate cone angles, but even over the whole range. (The good agreement for large angles may be due to the $\cos^2\beta$ term which $\rightarrow 0$ as $\beta \rightarrow 90°$.)

d. *Breakdown of the Axi-symmetric Cavity.* Conditions at the rear end of a cavity are important as they determine the rate at which the air contained in a detached cavity is dispersed, and hence the rate of collapse of the cavity. For fairly long cavities it was observed by Swanson and O'Neil (1951) that a pair of hollow core vortices was generated at the rear of the cavity (Fig. 19). A theoretical study of this problem by R. N. Cox and W. A. Clayden (1955) showed that the circulation round the vortices could be related to the buoyancy lift on the bubble. This flow model gives a more satisfying physical explanation for a resolution of the 'Brillouin paradox' than does the artificial—though extremely valuable—conceptions of the Riabouchinsky image body, or the re-entrant jet of Gilbarg-Rock.

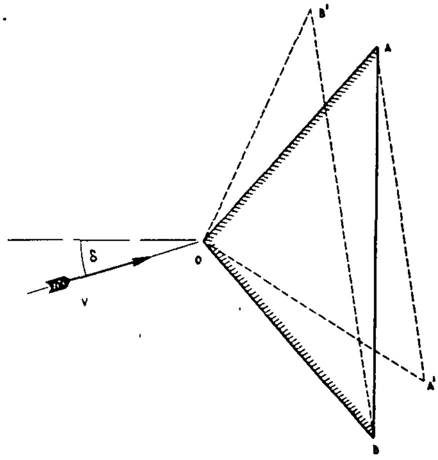

Figure 17. **Strip theory for flow past yawed cones.**

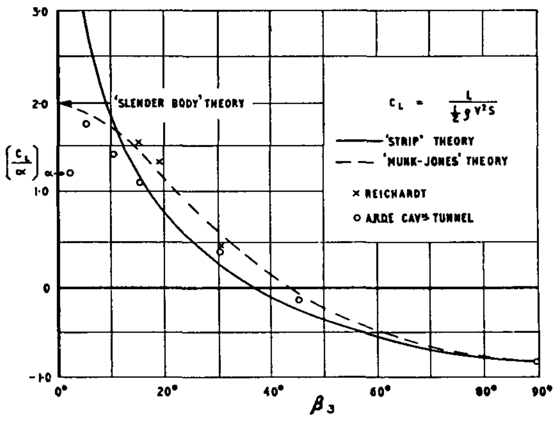

Figure 18. **Variation of lift curve slope with cone angle (β_3).**

V. Concluding Remarks

Having now complĕted our review of some of the recent advances in theoretical hydroballistics it may be of value to comment briefly on the present position of research in the several sections of the subject and to remark generally on some of the outstanding problems which need attention if the science is to continue its advance.

In this connexion it is clear that research on the steady cavity phase, especially when the body has straight line contours so that the point of separation is known, is much more highly developed than research on other aspects of the subject. Axisymmetric problems in the steady cavity phase have, at least with abrupt separation, become mainly a question of the labour involved; this should resolve itself once adequate programming techniques have been developed on high speed digital computers. Before we can attain this happy state with curved profiles it will be necessary to understand better the physical conditions at the point of separation. There is also a need for exact solutions with yawed bodies.

The entry phase remains a most difficult topic for basic research and calls for further elementary solutions, especially for oblique entry, which may help us to understand the flow mechanism during this phase. A direct theoretical attack on this essential practical problem is still a distant objective. The need for carefully controlled experimental data concerning this phase has been evident for several years and it is to be hoped that such information will be available before long from the experimental facilities in the U.S.A. and U.K.

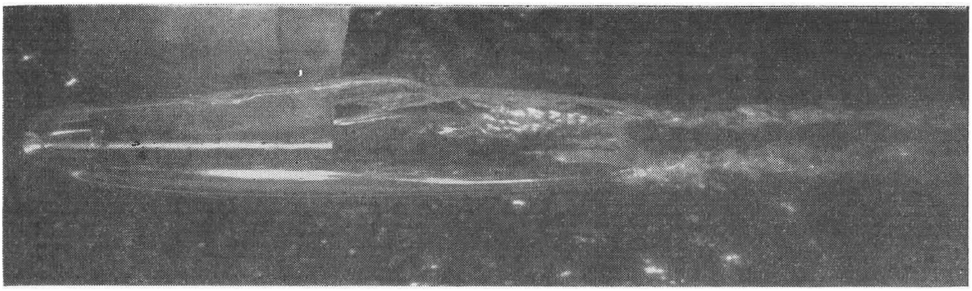

Figure 19. **Twin hollow core vortex-tubes at rear of steady cavity.**

Similar remarks can be made concerning conditions governing cavity closure. The principles involved in both surface closure and deep seal are now fairly well understood from war-time researches; there is, however, still scope for the development of improved theoretical techniques which would enable these features to be followed in greater detail.

As regards cavity breakdown, and the subsequent entrainment of the air within the cavity, the recent development of a theory which explains the effect of buoyancy on the generation of twin, hollow core, vortices at the rear end of the cavity is probably capable of theoretical extension.

BIBLIOGRAPHY

Armstrong, A. H. and Miss J. H. Dunham (1953).
Axi-symmetric Cavity Flow.
Ministry of Supply, A.R.E. Report 12/53.

Armstrong, A. H. [1953(a)].
Drag Coefficients of Wedges and Cones in Cavity Flow.
Ministry of Supply, A.R.E. Report 21/54.

Armstrong, A. H. [1953 (b)].
Abrupt and Smooth Separation in Plane and Axi-symmetric Cavity Flow.
Ministry of Supply, A.R.E. Memorandum 22/53.

Armstrong, A. H. and K. G. Tadman (1954).
Axi-symmetric Cavity Flow about Ellipsoids
Ministry of Supply, A.R.E., Theoretical Research Memorandum 5/54.

Birkhoff, Garrett (1950).
Hydrodynamics: a Study in Logic, Fact and Similitude.
Princeton University Press, 1950.

Birkhoff, Young and Zarantonello (1953).
Proc. Symposium on Applied Mathematics, *4*, 117-140, 1953.

Brodetsky, S. (1922).
Discontinuous Fluid Motion Past Circular and Elliptic Cylinders.
Proc. Roy. Soc. (A), 102, p. 542, 1923.

Coombs, A. (1956).
The Oblique Water Entry of a Cone.
A.R.D.E. Publication in preparation.

Cox, A. D. and W. A. Clayden (1956).
Cavitating flow about a Wedge at Incidence.
Ministry of Supply, A.R.D.E. Report (B) 3/56.

Cox, R. N. and W. A. Clayden (1955).
Air Entrainment at the Rear of a Steady Cavity.
Proceedings of National Physical Laboratory Symposium on Cavitation in Hydrodynamics, H.M.S.O., 1956.

Garabedian, P. R. (1955).
Calculation of Axially Symmetric Cavities and Jets.
Stanford University, Applied Mathematics and Statistics Laboratory, Technical Report No. 42, September 1955.

von Karman, Th. (1929).
The Impact on Seaplane Floats during Landing.
N.A.C.A. Technical Note No. 321, 1929.

Monaghan, R. J. (1949).
Theoretical Examination of the Effect of Deadrise in Seaplane-water Impacts, R.A.E. Technical Note Aero 1989, 1949.

Pierson, J. D. (1950).
Stevens Institute of Technology Report No. 381, July 1950.

Plesset, M. S. and P. A. Schaffer (1948).
Cavity Drag in Two and Three Dimensions.
J. of App. Phys., *19*, pp. 934-939, Oct. 1948.

Rouse, H. and J. S. McNown (1948).
Cavitation and pressure distribution: Headforms at Zero Angles of Yaw.
State University of Iowa, Studies in Engineering, Bulletin 32, 1948.

Schiffman, Max and D. C. Spencer (1951).
The Force of Impact on a Cone Striking a Water Surface (Vertical Entry).
Comm. Pure and App. Math., Vol. IV, No. 4, pp. 379-417, Nov. 1951.

Schmieden, C. (1929).
Die unstetige Strömung um einen Kreiszylinder.
Ing.-Arch I, p. 104, 1929.

Swanson, W. M. and J. P. O'Neil.
The Stability of an Air Maintained Cavity behind a Stationary Object in Flowing Water.
Unpublished Cal. Tech. report, 1951.

Trilling, L. (1950).
The Impact of a Body on a Water Surface at an Arbitrary Angle.
J. Appl. Phys., 21 (1950), 161-170.

Vandrey, F. (1951).
A Direct Iteration Method for the Calculation of the Velocity Distribution of Bodies of Revolution and Symmetrical Profiles.
Admiralty Research Laboratory Report, ARL/R1/G/HY/12/2, June 1951.
[Aeronautical Research Council Paper 14579 (FM.1665)]

Wagner, H. (1933).
Uber die Stoss-und Gleitvorgänge an der Oberfläche von Flussigkeiten.
Zeits. fur ang. Math. und Mech. *12*, pp. 193-215, 1933.

DISCUSSION

G. Birkhoff

My reactions to the preceding admirable review of recent contributions to hydroballistics tend to be those of the cautious critic.

A critical review of impact and water entry theory was initiated several years ago by E. Cooper, H. Wayland, and myself. A careful study revealed wider discrepancies between theory and recorded observations than might be inferred from the literature. Some of these discrepancies may be due to faulty instrumentation, but real further progress would seem to require that they be explained. Some of our conclusions are contained in Ref. [1], Ch. XI, §2, but N.O.T.S. lost interest in our project before it was completed. Also it was not clear how much experimental data should be declassified, especially as regards the influence of air viscosity.

Again, and in spite of laborious calculations by Marchet, Miss Vaisey, Brunauer, and Young (see Ref. [1], Ch. X, §9), the axi-symmetric potential-theoretic cavity flow past a disc has never been accurately computed, for any cavitation number Q. Though $C_D(Q)$ is now accurately known, the variation in cavity length l as a function $l(Q)$ of Q is not. In these calculations, the free streamline location seems very sensitive to small variations in pressure. Also, the singularity at the known separation point is extremely troublesome, and I wonder if a more careful analysis of the computational implications of this singularity should not take precedence over the still harder problem of locating the unknown separation point on streamlined missiles.

In the potential-theoretic approximation, this separation point should presumably be assumed to be determined by the axi-symmetric analog of the Brillouin-Villat Separation Condition (Ref. [1], Ch. VI, §6). But again, the numerical application of this method promises to be extremely difficult.—The situation is much more favorable in the plane case, and I confess to feeling great confidence in the calculations by Zarantonello and myself, referred to by Drs. Cox and Maccoll.

The theoretical analysis of cavitating flows past yawed solid bodies strikes me as very much harder, except perhaps in the case of long, nearly cylindrical missiles. Moreover it should be remembered that, in hydroballistic applications, the problem is to predict the angle of yaw α and cavitation number Q, whereas these are assumed known in most theoretical analyses. In practice also, l and hence $Q(l)$ depend on the complex mechanism of air entrainment at the rear end of the cavity, described by Drs. Cox and Maccoll. The amount of this air entrainment, and the degree of delay of separation due to boundary layer turbulence and air viscosity, all of which are neglected in the potential-theoretic model, impress me as being extremely difficult to predict.

In spite of the above difficulties, and the probability of other complications as yet unsuspected, I hope that a determined effort to solve impact and water entry problems will continue. My critical remarks are not intended to discourage further research, but to emphasize the need for distinguishing plausible ideas from reliable methods of prediction. In hydrodynamical theory, this distinction is always essential.

Ref. 1: G. Birkhoff and E. H. Zarantonello, "Jets, wakes and cavities," Academic Press, approx. Dec. 1956.

F. S. Burt

Dr. Maccoll has taken hydroballistics to cover not the whole field of the hydrodynamics of underwater weapons but rather the various aspects connected with the entry of an uncontrolled air launched weapon up to and including its fully wetted underwater run. I think this is a perfectly proper view to take and it certainly leaves a sufficiently wide and difficult field to be considered under the heading of hydroballistics.

In the water entry phase, even geometrically simple shapes like the sphere are extremely difficult to deal with theoretically. Unfortunately many of the applications we are interested in for naval use are concerned with the oblique entry of head shapes which are spherical, ogival, or a combination of both. With such head shapes the similarity principle that applies to the entry phase of a cone is no longer applicable. The cone is a rather special mathematical case in that during the initial entry phase the situation at a time t is very similar to that at a time $t + \delta t$. Nevertheless, I am very interested to see the data which Dr. Maccoll has presented on the oblique entry of cones.

At A.R.L. we had at one time extremely high hopes of Trilling's theory which Dr. Maccoll has mentioned but we now suspect that no theory which fails to take account of the deformation of the free surface can satisfactorily predict the horizontal component of force in oblique entry. We have recently done a recalculation for the oblique entry of a sphere using an approximation to an oblate spheroid rather than a prolate ellipsoid. The general trend of the drag coefficient for horizontal and vertical components seem to agree generally with that obtained by the Trilling method. That is to say that the vertical component is much the same as that obtained at vertical entry rising to a maximum of 0.8 to 0.9 at some time after an immersion of about $\frac{1}{10}$th of the radius below the undisturbed free surface, whilst the horizontal component of the force rises much more slowly.

Recently Hobbs, Backdon and Woodley at the National Bureau of Standards have produced some measurements of impact force on spheres at oblique entry. One of the results they obtained was for a 4 in. diameter sphere with an entry angle of 51°. Comparing the vertical and horizontal coefficients of drag they obtained with those predicted by Trilling's method, it can be seen that although the vertical components are of the right order, the horizontal components are way off in the time scale. At A.R.L. we are of the opinion that in calculating the horizontal force in oblique entry the free surface deformation has to be taken into account, whereas Trilling's method is only possible because the deformation of the free surface is ignored. Admittedly, satisfactory experimental data on impact forces during oblique entry are not numerous. In this connection, we at A.R.L. are in the early stages of a programme of whip measurements at model scale. A related programme of axial deceleration measurements, for which we have no spare effort at the moment, would ideally complement the whip measurements: between them they would provide much useful data on the impulse imparted to a weapon during the entry phase.

The recent work by A.R.D.E. on the lift on the nose of an inclined body on a fully developed cavity is extremely interesting. They are to be congratulated on their "strip theory" and their application of the Munk-Jones slender body theory to cavity-running missiles. A recent A.R.L. report No. ARL/R1/G/HY/18/1 entitled "Water Tunnel Boundary Effects on Axially Symmetric Fully Developed Cavities" by I. J. Campbell and G. E. Thomas, dealing with the special case of such a cavity near a solid boundary such as a water tunnel wall, may be of some interest. We are also hoping to devote some effort to such forces in connection with the problems of stability in the cavity stage and, in view of the difficulties of making such measurements in water tunnels of our type, we are planning an experimental programme for our large rotating arm which seems to offer a number of advantages for this type of work.

I can confirm that the application of Vandrey's method to the calculation of the pressure distribution on fully wetted bodies of revolution does indeed involve much complicated labour and at A.R.L. we are in the process of programming the method for a high speed computing machine. The evolution at A.R.D.E. of a similar method for application to cavities is very interesting.

J. D. Nicolaides

I had really only two remarks to make. One, I think that the timeliness and importance of this paper certainly should be recognizable to all who read the newspapers these days, and it is no breach of security to point out that the underwater weapons are beginning to gain on the pure air-flight weapons, and this problem of water entry and underwater motion is extremely important.

You will recognize with supersonic aircraft drops and rocket shoots, we are into areas in which we are now hardly thinking about in terms of water entry velocity.

Now I certainly acknowledge the many outstanding items which you included in your paper, particularly the lift force at angles of inclination, but these are indeed steady state phenomena, for the most part, and when one compares the accuracy of performance computations for missiles under water with missiles in the air, you will note we are in a very serious situation with regard to mils dispersion. The argument that you travel a short distance under water so that the errors, though large in mils, are not important, is not sound. When we talk about submarines at very deep depths, and the errors in mils begin to imply large errors in distance, the dynamic forces and moments acting may well be equal in importance to the static ones which we now consider, and, thus, will have to be included for calculations of satisfactory accuracy.

Recent tests in fully wetted flow indicate that the Magnus forces which are ordinarily completely neglected by the hydrodynamicist, are indeed quite important, and create quite large transverse displacements. It might be expected that in the cavity flow these forces will be important also.

In other words, what I am saying is we have come a long way, but the knowledge that we now have compared to what we need in weapons development is poor, and thus we are very far behind indeed.

Now I might say in that regard, and this my second point, that in reference to the high-speed computers, they may in a sense be our main hope. For example, in connection with the double disk cavity problem in recent work by the Naval Proving Ground (NPG Report No. 1413 by Young, et al.) this problem has been completely coded, and the free boundaries have been determined.

Now this problem was somewhat unchallenging to some of the people at the Naval Proving Ground, and they felt that it might be interesting to consider the case of a disk travelling at constant velocity in water, and then suddenly stopped, and then attempt, using that Naval Ordnance Research Computer to compute the transient development of the cavity. It is too early to indicate any results. More than a year has been spent just bringing up to date the coding required to attack this formidable problem.

I might also mention that once you determine your forces and moments on a hydroballistic missile, be they linear or non-linear, you have the problem of determining where the missile will go. Assuming we have these forces, let me say to you, the coding is now available for this same machine to predict the underwater trajectories.

In summary I would like to suggest two things about which there is an awful lot we don't know: first, transient or dynamic forces and moments, including those of the Magnus type; and secondly, flow and performance problems for which the high speed computing machine, in the final analysis, may be our only hope.

A. G. Fabula

This paper gives a valuable summary of recent results and new directions for hydroballistics research. This summary is of especial value because as the authors have indicated, the open literature on the subject is limited and the research establishment reports have limited distribution. In an attempt to thank the authors for their excellent work, I would mention certain additional reports—in the order as the subject material of the paper.

Shock Phase

In [1], Dergarabedian has summarized the work of Trilling, Owens, and Korkegi on effects of water compressibility in normal impact (two and three dimensions). Basic assumptions of these linearized theories are that the body is rigid and that the ratio of body speed to the speed of sound in the water is much less than one. The more general problem of an elastic body impacting on the surface of a compressible liquid has received some preliminary consideration in [2].

Normal Entry

In connection with the authors' mention of Monaghan's extension of Wagner's approximate treatment of normal symmetric wedge impact, it is interesting that related work was done by Bisplinghoff and Doherty [3]. Besides the effect of deadrise on the added mass coefficient, they included the effect on the rise of the free surface.

This was calculated as by Wagner, but computed with the velocity distribution along the line of zero potential of the prism instead of with that of the flat plate. Unfortunately an incorrect velocity distribution was used in [3]. Computations based on the correct velocity distributions have been made by the writer as part of a study of extensions of Wagner's method [4].

A new approach to the exact solution for normal symmetric wedge impact has been given very recently by Borg [5].

The authors tell how Coombs has utilized an expanding hollow wedge model for the wedge impact flow, and how this is an outgrowth of work (by Chou) with an expanding spherical bowl model of normal sphere impact. The direct two-dimensional analogue of Chou's work has also been considered, that is the expanding circular-arc model for the normal broadside impact of a circular cylinder [6].

Oblique Entry

In some recent work at the Naval Ordnance Test Station in Pasadena, miniature accelerometers inside two-inch diameter models have given data on axial and cross components of impact force during oblique inpact. For a hemisphere-cylinder configuration, the results have compared well with the predictions of Trilling's zero surface-rise, ellipsoid-fitting approximation, after some necessary rederivation of the expressions for the added mass of half-submerged spheroids. Reports are in preparation.

Water-Entry Modeling

While this facet of hydroballistics was not discussed by the authors, it seems worthy of note that recent work at NOTS under Dr. Waugh has demonstrated accuracy of the modeling technique wherein model cavitation index and Froude number are made the same as for the prototype. For a family of head shapes, the pitch versus penetration in diameters has been shown to be modeled accurately for at least several diameters of travel. Investigation is underway of the importance of gas-density scaling on transient cavities. This work is answering many of the questions raised by wartime research in modeling, e.g. [7].

In closing, this writer would like to mention two little known reports which may be of interest. The first of these, [8] by Birkhoff and Isaacs, expands in considerable detail the references to transient water entry cavities in Birkhoff's book "Hydrodynamics." The second [9] is a translation of Weible's summary of German wartime work on water impact loads, including Schmieden's extension to three dimensions of Wagner's approximation.

REFERENCES

1. Dergarabedian, P. "Compressibility Effects During Water Entry." NAVORD Report 3523 (NOTS 1159), June 1955.
2. DePrima, C. R. and F. E. Marble. "Impact of an Elastic Body Upon the Surface of a Compressible Liquid. Part I. Contributions of Previous Investigators to Solution of the Elastic Problem." California Institute of Technology unpublished report.
3. Bisplinghoff, R. L. and C. S. Doherty. "Some Studies of the Impact of Vee Wedges on a Water Surface." Jour. Franklin Inst., Vol. 253, No. 6, June 1952.
4. Fabula, A. G. "Ellipse-Fitting Approximation of Two-Dimensional Normal, Symmetric Impact of Rigid Bodies on Water." Proc. Fifth Midwest. Conf. Fl. Mech., Ann Arbor, Mich., 1-2 April 1957, Univ. of Michigan Press, 1957.
5. Borg, S. F. "Some Contributions to the Theory of Wedge-Water Entry." Proc. IXth Int. Congr. Appl. Mech., Brussels, 1956.
6. Fabula, A. G. and I. D. Ruggles. "Vertical Broadside Water Impact of Circular Cylinder: Growing Circular-Arc Approximation." NAVORD Report 4947 (NOTS 1268), October 1955.

7. Wayland, H. "Scale Factors in Water Entry." NAVORD Report 978 (NOTS 105), April 1947.
8. Birkhoff, G. and R. Isaacs. "Transient Cavities in Air-Water Entry." NAVORD Report 1490, January 1951.
9. Weible, A. (Forschunganstalt Graf Zeppelin). "Penetration Resistance of Bodies with Various Head Forms at Perpendicular Impact on Water." Naval Research Laboratory Translation No. 286, February 1952.

H. M. Berger

We have heard a very comprehensive review of the state of the art in the theory of water entry and it was interesting to hear in particular of the recent work at ARDE on steady state cavitation. I would like to point out that one of the problems in hydroballistics that was not mentioned and which has received practically no theoretical attention in the past is that of water exit. It seems to me that this is a fertile problem for a theoretical treatment. Many of the difficult problems that plague us when we attempt to analyze water entry are not present in water exit, and therefore, I believe that a theoretical approach could contribute a great deal to an analysis of the forces that arise during water exit.

I have just a few brief remarks to make about the state of the experimental work in this field. Concurrent with the effort to develop an adequate theory for water entry that has been outlined by Dr. Maccoll has been an extensive experimental program. However, early in the history of water entry studies, those working in the field realized that the modeling techniques they were using did not model important full scale effects such as the sudden accelerations experienced when the missile strikes the water. Consequently the experimental program divided into two parts—one, a full scale investigation of water entry characteristics of particular missiles—the other, an investigation of the scaling laws. Also, much attention was given to the development of adequate instrumentation for both full scale and model tests.

Recently, studies have been carried out by Levy at the California Institute of Technology [1] and Waugh at the Naval Ordnance Test Station [2] that have shown that with both Froude and cavitation number modeling, good agreement can be obtained with full scale water entry tests.

Levy concludes that for the range of Froude numbers and cavitation numbers that are within the prototype range:

1. "The behavior of the model with regard to trajectory, inclination, time to tail slap, and drag during cavity motion depends much more on the cavitation number than on the Froude number."

2. "Model tests in which the cavitation number is considerably greater than that of the prototype may yield a model behavior that is completely different from that of the prototype."

3. "Within the range of parameters that are encountered in prototype operation, the behavior of the missile is nearly independent of Froude number. This means that good results may be expected in most cases by using cavitation scaling alone. This is not meant to imply that the cavitation criterion is the only one that need be considered. It is known that under some conditions water entry phenomena are affected by gravity, viscosity and surface tension. However, cavitation scaling does not specify the operating conditions of the model except by establishing a functional relation between speed and pressure.

Therefore, it should be possible to avoid these other complications by proper choice of variables."

Dr. Waugh has told me that preliminary tests that he has made show that with gas density modeling, in addition to Froude and cavitation modeling, the cavity shape and closure can be accurately reproduced by model tests. The above results should give added impetus to experimental studies of water entry, especially in view of the difficulties Dr. Maccoll has pointed out that exist in the theory.

There are several questions that I would like to ask Dr. Maccoll. First, what test facilities does he consider best for studies of the various phases of hydroballistics that he has outlined? My second question arises from his mention of the gravity effects on the quasi-steady cavity phase—that is, the effects causing the rolling up of the rear of the cavity into two hollow-core vortices, I would like to know whether he knows of any tests that show a Froude number effect on the drag of a cavitating body, and if so, is the drag reduced or increased by such effects? My last question concerns the role of water entry theory in the development of hydroballistic missiles. I would like to hear Dr. Maccoll's opinion of the role water entry theory plays in such a development.

REFERENCES

1. Levy, J., "On Modeling the Oblique Entry Into Water of Air-Launches Missiles," Calif. Inst. of Tech. Hydrodynamics Lab. Rept. No. E-12.19, August 1956.
2. Waugh, J. G., "Water-Entry Pitch Modeling With One-to-One Froude and Cavitation Number Scaling," U. S. Naval Ordnance Test Station, Underwater Ordnance Department Internal Report, IDP 221, 8 July 1955.

J. W. Maccoll

These contributions represent a valuable extension to the topics discussed in the paper, and draw attention to some subjects, such as water exit, unsteady flow, and dynamics of an underwater body which it was not possible to deal with in the time available.

The remarks by Professor Birkhoff and Mr. Nicolaides serve to emphasise the computational difficulties of even the simplest potential-theoretic flows, and underline the need for a closer mathematical and physical understanding of the separation phenomenon.

The remarks by Dr. Berger and Mr. Fabula on the experimental work on water entry modelling at the California Institute of Technology and the Naval Ordnance Test Station are of great interest. An extension of this work to the high speeds associated with rocket entry would be of great practical use. Dr. Berger's question on test facilities is rather difficult to answer briefly, except perhaps to mention that in the experimental field there is a need for detailed measurements of pressure distributions during the water entry phase, and, in general, for work on high speed water entry, in which the main problem is that of controlling the entry conditions. For steady flow problems there is already an impressive and varied array of cavitation tunnels and rotating arm facilities in existence.

The authors know of no tests which show a marked Froude number effect on drag at the low cavitation numbers at which vortex rolling-up takes place; it is unlikely that the flow over the head of a body will be very much affected by conditions at the rear of a cavity. Security requirements unfortunately preclude a full answer to Dr. Berger's question on the role of water entry theory in the development of hydroballistic missiles; the remarks by Mr. Burt and Mr. Nicolaides point out the need for a great deal more work in this field before theory can be applied with confidence to the problem of water entry of missiles.

In conclusion the authors wish to thank Mr. Fabula, Dr. Berger and others taking part in the discussion for drawing attention to additional work which had escaped their attention.