III

REMARKS ON THE OCEAN WAVE SPECTRUM *

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ABSTRACT

A record of water level against time at a fixed point off La Jolla can be interpreted as the result of superposition from waves of all possible frequencies. A typical analysis into component frequencies shows five distinct bands of activity with frequencies higher than those of the tides: ripples around 20 c/s, ordinary gravity waves around 0.1 c/s, a narrow band of swell at 0.065 c/s, "surf beat" around 0.015 c/s (due to non-linear transformation of ordinary waves in shallow water), and surges with frequencies around 0.001 c/s. Quantitative estimates of energy density are given. The measurement of the directional properties of waves is discussed in some detail.

INTRODUCTION

"Remarkable progress would be attained if only naval architects and oceanographers would get together and study ship motion at sea." You may have come across statements like this; they frequently appear in the literature of the last hundred years. Offhand this sounds quite reasonable. But we doubt whether the proposed marriage of the two professions would have resulted in a healthy offspring if it had taken place prior to the last decade. The oceanographer simply could not offer an *adequate description* of the sea surface.

The latest and most successful of matchmakers is J. King Couper of the Bureau of Ships. We think his success is due, at least in part, to the circumstance that we are now more than halfway towards an adequate description of the surface in terms of the energy spectrum. The first step was to resolve the sea surface at a point in terms of elementary wave trains of various frequencies, but regardless of wave direction. This has been done. The next step is to obtain the directional pattern as well. Work on this problem is underway. When it has succeeded, an important goal will have been attained: the two-dimensional spectrum of the sea surface.

Two reviews have recently been published: Pierson (1955) emphasizes how much is known about what waves are like; and Ursell (1956) insists that nothing is known about how they are generated. Our remarks will be confined to a few selected topics: first a few basic (and well known) features of ocean wave spectra; then low frequency waves, because we find them interesting; and finally, the directional spectrum, because it is an urgent topic.

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DISCRETE AND CONTINUOUS SPECTRA

Suppose we were to convert the fluctuating water level above a point into a fluctuating voltage, pass this through a filter which responds fully to frequencies between $f - \frac{1}{2}\delta f$ and $f + \frac{1}{2}\delta f$ and not at all to others; then square the filter output and average. Designate the final output by $E(f)\ \delta f$. If the ocean waves consisted of one or several *discrete* frequencies, f_1, f_2, . . ., then we should expect a large output $E(f_1)$ $E(f_2)$ . . . when the filter is tuned to these frequencies, and a very low output otherwise. What we find in fact is that $E(f)$ varies more or less smoothly with f, as if energy were distributed continuously among all possible frequencies. No matter how narrow we design the width δf of the filter, the result is the same. This leads us to suspect that the ocean wave spectrum is a continuous and not a discrete spectrum.

In practice we must always deal with records of finite length, T, and the best we can do is to analyze for harmonics of this record length. This means that we can sample only for frequencies separated by T^{-1}. But this discreteness in sampling has nothing whatever to do with whether the true spectrum is discrete or continuous. In the former case we would find that the energy content is insignificant except for just two or three lines in the immediate vicinity of the discrete frequencies present; in the latter case we would find individual energy values to be badly scattered, but if T is sufficiently long so that we can crowd (say) 10 or more values into the frequency range to be analyzed, $T = 10/\delta f$, then the average of these ten values is a meaningful measure of the energy density in this range.

The important thing is that all pertinent measurements of ocean waves with periods shorter than the tides have been in agreement with the expected results from continuous spectra, and none have given evidence for discrete spectral lines.

One observation all of us have made suffices to dispose of the idea that the ocean surface can be represented by a single-frequency, single-directional wave train. Consider sun glitter on the sea surface (Cox and Munk, 1956). The glitter pattern is composed of millions of tiny images of the sun, an image occurring wherever the sea surface has the slope (tilt and azimuth) required to reflect the sun's rays into the observer's eye. The radial grid lines in Fig. 1 give the required azimuth of tilt (to the right and left of the sun), the quasi-elliptical lines the required tilt, drawn for a solar elevation of 30°. Suppose the water pattern were represented by a train from a direction 60° to the right of the sun, with a maximum slope of 20°. The resulting glitter pattern is shown in the diagram on the left of Fig. 1. It lies entirely along the 60°

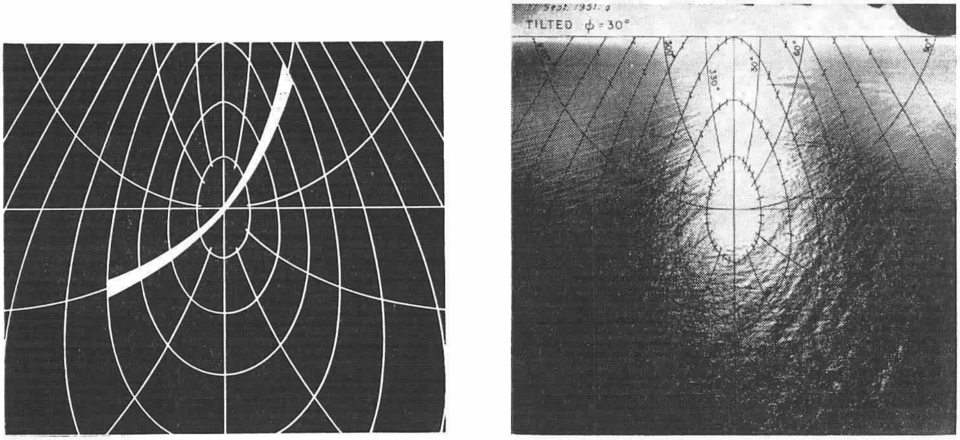

Figure 1. **The glitter pattern due to a single sinusoidal wavetrain approaching from a direction 60° to the right of the sun, and a photograph of a real glitter pattern.**

azimuth line. The glitter is brightest where the slope reaches the maximum values, these being the most probable slopes of a sinusoidal wave. Actual conditions are just the opposite. A real glitter pattern as shown in the photograph does not follow any particular azimuth line, and it is brightest in the center, where the required tilt is zero. The observed result is consistent with a continuous two-dimensional spectrum.

THE SPECTRUM FROM .2 to 200 C/KS

Most of our work deals with frequencies that are dismissed as D. C. by the radio engineer. For unit frequency we use cycles per 1000 sec., or c/Ks. Fig. 2 shows a typical La Jolla frequency spectrum (non-directional). The ordinate E in cm^2 per c/Ks is referred to as the "Energy density." The range between 50 and 200 c/Ks comprises the ordinary sea and swell spectrum, and has been extensively studied. The remaining part of the spectrum is based on recent work which has not been presented

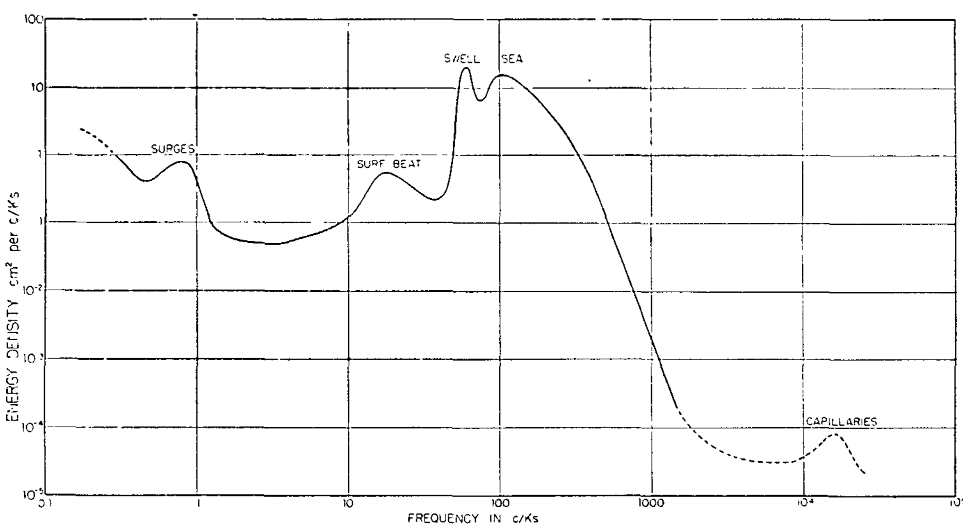

***Figure 2.* A typical frequency spectrum of the variations in the level of the sea surface above a fixed point at La Jolla, California.**

previously. The low frequency records were obtained by means of a sensitive pressure transducer, the Vibrotron. Some analyses have been performed with an analogue computer in England, others with a high speed digital computer in the United States.[1] It is encouraging that the results agree.

Apparently there are five separate bands of high activity. This is a remarkably complex pattern. For comparison, the spectrum of sound background in the sea decreases monotonically between 10 c/s and 100,000 c/s by roughly 6 db per octave.

Of course, Fig. 2 is a description, not an explanation. But we have reasons to believe that each of the five peaks involves different physical factors, and that the resolution by frequency is a fruitful one. Also, that each of these bands is a factor in Naval Hydrodynamics when interpreted in its broadest sense.

The usefulness of the spectral representation has been limited largely to problems where each frequency could be considered as being independent of, and superimposed on, other frequencies; in other words, linear problems. The topic of ocean waves pre-

[1] We are greatly indebted to B. Ferber, D. Shumway and M. Covher of CONVAIR. San Diego, for their part in analyzing many records.

sents important non-linear problems, such as the breaking of ocean waves. The previous speaker, Dr. Lighthill, has been particularly successful in coming to grips with non-linear problems. But as far as we know, all such work has been confined to monochromatic, one-directional waves. What is the situation for a continuous spectrum? Not even the limits for the linear approximation have been stated explicitly. For the monochromatic case one simply assumes that the height-length and height-depth ratios must be small. Is the equivalent requirement that the rms slope must be small, or would infrequent occurrence of steep slopes change the situation materially. These remarks are in part directed at Professor Stoker, the chairman of this session. Is it safe to apply any of the work on the formation of bores to the actual ocean unless we understand more about the effect of non-linearity for continuous spectra?

RIPPLES

The tiny peak to the right of Fig. 2 represents enhanced activity for waves a mm or so in length. The result is preliminary, and numerical values of energy density are not yet available. It is based on work by Charles Cox. In this range of the spectrum, capillary forces are more important than gravity. Some of this activity must be related to tiny wavelets on the lee side (forward) of wave crests. An oil slick will wipe out this part of the spectrum.

It has been shown by Eckart (1953) and others that sound waves or electromagnetic waves impinging on the sea surface will be backscattered by ocean waves of about half the wavelength of the incident radiation. The problem of RADAR sea return and acoustic surface reverberation involves this high-frequency spectrum.

The visibility of objects beneath the surface from an observer above the surface also appears to be critically related to this part of the spectrum (Duntley, 1950).

SEA AND SWELL

This part of the spectrum is closely related to meteorologic activity. Sea is generated by local winds, swell by distant winds. In La Jolla we usually have a peak at about 65 c/Ks (period 15^s), presumably due to swell arriving from generating areas 5000 miles to the south. More often than not there is a second peak due to meteorologic activity in the Northern Hemisphere. Numerical values in the figure represent typical conditions. On occasion the energy might be 10 times higher or 10 times lower. The cutoff to the right is drawn as $E \propto f^{-6}$ in accordance with Neumann's proposed formula (Neumann, 1954) for the spectrum of a fully developed sea.

It is a strange, if you wish an unhappy circumstance, that this peak is associated with wave lengths of dimensions commensurate with those of ships. We shall return to this subject later.

SURF BEAT

This band was discovered several years ago by means of wave recorders tuned to low frequency (Munk, 1949; Tucker, 1950). At a depth of 20 to 30 feet the energy of the surf beat band is about 1 per cent of the sea-swell band; at greater depths and large distances offshore this band appears to be considerably reduced. The mechanism of generation is not clear, but some kind of non-linear interaction of the sloping bottom with the incoming groups of waves appears to be involved. Obviously our pressure mines, which are tuned to frequencies lower than the swell, must not be triggered by surf beat.

SURGES

These differ greatly from locality to locality, depending critically on offshore topography. Some of the activity is in the form of trapped modes, with the continental shelf acting as a wave guide (Munk, Snodgrass and Carrier, 1956). Crests and troughs are normal to shore, and the direction of propagation parallel to shore. Amplitudes decrease rapidly seaward, and are insensible at a distance of one wave length from shore. Such wave motion was theoretically described by Stokes more than one hundred years ago, and referred to as "edge waves." (see also Eckart, 1951; Ursell, 1952; Isaacs, Williams and Eckart, 1951.)

Edge wave activity is related to meteorologic activity, but not in the same sense as sea and swell. There is some indication that these waves are generated by traveling disturbances of atmospheric pressure, such as result from internal waves in the atmosphere. If so, they represent a curious coupling of the height and sharpness of the atmospheric inversion layer, on one hand, to the slope and width of the continental shelf on the other hand.

All our measurements so far have been confined to the continental shelf and borderland. One week from today we plan to occupy a station off the coast of Mexico and beyond the borderland to see whether such activity can be found also in the open sea. If so, this would open up the possibility of an old dream; could deep sea surges, which travel at speeds of about 400 knots, serve as a storm warning?

SHIPS AND WAVES

The aspect of waves which is probably of greatest interest to the present gathering is their effect on ships. To a first order of accuracy, each degree of freedom of the ship may be regarded as an independent filter which responds in a prescribed way to the various wave components. For example, in its rolling motion a ship behaves rather like a pendulum and responds only to those components of the complex wave pattern which pass the ship with more-or-less the resonant frequency. It is obvious, however, that wave frequency is not the only factor since we all know that the rolling of a ship depends greatly on the direction of approach of the waves, usually being greatest when the waves approach from 2 or 3 points abaft the beam. The frequency at which the waves pass the ship will also depend on the ship's speed and its direction relative to the direction of wave travel.

If we regard the sea as being composed of a large number of simple sinusoidal, unidirectional wavetrains (see below), it is in principle possible to calculate the response of the ship to each of these, and the mean square response will be the sum of the mean square amplitudes of each individual wavetrain multiplied by the appropriate response factor. When generalized to a continuous spectrum, this is equivalent to regarding the ship as a filter with a two-dimensional filter function depending on the ship's geometry and velocity. The spectrum of the ship's motion is the integral of the appropriate product of this function with the two-dimensional spectrum of the waves, and can be calculated, at least in principle. From this, many useful statistical parameters can be calculated by methods developed by Rice (1944) for one-dimensional problems, and by Longuet-Higgins (1957) for two-dimensional problems. For example, r.m.s. amplitude, mean period, greatest expected amplitude during an hour's period of observation, number of times a given amplitude will be exceeded and whether the cook will be seasick can all be predicted. A convenient summary of these relationships is given by Cartwright and Rydill (1956).

A great deal of theoretical work has been done on the response of ships to waves, the effect of a simple sinusoidal wave train traveling in a single direction having received most attention, though more complicated wave patterns have recently been studied by St. Denis and Pierson (1953), by Cartwright and Rydill (1956) and others. (See, for example, several papers in Ships and Waves, 1954.) Considerable progress has been made in checking these theoretical results using models in tanks, but the acid test will be measurements using real ships in a real seaway. For these tests it will be necessary to measure the characteristics of the wave pattern in both frequency and direction of travel, and a knowledge of typical sea conditions based on such measurements is also necessary to enable the naval architect to design ships with the best performance in waves.

What is the present position with regard to the measurement of waves at sea? A shipborne wave recorder is available (Tucker, 1955) which enables the frequency composition and the mean direction of wave travel to be measured (Cartwright, 1956), but as far as the authors are aware no one has yet succeeded in measuring the complete wave characteristics in frequency and direction with anything approaching the necessary detail, and this remains one of the most important problems in wave studies. We shall now discuss the problem in more detail, and then some possible methods of measurement.

THE 2-DIMENSIONAL WAVE SPECTRUM

We have seen that the wave spectrum is continuous in direction. However, for most problems, both the conception and the mathematics become considerably simpler if we start by considering the sea as being made up of a large number of simple wavetrains, so that the elevation $\zeta\ (x,y,t)$ above a point in the mean sea surface with coordinates (x,y) is

$$\zeta(x,y,t) = \sum_n C_n \cos(\omega_n t - k_n x \cos\theta_n - k_n y \sin\theta_n + \epsilon_n) \qquad (1)$$

where $k_n = 2\pi/\text{wavelength}$, $\omega_n = 2\pi f_n$ and ϵ_n is a phase angle which is assumed to be random and usually disappears when appropriate averages are taken. At some stage in the problem the expressions can be written in terms of $\frac{1}{2}C_n^2$, which is proportional to the energy per unit area of the sea surface associated with the n^{th} wavetrain. If the energy of all the wavetrains contained in an interval $\delta\theta$, δf is summed, then

$$\sum_{\delta\theta\delta n} \tfrac{1}{2}C_n^2 \simeq E(f,\theta)\delta\theta\delta f, \qquad (2)$$

where $E(f,\theta)$ is the energy per unit frequency range and per unit angle. The number of elementary wavetrains can now be considered to be infinite, so $\delta\theta$ and δf to tend to zero, so that

$$\sum_n \tfrac{1}{2}C_n^2 \rightarrow \int E(f,\theta)d\theta df. \qquad (3)$$

This process was used, for example, by Rice (1944) to solve one-dimensional problems. $E(f,\theta)$ is called the spectral density function, and is the characteristic of the waves which we desire to measure.

For a given f, $E(f,\theta)$ is periodic in θ with a period of 2π, and may therefore be represented as a Fourier series of harmonic components:

$$E(f,\theta) = a_0(f) + \sum_n [a_n(f) \cos n\theta + b_n(f) \sin n\theta] \qquad (4)$$

where n has integral values from 1 to ∞. $a_n(f)$ and $b_n(f)$ define the n^{th} harmonic and are given by

$$a_n(f) = \frac{1}{\pi}\int_0^{2\pi} E(f,\theta) \cos n\theta d\theta \qquad (5)$$

$$b_n(f) = \frac{1}{\pi}\int_0^{2\pi} E(f,\theta) \sin n\theta d\theta \qquad (6)$$

Most methods for measuring $E(f,\theta)$ do not give a direct estimate of its value (smudged by the instrumental resolving power) but rather values of the individual angular harmonics a_n, b_n. For example, pressure fluctuations at one point give a_0, the total energy; the shipborne wave recorder (see below) gives a_0, a_1 and b_1; a current meter and pressure recorder give a_0, a_1, b_1, a_2 and b_2. From a_0, a_1 and b_1 one could determine, at least in principle, three parameters of a wave pattern, such as energy, mean direction and angular spread (Longuet-Higgins, 1954). In practice these values are subject to instrumental and statistical uncertainties, and it will be difficult to distinguish between a narrow beam and a unidirectional beam.

The problem can be put in another way. A unidirectional wavetrain of unit energy has all its harmonics equal, so that if it comes from direction $\theta = 0$, an instru-

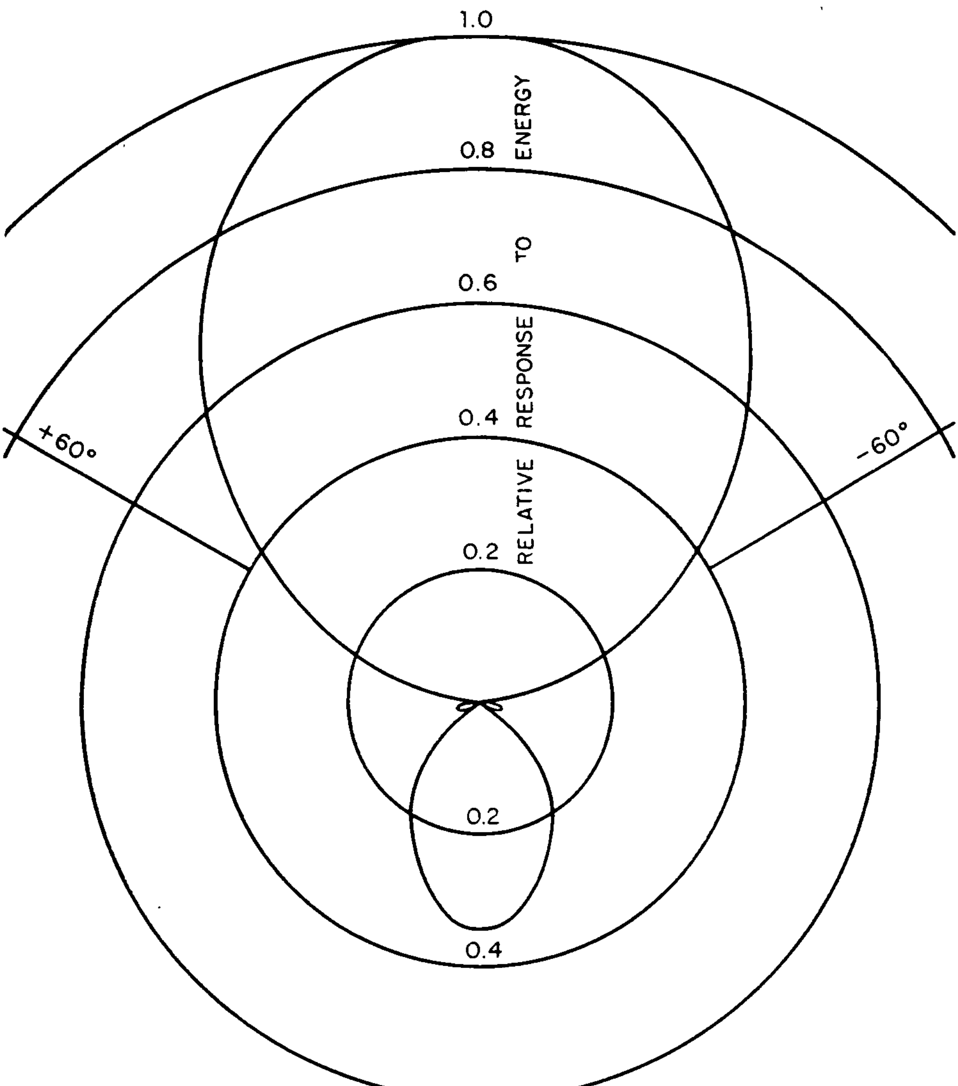

Figure 3. **The directional sensitivity pattern of a device measuring the zero order and first two angular harmonics of the wave spectrum. The equation of the curve is $r = 1 + \cos\theta + \cos 2\theta$.**

ment measuring the zero order and first two harmonics will show $a_0 = a_1 = a_2 = 1$, $b_1 = b_2 = 0$. Recombining these,

$$R(\theta) = 1 + \cos\theta + \cos 2\theta. \tag{7}$$

This is plotted in Fig. 3, and is the exact equivalent of the directional sensitivity pattern of a radio aerial or sound receiver. It shows a lobe covering approximately ±60°, and the accuracy of detecting mean direction and beam width of the waves is determined by this lobe. A beam width which is narrow compared to this lobe will be difficult to distinguish from a unidirectional beam.

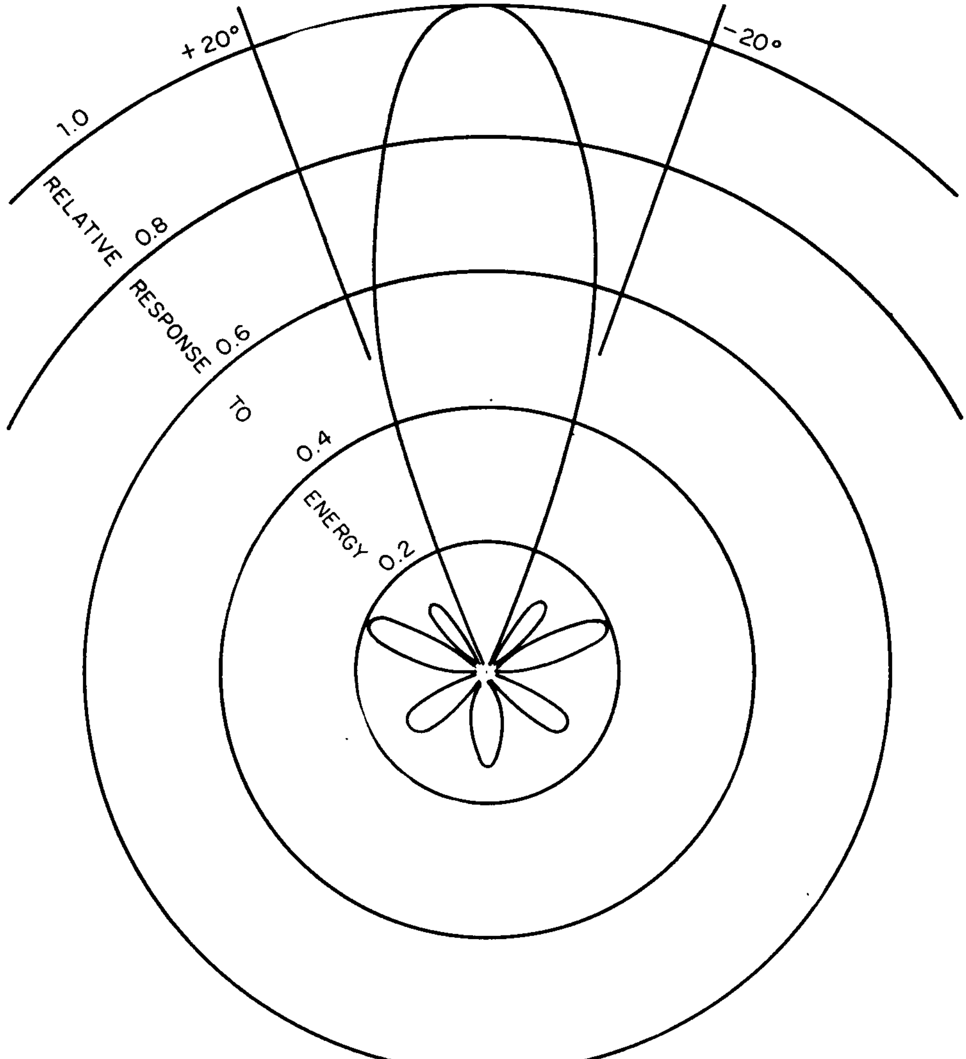

Figure 4. **The directional sensitivity pattern of a device measuring the zero order and first six angular harmonics of the wave spectrum. The equation of the curve is $r = 1 + \cos\theta + \cos 2\theta + \ldots + \cos 6\theta$.**

Instruments measuring higher harmonics have a narrower lobe. The directional sensitivity pattern for an instrument measuring the first 6 harmonics is shown in Fig. 4.

A locally generated sea is unlikely to contain important harmonics above the 5th or 6th, and an instrument giving the first six harmonics would therefore be adequate for measuring storm waves. Information approximately equivalent to this can be obtained for a limited range of frequencies using an array of six conventional wave recorders suitably spaced in an L-shaped pattern. The calculation of the spectrum from the recorder outputs requires a rather formidable amount of computation unless automatic equipment is used (see below).

Swell from a small distant storm may be a very narrow beam, and harmonics up to the 20th or 30th might be required to define it fully. For most purposes, however, a knowledge of the first 5 or 6 angular harmonics will be sufficient.

The first estimate of directional distribution was made by Arthur (1949). He observed that the high surf activity experienced during the landings in Sicily during World War II could not be accounted for if the wind-generated waves travelled only in the direction of the mean wind, but required that substantial energy be radiated up to 30° to each side of the mean wind direction.

PRACTICAL METHODS

At the present time the most practicable method of gaining some directional information is probably the shipborne wave recorder (Tucker, 1955; Cartwright, 1956), which is the basis of some very useful comparisons of ship and wave motion by Cartwright and Rydill (1956). This instrument may be regarded as measuring the height which the water surface would have near the center of the ship were the ship not there. The directional information is obtained by steaming the ship on a series of different courses (e.g., 12 courses at 30° spacings) and measuring the Doppler shifts of the various frequencies recorded. It is possible to measure only the first harmonic of the angular distribution integrated over all frequencies present, though in the special case where there are two or more separate bands of frequencies (e.g., long period swell and short period locally-generated sea), these can be studied separately. In principle more detail can be obtained, but impossibly long records are required to give sufficient statistical reliability.

Methods depending on the correlation of the outputs of various measuring devices are practicable with present techniques. The combination of a pressure measuring device with a current meter measuring the two horizontal componnts of orbital velocity gives a_0, a_1, b_1, a_2, b_2. The same information could be obtained by other similar methods, such as a free floating buoy measuring the three components of its acceleration (Barber, 1946). Barber (1954) has described a method for measuring the two-dimensional spectrum using a number of correctly spaced wave recorders and simple correlating equipment which works directly on the recorder outputs. Using three wave recorders in a limited fetch, Barber observed a "beam width" of ±45°. Barber and Doyle (1956) have described a simplified version of this which measures the mean direction of a narrow beam using just two measuring heads and which is suitable for measuring the direction of approach of swell.

Another series of methods depends on photography of the sea surface. Since these are instantaneous measurements, they all have a 180° ambiguity, and energy traveling in directions opposed by 180° is added together. Barber (1949) used a photograph of the sea surface as an optical diffraction grating. If the optical density at each point of the photograph were proportional to the elevation of the sea surface, the diffraction pattern would be a 2-dimensional analysis of the wave pattern, with the radius from the center of the pattern being proportional to 1/wavelength and the intensity being related to $E(\theta,f)$ in a simple manner. The difficulty is, of course, that the optical density of the photograph is a function of the slope of the surface, the angle of observation and of the distribution of the brightness over the sky. Cox (1955) has examined this problem in more detail, and finds that if a photograph of the sun's glitter pattern on the sea surface is taken, it is possible to deduce the angular distribution of the long wave components from diffration analyses of two small areas of the photograph. (In practice, two independent photographs of the required areas would be taken.) This technique has some theoretical and practical limitations, perhaps the most important being that it is only applicable to the long-wave part of the spectrum. It has not so far been applied to a practical example.

It is possible that Barber's simple diffration analysis could be applied to an artificial "photograph" of the sea surface prepared from stereoscopic photographs in such a way that the optical density is proportional to the corresponding surface elevation.

Pierson (brief remark in his 1955 reference) is analysing a number of stereoscopic photographs of the sea surface. Perhaps he will tell us later how he is analysing the measurements and something about the results. The first step is to convert the photograph into "maps" of the elevation of the sea surface. Marks (1954) has described one way in which these maps may be analysed by averaging surface elevations along a series of parallel lines to produce a one-dimensional record which contains contributions from components traveling in a known narrow range of directions. By altering the direction of the lines of averaging, a 2-dimensional analysis may be made. Methods using stereoscopic photographs are unlikely to become routine, since the labor of analysis is great.

A related method is based on recording surface profiles along different courses using an airborne radio altimeter. Perhaps Dr. Longuet-Higgins can inform us about the present status of this technique?

Cox and Munk (1954) have shown that it is possible to obtain some directional information from a simple photometric analysis of an out-of-focus photograph of the sun's glitter pattern (see Fig. 1). Their method gives only the second harmonic of the spectrum of the surface slopes integrated over all frequencies, and as such is of limited utility. (Their principal aim was to determine the statistical distribution of slopes.) They found a beam width of $\pm 45^\circ$.

Another class of methods depends on the use of directional primary measurement. Eckart (1953) and Crombie (1955) have shown that if a beam of radio waves is directed at the sea surface at almost glancing incidence, the back-scattering is due to the component of the sea waves with half of wave-length of the incident radiation and which is traveling in the direction of the receiver. Since the sea-waves are moving, the back-scattered radiation has a Doppler shift in frequency which enables it to be separated from direct coupling between the transmitter and receiver and from back-scattering due to static objects. The range of directions included in the back-scattering depends on the beam-width of the transmitting aerial, and this can only be made small if the aerial is several wavelengths long. The method is therefore probably practical only for short waves.

Groves (private communication) is examining the possibility of using towed electrodes as a directional wave measuring device. The use of towed electrodes for measuring ocean currents is now a routine matter (Longuet-Higgins, Stern and Stommel, 1954). The average value of the recorded e.m.f. is due to the average motion of the cable in the Earth's magnetic field. Wave frequency oscillations, now smoothed, might give useful information. Groves finds that with normal electrode separations the results are rather difficult to interpret, particularly when the device is near the magnetic equator. If a cable with a length of several wavelengths is used the results become easier to interpret, but the device may then be impracticable. The cable has to be kept near the water surface and sufficiently slack to enable it to move with the water particles, but at the same time it must be kept in a straight line.

To sum up, through several ways of obtaining information about the direction of wave travel has been suggested, the more practicable methods give very limited information, and no satisfactory way of obtaining a detailed 2-dimensional spectrum is yet available.

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DISCUSSION

M. S. Longuet-Higgins

The paper presented by Dr. Munk has rightly stressed the importance of measuring the whole two-dimensional spectrum of the sea surface. I would like to describe to you more fully the method he has mentioned that we are developing at the National Institute of Oceanography, and to point out some of its advantages.

The principle of the method is as follows. Suppose the surface elevation is represented in the form

$$\zeta(x,y,t) = \sum_n C_n \cos(u_n x + v_n y + \sigma_n t + \epsilon_n), \qquad (8)$$

where (x, y) are horizontal co-ordinates, and t is the time. The wave-numbers (u_n, v_n) are assumed to be distributed densely throughout the (u, v)-plane, and the frequency σ_n is a function only of the wavelength;

$$\sigma_n = \sigma(u_n^2 + v_n^2)^{1/2} \qquad (9)$$

as in free gravity waves. The phases ε_n are randomly distributed between 0 and 2π and the amplitudes C_n are such that over any small interval of wave-number $du\, dv$

$$\sum_{du dv} \tfrac{1}{2} C_n^2 = E(u,v)du dv, \qquad (10)$$

where $E(u, v)$ represents the spectrum of the surface. If, by some means, we take the intersection of the surface with a vertical plane $x \sin\theta = y \cos\theta$ then the resulting curve will have an autocorrelation function $\psi(R)$ given by

$$\psi(R) = \overline{\zeta(x,y,t)\zeta(x + X,y + Y,t)} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E(u,v) \cos(uX + vY)du dv, \qquad (11)$$

where

$$X = R\cos\theta, \qquad Y = R\sin\theta \qquad (12)$$

Assuming this function to be obtained for all directions θ and hence all values of X and Y, we may, by taking the cosine transform, obtain the function

$$\tfrac{1}{2}[E(u,v) + E(-u, -v)], \qquad (13)$$

which is the even part of the energy spectrum.

Suppose also that we are able to measure $\partial\zeta/\partial t$ at all points along the plane section. Then from the relation

$$\overline{\frac{\partial\zeta}{\partial t}(x,y,t)\zeta(x + X,y + Y,t)} = -\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \sigma(u,v)E(u,v) \sin(uX + vY)du dv \qquad (14)$$

we may find, by taking the sine-transform, the function

$$\tfrac{1}{2}[\sigma(u,v)E(u,v) - \sigma(-u,-v)E(-u,-v)] \tag{15}$$

But $\sigma(u, v)$ is known, even function. Thus on division we find

$$\tfrac{1}{2}[E(u,v) - E(-u,-v)] \tag{16}$$

and hence, by addition of (13) and (16), the spectrum $E(u, v)$ itself.

It is planned to measure $\zeta(x, y, t)$ along an arbitrary straight line $x \sin\theta = y\cos\theta$ by means of a radar altimeter mounted in an aircraft and directed vertically downwards to the sea surface. A second altimeter, mounted directly aft of the first, and therefore passing over the position occupied by the first a short time before, would provide a measure of $\partial\zeta/\partial t$ along the same path.

The method has certain advantages over stereophography. In the first place a large number of wavelengths may be included in each straight cross-section of the surface, so enabling a more accurate determination of the autocorrelation function to be made. In the second place the complete directional spectrum $E(u,v)$ is obtained, not merely its even part $\tfrac{1}{2}[E(u,v) + E(-u,-v)]$.

Even without performing the correlation analysis, the data may be made to yield some useful information. For example, it can be shown (see reference [1]) that the average number N_o of zero-crossings of the surface $\zeta(x,y,t)$ along a plane section in direction θ is given by

$$N_0^2 = \frac{1}{\pi^2 m_{00}}(m_{20}\cos^2\theta + 2m_{11}\cos\theta\sin\theta + m_{02}\sin^2\theta), \tag{17}$$

where

$$m_{pq} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E(u,v)u^p v^q du dv. \tag{18}$$

Thus N_o is a maximum and a minimum in two directions at right-angles. The ratio

$$\frac{N_{0\,\min}}{N_{0\,\max}} \tag{19}$$

is equal to the r.m.s. angular spread in direction of the energy about the mean direction, provided this is small (see reference [1]). So by measuring N_o as a function of θ the r.m.s. spread in direction may be found.

Similarly, the number N_1 of crests and troughs per unit distance in the direction θ can be shown to be given by

$$N_1^2 = \frac{1}{\pi^4 N_0^2 m_{00}}(m_{40}\cos^4\theta + 4m_{31}\cos^3\theta\sin\theta + \cdots + m_{04}\sin^4\theta). \tag{20}$$

From this the moments $m_{40}, m_{31} \ldots m_{04}$ can be determined. If we write

$$\Delta_4 = \begin{vmatrix} m_{40} & m_{31} & m_{22} \\ m_{31} & m_{22} & m_{13} \\ m_{22} & m_{13} & m_{04} \end{vmatrix} \tag{21}$$

and

$$3H = m_{40}m_{04} - 4m_{31}m_{13} + 3m_{22}^2 \tag{22}$$

then it can be shown that the ratio

$$\frac{\Delta_4^2}{H^3} \qquad (23)$$

vanishes only when the wave system consists of two intersecting systems of long-crested waves, and equals unity when the surface is isotropic. So this ratio is a measure of the "bimodality" of the spectrum with regard to direction. Upon the same parameter depend also the distribution of the "total" curvature of the surface and the number of maxima, minima and saddle-points per unit horizontal area.

Reference 1.

Longuet-Higgins, M. S. "The statistical analysis of a random, moving surface." Phil. Trans. Roy. Soc. A. *249*, p. 321 (1957).

R. W. L. Gawn

The determination of the frequency and amplitude of the complex undulations of the ocean surface is of fundamental importance to those who design ships, those who operate them and to those who plan operations.

Dr. Munk mentioned the importance of the information as regards ship motion but the designer's interest is much wider. The ship must stand up to all the hazards of rough weather and I would like to make a few remarks on the designer's problems in this connection. Gravity waves impose heavy stresses and slamming, objectionable motion and speed loss and further limit efficiency if operational positions, e.g. gun positions in a naval ship are exposed to water or spray.

In the past and that not far distant the information available on ocean waves was scanty and it is remarkable that the designer was able to achieve a good measure of success. He assumed for strength calculations that the height of an ocean wave was one twentieth the length, although later thanks to the observations of Vaughan Cornish and others he could accept a smaller ratio for longer waves.

Until a few years ago that was about all that was known of use to the designer and it is refreshing now to hear of the enterprising work and the ingenious and painstaking analysis which Dr. Munk and others are carrying out.

There is scope for a closer identification of the length or period of the wave and the related height with the probability of occurrence in different oceans. While height is the dominant term in the energy estimates mentioned in the paper, period or length is dominant for ship motion since it influences both the excitation and the magnification. The Authors mention that the gravity waves are centered around 10 seconds period or a length of about 500 feet. It is emphasized that the range of interest to the ship designer is really from about 100 to 1500 feet length or about 4 to 17 seconds period and it is hoped that the Authors' further investigations will include a detailed study of this particular field accordingly. It looks as though from the wonderful instruments described by Dr. Munk and the work of the last speaker this should be realized.

As regards instrumentation, one requirement is a handy apparatus that can be taken to a ship and dropped in the water as necessary. As a matter of fact thinking on these lines we have made a simple buoy at Haslar which transmits radio signals of its vertical movement. Whether it is effective or not I am not yet in a position to say but a preliminary trial was promising. I am hoping that it is at sea just about now so that we should soon know if it is at all promising.

J. D. Pierson

Before proceeding to a few comments on this excellent paper, I would like to bring a third party to the "marriage" of the oceanographers and ship builders. The designers of aircraft also have a great interest in the surface of the sea. Water-based aircraft are moving out of the bays and rivers further into the open sea.

Generally, aircraft are too fragile to operate in all sea conditions. The sea becomes too rough (both in a local and overall sense) once in a while, but on the other hand the hybrid nature of the water-based aircraft enables us to avoid the worst conditions. On the water, at rest or slowly taxiing, the seaplane experiences a long sequence of waves representing the full spectrum of the sea. However, in the transition conditions of landing and take-off, which are critical for the aircraft design, only a very small portion of the whole sea wave spectrum is encountered.

Thus, we may be looking for somewhat different information than ship designers. We are both concerned with the occurrence of high water loads and seaworthiness as they may be related to the energy spectrum for the seaways. Yet because of the short transient nature of the seaplane takoff and landing, a further analysis of the sea spectrum may be profitably employed.

For example, as the various components of the sea add to give the larger waves —do high crests generally follow immediately the deep troughs? The extent to which this may occur on a probability basis would be helpful in determining the conditions for which the hull loads should be obtained.

Another problem suggested by the type of operation of water-based aircraft is the description of the sea so that small areas at a time could be defined in terms of the overall spectrum existing at the time. Along this line we may be able to contribute through the measurement of the sea surface from the air. Although airborne direct wave measuring systems have not yet been satisfactorily demonstrated, there are several now under development which have considerable promise. Measurements of the sea roughness by aircraft will not only provide a tremendous increase in data over what can be obtained at fixed locations but also provide quick data for portions of the total sea spectrum for small transient areas.

W. J. Pierson, Jr.

I should like to add to Dr. Munk's remarks a very brief summary of Project SWOP (Stereo Wave Observation Project), a joint effort of the Office of Naval Research, U. S. Navy Hydrographic Office, Woods Hole Oceanographic Institution, and many operating arms of the Navy. A final report on the project is soon to appear under the title "The directional spectrum of a wind-generated sea as determined from data obtained by the Stereo Wave Observation Project."

The plan of the project was to take simultaneous stereo photographs and wave pole records of the sea surface. The directional spectrum could be computed from the stereo photographic data, and the frequency spectrum could be computed from the wave pole data. Theoretical considerations led to the conclusion that the total volume under the directional spectrum should equal the total area under the frequency spectrum and thus a check of the two methods would be obtained. After the computation of the directional spectrum and the wave pole spectrum, it was discovered that the two quantities given above did not agree. Several sources of discrepancy have been discovered and eliminated from the data. First of all, the wave pole spectrum did not have enough area under it to represent the true wave record. The spectral components for many important frequencies were attenuated because the wave pole had a tendency to move up under the crest of passing waves and move down in the troughs. A correction was derived for this effect, and brought the two results into closer agreement.

The stereo data unfortunately were obtained from topographic-base film instead of stereo-base film and there was some differential shrinkage in the record. The errors in the stereo data are due to this differential shrinkage, to a background curvature of the plane of the sea surface, and to spot height reading error. They all combine to make the volume under the directional spectrum quite a bit greater than the area under the frequency spectrum. The errors due to the spot height reading and to background curvature have been eliminated, and at the moment work is under way to eliminate the error due to differential shrinkage. The directional spectra have been computed and indications are that such a correction can be made and that the results of the wave pole observations and the stereo observations can be made consistent.

Some tentative conclusions can be made even with the data in their present form; and even if no further attempt were made in analyzing the data, many valuable conclusions could be drawn. The error due to differential shrinkage shows a definite pattern which can be removed qualitatively from the spectrum. The conclusions would then be that the theoretical Neumann spectrum is well verified by the corrected wave pole spectrum and that the directional spread of the spectrum is symmetric about the wind direction and falls to half power at $\pm 30°$ to the wind direction.

Unfortunately none of the slides and drawings to accompany the final report are at present in a form ready for inclusion in this report. They need correction (in some cases) and further checking. Therefore the reader is requested to write for the forthcoming report which will describe this work in complete detail.

R. C. Vetter

It may be appropriate to add, to the various ingenious methods mentioned by Dr. Munk, one more method for determining some two-dimensional characteristics of ocean waves. If two identical transparent copies of a vertical photograph of the sea surface are super-imposed and illuminated by a beam of light, one may determine the correlation function of the density variations in the transparencies by recording the change in light intensity as one transparency is translated with respect to the other. The two dimensional power spectrum of the density patches in the transparencies can then be derived from this correlation function. This method is being used with some success by Dr. Uberoi of the University of Michigan in studies of turbulence in gases. Some of the same transparencies that were used in Project SWOP (see Dr. Willard Pierson's comments) have been sent to Dr. Uberoi for analysis. It seems highly unlikely that the two two-dimensional power spectra obtained from Dr. Pierson's SWOP Project and Dr. Uberoi's method of analysis will be alike since there is no simple relationship between the brightness of the sea surface and the height of waves. However, it will be interesting to see in what respects the two spectra are different or alike.

A tremendous effort has gone into the determination of a two-dimensional power spectrum by Project SWOP. I am sure that the results will justify this effort many times over. One wonders if an easier way can be found to give similar information. At present, none of the optical methods appear to be satisfactory. The value of more information about the two-dimensional characteristics of real ocean waves is considerable and I am sure that everyone, including Dr. Pierson, will thank the ingenious person who finds an easier way of getting it.

Walter H. Munk

Vetter's suggestion is closely related to Barber's diffraction method (PRACTICAL METHODS, p. 3 of our paper). In particular, the reference to Cox (1955) is pertinent.