SIMILARITY RELATIONS IN AERODYNAMIC NOISE MEASUREMENTS

E. Mollo-Christensen and H. W. Liepmann
California Institute of Technology

INTRODUCTION

Turbulence still remains the least understood and most interesting field of fluid mechanics. The interaction of sound and turbulent flow thus presents a problem for which one cannot hope at the present time to obtain a complete analytical solution, since besides our ignorance about the structure of turbulent flow in general the added feature of a non-linear interaction between the two fields enters. Consequently the need for analytical experiments in the field is obvious. Experimentation in aerodynamic acoustics is anything but easy, not because of any insurmountable difficulties with the measuring technique but rather because it is quite difficult to set up a clean and simple problem, free of parasitic effects, and in addition to decide what the really important quantities to be measured are. Experimental studies of turbulence, even in the absence of interaction with sound, have given ample evidence of these difficulties.

In the following we will illustrate some very simple similarity considerations with experimental results obtained at GALCIT in the course of a study of aerodynamic noise sponsored by the NACA.

The experimental results which will be used for illustration have been obtained by Willmarth (1), Weyers (2), and Narasimha and Mollo-Christensen. Willmarth (1) measured power spectral densities and space-time correlations of pressure fluctuations on the wall under a turbulent boundary layer. Weyers (2) measured the pressure fluctuations on the wall and in the emitted sound field of a pipe with extremely thin walls with turbulent flow inside. Narasimha and Mollo-Christensen measured noise from subsonic jets.

In all these experiments, great care was taken to avoid parasitic effects, such as noise from unknown sources, diffraction, and scattering.

PARAMETERS OF THE PROBLEM

In the study of turbulence-produced sound, the fluctuating pressure $p(\vec{R}, t)$ is measured. Its statistical properties determine the structure of the sound field and its relation to the original turbulence. The simplest and most important quantities to be measured are the mean-square pressure $\overline{p^2(\vec{R}, t)}$ and the power spectral density $\varphi(\omega, \vec{R})$. These are related by

$$\overline{p^2} = \int \varphi \, d\omega .$$

The fluid properties and the sound-producing turbulent field are specified by a set of parameters which involve at least the following: density ρ, viscosity ν, and velocity of sound a; a characteristic flow velocity U and a characteristic length D. One can of course think up many other, parasitic, parameters, such as: the dimension of the pressure transducer, the turbulence level and the sound level of the stream used, the cutoff frequencies of the recording equipment, the length of time averaging, the dimension of the receiving volume.

Indeed, a very important and in some cases decisive part of an experimental study of aerodynamic noise lies with the identification and elimination of as many parasitic parameters as possible. This is obvious for any experiment; it is emphasized here for the case of aerodynamic noise since the nature of the problem, namely the study of a random wavefield of small intensity without a priori knowledge of what to expect, often makes it very difficult to know whether one has succeeded in measuring what was intended or not.

It is therefore especially important to check the similarity properties of the experimental data before any detailed conclusions can be drawn.

DIMENSIONAL ANALYSIS

If it is assumed that the problem is defined in the minimum number of parameters listed, dimensional analysis yields for the form of p^2 and $\varphi(\omega)$:

$$\overline{p^2} = \rho^2 U^4 f(\vec{R}/D;\ Re,\ Ma)$$

where $Re = UD/\nu$ and $Ma = U/a$ are Reynolds number and Mach number. For the power spectral density it follows:

$$\varphi(\omega,\vec{R}) = \rho^2 U^3 D\ g(\vec{R}/D,\ \omega D/U;\ Re,\ Ma)\ .$$

Any measurement which cannot be represented in this form must be influenced by at least one additional parameter, and the above relations can then be used to eliminate such a parameter by a limiting procedure.

As examples, measurements by Weyers (2) and Willmarth (1) will be cited. Figures 1 and 2 taken from Weyers' paper demonstrate the similarity relations in intensity and spectrum. Figure 3 shows a plot of spectra of wall-pressure fluctuations under a turbulent boundary layer obtained by Willmarth (1), in which the ratio of the diameter of the pressure transducer d to the boundary layer thickness δ^* is the parameter which is different for the different spectra. The effect of a finite transducer size can thus be estimated, and eliminated by an extrapolation to $d/\delta^* \longrightarrow 0$.

In both sets of measurements the influence of Reynolds number and Mach number were found to be small; this agrees with the expectation that the turbulence at not too high Mach numbers is unaffected by compressibility and that at sufficiently high Reynolds number no viscous effects remain.

We have not investigated the near field of a jet or wake, for which similar relations are bound to hold. However, in measurements of the far field it becomes quite obvious that microphone size and orientation can cause very large deviations from the proper similarity laws due to diffraction effects.

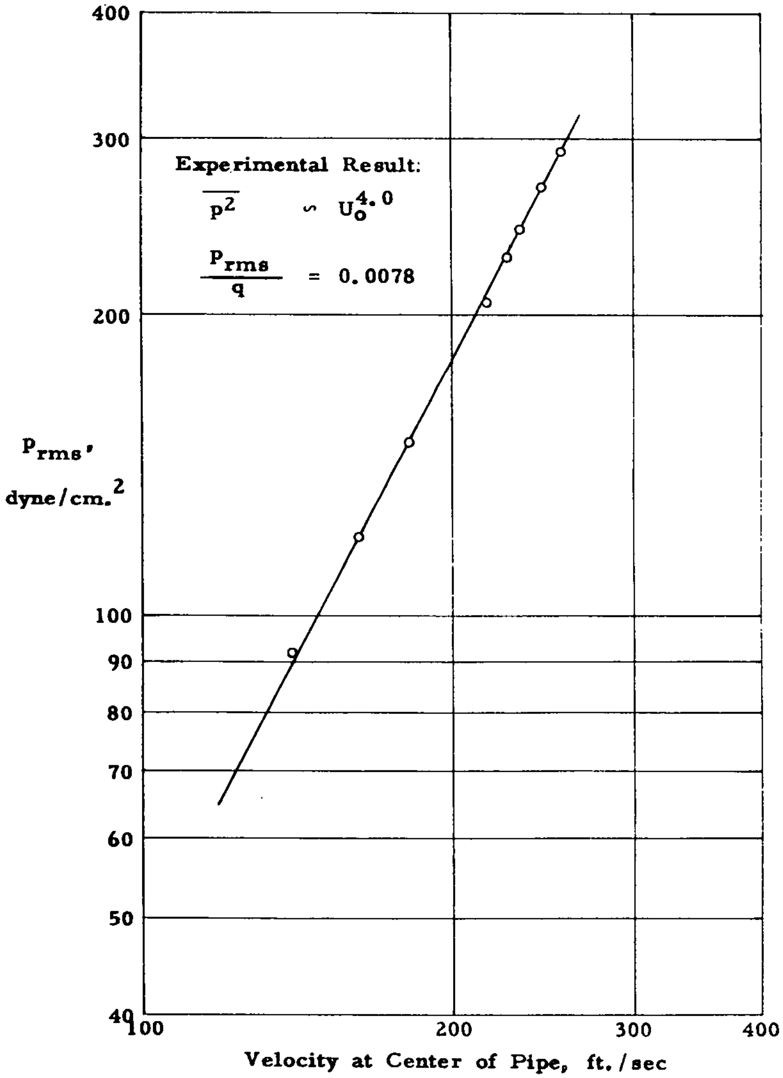

Fig. 1 - Variation of p_{rms} with velocity. $\tilde{t}$ = 0.0005 inch, d = 1 inch, r/d = 1/2

SIMILARITY

If we supplement the bookkeeping of dimensional analysis with some insight into the physics of the problem, such as can be obtained from general conservation laws or the differential equations of the problem, one can attempt to reduce the number of variables further specifying the forms of $\overline{p^2}$ and ϕ more closely. One may also turn the process around and discuss the physical mechanism which must be involved to give a certain experimentally obtained similarity law.

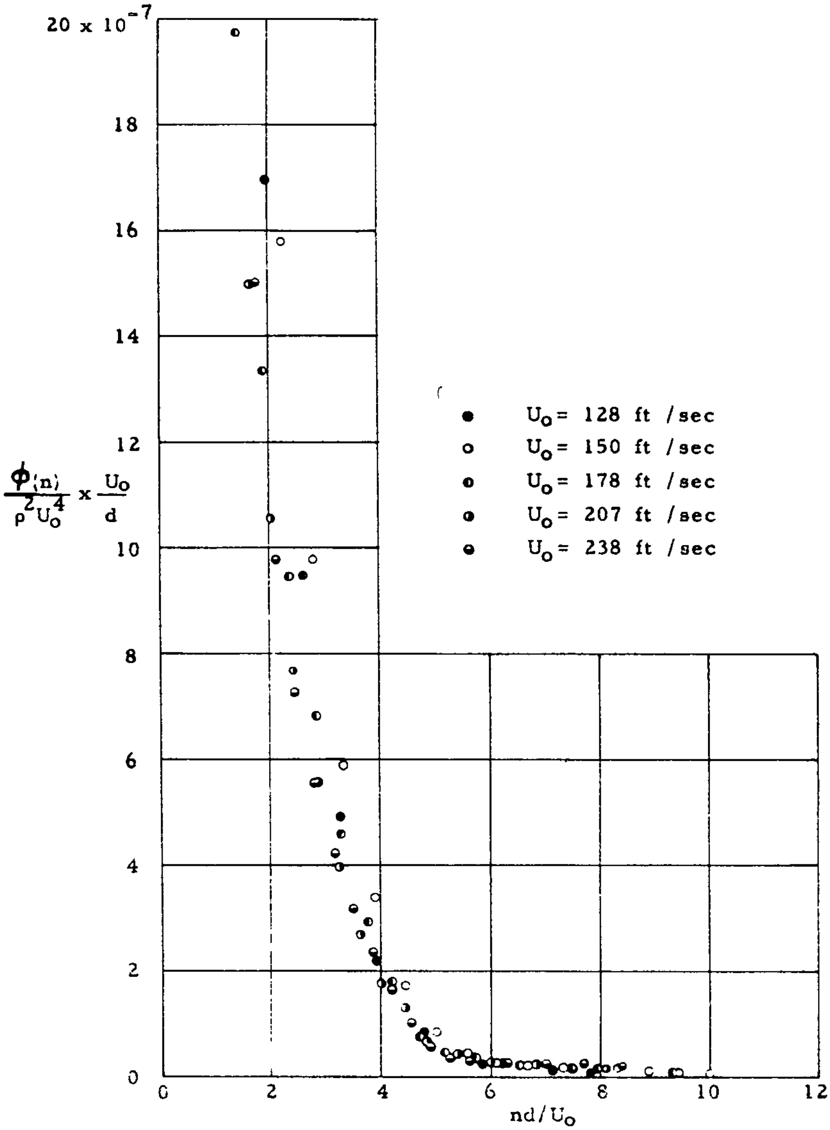

Fig. 2 - Similarity of power spectra of p(t). $\tilde{t}$ = 0.0005 inch, d = 1 inch, r/d = 1/2.

In the far field of a limited-size emitter, the intensity must fall off as $1/r^2$; consequently it follows that

$$\overline{p^2} = \rho^2 U^4 \frac{D^2}{r^2} f(\theta ; \text{Ma}, \text{Re})$$

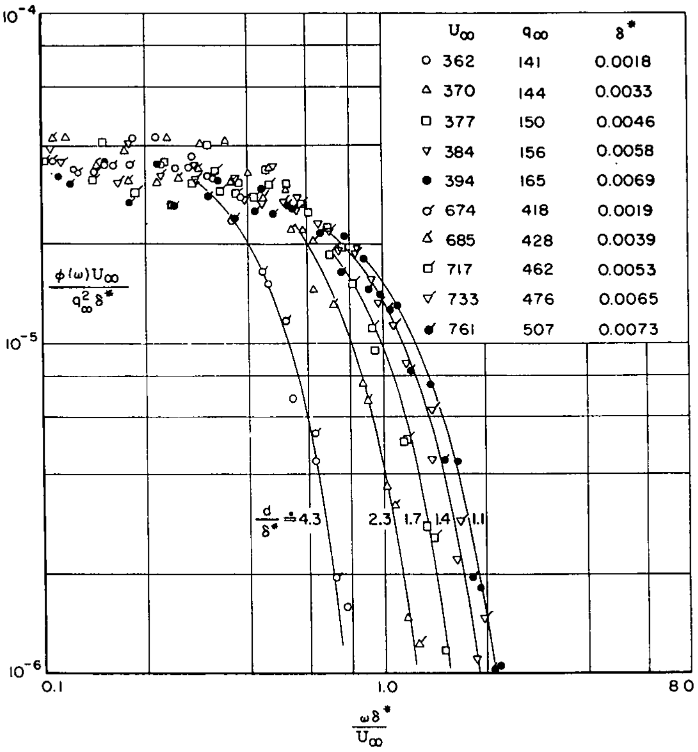

Fig. 3 - Power spectra of the fluctuating wall pressure in a turbulent boundary layer

and

$$\varphi = \rho^2 U^3 \frac{D^3}{r^2} g\left(\frac{\omega D}{U}, \theta ; \text{Ma}, \text{Re}\right)$$

where the vector $\vec{R}$ has been replaced by its magnitude r and its inclination θ to the jet axis.

To go beyond this stage, more and more detailed physical models must be adopted, or else the emphasis is thrown entirely on the measurements per se. For example, if the emitter contains travelling sources, the combination Ma cos θ will be characteristic, rather than Ma and θ by themselves.

INTERPRETATION OF JET NOISE

While the measurements of wall-pressure fluctuations under a turbulent boundary layer and turbulent pipe flow are measurements of noise from a practically homogenous surface emitter, jet noise is emitted from highly nonhomogenous turbulence which occupies a small region.

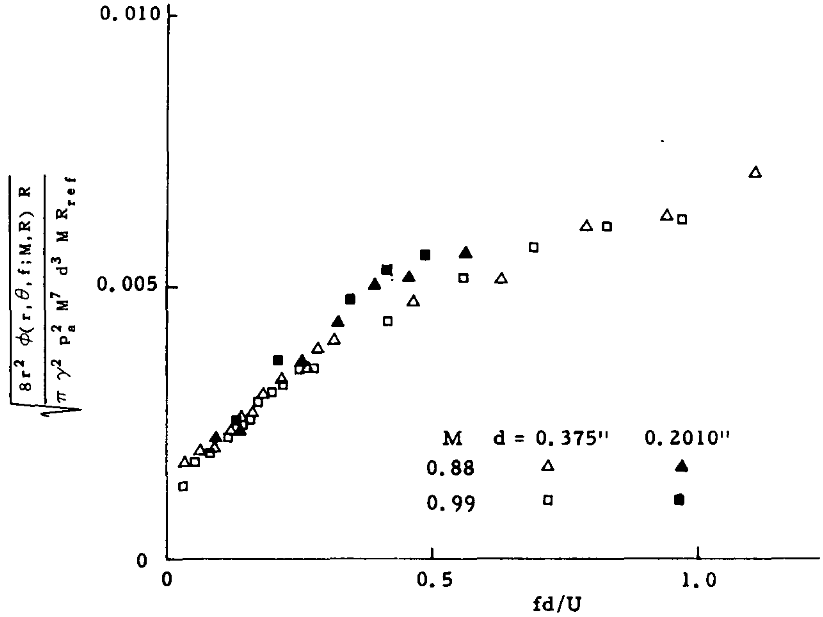

Fig. 4 - Power spectra of jet noise at $\theta = 90°$

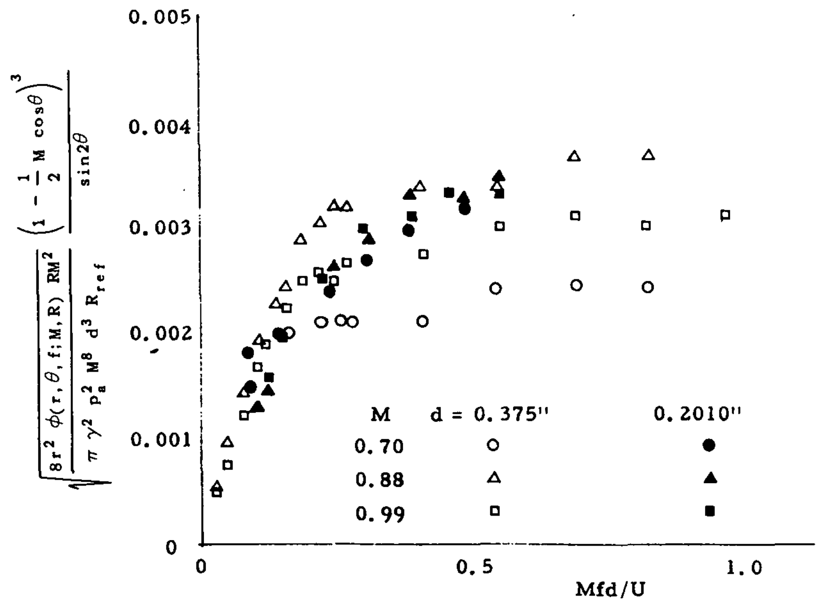

Fig. 5 - Power spectra of jet noise at $\theta = 40°$

It is usually attempted to interpret measurements of jet noise in terms of the concepts introduced by Lighthill (3) in his now famous paper. It may still be true that this is still too ambitious an undertaking, since a true comparison with Lighthill's theory demands a detailed knowledge of the flow field far beyond what has so far been obtained. Lighthill's theory attempts to determine the overall sound field of a turbulent flow in terms of elementary, characteristic sound sources, the quadrupole field of turbulence. Between such an approach and the still comparatively crude measurements, one may well interpose a more naive and restricted interpretation, in which for example the jet is considered as an axially symmetric line emitter, as far as the far field is concerned. The gross features of this emitter can be determined from experiment. An application of this intermediate approach will be given in the forthcoming paper by Mollo-Christensen and Narasimha; a few features will be illustrated here.

The most surprising result of our measurements of jet noise was the marked Reynolds number influence both on intensity and spectrum. This we believe can only be explained in terms of transition to turbulence in the free shear layer. Since a turbulent flow induces more influx than a laminar shear layer, one may interpret the corresponding sound field as produced by a source on the axis if the transition is axially symmetric. If the transition is antisymmetric about the jet axis, the jet is whipping around, one can describe the sound emission as that due to dipoles perpendicular to the axis. If the whipping of the jet is strongly anticorrelated along the jet axis for one-half wavelength, the field will be that of a lateral quadrupole.

In part, emitted sound may have wavelengths comparable to the size of the emitter, and to determine the spectra one then has to consider the antenna problem involved. In any case the gross motion of transition and its influence upon the sound field is a feature which is relatively easily tractable.

An example of the effect of Reynolds number on the sound field is shown in Fig. 4, where spectra from two jets from geometrically similar nozzles of two diameters are shown, with a Reynolds number factor included in the spectra. In Fig. 5, similarly, we show spectra measured at $\theta = 40°$, at three Mach numbers, and two nozzle diameters. In addition to a Reynolds number correction, we have attempted to include a factor to account for the possibility of source convection. The same spectra, without the Reynolds number correction, are shown in Fig. 6. As can be seen, there appears to be a strong Reynolds number dependence. Finally, in Fig. 7, some spectra obtained at $\theta = 25°$ are shown.

CONCLUSIONS

It seems to be imperative to utilize dimensional analysis and similarity relations to the extent possible in experiments on aerodynamic noise. Only by this approach can one identify and eliminate spurious effects; also, it appears to be the only possible framework for analysis of experimental results and synthesis of possible mechanisms of emission, in short, for understanding.

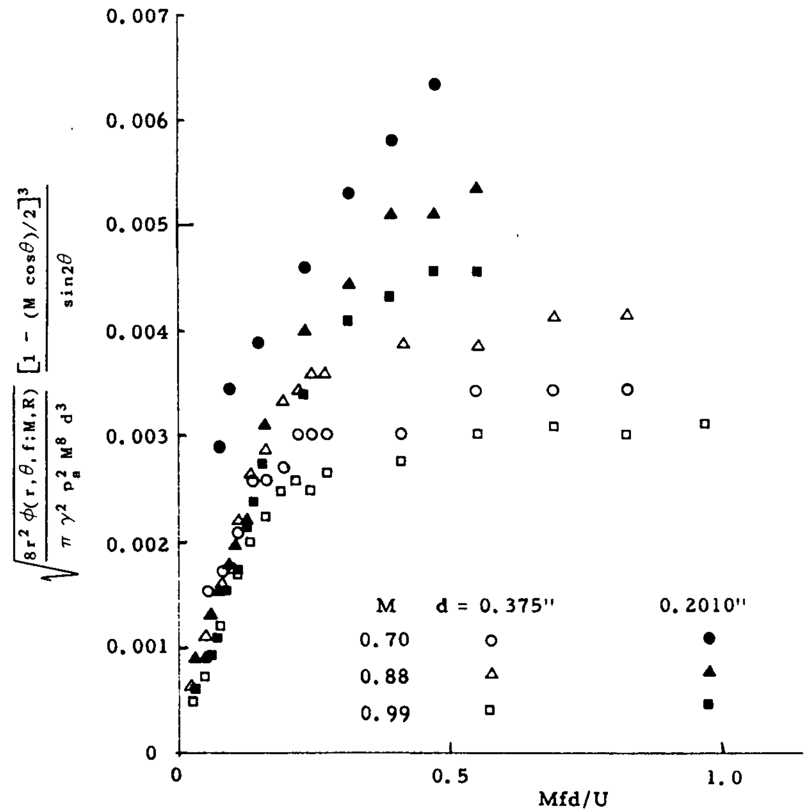

Fig. 6 - Power spectra of jet noise at θ = 40°

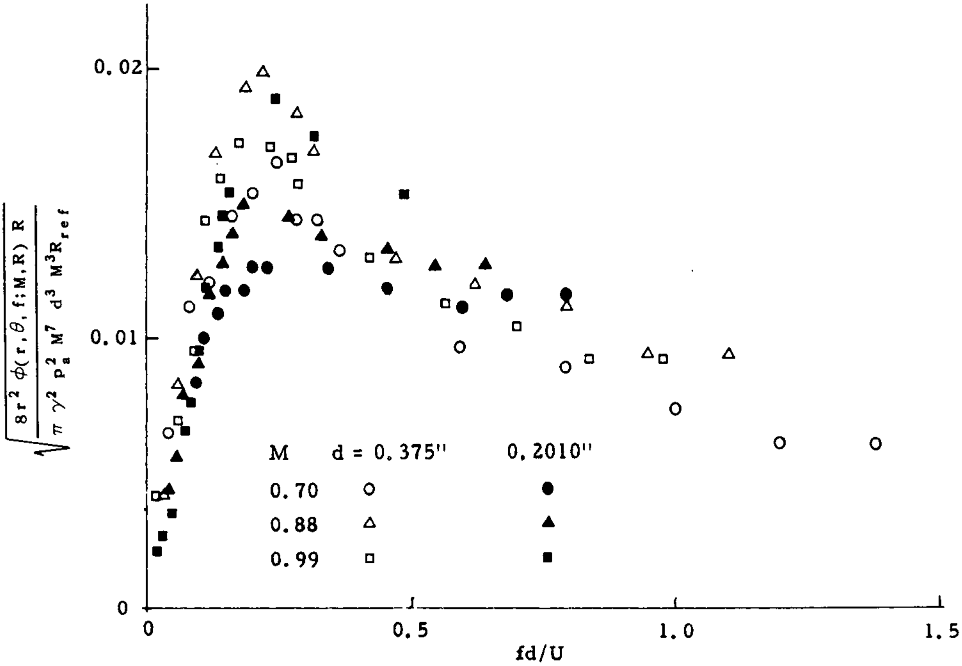

Fig. 7 - Power spectra of jet noise at $\theta = 25°$

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1. Willmarth, W.W., "Space-Time Correlations and Spectra of the Wall Pressure in a Turbulent Boundary Layer," NACA Technical Note in print

2. Weyers, Paul F.R., "The Vibration and Acoustic Radiation of Thin-Walled Cylinders Caused by Internal Turbulent Flow," submitted to NACA, Aug. 1958

3. Lighthill, M.J., "On Sound Generated Aerodynamically - I. General Theory," Proc. Roy. Soc. A211:564-587 (1952)

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DISCUSSION

M. Strasberg (David Taylor Model Basin)

At the David Taylor Model Basin we have performed measurements of pressure fluctuations. For those measurements involving essentially incompressible flow, that is, pressure fluctuations in the near field (at distances from the source that are less than a quarter wavelength of whatever frequency is of interest), we have found that similarity is maintained. These include the near field of a turbulent jet at low Mach numbers and the pressure fluctuations in the wake of a cylinder. In those cases, similarity was maintained, that is, when we plotted a non-dimensional, spectral density versus a Stroudhal number, we would get a spectral curve that applied at all velocities. We were not surprised by this. We would have been more surprised if this hadn't been the case.

I would like to mention a convenient way for expressing spectral distributions. It has been common to express the spectrum in terms of spectral density, which is the mean square pressure in a narrow band divided by the bandwidth, but for non-dimensional parameters one might consider expressing the spectrum in a slightly different way as the mean square pressure itself, which remains proportional to the frequency. This is just a matter of convenience, the advantage being that when going from one scale to another it is not necessary to correct for the bandwidth. The bandwidth automatically changes in the right way. For example, one can give the mean square pressure in a half octave band. The reason I am suggesting this is that I have seen many cases where people scale from one dimension to another, and forget to change the bandwidth proportionally.

E. Mollo-Christensen

In response to Dr. Strasberg's remark that similarity should be expected in individual measurements, I would like to point out that if only one parameter is varied it may be easy to obtain apparent similarity, but no evidence has been obtained that the phenomenon is independent of the variables which were not changed during the experiment.

One has not demonstrated similarity, for example, when measuring the pressure fluctuations in a turbulent boundary layer if a part of the observed fluctuations could be due to turbulent flow in the tunnel. Such pressure fluctuations would probably scale with velocity the same way as the pressure fluctuations due to boundary layer turbulence. Thus, parasitic effects may follow the same similarity law as does the process one intended to observe. To avoid this, one must either eliminate or change the wind tunnel turbulence. I believe the disagreement between the results obtained by Dr. Willmarth and those obtained by Dr. Strasberg may be due to the fact that Willmarth's measurements were performed with virtually zero free-stream turbulence, while Strasberg's were obtained in an existing wind tunnel where there apparently was some free-stream turbulence, judging from the difference in spectra and space-time covariance observed in the two investigations.

About the suggestion that one plot constant-Q spectra: I feel it may take some time to become accustomed to the relation for example, between such spectra and the correlation function.

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