SOME EXPERIMENTAL AND THEORETICAL RESULTS RELATING TO THE PRODUCTION OF NOISE BY TURBULENCE AND THE SCATTERING OF SOUND BY TURBULENCE OR SINGLE VORTICES

Ernst-August Müller
Max-Planck-Institut für Strömungsforschung, Göttingen

EXPERIMENTS WITH REDUCING VALVES

In the piping system of steam power plants, reducing valves (see Fig. 1) are often installed to control the flow rate of the steam and to reduce the steam from a high pressure to a lower pressure under certain operating conditions of the plant. In many cases the flow rate of the steam and the pressure difference are large; for example, a maximum flow rate of 200 tons per hour may have to be reduced from 150 atm at the high pressure side of the valve (left) to 20 or 30 atm at the low pressure side (right). The physical process of pressure reduction consists in the conversion of kinetic energy, which is produced by the valve in its smallest cross section, to heat by turbulent mixing in the region R, a process in which the enthalpies (per unit of mass) before and after the valve are approximately equal because the losses by heat-conduction and heat- and sound-radiation are small. In the above example, the energy being converted per second is of an order of magnitude of 10 MW (megawatts), but sometimes it is even higher (up to 100 MW). Of course, turbulent mixing at such a high power produces noise in the vicinity of the valve of a strength which often lies beyond the threshold of pain. In addition, strong vibrations of the valve and the piping system are caused which lead to vibration breakages of screws, welds, etc. (1) and thereby threaten the safety of operation. In Fig. 2 one sees a typical broad noise spectrum with some dangerous high peaks which obviously are due to resonance phenomena.

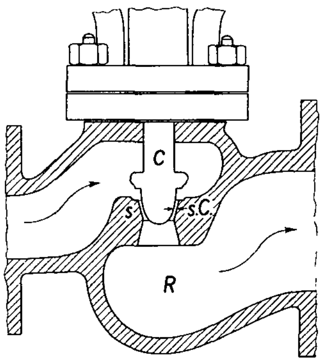

Fig. 1 - Schematic diagram of a typical reducing valve. S seat, C cone, s.C. smallest cross section, R mixing region.

Because of these difficulties, an association of power plant operators asked the Max-Planck-Institut für Strömungsforschung to design valves which would work reliably even if the flow rates, the pressures and the temperatures increase more and more, as seems to be the general trend today. At first, model experiments were

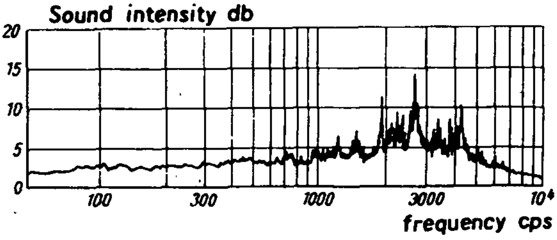

Fig. 2 - Sound intensity in the vicinity of a reducing valve (with arbitrary reference intensity)

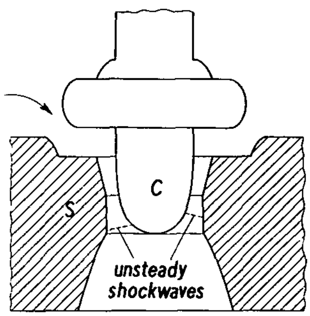

Fig. 3 - Unsteady shock waves at cone and seat of the valve

made with air with a pressure reduction from 4 to 1 atm. Attention was focused on two regions of the valve:

1. The surroundings of the valve cone and the valve seat (Fig. 3). Here the flow is supersonic; unsteady shock waves occur and cause vibrations, especially of the valve cone, and, thus, of the whole reducing valve.

2. The mixing region R (see Fig. 1) and its shape.

The vibrations were measured in the following way (Fig. 4): Different valve cones C (Fig. 4b) and different bottoms B (Fig. 4c) were fixed on thin iron plates. The plate with the bottom could be mounted at a variable distance from the seat. Opposite the outer sides of the plates electromagnetic microphones were set up in which voltage oscillations were induced by the vibrations of the cone and the bottom. (The characteristic frequencies of the fixing plates did not play a role as was checked by earlier experiments.) The experiments gave the following results (details in Ref. 2):

1. The surroundings of cone and seat are not of great importance for the vibrations of the valve. The vibrations are of minimum strength if the nozzle contour formed by the seat and the cone has a ratio of throat cross section to exit cross section (into the region R) which is equal to the well-known ratio one has to use for the design of supersonic nozzles at a given pressure ratio. This result corresponds to results found by other authors (see Ref. 3, for instance) using convergent-divergent nozzles.

2. The main source of the vibrations and the noise is the mixing region R. Here, a change in the shape of R by use of a smooth contour a certain distance from the valve seat gave a reduction in the strength of the vibrations of 12 db. However, if the mixing region was changed so as to completely eliminate the bend after the valve, the performance was almost as bad as in the original case.

The reduction of 12 db was not yet large enough for practical purposes. Therefore, two other possibilities were considered:

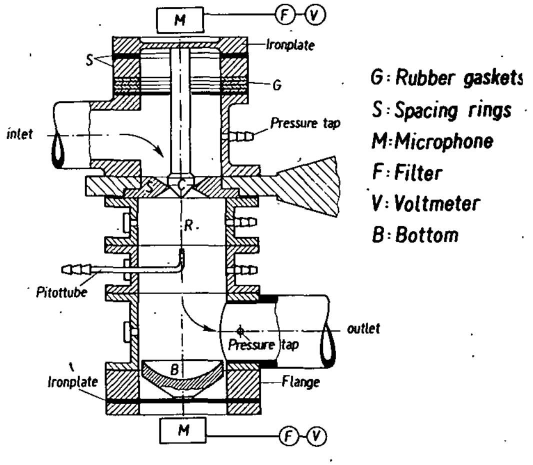

Fig. 4a - Schematic diagram of the model valve

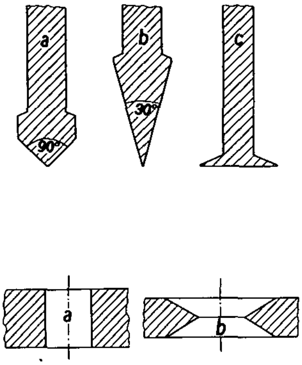

Fig. 4b - Shapes of the cones and seats

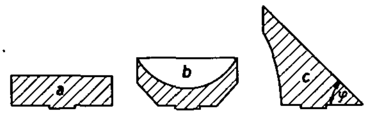

Fig. 4c - Bottom shapes

1. A reduction of the flow velocity from supersonic to subsonic values. Such a reduction would result in the disappearance of the unsteady shock waves. Further, the noise level would decrease according to the U^8 law (or a similar power-law of a characteristic velocity U) which governs the production of noise by turbulent mixing. Of course, this decrease of velocity is limited by the demand for a small cross section of the valve.

2. A great decrease in the size of the turbulent eddies. The physical argument for this device is not only the shifting of the dominant frequencies toward the ultrasonic domain, but also the influence on the turbulent mixing process itself. This can be understood if one recognizes that the conversion of kinetic energy into heat is performed by a kind of cascade-type mixing process in which the energy is transferred from the energy containing wave numbers (eddies in the size of the mechanism producing the turbulence) through a region of mean wave numbers (mean-size eddies) up to the dissipative domain of wave numbers (small eddies). Qualitatively, one may say that during this transfer, the turbulent eddies strike one another and thereby produce noise. If the process of energy transfer from large eddies (small wave numbers) to small eddies (large wave numbers) is cut off in such a way that the initial size of the eddies is smaller than before, a part of the transfer process does not take place, and the noise production decreases. (This process will be discussed in more detail in the next section.)

In order to test the possibilities mentioned above, a "labyrinth system" (Fig. 5) was designed which was set directly below the valve seat. It consisted of a set of plates with many small holes (1 to 4 mm in diameter) which were placed so that each hole of one plate lay opposite a closed part of the next plate. The spacing between subsequent plates was also very small. The flow velocity through the system could be controlled by varying the number and the size of the holes and the spacing of the

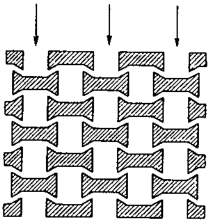

Fig. 5a - Schematic diagram of labyrinth set

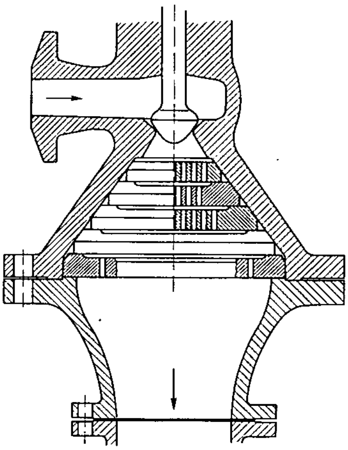

Fig. 5b - Valve with labyrinth set

plates. The degree of turbulence was increased by sharp edges. In this way a drag was produced which was large enough to reduce the pressure at subsonic velocity, at least at maximum flow rate. This type of reducing valve worked very well and gave an improvement of 24 db in the model experiments (measured again by the vibrations of a bottom mounted below the labyrinth system).

On the basis of these experiments, several full-scale valves (Fig. 6) with labyrinth sets were designed and installed in some heating and electric power plants. They worked very satisfactorily. At one plant, for instance, the inhabitants of the surrounding apartment houses had filed suit, charging that the company was disturbing the peace with the noise caused by the old-type valve. After the new valve was installed the noise outside the engine house was very low and no longer disturbed the inhabitants. Inside the engine room the noise production of the other machines placed there was larger than that of the valve. Measurements gave a noise level of 91 phons in this room, whereas the noise level produced by the valve alone was about 85 phons. The acceleration of the material, which is important for the vibration breaking strength, was determined at different points of the valve surface. The measured values ranged from 10 to 20 m/sec^2, which was several orders of magnitude below the critical acceleration for vibration breakage.

In another application, a valve was also used for pressure reduction at extreme pressure ratios. A flow rate of 64 tons per hour had to be reduced from 116 to 3 atm. Even at this supercritical ratio the valve performed very satisfactorily, both from the standpoint of noise and from that of vibrations.

CALCULATION OF THE INFLUENCE OF THE EDDY SIZE ON NOISE PRODUCTION

In the last section the supposition was made that noise generation is influenced by the eddy size of the turbulent motion. In the language of statistical analysis this means a dependence on the spectral distribution of the turbulent properties. In order to check this supposition, the noise output was calculated for several different distributions of turbulent energy in wave number space for the case of decaying, homogeneous, isotropic turbulence with zero mean velocity. This flow may be considered as representing the flow in a jet, in which the turbulence of the different fluid volume elements (in the moving system) also decays.

The following assumptions were made: At the time $t = t_o$ a homogeneous, isotropic turbulent motion exists in the volume V. The mean velocity is zero. The spectral energy distribution at $t = t_o$ is given by the function F so that

$$e_o = \int_0^\infty F(k, t_o)\, dk \tag{1}$$

where e_o is the kinetic energy per unit mass at $t = t_o$ and k is the wave number. The shape of $F(k, t_o)$ is rectangular (see Fig. 7):

$$
\begin{aligned}
F &= 0 && \text{for} \quad 0 \leq k \leq k_o \\
F &= F_o = \frac{e_o}{k_o(a-1)} && \text{for} \quad k_o \leq k \leq ak_o \qquad (2) \\
F &= 0 && \text{for} \quad ak_o \leq k
\end{aligned}
$$

$(a > 1)$. At $t = t_o$ the turbulence begins to decay. The total sound energy E radiated from the volume V during the decay process is to be calculated, especially its dependence on the initial condition Eq. (2).

The initial condition, Eq. (2) contains three parameters e_o, a and k_o. For the purposes of this calculation, only k_o is of interest, because $1/k_o$ is proportional to the longitudinal scale

$$L_o = \int_0^\infty f(r, t_o)\, dr$$

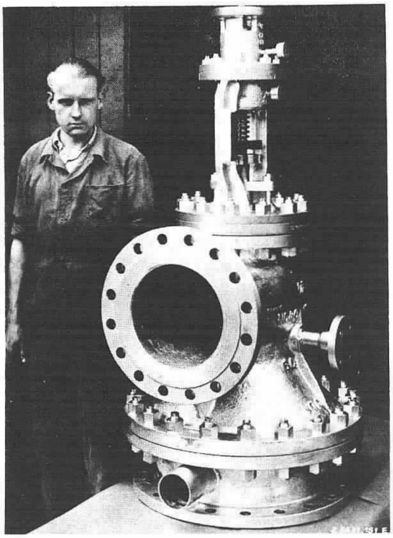

Fig. 6 - Full-scale valve with labyrinth set (after Pontow (4))

which is a measure of the eddy size (f is the longitudinal correlation function, r the distance). Therefore, constant values of e_o and a were assumed and the noise output for different values of k_o was calculated. The parameter a was taken equal to 2. This value of a means that the energy is contained in an octave of wave numbers at the beginning of the decay.

With a derivation similar to that in I. Proudman's paper (5) the energy E was found to be (for details see Ref. 6)

$$E = \frac{\rho_o V}{c_o^5} \int_{t_0}^{\infty} \gamma(t) \frac{\left[\overline{v^2}^4\right] t}{L(t)}\, dt\,, \qquad (3)$$

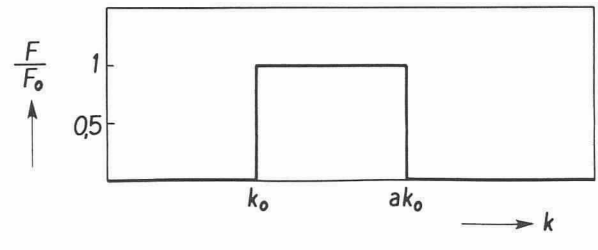

Fig. 7 - Spectral energy distribution F at $t = t_o$ vs wave number k

where ρ_o and c_o are the density and the speed of sound at infinity, respectively, $\overline{v^2}$ is the instantaneous mean square value of any velocity component (arbitrary because of isotropy),

$$L = \int_0^\infty f(r,t)\,dr$$

is the longitudinal scale, and $\gamma(t)$ is a function dependent on the shape of f only. The dependence of the integrand on t was calculated by Heisenberg's integro-differential equation for the decay of homogeneous, isotropic turbulence, which was solved as an initial value problem. The results are given in Fig. 8, in which the energy E, divided by M_o^5 and by the total initial turbulent energy $\rho_o e_o V$ contained in V, is plotted against Re_L, the initial Reynolds number of the turbulence. The following definitions apply:

$$M_o = \frac{(\overline{v_o^2})^{1/2}}{c_o}$$

and

$$Re_L = \frac{(\overline{v_o^2})^{1/2}\,L_o}{\nu}.$$

$\overline{v_o^2}$ is the mean square of one velocity component at $t = t_o$ and ν is the kinematic viscosity.

Figure 8 shows that the expected general trend indeed exists. At low Reynolds numbers (small eddies) less noise is produced than at high Reynolds numbers (large eddies), the other parameters remaining unchanged. For example, a decrease in Re_L from 3×10^4 to 3×10^1 gives a decrease in the noise output of about 7 db. Quantitatively, the dependence of the noise output on Re_L is small at high Reynolds numbers (in agreement with Proudman's considerations (5)) and larger at lower Reynolds numbers.

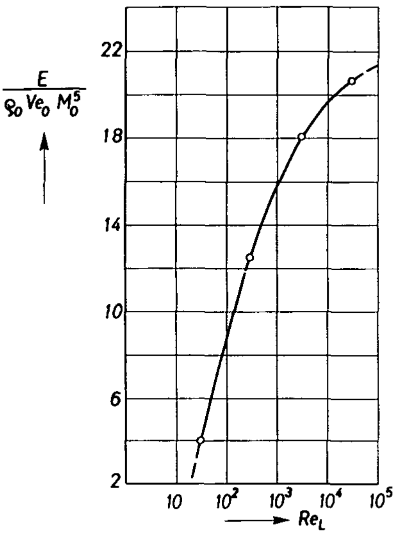

Fig. 8 - Dimensionless total sound energy vs Reynolds number Re_L. o calculated points for a = 2.

The result can easily be understood by means of a dimensional analysis. For this purpose, the integration in Eq. (3) is replaced by multiplying the initial sound power, which is proportional to

$$\frac{\rho_o V}{c_o^5} \cdot \frac{\left[\overline{v_o^2}\right]^4}{L_o},$$

with the time of decay of the turbulence. For large Reynolds numbers, this time is proportional to

$$\frac{L_o}{(\overline{v_o^2})^{1/2}}.$$

Hence, E is independent of L_o. For smaller Reynolds numbers, a part of the initial energy e_0 is contained in the dissipative domain of wave numbers, and another characteristic time becomes important which involves the viscosity, namely the time L_0^2/ν (this also follows from Heisenberg's equation). Therefore, if this time becomes significant, the energy E becomes proportional to L_0 and decreases with decreasing L_0. The curve shown in Fig. 8 covers the upper part of the transition region between the two laws.

What practical conclusions can be drawn from these results? The calculations were only concerned with homogeneous, isotropic turbulence, but one can assume that the results are also qualitatively correct for inhomogeneous, non-isotropic turbulence. Hence, one can say that the reduction in the size of the turbulent eddies is an effective means of noise suppression if the size of the eddies can be made very small. In the case of a reducing valve this is easily possible. In the investigations of the last chapter, for instance, the Reynolds number Re_L lay between 10^2 and 10^4, but even smaller values could have been attained. For jet engines, the overall benefit incurred in reducing the eddy size is problematic, because the reduction may lead to a too large increase of weight and drag and also to thrust losses.

Some remarks concerning some devices for the reduction of jet noise are perhaps in order. The above calculations show the effect of changing eddy size on sound output and thereby offer a possible explanation for the reduction of overall sound power effected by some jet noise suppressors. For instance, the reduction of about 7 db achieved by the slit-type nozzle proposed by E. J. Richards (7) can be due - at least partially - to the decreased eddy size. Further, one can expect that certain other changes in the distribution of turbulent properties also reduce the rate of noise production. For example, the avoidance of strong inhomogeneities or nonisotropies may result in a reduction of the sound level by eliminating the eddy exchange processes needed to smooth out a region of strong gradients. This seems to be the situation with some suppressors for jet planes (at the jet edge of the corrugated nozzles, for example). These proposed explanations ought to provide a basis for further experimental investigations in which particular care should be taken regarding measurements in the higher frequency bands.

SCATTERING OF SOUND BY A SINGLE VORTEX AND BY TURBULENCE

Another subject of the investigations of the Max-Planck-Institut für Strömungsforschung concerning the interaction of sound and turbulence is the scattering of external sound by a turbulent medium.* This research is still going on and, hence, only preliminary results can be given here. Both theoretical and experimental investigations have been made. First the theoretical considerations will be described.

Scattering by a Single Vortex

General studies of the scattering of light or sound have already been made by various authors who have considered wave propagation in a medium with fluctuating refraction coefficient (see the references listed in (8), also (9,10) and others). Special theories for the scattering of sound by turbulence have been developed by Lighthill (11), Kraichnan (12), and Batchelor (13). These theories, while having general validity, have been applied specifically to homogeneous, isotropic turbulence. In addition the noise generated by the interaction of shock waves and turbulence has been considered (11,14).

*These investigations are being performed under Contract No. AF-61(514)-1143. The theoretical part of the investigations is published in detail in (15).

The starting point for the investigations of the Max-Planck-Institut was the problem of the scattering of a sound wave as it passes through a single vortex. This single-vortex scattering may be considered as a kind of elementary model for the process of scattering by turbulent eddies. As will be shown, the knowledge gained from a study of this elementary model provides an understanding of the scattering caused by both extreme non-isotropic and by isotropic turbulence.* Of course, scattering by a single vortex is also in itself a problem of mathematical and physical interest.

The problem was formulated as follows: Consider a steady potential vortex of circulation Γ'. The axis of the vortex coincides with the z-axis of a cylindrical coordinate system r, φ, z and extends from $-\infty$ to $+\infty$. In the r-φ-plane the vortex extends from $r = r_i$ to $r = r_a$. In the domains $r < r_i$ and $r > r_a$ the fluid is at rest. This vortex interacts with a plane sound wave of wave length λ. The projection of the normal of wave propagation into the r-φ-plane is taken as the line $\varphi = 0°$. The direction of wave propagation includes an angle θ with the axis of the vortex (see Fig. 9). The polar diagram of scattering intensity and the scattering power are required as functions of the five parameters of the problem Γ', λ, r_i, r_a, θ.

The problem was solved by applying the method of small perturbations to the frictionless equations of motion, the equation of continuity and the equation for the conservation of entropy of a mass element. This means that terms involving the square of the amplitude of the sound wave were neglected.

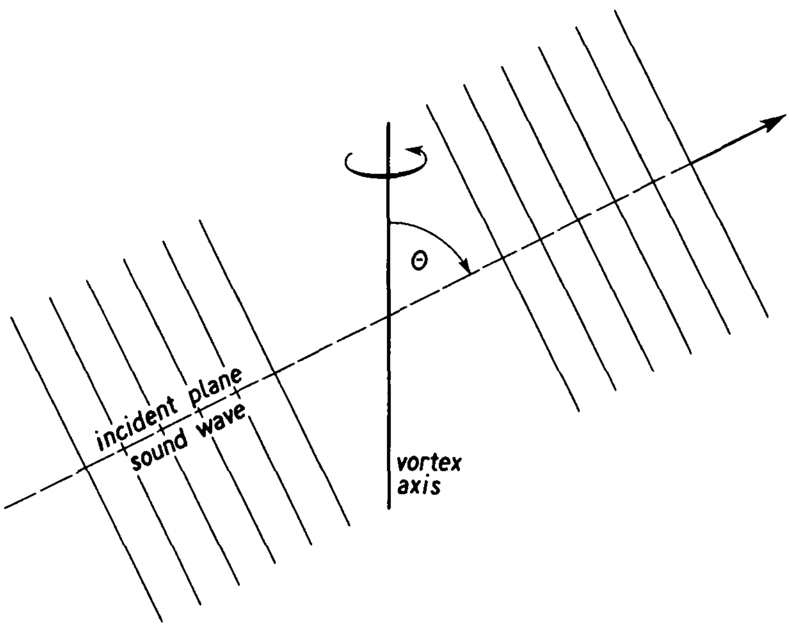

Fig. 9 - Schematic drawing of obliquely incident sound wave

*It may be mentioned that a related problem - the interaction of a shock wave and a single vortex - has been treated in several papers (16,17). A kinematical consideration of the sound wave-vortex interaction is given in (18).

The five parameters of the problem can be reduced to four by defining new dimensionless parameters $\mu = \Gamma/\lambda c_o$, $R_a = r_a/\lambda$, $R_i = r_i/\lambda$ ($\Gamma = \Gamma'/2\pi$ and c_o is the speed of sound). $|\mu|$ is assumed to be small compared with 1. Therefore, if all variables are expanded in power series in μ, terms of order μ^2 and smaller may be neglected. With these approximations, the potential of the sound reads $\phi = \phi_o + \mu\phi_1$. ϕ_o is the potential of the original plane sound wave.

$$\phi_o = B\, e^{2\pi i(\tau - R \sin\theta \cos\varphi - Z\cos\theta)}, \tag{4}$$

where B is a constant, $R = r/\lambda$, $Z = z/\lambda$, $\tau = tc_o/\lambda$ are the dimensionless independent variables and t is the time. $\mu\phi_1$ is the potential of the scattered sound. For ϕ_1 one obtains the inhomogeneous wave equation

$$\phi_{1\tau\tau} - \Delta\phi_1 = -\frac{2}{R^2}\cdot\phi_{o\tau\varphi} \tag{5a}$$

in the region $R_i \leq R \leq R_a$, and the homogeneous wave equation

$$\phi_{1\tau\tau} - \Delta\phi_1 = 0 \tag{5b}$$

in the regions $0 \leq R \leq R_i$ and $R \geq R_a$. Δ is the Laplacian operator

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial Z^2}.$$

Equation (5) can be solved by the well-known method of retarded potentials.

Special care has to be taken regarding the discontinuities in the potential ϕ_1 and its derivatives at $R = R_a$ and $R = R_i$. From dynamical considerations it follows that the sound pressure for $R \to R_a - 0$ and $R \to R_a + 0$ has to be continuous. This requirement leads to the jump condition

$$[\phi_1]_{R_a+0} - [\phi_1]_{R_a-0} = \frac{1}{2\pi i\, R_a^2}\left[\frac{\partial\phi_o}{\partial\varphi}\right]_{R_a} \tag{6a}$$

for all τ, φ, Z. Further, from the requirement that the flow velocities normal to the interface between the vortex and the outer flow be continuous, a second jump condition results:

$$\left[\frac{\partial\phi_1}{\partial R}\right]_{R_a+0} - \left[\frac{\partial\phi_1}{\partial R}\right]_{R_a-0} = -\frac{1}{2\pi i\, R_a^2}\left[\frac{\partial^2\phi_o}{\partial R\,\partial\varphi}\right]_{R_a} \tag{6b}$$

for all τ, φ, Z. With this condition, the ripples at the interface due to the original sound wave are taken into account.* At $R = R_i$ corresponding conditions apply. Conditions (6a) and (6b) require doublet layers and source layers respectively at both $R = R_a$ and $R = R_i$.

The solution of Eqs. (5) and (6) gives the following formula for the total scattering potential in the far zone ($R \sin\theta >> R_a$) for the case $R_i \to 0$:

*Compare the papers (19,20) that treat other but similar problems in which these ripples are of great importance.

$$\mu\varphi_1 = \frac{\mu \pi B}{\sqrt{R \sin \theta}} e^{i\pi/4} e^{2\pi i(\tau - R\sin\theta - Z\cos\theta)} \left[J_o\left(4\pi R_a \sin\theta \sin\frac{\varphi}{2}\right) - 1\right] \times$$
$$\times \left[-1 + 2\sin^2\theta \sin^2\frac{\varphi}{2}\right] \cot\frac{\varphi}{2}, \tag{7}$$

where J_o is the zero-order Bessel function of the first kind. For $\theta = 90°$ (sound wave falling perpendicularly on the vortex) this is a cylindrical wave, while for $\theta \neq 90°$ (sound wave falling obliquely on the vortex) it is a conical wave. This means that the surfaces of constant phase are cones moving outward as indicated in Fig. 10.

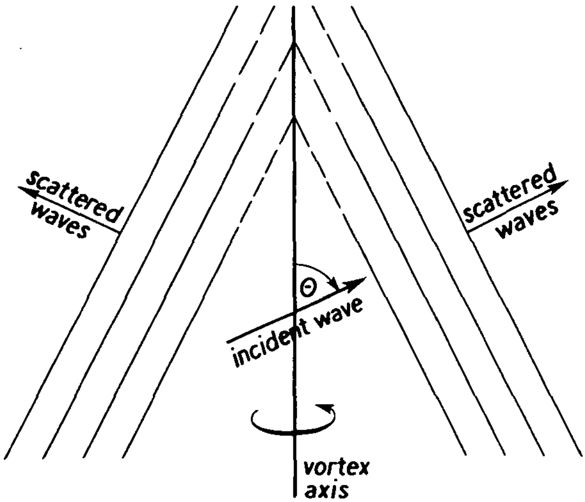

Fig. 10 - Cross-sectional view of the phase surfaces of the scattered sound waves

With the help of Eq. (7) polar diagrams of the scattering intensity

$$I_{sc} = \overline{pv_n} = \frac{\mu^2\pi^2}{R \sin\theta} F(R_a, \varphi, \theta) I_o \tag{8}$$

can be drawn. Here, p is the sound pressure, v_n is the particle velocity normal to the phase surfaces of the scattered sound, and I_o is the sound intensity of the incident wave. The bar indicates mean value and F is equal to

$$F = \left[J_o\left(4\pi R_a \sin\theta \sin\frac{\varphi}{2}\right) - 1\right]^2 \left[1 - 2\sin^2\theta \sin^2\frac{\varphi}{2}\right]^2 \cot^2\frac{\varphi}{2}. \tag{8a}$$

Such polar diagrams were constructed for four different values of θ (90°, 60°, 45°, 15°) and three different values of R_a (10, 1, 0.1) and are shown in Fig. 11. I_{sc} is referred to $(I_{sc})_{max}$, the value of maximum intensity. The waviness of the curves for $R_a = 10$ is due to the oscillations of the Bessel function or, physically, to interference effects.

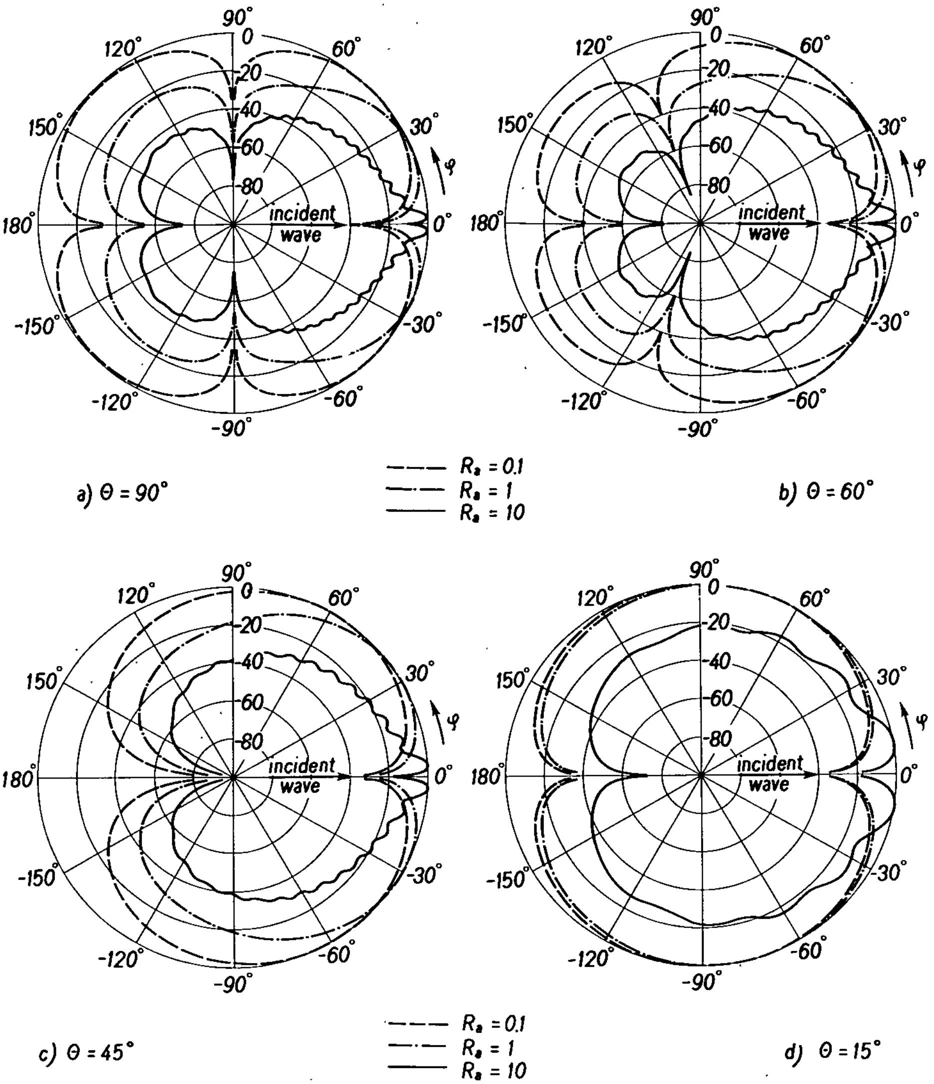

Fig. 11 - Polar diagrams of scattering intensity for different values of R_a and θ. Plotted is

$$10 \log_{10} \frac{I_{sc}}{(I_{sc})_{max}}$$

For $\theta = 90°$ the intensity is zero in four directions ($\varphi = 0°, \pm 90°, 180°$), but only for $R_a \rightarrow 0$ do the four leaves of the characteristic appear symmetrical and have the same maximum intensity (at $\varphi = \pm 45°, \pm 135°$). For $R_a \gtrsim 1$ the intensity of the two leaves in the half circle $-90° \leq \varphi \leq 90°$ is much greater than that of the other two leaves. Therefore, scattering by vortices with diameters of the order of magnitude of the wave length or larger is a forward scattering. Further, for $R_a >> 1$, the intensity is contained in a very sharp angle interval (see $R_a = 10$ in Fig. 11) and the angle φ_{max} of maximum intensity is proportional to $1/R_a$. It should be mentioned here that for $R_a << 1$ the angular distribution F (Eq. (8a)) agrees exactly with Lighthill's distribution for the case of homogeneous, isotropic turbulence (see Ref. 11, Eq. (25) with $E(k) \sim k^4$ for $k << 1$).

For $\theta \neq 90°$ (see Fig. 11b, c, d) the zeros at $\theta = 0°$ and $180°$ remain, while the zeros at $\pm 90°$ are shifted toward $\varphi = 180°$, as one can also see from Eq. (8a). For $\theta \leq 45°$ only two leaves can exist, because the second term in the second bracket of Eq. (8a) can no longer attain the value 1. For fixed R_a and $\theta \rightarrow 0°$ the angle φ_{max} approaches the value $90°$.

By use of Eq. (8) one can easily calculate the total scattering power L_{sc} emitted per unit length of the vortex. One obtains

$$\frac{L_{sc}}{L_o} = \frac{1}{2} \frac{\mu^2 \pi^2}{R_a \sin\theta} \int_0^{2\pi} F \, d\varphi \ . \tag{9}$$

$L_o = 2 r_a \sin\theta \, I_o$ is the sound power falling on the vortex per unit length. Because the most interesting question is the dependence on the frequency ν of the incident sound wave - the other parameters remaining unchanged - this ratio is plotted in Fig. 12 against R_a which is proportional to ν. (For this purpose μ is written

$$\mu = \frac{\bar{v}}{2c_o} R_a = \frac{1}{2} \bar{M} R_a \ , \tag{10}$$

where

$$\bar{v} = \frac{1}{\pi r_a^2} \int_0^{r_a} \int_0^{2\pi} v \, df$$

is the mean value of the azimuthal velocity v of the vortex and $\bar{M}$ is the mean Mach number.) For $R_a << 1$, L_{sc}/L_o varies with the 5th power of R_a, for $R_a >> 1$ with the 2nd power. In the vicinity of $R_a = 1$ the R_a^5 law changes to the R_a^2 law.

The scattering power per unit volume is $\ell = L_{sc}/\pi r_a^2$ which has the limiting values

$$\ell = \mu^2 \pi^6 R_a^2 \sin^4\theta (\sin^4\theta + 4\cos^4\theta) \frac{I_o}{\lambda} \qquad R_a << 1 \tag{11a}$$

and

$$\ell = 32 \, \mu^2 \pi^2 \left(1 - \frac{2}{\pi}\right) \sin\theta \, \frac{I_o}{r_a} \qquad R_a >> 1 \ . \tag{11b}$$

The last formula can also be written (by use of Eq. (10)) as

$$\ell = 8\pi^2 \frac{\bar{v}^2}{c_o^2} \left(1 - \frac{2}{\pi}\right) \frac{r_a \sin\theta}{\lambda^2} I_o \ , \tag{11'b}$$

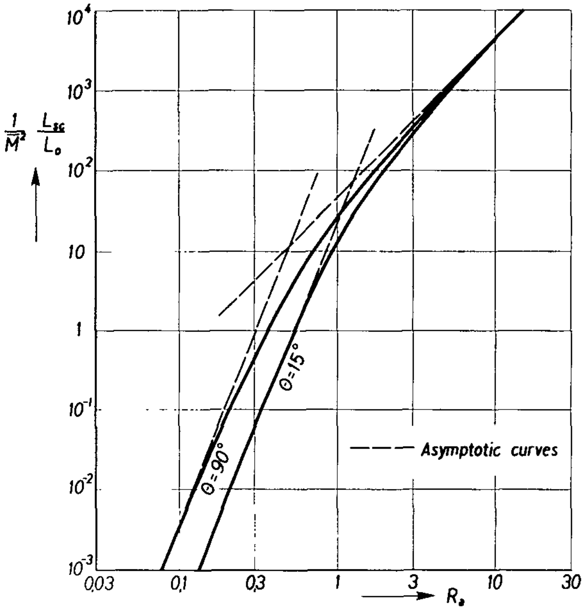

Fig. 12 - L_{sc}/L_0 vs R_a for θ = 90° and θ = 15°

which has exactly the same structure as Lighthill's formula (20) in Ref. 11 for the same limiting case. This again shows, as expected, that scattering by a single vortex contains the essential features of the scattering process in a turbulent medium and may be considered as a kind of elementary process.

Scattering by Turbulence

In order to apply the above results to turbulence it is assumed that the turbulence can be described by a statistical superposition of vortices of different radii r_a, circulations Γ', and inclinations θ with respect to the incident sound wave. Because the different vortices (eddies) are statistically independent of one another, the intensities of the scattered sound produced by them can be simply added. Further, it is assumed that the time required for the external sound wave to pass through one vortex is small as compared with the time in which the vortex undergoes any sensible change. This assumption is equivalent to the requirement that $R_a >> \mu$ (or $\bar{M}/2 << 1$). One may then proceed as follows:

Let $W(R_a, \theta, \mu; x, y, z)\, dR_a\, d\theta\, d\mu$ be the probability that the volume element $dV = dxdydz$ (x, y, z are the coordinates of a cartesian system) belongs to a vortex having a radius between R_a and $R_a + dR_a$, a dimensionless circulation between μ and $\mu + d\mu$, and an angle θ (see Fig. 9) between θ and $\theta + d\theta$. W has to satisfy the condition

$$\int_{-\infty}^{+\infty}\int_0^{\pi}\int_0^{\infty} W(R_a, \theta, \mu;\ x, y, z)\, dR_a\, d\theta\, d\mu = 1.$$

With this probability function, the total sound radiation L from the turbulent volume V is

$$L = \int_V \int_{-\infty}^{+\infty} \int_0^{\pi} \int_0^{\infty} W(R_a, \theta, \mu; x, y, z)\, \ell(R_a, \theta, \mu)\, dR_a\, d\theta\, d\mu dV . \tag{12}$$

This general expression was evaluated for two different cases, namely extreme non-isotropy (applicable for boundary layers or jet edges, for instance) and isotropy. (Non-isotropy is a case treated only with difficulty by the method given in Ref. 11.) For simplicity, it was assumed that all vortices had the same radius R_a and the same absolute value of circulation μ, and that the field was homogeneous (W independent of x, y, z). Then, non-isotropy means that all vortices have parallel axes making an angle θ with the direction of external wave propagation and can turn clockwise or counterclockwise. In the isotropic case W was taken proportional to the differential element of solid angle. The results are plotted in Fig. 13. This figure shows that the greatest scattering power occurs for $\theta = 90°$ (normal incidence) and that it decreases as $\theta \to 0$ (where it becomes zero). The scattering power in the isotropic case is approximately equal to that in the non-isotropic case with $45° \leq \theta \leq 60°$. Hence, one can say that there is no significant difference between isotropic and non-isotropic scattering.

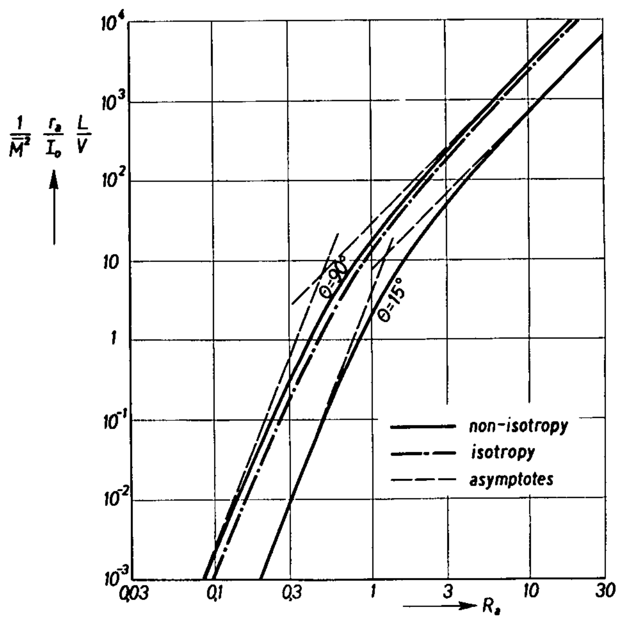

Fig. 13 - Dimensionless total scattering power radiated from the volume V, for two cases of non-isotropy ($\theta = 90°$ and $15°$) and for the case of isotropy

Experimental Investigations

Figure 14 shows the apparatus which is being used for the experimental investigations. The laminar air flow within the entrance region of a circular duct 300 mm in diameter is made turbulent by a grid consisting of parallel circular rods, 5 mm in

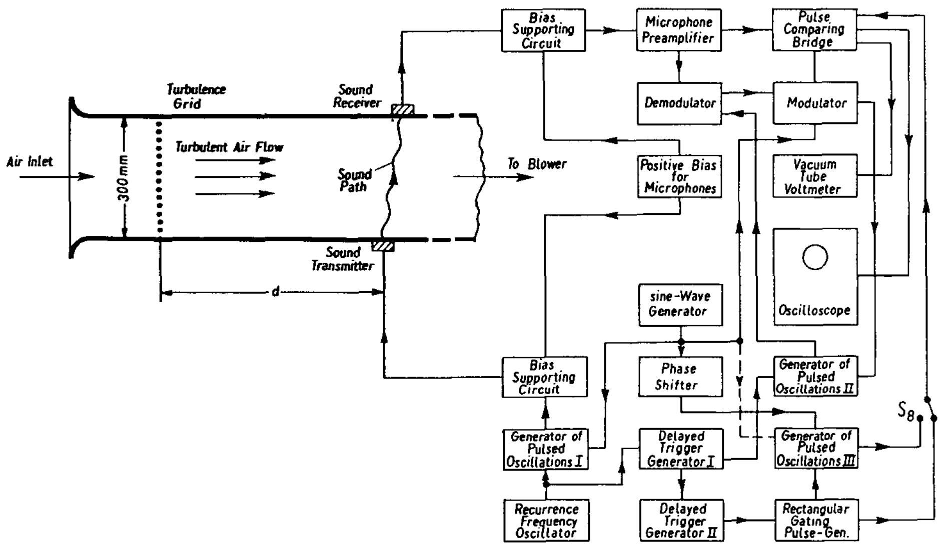

Fig. 14 - Block diagram of equipment used for tests of scattering of sound in a turbulent medium

diameter and 12.8 mm distant from one another. The flow velocity is 25 m/s. Short sound pulses are transmitted perpendicular to the flow direction from one side of the duct and are received at the opposite side.* The degree of turbulence of the flow traversed by the sound waves can be changed by varying the distance d of the turbulence grid from the test section. For the exact measurement of the intensity of the received sound pulses, electronic equipment is used which is shown in the right part of the figure. The main element, a "pulse comparing bridge," contains a bridge consisting of two condensers and two electron tubes controlling the charge of the condensers. In this bridge the effect caused by the received sound pulses on one arm is compensated by the effect of electrically produced pulses of the same shape on the other arm. The height of these electrically produced pulses can be measured easily and, if the bridge is balanced, is equal to the height of the sound pulses. The other parts of the electronic system shown in the figure are needed to compensate for the delay time incurred in the transmission of the acoustical pulses through the flow and to compensate for the amplitude modulation imposed on these pulses by disturbing noise and other influences.

Some experimental results concerning the dependence of scattering intensity on the frequency of sound are given in Fig. 15. The measurements were made in the following manner: At first, by shifting the receiving microphone, the position of the maximum intensity of the sound at the wall opposite to the transmitter was determined without scattering (that is to say, with the grid far removed from the test section or with laminar flow in the duct). The microphone was then left in this position and the grid was brought closer to the test section. In this way one obtains the intensity I^{lam}

*The sound transmitter and the receiver are identical condenser type microphones with solid dielectric, which have been developed by W. Kuhl, G. R. Schodder and F.-K. Schröder in the III. Physikalisches Institut of the University of Gottingen (21).

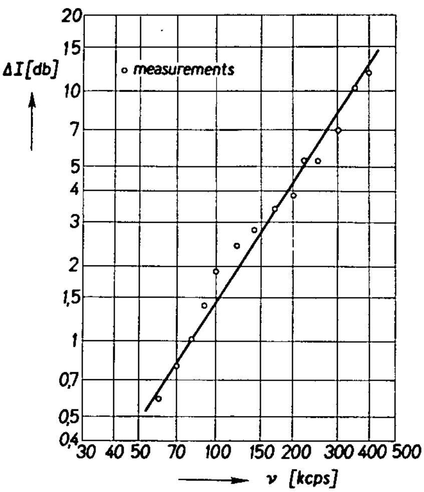

Fig. 15 - $\Delta I = 10 \log_{10} I^{lam}/I^{turb}$ vs frequency of sound. Distance d = 120 mm.

(without scattering) and the intensity I^{turb} (with scattering) for each frequency ν of the sound. The quantity $\Delta I = 10 \log_{10} I^{lam}/I^{turb}$ is plotted against ν in Fig. 15. ΔI must be proportional to L_{sc}/L_0 given by Eq. (9), since it represents the attenuation of the intensity due to scattering (see Ref. 15).

The calculations gave asymptotic power laws $L_{sc}/L_0 \sim \nu^n$ with $n = 5$ for the lower frequency range and $n = 2$ for high frequencies. The measuring values obtained until now seem to obey the ν^2-law. Further measurements are being made and will be published later.

ACKNOWLEDGMENTS

The author wishes to express his thanks to Prof. Dr. W. Tollmien for his steady encouragement, to his colleagues in these investigations, Dr. E. Koppe, Dr. K. Matschat, and Dipl. Phys. D. Schmidt, for their constant enthusiasm, to Mr. W. G. Nelson for his valuable help with the problems of English technical composition. Further, the author is grateful to the sponsors of this symposium for giving him the opportunity to present this paper.

REFERENCES

1. Buhr, W., "Erfahrungen mit Hochdruck-Reduzieranlagen bei den Hamburgischen Electricitäts-Werken," Mitteilungen der Vereinigung der Grosskesselbesitzer 41:93-99, April 1956

2. Koppe, E., und Müller, E.-A., "Modellversuche zur Klärung von Geräusch- und Vibrationsfragen an Reduzierventilen," Mitteilungen der Vereinigung der Grosskesselbesitzer 41:65-83, April 1956

3. Callaghan, E.E., Sanders, N.D., and North, W.J., "Recent NACA Investigations of Noise-Reduction Devices for Full Scale Engines," Aeron. Engin. Review 14(6): 66-71 (1955)

4. Pontow, W., "Wege zur Beherrschung von Geräuschen und Vibrationen bei der Dampfdruckminderung durch Reduzierventile," Mitteilungen der Vereinigung der Grosskesselbesitzer 41:83-93, April 1956

5. Proudman, I., "The Generation of Noise by Isotropic Turbulence," Proc. Roy. Soc. A214:119-132 (1952)

6. Müller, E.-A., und Matschat, K., "Zur Lärmerzeugung durch abklingende homogene isotrope Turbulenz," Zeitschrift für Flugwissenschaften 6:161-170 (1958)

7. Richards, E.J., "New Type of Tip Jet to Reduce Noise," J. Helicopter Assoc. Great Britain 9:404-413 (1956)

8. Koppe, E., und Müller, E.-A., "Strahllärm, ein wissenschaftliches und technisches Problem der Luftfahrtforschung," Jahrbuch der Wissenschaftlichen Gesellschaft für Luftfahrt, pp. 155-164 (1956)

9. Obuchow, M.A., "On the Influence of Weak Inhomogeneities of the Atmosphere on the Propagation of Sound and Light," Iswestija Akademii Nauk 2:155-165 (1953) (in Russian)

10. Potter, D.S., and Murphy, S.R., "On Wave Propagation in a Random Inhomogeneous Medium," J. Acoust. Soc. Am. 29:197-198 (1957)

11. Lighthill, M.J., "On the Energy Scattered from the Interaction of Turbulence with Sound or Shock Waves," Proc. Cambridge Phil. Soc. 49:531-551 (1953)

12. Kraichnan, R.H., "The Scattering of Sound in a Turbulent Medium," J. Acoust. Soc. Am. 25:1096-1104 (1953); errata: J. Acoust. Soc. Am. 28:314 (1956)

13. Batchelor, G.K., "Wave Scattering due to Turbulence," Proceedings 1st Symposium on Naval Hydrodynamics, Nat'l Academy of Sciences - National Research Council, Publication 515, Washington, pp. 409-430 (1956)

14. Ribner, H.S., "Shock-Turbulence Interaction and the Generation of Noise," NACA Report 1233 (1955)

15. Müller, E.-A., and Matschat, K.R., "The Scattering of Sound by a Single Vortex and by Turbulence," Tech. Rpt., Max-Planck-Institut für Strömungsforschung, Göttingen, Jan. 1959

16. Ram, G.S., and Ribner, H.S., "The Sound Generated by Interaction of a Single Vortex with a Shock Wave," (preprint)

17. Hollingsworth, M.A., and Richards, E.J., "On the Sound Generated by the Interaction of a Vortex and a Shock Wave," ARC 18:257, F.M. 2371 (1956)

18. Lindsay, R.B., "Compressional Wave Front Propagation through a Simple Vortex," J. Acoust. Soc. Am. 20:89-94 (1948)

19. Miles, J.W., "On the Reflection of Sound at an Interface of Relative Motion," J. Acoust. Soc. Am. 29:226-228 (1957)

20. Ribner, H.S., "Reflection, Transmission and Amplification of Sound by a Moving Medium," J. Acoust. Soc. Am. 29:435-441 (1957)

21. Kuhl, W., Schodder, G.R., and Schröder, F.-K., "Condenser Transmitters and Microphones with Solid Dielectric for Airborne Ultrasonics," Acustica 4:519-532 (1954)

* * * * *

E.-A. Müller

DISCUSSION

W. Willmarth (University of Michigan)

I wish to compliment the author on his excellent experimental and theoretical work. The experiments on sound scattering are very well done. In particular, the use of sound pulses produced and detected by identical transducers eliminates difficulties with sound reflection and transducer calibration. It is gratifying to see that the simple, or relatively simple, results of scattering from a single vortex can be superimposed to give a description of the sound scattering from turbulence.

With regard to the noise produced by a high-pressure steam valve: by the use of a set of small passageways the turbulence and shock waves within the valve were reduced in scale. The small scale motion which is produced probably does not vibrate the pipe as much because the mechanical impedance of the walls for the high-frequency, small-scale fluctuations is greater.

A related problem of valve noise is being studied at the Stanford University Medical Center. They are working on the valve noise produced by a heart valve, and would like to know what types of obstructions in the heart valve produce what types of noise. Of course, the sound intensity for the heart valve is very much lower than that for the steam valve discussed by Professor Müller.

R. H. Kraichnan (New York University)

The comment I wish to make concerns the dependence of sound-radiation-efficiency upon the turbulence parameters. Work which we have done recently on the radiation from isotropic turbulence at high Reynolds numbers has yielded the formula

$$W \sim \epsilon (v/c)^5 \ln R,$$

where W is the radiated power per unit mass, ϵ is the power dissipated by shear viscosity per unit mass, v is the r.m.s. fluctuation velocity, c is the velocity of sound, and R is the Reynolds number defined by $R = vL/\nu$, where L is the length scale of the energy-containing range of the turbulence and ν is the kinematic viscosity. This formula differs from the earlier result of Proudman only in the presence of the $\ln R$ factor, which arises from a slowly-decreasing, high-frequency component to the radiation spectrum suggested by our work.

It is well-known empirically that

$$\epsilon \sim v^3/L.$$

Thus, W depends on the basic parameters according to

$$W \propto (v^8/L) \ln(vL/\nu)$$

and varies, approximately, inversely with L. However, the efficiency of the conversion of turbulent energy into sound is given by

$$W/\epsilon \propto v^5 \ln(vL/\nu)$$

and, apart from the very-slowly-varying logarithmic factor, is unaffected by a change of length scale, provided v is fixed.

A. Regier (National Advisory Committee for Aeronautics)

I am rather interested in relating the scattering from a large vortex to the valve-noise problem. At Langley we have a test facility for producing turbulent noise for testing aircraft and missile components, and we are interested in making as much noise as possible from a jet. We found that by turning the air supply pipe through a 90-degree turn we can raise the jet noise levels about 5 decibels, possibly because of the large vortices generated in the turns. Actually, we can shape our spectrum and increase the lower frequencies by these large pipe bends. In our test set-up we are using a 12-inch pipe which has four 90-degree turns near the jet exit, and have succeeded in increasing our noise considerably and in bringing our frequencies down.

I. Dyer (Bolt, Beranek, and Newman, Massachusetts)

Several years ago I made an analysis of scattering from a single vortex. The flow field I used started at the origin with zero velocity, increased linearly, and then, at a given value, decreased as the inverse distance law. I, too, obtained good agreement with Lighthill's theory. This seems to be a very different flow model compared to that of Müller's, in the sense that the velocity was not divergent at the origin and contained a central core as a true single vortex might. Thus the model doesn't appear to be too critical in the computation of the scattering.

H. S. Ribner (University of Toronto)

Dr. Müller attributes the noise reduction in the large steam valves to the great reduction in scale; he supports the idea by a theoretical analysis of the noise radiation of homogeneous, isotropic turbulence during its decay as a function of the initial scale. This reviewer believes, on the other hand, that the dominant factor is the reduction of the turbulent velocities in the labyrinth: note that v enters as the eighth power in Eq. (3), scale L as the minus first power. A similar argument applies to jet-muffler silencing. It is to be noted that, according to Fig. 8, a factor of 10^3 in L—a wholly implausible amount—is required to explain the 7-db noise reduction attributed to Richards' slit-type nozzle.

* * * * *