SOUND RADIATION INTO A CLOSED SPACE FROM BOUNDARY LAYER TURBULENCE

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A theory of noise generation by the action of boundary layer turbulence on an elastic system has been developed, with particular reference to underwater application. The elastic system considered is a rectangular plate closed off by a rectangular liquid-filled volume. A simple yet physically meaningful form is assumed for the boundary layer pressure correlation. The resulting plate vibration and sound radiation are derived, taking into account coupling between the plate motion and the sound field in the liquid. In the theory, coupling is manifested by radiation-induced masses and viscous damping which, in effect, may be added to the mass and damping of the plate considered to be vibrating in vacuo. It is shown that the noise in the closed space decreases with decreasing free-stream speed and boundary layer thickness. Also the noise decreases with increasing mass and damping of the plate, and with increasing distance from the plate. The use of plate damping as a means of noise reduction is predicted to be limited on the one hand by radiation damping, and on the other hand by correlation decay in the assumed turbulence pressure field.

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INTRODUCTION

Noise caused by boundary layer turbulence is important in several fields of application. Boundary layer turbulence is believed to be an important factor in the generation of noise by underwater devices, and is the prime motivation for the present study. In addition, the noise in aircraft cabins at high speeds and at high frequencies is determined mainly by boundary layer turbulence. Also the vibration caused by boundary layer turbulence may have an important bearing on problems of equipment damage and structural fatigue in missiles and aircraft.

There are two mechanisms of noise generation by boundary layer turbulence. First, the turbulence pressure field may excite the plate adjacent to it; the plate vibrations then radiate noise to the surrounding medium. Second, the boundary layer turbulence may radiate noise directly by virtue of the fluctuations in the fluid properties. For nonrigid plates such as met in practice the first mechanism is more important, and consequently we exclude consideration of the second mechanism in the present investigation.

We wish to study the mechanics of excitation of plates exposed to boundary layer turbulence and the consequent transmission and radiation of the vibrations. In particular, we wish to treat the case in which the excited plate forms one of the surfaces of a closed space. Figure 1 shows the idealized geometry we have chosen for study. The boundary layer is assumed to be in contact with a large flat plate, the other side of which is in part a closed volume.

The problem may be thought to consist of three elements: determination of the boundary layer pressure field, calculation of the vibration in the plate adjacent to the boundary layer, and calculation of the sound field radiated by the plate vibrations. It is tempting to assume that these elements can be investigated independently, but only in the case of a plate immersed in a gas is this assumption likely to be acceptable. For a plate in contact with a liquid, it is well known that the sound radiation will greatly influence the vibrations. On the other hand, it is probably acceptable to assume that the boundary layer pressure field can be investigated independently of the plate vibrations, provided that the resulting plate displacement is small compared with the boundary layer displacement thickness. In what follows we shall assume that the boundary layer pressure field can be specified independently, while the plate vibrations and consequent radiation are coupled phenomena.

In the present investigation we are not primarily concerned with the basic properties of the boundary layer pressure field but rather with the mechanics of excitation and consequent transmission and radiation. We are primarily concerned with the understanding of the structural and acoustical elements of the problem, with the ultimate objective of devising geometries and treatments that may lead to boundary layer noise control.

Several recent works have direct bearing on the present investigation. Kraichnan (1), Ribner (2), and Corcos and Liepmann (3) computed the noise radiated from plates excited by boundary layer turbulence. Because their work concerned plates immersed in air, no account was made of possible plate vibration and sound radiation coupling, such as may be important in liquids. Kraichnan considered the radiation from finite square plates while Ribner and Corcos and Liepmann considered radiation from large plates.

Lyon (4) made theoretical and experimental studies on the excitation of strings by a random pressure field resembling that of boundary layer turbulence. Eringen (5) studied the excitation of beams and plates to a purely random pressure field, without particular reference to boundary layer excitation. Strasberg (6) used boundary layer pressure measurements by Harrison (7) to compute the excitation of both finite membranes and finite plates.

In addition to Harrison, Willmarth (8) has made spectral and correlation measurements on boundary layers in wind tunnels. Kraichnan (9) studied the boundary layer pressure field theoretically as a preliminary to his later work on plate excitation (1). Kraichnan's (10) most recent work on turbulence holds promise of providing additional information on boundary layer turbulence. In addition to the foregoing, there is a very rich literature on turbulent boundary layers and their more general aspects.

Boundary layer pressure fields have also been measured on airplane wings by Mull and Algranti (11). Their results are subject to some uncertainty, however, because the microphone used was not small compared with the boundary layer displacement thickness. Earlier, Rogers and Cook (12) made measurements relevant to the boundary layer noise problem in aircraft spaces. Also Dyer (13) has reported boundary layer pressures as determined indirectly from noise measurements within aircraft cabins. These latter two studies are largely heuristic in nature.

GENERAL EQUATIONS FOR PLATE EXCITATION

The vibration on a plate exposed to an external pressure field $f(x,y,t)$ is assumed to obey the classical thin-plate equation:

$$B\nabla^4 w + M\frac{\partial^2 w}{\partial t^2} + \beta\frac{\partial w}{\partial t} = -f - (p_1 - p_2)_{z=L_z} = -F(x,y,t) \tag{1}$$

where $w(x,y,t)$ is the displacement of the neutral plane of the plate, B the bending stiffness, M the mass per unit area of the plate, β the damping coefficient, and p_j the sound pressures on either side of the plate (at $z = L_z$). Figure 1 shows the geometry relevant to the problem. The bending stiffness is given by

$$B = \frac{Eh^3}{12(1-\sigma^2)} \tag{2}$$

where E is Young's modulus, h the plate thickness, and σ Poisson's ratio. The damping coefficient is assumed to include both viscous and hysteretic damping, a step that is possible only if we make a special assumption on the time dependence in Eq. (1), as we shall do later.

Boundary conditions (assumed time-independent) and initial conditions for Eq. (1) are specified in a later section.

The sound field on either side of the plate is assumed to obey the non-dissipative linear wave equation of acoustics

$$\nabla^2\psi_i - \frac{1}{c^2}\frac{\partial^2\psi_i}{\partial t^2} = 0 \tag{3}$$

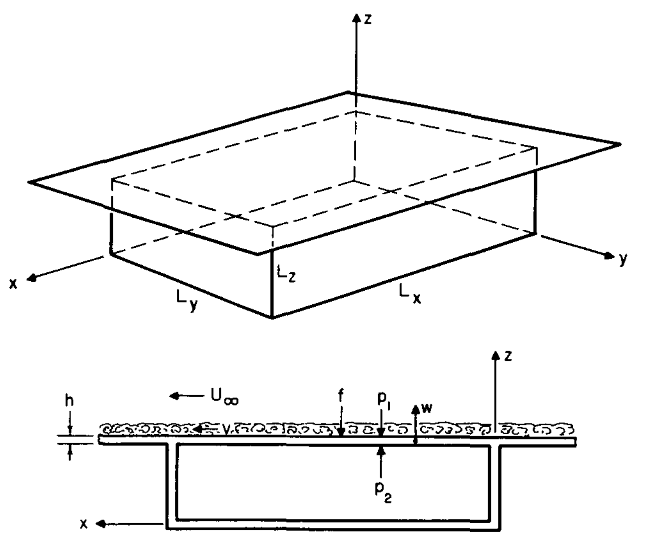

Fig. 1 - Geometry and coordinate system for boundary layer excitation

where ψ_j is the velocity potential and c the sound speed. We have assumed that the fluid is the same on both sides of the plate, but it will prove simple to consider the more general case should the need arise. The velocity potential is related to the sound pressure and sound particle velocity $\underline{v}$ by

$$p_j = \rho \frac{\partial \psi_j}{\partial t} \qquad \underline{v} = -\nabla \psi_j \tag{4}$$

where ρ is the fluid density. The space $j = 1$ is taken to be free from boundaries, except at $z = L_z$. The space $j = 2$ is a closed space with reflective boundaries. More precise boundary and initial conditions must also be specified with Eq. (3) but we shall leave this also to a later section.

Equation (3) is coupled to Eq. (1) by virtue of the continuity condition on velocity:

$$\frac{\partial w}{\partial t} = - \left.\frac{\partial \psi_1}{\partial z}\right|_{z=L_z} = - \left.\frac{\partial \psi_2}{\partial z}\right|_{z=L_z} . \tag{5}$$

We may associate with Eq. (1) the corresponding equation for the impulse response $g(x,y,t|x_0,y_0,t_0)$ of the plate:

$$B\nabla^4 g + M \frac{\partial^2 g}{\partial t^2} + \beta \frac{\partial g}{\partial t} = - \delta(x - x_0) \cdot \delta(y - y_0)\ \delta(t - t_0) \tag{6}$$

where δ is the Dirac delta function. Then an integral equation for w may be written (14):

$$w(r,t) = \int_{-\infty}^{t} dt_0 \int_s dS_0\ g(r,t|r_0,t_0)\,F(r_0,t_0) \tag{7}$$

where we have abbreviated r for x,y and dS_0 for dx_0dy_0. We note that Eq. (7) is a sum of all responses of the plate for every time t_0 before time t. For the uncoupled case in which F is independent of w, Eq. (7) is a solution for w. For both the coupled and uncoupled cases, however, we require a representation of the impulse response g.

We may obtain a formal representation of the impulse response in terms of the eigenfunction or normal modes of oscillation of the plate. The normal mode W_{mn} for a mode designated by the two order numbers m and n, is of the form

$$W_{mn}(x,y,t) = \phi_{mn}(x,y)\ \exp[-a_{mn}t - i\omega_{mn}t] \tag{8}$$

where a_{mn} is the modal damping and ω_{mn} the damped resonance frequency. Both a_{mn} and ω_{mn} are positive and real. The normal mode satisfies the equation

$$B(1 - i\eta)\nabla^4 W_{mn} + M \frac{\partial^2 W_{mn}}{\partial t^2} + \beta_0 \frac{\partial W_{mn}}{\partial T} = 0 \tag{9}$$

where we have made explicit division of the damping into hysteretic damping (determined by the loss factor η) and viscous damping (determined by the resistance coefficient β_0).

Substitution of Eq. (8) into Eq. (9) gives the equation for the eigenfunctions

$$\nabla^4 \phi_{mn} - \Gamma^4_{mn}\, \phi_{mn} = 0 \tag{10}$$

where the eigenvalues Γ_{mn} are taken to be real. It is convenient to have the eigenfunctions normalized such that

$$\int_s \phi_{mn}\, \phi_{pq}\, dS = \delta_{mp}\, \delta_{nq} \tag{11}$$

where $\delta_{\mu\nu}$ is the Kronecker delta. Solutions of Eq. (10) are to obey the same boundary conditions as those of Eq. (1). The eigenvalues Γ_{mn} are then determined by the particular boundary conditions of interest.

The modal damping and damped resonance frequency are determined by the damping coefficients and eigenvalues as follows: We assume that the damping meets the inequalities

$$\eta \leq \frac{1}{3} \quad \text{and} \quad \frac{\beta_0}{2\omega_{mn} M} \leq \frac{1}{3}. \tag{12}$$

These restrictions insure that the damping is not too large, but they do not exclude cases of practical interest. Then the modal damping is given by

$$\begin{aligned} a_{mn} &= \frac{\beta_0}{2M} + \frac{\Gamma^4_{mn} B\eta}{2M\omega_{mn}} \\ &\doteq \frac{\omega_{mn}}{\eta}\left[\left(1 + \frac{\eta\beta_0}{\omega_{mn} M} + \eta^2\right)^{1/2} - 1\right] \end{aligned} \tag{13}$$

which follows as a consequence of Eqs. (8), (9), and (10). Important limiting values of a_{mn} are

$$a_{mn} \doteq \frac{\eta\omega_{mn}}{2}, \qquad \frac{\beta_0}{\omega_{mn} M} \ll \eta \tag{14}$$

$$a_{mn} \doteq \frac{\beta_0}{2M}, \qquad \eta \ll \frac{\beta_0}{\omega_{mn} M} \tag{15}$$

$$a_{mn} \doteq \frac{\eta\omega_m}{2} + \frac{\beta_0}{2M}, \qquad a^2_{mn} \ll \omega^2_{mn}. \tag{16}$$

The damped resonance frequency is also determined from Eqs. (8), (9), and (10),

$$\omega^2_{mn} \doteq \frac{B}{M}\Gamma^4_{mn} - a^2_{mn} \tag{17}$$

and requires simultaneous solution with Eq. (13). However, for most cases of practical interest ω_{mn} may be closely approximated by

$$\omega_{mn} \doteq \left(\frac{B}{M}\right)^{1/2} \Gamma^2_{mn} \tag{18}$$

and is independent of damping.

We now turn to the representation for the impulse response. We introduce the Green's function G, which is the Fourier transform of g,

$$g(r,t|r_0,t_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(r,r_0,\omega)\,\exp[-i\omega(t-t_0)]\,d\omega\,, \tag{19}$$

substitute into Eq. (6), and obtain

$$\nabla^4 G - \Gamma^4 G = -\delta(x-x_0)\,\delta(y-y_0) \tag{20}$$

where

$$\Gamma^4 = \frac{\omega^2 M + i\omega\beta_0}{B(1-i\eta)}\,. \tag{21}$$

The Green's function G can be expanded in terms of the eigenfunctions φ_{mn}:

$$G(r,r_0,\omega) = \sum_{m,n} A_{mn}(r_0,\omega)\,\varphi_{mn}(r)\,. \tag{22}$$

We evaluate the coefficients A_{mn} by placing Eq. (22) in Eq. (20), using Eq. (10), multiplying by φ_{pq}, and integrating over S. The result is

$$G(r,r_0,\omega) = \sum_{m,n} \frac{\varphi_{mn}(r)\,\varphi_{mn}(r_0)}{\Gamma_{mn}^4 - \Gamma^4}\,. \tag{23}$$

We then use Eq. (19), which is evaluated by the calculus of residues at the poles $\omega = \omega_{mn} - i a_{mn}$, to obtain (14)

$$g(r,t|r_0,t_0) = \sum_{m,n} \frac{\varphi_{mn}(r)\,\varphi_{mn}(r_0)}{\omega_{mn}M}\,\exp[-a_{mn}(t-t_0)]\cdot\sin\omega_{mn}(t-t_0)\,U(t-t_0) \tag{24}$$

where $U(t-t_0)$ is the unit step function.

Equations (7) and (24) complete the formal representation for the plate displacement w.

THE BOUNDARY LAYER PRESSURE FIELD

Although it is not our purpose to investigate the basic structure of a turbulent boundary layer, we must obtain an appropriate description of the field to understand satisfactorily the mechanics of excitation and the possibilities for noise control. We assume that the boundary layer pressure field has the following characteristics:

1. The pressure field is a stationary random process.
2. The pressure correlation decays with time.
3. The pressure correlation has a spatial extent small compared with the plate size of interest.
4. The pressure correlation is convected along the surface of the plate, in the direction of the free-stream velocity.

5. The spatial correlation is homogeneous, depending upon the difference in the spatial coordinates in a frame of reference moving with the mean convection speed.

Accordingly, we adopt the form given in Eq. (25) as a possible phenomenological description of the boundary layer pressure field:

$$\begin{aligned} \langle f(x,y,t) f^*(x',y',t')\rangle &= f^2 \exp\left[-\kappa\sqrt{(\zeta - v\tau)^2 + \xi^2} - \frac{|\tau|}{\theta}\right] \\ &= f^2 R(\zeta,\xi,\tau) \end{aligned} \tag{25}$$

where

$\langle\ldots\rangle$ symbolizes the time average operation

$*$ denotes the complex conjugate

R is the normalized pressure correlation

f^2 is the mean-square boundary layer pressure

$\zeta = x - x'$

$\xi = y - y'$

$\tau = t - t'$

v is the mean convection speed, taken to be in the positive x direction

κ is a measure of the inverse radius of the turbulence "eddy"

θ is the mean statistical lifetime of the turbulent state.

The correlation area A is obtained by integration of Eq. (25) over ζ and ξ:

$$A = \frac{2\pi}{\kappa^2}. \tag{26}$$

Recent measurements by Willmarth (8) show that the root-mean-square boundary layer pressure is given by

$$f_{rms} \doteq 6 \times 10^{-3}\,\frac{1}{2}\rho U_\infty^2 \tag{27}$$

where U_∞ is the free-stream velocity. Harrison (7) obtained a numerical coefficient somewhat greater than that given in Eq. (27), namely 9.5×10^{-3}. (Earlier results by Dyer (13) from aircraft cabin noise measurements yielded a value very close to that of Eq. (27).) We shall use the value given in Eq. (27) in examples later in the report.

The normalized pressure correlation of Eq. (25) is shown in Fig. 2 for an arbitrary value of the product $\kappa v\theta$. We see that the correlation decreases as the time delay increases, and that the correlation peaks for particular values of ζ. Measurements on pressure correlation generally exhibit these characteristics (as do measurements on velocity correlation (15)). However, the measurements also show that the correlation is somewhat less peaky than that shown, and tends to become slightly

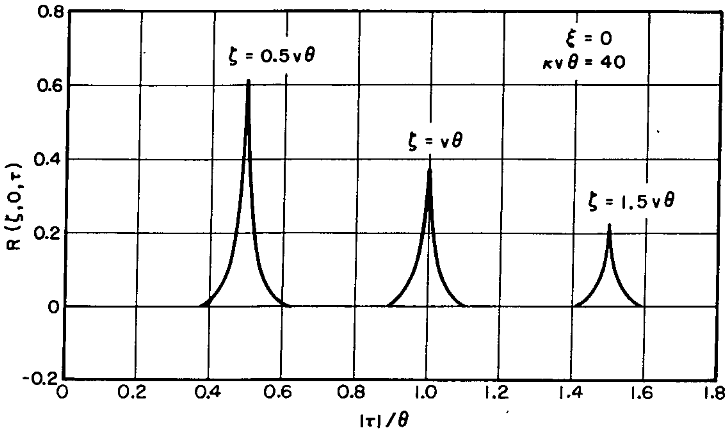

Fig. 2 - Normalized boundary layer pressure correlation for an arbitrary value of $\kappa v\theta$

negative and oscillatory beyond the major peak. Thus, Eq. (25) is an idealization of the pressure field, although it contains all the elements assumed to be of importance. The added complexity required to obtain a more exact fit with measurements does not appear to be warranted for the present investigation.

The spectral density $S(\omega)$ may be determined from the pressure correlation by the Fourier inversion

$$S(\omega) = 2 \int_{-\infty}^{\infty} \langle f(r,t) f^*(r,t') \rangle \exp[i\omega\tau]\, d\tau. \tag{28}$$

With the use of Eq. (25), the spectral density is found to be

$$S(\omega) = \left(\frac{4f^2}{\omega_0}\right) \frac{1}{1 + \left(\frac{\omega}{\omega_0}\right)^2} \tag{29}$$

where

$$\omega_0 = \kappa v + \frac{1}{\theta} \tag{30}$$

ω_0 is a characteristic frequency determined by the sum of the "eddy" convection frequency and the inverse time constant of the decay. $S(\omega)$ is plotted in Fig. 3 as a function of ω/ω_0. Measurements by Harrison (7) and Willmarth (8) are in general accord with Fig. 3, except that the data fall off more rapidly with frequency above ω/ω_0 equal to about 3.

Comparison of Eqs. (25) and (28) with measurements (7,8) lead to the following self-consistent estimates for the parameters describing the boundary layer pressure field:

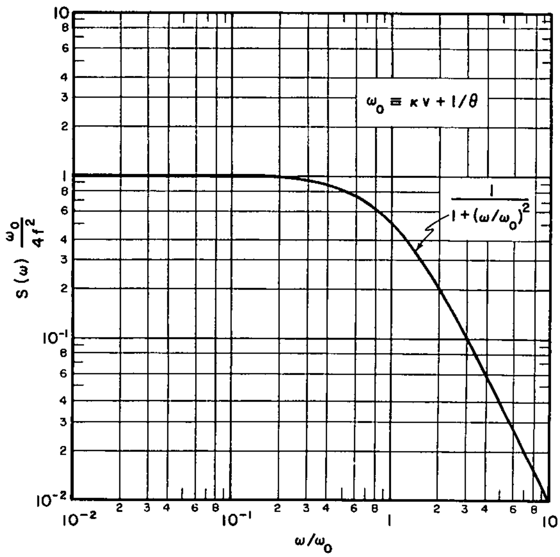

Fig. 3 - Spectrum of the boundary layer pressure field corresponding to the assumed pressure correlation

$$\begin{aligned} \kappa d &\doteq 2 \\ \theta &\doteq 30\,\frac{d}{U_\infty} \\ v &\doteq 0.8\,U_\infty \\ \omega_0 &\doteq \kappa v \end{aligned} \tag{31}$$

where d is the displacement boundary layer thickness. We see that the characteristic frequency ω_0 appears to be determined mainly by the eddy passage frequency, a result obtained theoretically by Kraichnan (1). Thus, the spectrum $S(\omega)$ for boundary layer turbulence is nearly independent of the lifetime θ, although the correlation $R(\zeta,\xi,\tau)$ is strongly θ dependent.

The correlation area A is of the order of d^2, as may be seen from Eqs. (26) and (31). If we make the relatively unrestrictive assumption that A is much smaller than the plate area L_xL_y, we may simplify Eq. (25) to the form

$$\langle f(r,t)f^*(r',t')\rangle \doteq Af^2\delta(\zeta - v\tau)\,\delta(\xi)\exp -\left[\frac{|\tau|}{\theta}\right]. \tag{32}$$

In essence, Eq. (32) is a valid simplification of Eq. (25) for $\kappa L >> 1$, where L is the smaller plate dimension. Equation (32) gives analytical simplicity to the boundary layer pressure field, while at the same time retaining the essential properties of a decaying and convecting field of small scale. Lyon (4) used a correlation of the form of Eq. (32) to compute the response of strings in a convecting turbulent field.

It is of interest to note that, in the limit of very short lifetime, Eq. (32) approaches

$$<f(r,t)f^*(r',t')> \rightarrow 2A\theta f^2 \; \delta(\zeta - v\tau) \; \delta(\xi) \; \delta(\tau) \; . \tag{33}$$

Also, for $v\tau$ small compared with ζ, the correlation approaches that of a purely random field, sometimes described as "rain on the roof," for which Eringen (5) has computed the excitation of finite plates. On the other hand, in the limit of very long lifetime, the exponential factor in Eq. (32) is unity for nearly all τ, corresponding to the case of convection of a frozen pattern in the turbulent field.

In what follows we shall use the form of Eq. (32) and the estimates of the field parameters given by Eq. (31) as a description of the boundary layer pressure field. It is important to emphasize that this description is not precise, but is intended to display in a simple way the essential features of boundary layer excitation.

RESPONSE OF A FINITE UNLOADED DAMPED PLATE

We consider here the case of a plate immersed in a low-density fluid* such that the radiation reaction on the plate may be neglected (i.e., $p_1 - p_2 << f$). Then the sound field external to the plate and the vibration field on the plate can be considered independently. Actually we shall find that the effect of fluid coupling does not change the form of the results, so that much of what is done here can be carried over with appropriate modification to the more general case. Thus, we proceed to the limit of zero fluid load, and the plate displacement becomes

$$w(r,t) = \int_{-\infty}^{t} dt_0 \int_s dS_0 g(r,t_0|r_0,t_0) f(r_0,t_0) \; . \tag{34}$$

Also, Lyon (16) has shown that the correlation may be obtained from an ensemble average, and may be written as

$$\begin{aligned} <w(r,t)w^*(r',t')> = \int_{-\infty}^{t} dt_0 \int_{-\infty}^{t'} dt_0' \int_s dS_0 \int_s dS_0' g(r,t|r_0,t_0) \\ \cdot \; g^*(r',t'|r_0',t_0') \; <f(r_0,t_0)f^*(r_0',t_0')> \; . \end{aligned} \tag{35}$$

The source correlation required in Eq. (35) is given by Eq. (32); we now proceed to the explicit form for the eigenfunctions required in the impulse response g.

For simplicity, let us assume simply supported boundaries at the plate edges:

$$w = \frac{\partial^2 w}{\partial x^2} = 0 \quad x = 0, \; L_x \; ; \qquad w = \frac{\partial^2 w}{\partial y^2} = 0 \quad y = 0, \; L_y \; . \tag{36}$$

These boundary conditions are not apt to be met precisely in practice, but they lead to relatively simple eigenfunctions. Furthermore, the use of Eq. (36) is likely

*The quantitative requirement on the density will be evident from the results of the next section.

to lead to error only for the very lowest modes; the eigenfunctions and eigenvalues are not too sensitive to the boundary conditions for higher modes. Normalized solutions of Eq. (10) that obey Eq. (36) are

$$\phi_{mn}(r) = \frac{2}{(L_x L_y)^{1/2}} \sin\frac{m\pi x}{L_x} \sin\frac{n\pi y}{L_y} \tag{37}$$

with

$$\Gamma_{mn}^2 = \left(\frac{m\pi}{L_x}\right)^2 + \left(\frac{n\pi}{L_y}\right)^2 . \tag{38}$$

Equation (35) involves the product of two doubly infinite sums, a typical term of which is the cross-product of two modes (m, n) and (p, q):

$$\begin{aligned} \langle w(r,t)w^*(r',t')\rangle_{mn,pq} &= Af^2 \frac{\phi_{mn}(r)\ \phi_{pq}(r')}{\omega_{mn}\omega_{pq}M^2} \int_{-\infty}^{t} dt_0 \int_{-\infty}^{t'} dt_0' \\ &\cdot \int_S dS_0 \int_S dS_0'\ \phi_{mn}(r_0)\ \phi_{pq}(r_0')\ \exp\left[-a_{mn}(t-t_0) - a_{pq}(t'-t_0') - \frac{|\tau_0|}{\theta}\right] \\ &\cdot \sin\omega_{mn}(t-t_0)\ \sin\omega_{pq}(t'-t_0')\ \delta(\zeta_0 - v\tau_0)\ \delta(\xi_0)\ . \end{aligned} \tag{39}$$

The y_0 integration is simple because of the delta function $\delta(y_0 - y_0')$. Thus the y_0' integration follows readily and yields $\delta_{nq}L_y/2$. The x_0 integration is equally simple if we assume

$$v\theta \ll L_x . \tag{40}$$

The quantity $v\theta$ is essentially the greatest distance over which the source is correlated. From Eq. (31) we estimate $v\theta$ to be about 24 d. Consequently we require the plate length to be much greater than about 24 displacement thicknesses, or about 3 boundary layer thicknesses,* a condition that includes all but the very thick boundary layers or very small plates.

With the use of Eq. (40) the x_0 and x_0' integrations yield

$$\frac{L_x}{2}\ \delta_{mp}\ \cos\alpha\tau_0$$

where

$$\alpha = \frac{m\pi v}{L_x} . \tag{41}$$

α may be termed the convection frequency, and is interpreted as the frequency at which the turbulent field is convected past a length of plate equal to the m-modal wavelength. We see that by virtue of the Kronecker deltas, δ_{nq} and δ_{mp}, the plate modes are statistically independent. Physically this result is due in essence to the largeness of the plate compared with the correlation size of the source.

*Inasmuch as the boundary layer thickness is about 10 times the displacement thickness d.

We now turn to the time integrals of Eq. (39). To facilitate integration we introduce the variables

$$\begin{aligned} \gamma &= (t' - t_0') - (t - t_0) = \tau_0 - \tau \\ \mu &= (t' - t_0') + (t - t_0) . \end{aligned} \tag{42}$$

The new coordinate system γ, μ, is shown in Fig. 4 in relation to the coordinates t_0, t_0'. By use of the Jacobian, the differentials become

$$dt_0 dt_0' = \frac{1}{2} d\gamma d\mu \tag{43}$$

and the limits are, as can be seen from Fig. 4,

$$-\mu \text{ to } \mu, \text{ for } \gamma$$

$$0 \text{ to } \infty, \text{ for } \mu .$$

Then Eq. (39) becomes

$$\langle w(r,t) w^*(r',t') \rangle_{mn} = \frac{Af^2}{4\omega_{mn}^2 M^2} \phi_{mn}(r)\ \phi_{mn}(r')\ I_{mn}(\tau) \tag{44}$$

where the time correlation integral $I_{mn}(\tau)$ is given by

$$\begin{aligned} I_{mn}(\tau) = \int_0^\infty d\mu \int_{-\mu}^{\mu} d\gamma \exp\left[-a_{mn}\mu - \frac{|\gamma+\tau|}{\theta}\right] \\ \cdot \cos \alpha(\gamma+\tau)\,[\cos \omega_{mn}\gamma - \cos \omega_{mn}\mu], \quad \tau \geq 0 . \end{aligned} \tag{45}$$

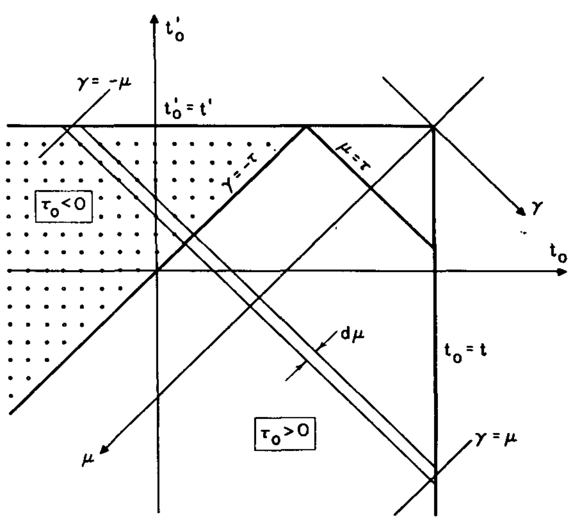

Fig. 4 - Region of integration and coordinate systems the time domain

Equation (45) is similar to an integral used by Lyon (4) in the string response problem, except that he specialized to the case $\tau = 0$. Because of the absolute value sign in the integrand we must integrate Eq. (45) in separate regions, depending in part upon whether γ is greater than or less than $-\tau$ (i.e., $\tau_0 > 0$ or $\tau_0 < 0$). Figure 4 shows the regions within which the integral must be evaluated. Thus, Eq. (45) becomes

$$I_{mn}(\tau) = \left\{ \int_{\tau}^{\infty} d\mu \int_{-\mu}^{-\tau} d\gamma \exp\left[\frac{\gamma+\tau}{\theta}\right] + \int_{\tau}^{\infty} d\mu \int_{-\tau}^{\mu} d\gamma \exp\left[\frac{-\gamma-\tau}{\theta}\right] \right.$$
$$\left. + \int_{0}^{\tau} d\mu \int_{-\mu}^{\mu} d\gamma \exp\left[\frac{-\gamma-\tau}{\theta}\right] \right\} \exp[-a_{mn}\mu] \tag{46}$$
$$\cdot \cos \alpha(\gamma+\tau)[\cos \omega_{mn}\gamma - \cos \omega_{mn}\mu], \qquad \tau \geq 0 .$$

The integral for $\tau < 0$ is identical to Eq. (46) except that τ is replaced by $-\tau$ throughout. Although the integrals of Eq. (46) involve elementary functions, they are exceedingly laborious to evaluate in generality. Rather it is more convenient to give the results for special cases, in order to avoid horrendously long expressions and to maintain physical clarity.

It is of interest to note that the results expressed by Eqs. (44) and (46) can be obtained in a somewhat different but equivalent way, using the formalism of Powell (17).

Mean-Square Response at Coincidence

We consider first the case of great importance in aircraft and missile applications, where the mean convection speed v of the boundary layer pressure field can be the same order as the free flexural phase velocity c_B in the plate.*

Integration of Eq. (46) shows that for $\alpha\theta > 1$ maximum response occurs when the convection frequency equals the resonance frequency (i.e., $\alpha = \omega_{mn}$). The foregoing condition $\alpha\theta > 1$ (or $\omega_{mn}\theta > 1$) corresponds to the requirement that the boundary layer pressure field be correlated over a distance greater than the m-modal wavelength. Inasmuch as the correlation distance is also taken to be smaller than the plate length see Eq. (40), we find that for maximum response at $\alpha = \omega_{mn}$ the mode number m is restricted by

$$m\pi > \frac{L_x}{v\theta} >> 1 . \tag{47}$$

With the use of Eqs. (18), (38), and (41), we can see that α is equal to ω_{mn} for a particular speed v_0 given by

$$\frac{v_0}{c_B} = \left[1 + \left(\frac{nL_x}{mL_y}\right)^2\right]^{1/2} \tag{48}$$

The speed v_0 is termed the hydrodynamic coincidence speed. We see that for modes $n << m$, v_0 is approximately equal to c_B, corresponding to the situation expected in one dimension (2,13). For n the same order or greater than m, the coincidence speed

*The free flexural phase velocity for thin plates is given by $c_B^4 = \omega^2 (B/M)$.

is increased by the requirement that matching occur with the component of v projected on the direction of the standing wave. This result is similar to the result of Ribner (2) for infinite plate excitation.

The time correlation integral $I_{mn}(\tau)$ is now specialized to $\tau = 0$ as well as $\alpha = \omega_{mn}$ (i.e., $v = v_0$). We find

$$I_{mn}(0) \doteq \frac{\theta}{a_{mn}(a_{mn}\theta + 1)} + \frac{\theta^2}{(a_{mn}\theta + 1)^2 + 4\omega_{mn}^2\theta^2}\left[\frac{2 - a_{mn}\theta}{4\omega_{mn}^2\theta^2}\right], \quad \omega_{mn}\theta > 1. \tag{49}$$

Equation (49) plus Eq. (44) gives us the mean-square displacement w^2 at hydrodynamic coincidence ($v = v_0$).

We may specialize Eq. (49) further. Consider the case of low damping. Then

$$I_{mn}(0) \doteq \frac{\theta}{a_{mn}}, \quad a_{mn}\theta << 1 . \tag{50}$$

As expected, the mean-square response in this case increases with increasing lifetime and decreasing damping.

Next consider the case of very high frequencies. Then we get

$$I_{mn}(0) \doteq \frac{\theta}{a_{mn}(a_{mn}\theta + 1)}, \qquad \omega_{mn}\theta >> 1 . \tag{51}$$

Note that the mean-square response may be proportional to a_{mn}^{-2} if $a_{mn}\theta$ is much greater than unity, corresponding to the situation in which damping decreases the displacement at resonance without appreciably broadening the bandwidth of the resonance.

Mean-Square Response Below Coincidence

In underwater applications the mean convection speed is much smaller than the coincidence speed (i.e., $v << v_0$). Consequently, we investigate Eq. (46) for $\alpha << \omega_{mn}$. Also we specialize to the case $\tau = 0$ in order to obtain the mean-square response. The result is

$$\begin{aligned} I_{mn}(0) = {} & \frac{2\theta^2}{1 + \omega_{mn}^2\theta^2}\left[\frac{\omega_{mn}^2\theta^2 - (a_{mn}\theta + 1)}{\omega_{mn}^2\theta^2 + (a_{mn}\theta + 1)^2} + \frac{1}{a_{mn}\theta}\right] \\ & + \frac{2\theta^2}{1 + \alpha^2\theta^2}\left[\frac{a_{mn}\theta + 1}{\omega_{mn}^2\theta^2 + (a_{mn}\theta + 1)^2} - \frac{a_{mn}}{\omega_{mn}^2\theta}\right] . \end{aligned} \tag{52}$$

For ease in interpretation, we consider two special cases of Eq. (52), for low damping and for high frequencies, respectively:

$$I_{mn}(0) \doteq \frac{2\theta}{a_{mn}} \frac{1}{1 + \omega_{mn}^2\theta^2}, \qquad a_{mn}\theta << 1 \tag{53}$$

and

$$I_{mn}(0) \doteq \frac{2}{\omega_{mn}^2}\left[1 + \frac{1}{a_{mn}\theta} + \frac{1}{1+\alpha^2\theta^2}\right], \qquad \omega_{mn}\theta >> 1. \tag{54}$$

Note that for underwater problems we cannot apply these results directly because we have assumed the plate to be immersed in a low-density fluid. However, we may anticipate a result of the next section that shows the correct result to be of the same form. Consequently, we may proceed with a discussion in the frame of underwater application.

For definiteness, let us make β_0 small, so that $a_{mn} = \eta\omega_{mn}/2$. The resulting value of $I_{mn}(0)$ is plotted in Fig. 5 for various values of η. Several important conclusions and comments may be made from Fig. 5 and the foregoing:

1. For usual underwater conditions the plot is estimated to cover the frequency range from about 10 to 10,000 cps, with some extension below 10 and 10,000 cps possible, depending on θ.

2. $I_{mn}(0)$ is proportional to the mean-square modal velocity. This quantity has a varying slope as a function of ω_{mn}. At low frequencies the slope is about -3 db/octave, at intermediate frequencies about -10 db/octave, and at high frequencies about -6 db/octave.

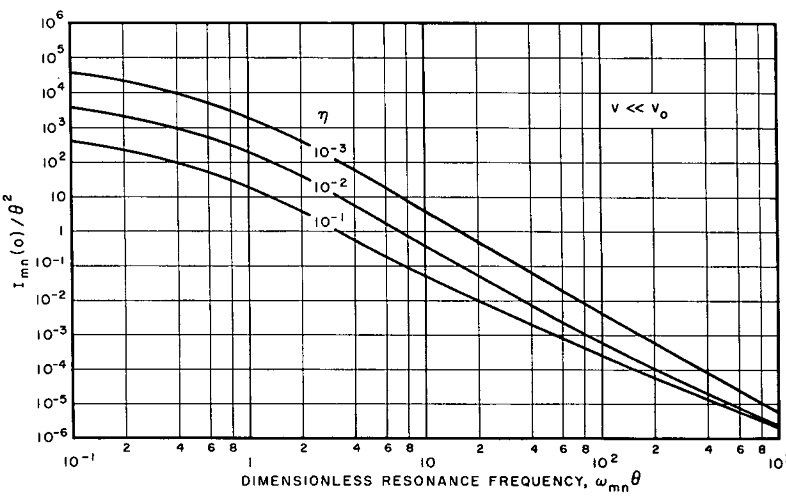

Fig. 5 - Time correlation integral below coincidence for zero time delay

3. With the use of information presented previously in the Section on Boundary Layer Pressure Field it is found that $I_{mn}(0)$ varies with about the 3rd power of U_∞ at low frequencies and about the 5th power of U_∞ at high frequencies. Also the dependence on boundary layer thickness is the 3rd power at low frequencies and the 1st power at high frequencies.

4. Hysteretic damping is an important means of noise reduction. Above a certain frequency, however, damping is seen to have a decreasing influence on the response.

We wish to expand upon the last comment. Damping appears to be an important mechanism for boundary layer noise reduction. Recent researches on the mechanisms of damping by applied treatments (18-21) are therefore of major interest in the present problem. As implied above, however, there is a transition frequency ν_t above which increased hysteretic damping brings diminishing returns. This frequency is given by the condition $a_{mn}\theta = 1$ from Eq. (54), and is

$$\nu_t = \frac{1}{\pi \eta \theta} . \tag{55}$$

Equation (55) is charted in Fig. 6. A typical small value of θ for underwater applications is estimated to be about 3×10^{-2} sec (corresponding to $U_\infty \doteq 20$ fps $\doteq$ 12

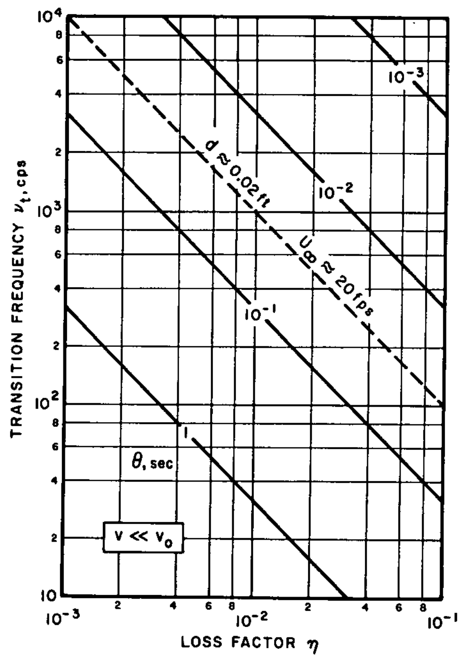

Fig. 6 - Transition frequency for hysteretic damping below coincidence

knots and $d \doteq 0.02$ ft). We see, for example, that if η is 10^{-2} (a quality factor Q of 10^2) there would be diminishing returns in increasing η above 1000 cps, although great benefit would be derived below 1000 cps. Qualitatively, we may conclude that damping of a plate and decay of turbulence play analogous roles in the plate response below coincidence, and that usually, but not always, damping is the dominant factor.

We may understand the foregoing result on damping effectiveness by appeal to the following physical arguments. In general, the response of a plate to an arbitrary spatially extended driving force is a combination of the "forced motion" (which would result if the plate were unbounded) and the "free motion" (which takes into account boundary conditions and is manifested by resonances). For short lifetimes (θ small) the plate response is largely that of the "free motion" of the plate, and is therefore damping controlled. For longer lifetimes (θ larger) the plate response contains more of the "forced motion," and is therefore controlled less by damping. Thus, for frequencies less than the transition frequency, the plate response is largely that of "free motion," and the plate vibration is controlled by hysteretic damping. Above the transition frequency, the reverse situation is obtained.

Displacement Correlation Below Coincidence

With reference again to underwater applications, we consider $\alpha \ll \omega_{mn}$ for τ different from zero. For the case of low damping, integration of Eq. (46) yields

$$I_{mn}(\tau) \doteq \frac{2\theta}{a_{mn}} \frac{\exp[-a_{mn}|\tau|] \cos \omega_{mn}\tau}{1 + \omega_{mn}^2 \theta^2}, \qquad a_{mn}\theta \ll 1 \tag{56}$$

from which, of course, we could have obtained Eq. (53). Then from Eq. (44) we get

$$\langle w(r,t)w^*(r',t')\rangle_{mn} = \frac{A\,\theta\, f'^2}{2\omega_{mn}^2 M^2 a_{mn}} \phi_{mn}(r)\, \phi_{mn}(r') \cdot \frac{\exp[-a_{mn}|\tau|] \cos \omega_{mn}\tau}{1 + \omega_{mn}^2 \theta^2}, \qquad a_{mn}\theta \ll 1. \tag{57}$$

In general, the plate correlation is required for the computation of the radiated sound field. Eq. (57) reduces to the result obtained by Eringen (5) for the case of zero lifetime.

SOUND RADIATION INTO A CLOSED SPACE

We shall investigate the radiation of boundary layer noise into a closed space with reference to the geometry of Fig. 1, and in particular to the underwater case. Our major concern is the sound within the closed space. But we shall see that the radiation into free space influences the resulting sound field in the closed space as well, and hence must be considered.

Influence of the Closed Space on Plate Vibrations

Consider first the closed space ($j = 2$). The Fourier transform Ψ_2 of the velocity potential ψ_2 satisfies the Helmholtz equation

$$\nabla^2\Psi_2 + k^2\Psi_2 = 0 \tag{58}$$

where $k = \omega/c$. Assume all the interior surfaces with the exception of the exposed plate are pressure release surfaces, i.e.,

$$\Psi_2 = 0, \quad \begin{cases} x = 0,\ L_x \\ y = 0,\ L_y \\ z = 0 \end{cases} \tag{59}$$

These boundary conditions correspond approximately to an interior lining of, say, air-filled rubber, or adjacent air-filled compartments, when the closed space is liquid filled.

A general solution of Eq. (58) that obeys Eq. (59) is

$$\Psi_2(x,y,z,\omega) = \sum_{p,q} D_{pq}\, \varphi_{pq}(r)\, \sin k_{pq} z \tag{60}$$

where

$$k_{pq}^2 = k^2 - \Gamma_{pq}^2 \tag{61}$$

and where φ_{pq} and Γ_{pq} are the eigenfunctions and eigenvalues given by Eqs. (37) and (38). Because of the coupling condition given in Eq. (5),

$$\sum_{p,q} k_{pq} D_{pq} \varphi_{pq}(r) \cos k_{pq} L_z = -\sum_{m,n} H_{mn} \varphi_{mn}(r) \tag{62}$$

where $H_{mn}\varphi_{mn}$ is the modal plate velocity. Consequently, the modal coefficients are related by

$$D_{mn} = - \frac{H_{mn}}{k_{mn} \cos k_{mn} L_z}\, . \tag{63}$$

The pressure transform is equal to $-i\omega\rho\Psi_2$. We form the ratio of the coefficients of the modal pressure to the negative of the modal velocity; at $z = L_z$ this ratio is the impedance Z_{mn} felt by the plate as a result of radiation into the closed space. Thus

$$Z_{mn} = \frac{-i\omega\rho}{k_{mn}} \tan k_{mn} L_z \equiv -i\omega M_2 \tag{64}$$

where M_2 is the effective mass per unit area associated with coupling to sound waves in the closed space. We see that M_2 may be positive or negative, corresponding to the situation where the radiation load is truly a mass, or a stiffness, respectively.

Equation (64) tells us that we can include the influence of the closed space on the plate response by adding M_2 to the mass M appearing in Eq. (44). It is important to note that for liquids, M_2 may be the order of or appreciably greater than M.

Let us define a sound coincidence frequency, ω_c, such that $c = c_B$. Then from Eq. (61), k_{mn} is imaginary for $\omega_{mn} < \omega_c$, and real for $\omega_{mn} > \omega_c$. Thus

$$M_2 = \begin{cases} \dfrac{\rho}{|k_{mn}|} \tanh |k_{mn} L_z|, & \omega_{mn} < \omega_c \\[2ex] \dfrac{\rho}{k_{mn}} \tan k_{mn} L_z, & \omega_{mn} > \omega_c\, . \end{cases} \tag{65}$$

A 1-inch steel plate in water, for example, will have a coincidence frequency $\nu_c = \omega_c/2\pi \doteq 10{,}000$ cps. For thinner plates the coincidence frequency will be correspondingly higher inasmuch as ω_c is proportional to h^{-1}. For the frequency range of interest in most practical applications, ω_{mn} will be less than ω_c, corresponding to M_2 positive (i.e., massive).*

It is of interest to add that Berry and Reissner (22) have computed the analogous case of radiation into a cylindrical enclosure by vibration of the cylinder surface. Dyer (23) has applied this work to the case of missile vibration.

Influence of the Free Space on Plate Vibration

The situation for radiation into free space is not as clear cut. A rigorous treatment of radiation from two-dimensional standing waves on a plate is not yet available. However, Morse (24), Goesele (25), and Westphal (26) have studied the case of one-dimensional standing waves. Their results show that for frequencies higher than ω_c, the radiation reaction on the plate is purely resistive, corresponding to the propagation of energy away from the plate. On physical grounds we may take this to be the case for two-dimensional standing waves also.

For frequencies less than ω_c it was found that the resistive reaction became small, corresponding to very little radiation of energy away from the plate. Also, Morse found that the reaction was primarily massive such that a fraction of the plate length

$$1 - \frac{\sqrt{8}}{m\pi}$$

behaved as if it were an infinite plate having a mass impedance determined (in present notation) by

$$M_1 = \frac{\rho}{|k_{mn}|}, \qquad \omega_{mn} < \omega_c . \tag{66}$$

Inasmuch as we are primarily concerned with frequencies below the sound coincidence frequency, Eq. (66) may be used as an approximation to the present case, provided that the mode number is not too low.

Correspondingly, when the mode number is low, we must expect that radiation of energy away from the plate is the important phenomenon, giving rise to a resistive impedance (viscous damping):

$$\beta_1 = \rho c s \tag{67}$$

where s is the radiation efficiency. For $\omega_{mn} << \omega_c$ and for infinite plates,† s is zero because of complete destructive wave interference in the fluid. For finite plates, s may be greater than zero because of incomplete destructive wave interference. The higher the mode number for a given plate size, the lower s becomes because of increased destructive interference.

*For curved plates this simplification may not always be possible.
†For $\omega_{mn} >> \omega_c$, s approaches the radiation efficiency of a piston radiator.

As a rough approximation formula for the radiation efficiency we may take

$$s \doteq \left[\frac{2}{\pi m} + \frac{2}{\pi n}\right] \frac{\left(\frac{\omega_{mn} L}{c}\right)^2}{1 + \left(\frac{\omega_{mn} L}{c}\right)^2}, \qquad \omega_{mn} < \omega_c \tag{68}$$

where

$$L = \frac{1}{2} (L_x L_y)^{1/2} .$$

The first term in Eq. (68) is suggested by the aforementioned work on one-dimensional standing waves; the second term, by the classical radiation efficiency of a piston in a flat plane.

For frequencies less than ω_c the influence of radiation to free space is thus that of viscous damping (β_1) at low mode numbers and mass reaction (M_1) at high mode numbers. Unfortunately, these quantities have not as yet been determined rigorously, but the rough approximations presented above will be of use in estimating boundary layer excitation.

Plate Excitation

Equation (44) gives the modal displacement correlation for a plate in a low-density medium. We have seen that the effect of radiation into a medium of arbitrary density is to add to the mass per unit area and to the viscous damping. Thus, Eq. (44) may be applied directly to the case of liquid coupling with the use of a new mass M' and damping a'_{mn} such that

$$M' = M + M_1 + M_2 \tag{69}$$

and

$$a'_{mn} = \frac{\beta_0}{2M'} + \frac{\Gamma_{mn}^4 B\eta}{2M'\omega_{mn}} + \frac{\beta_1}{2M'} . \tag{70}$$

Liquid coupling below sound coincidence thus influences the modal plate response in three ways: It tends to increase the effective mass of the plate, thus reducing the motion. Because of the increased mass, coupling tends to reduce the resonance frequencies, as may be seen from Eq. (17). And it changes the damping; coupling adds the term in β_1, but reduces the original terms because of the increased mass. On the other hand, the forms of the plate response solutions discussed previously are unchanged.

For most cases we can set the plate viscous damping coefficient β_0 equal to zero. Then we see from Eqs. (16) and (70) that hysteretic damping is an important noise-reduction mechanism if

$$\eta \geq \frac{\beta_1}{\omega_{mn} M'} . \tag{71}$$

The equality sign in Eq. (71) defines a set of conditions on either ω_{mn} or m, n for which noise reduction by hysteretic damping is not efficient. These conditions can be estimated for a particular case with the use of Eq. (68). However, noise reduction by hysteretic damping can be achieved efficiently for high enough mode numbers, inasmuch as β_1 tends to zero in this range. Hence we see that there are two factors limiting the use of hysteretic damping for noise reduction; one the limit of energy radiation given by Eq. (71), and the other the limit of turbulence decay given by Eq. (55).

Sound in the Closed Space

In general, we desire the sound pressure correlation to describe the sound field within the closed space. The pressure correlation is determined by the plate displacement correlation, as in Eq. (57), plus the velocity potential field, as in Eq. (60). Dyer (27) has recently worked out an analogous problem, that of noise propagation in a circular space, the methods of which apply to the present case. We shall therefore pass on to a simpler problem, that of determining the mean-square pressure adjacent to the excited plate.

The modal mean-square pressure p_{mn}^2 at the plate is simply determined by Eqs. (44) and (64):

$$\begin{aligned} p_{mn}^2 &= w_{mn}^2 \, \omega_{mn}^4 \, M_2^2 \\ &= f^2 \frac{A}{L_x L_y} \left(\frac{M_2}{M'}\right)^2 \sin^2 \frac{m\pi x}{L_x} \sin^2 \frac{n\pi y}{L_y} \, \omega_{mn}^2 \, I_{mn}(0) \, , \end{aligned} \tag{72}$$

and the spatial average of p_{mn}^2 is simply

$$\overline{p_{mn}^2} = f^2 \frac{A}{4 L_x L_y} \left(\frac{M_2}{M'}\right)^2 \omega_{mn}^2 \, I_{mn}(0) \, . \tag{73}$$

As an example, consider the following case of a steel structure in water:

$h = 1/2$ in.

$L_x = L_y = 5$ ft

$L_z = 1$ ft

$U_\infty = 20$ fps $\doteq 12$ knots

$d = 0.02$ ft

$\theta \doteq 3 \times 10^{-2}$ sec

$\eta = 10^2$ ($Q = 100$)

In this case the sound coincidence frequency is about 20,000 cps. Let us restrict attention to the case $\omega_{mn} < \omega_c$. Then for frequencies less than 10,000 cps we may approximate $|k_{mn}|$ by Γ_{mn}. Furthermore, for simplicity, let us restrict attention also to frequencies greater than about 200 cps, in which case $M' \doteq M + 2\rho/\Gamma_{mn} \doteq M$, and $\beta_1 \doteq 0$, as can be calculated from Eqs. (69) and (67). Under these conditions,

$$\left(\frac{M_2}{M'}\right)^2 \doteq \frac{1}{\sqrt{12}} \; \frac{\rho^2 c_L h}{\omega_{mn} M^2} \tag{74}$$

and ω_{mn} is the uncoupled resonance frequency given approximately by Eq. (18).* c_L is the longitudinal bar velocity in steel ($\doteq$ 17,000 fps).

Equation (73) can be evaluated with the use of Eq. (52) (or Fig. 5) and the information given above. It is customary, however, to measure noise in frequency bands of width $\Delta\nu$. The average number of modes ΔN in a band $\Delta\nu$ may be determined from Eq. (18), and is approximately

$$\Delta N \doteq \frac{\sqrt{3}\ L_x L_y \, \Delta\nu}{c_L h} . \tag{75}$$

Thus, the mean-square pressure as measured by frequency analysis in bands of width $\Delta\nu$ will be on the average

$$\overline{p^2} = f^2 \frac{A}{8h^2}\left(\frac{\rho}{\rho_s}\right)^2 \omega_{mn} I_{mn}(0)\,\Delta\nu \tag{76}$$

where we now consider ω_{mn} to be a continuous frequency variable, and where ρ_s is the density of the plate. Note that the plate area does not appear in the expression for $\overline{p^2}$. Equation (76) is plotted in Fig. 7 for the conditions given above, in terms of the Spectrum Level (SL) defined by

$$\mathrm{SL} = 10 \log_{10} \frac{(\overline{p^2})}{(p_0{}^2)} \text{, db,} \qquad \text{for } \Delta\nu = 1 \text{ cps ,} \tag{77}$$

where the reference pressure p_0 is 1 microbar (1 dyne/cm^2).

Of course, measured spectra would tend to appear less regular than that shown in Fig. 7, particularly at the lower frequencies where the measurement bandwidth may be small compared with the frequency spacing of the resonances. Also a hydrophone spaced out somewhat from the excited plate would be expected to record considerably less than that predicted in Eq. (76). From Eq. (60) we see that the mean-square pressure for a location away from the excited plate would be reduced by the factor

$$\left[\frac{\sinh \Gamma_{mn} z}{\sinh \Gamma_{mn} L_z}\right]^2 . \tag{78}$$

As an example, if $z = 0.9\ L_z$, the above factor is about -2 db at 100 cps and -17 db at 1000 cps. Even larger reductions are predicted at frequencies larger than 1000 cps, but these are not likely to be realized in practice because of the presence of non-ideal boundaries, dissipation in the water, and other noise sources.

On the basis of Eqs. (76) and (78) we may reach the following tentative conclusions on possibilities for the reduction of boundary layer noise associated with underwater devices:

1. The slower the speed and the thinner the boundary layer, the less the noise.

2. The greater the mass per unit area of the excited plate, the less the noise.

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*The fundamental resonance of the plate considered to be vibrating in vacuo is at about 25 cps.

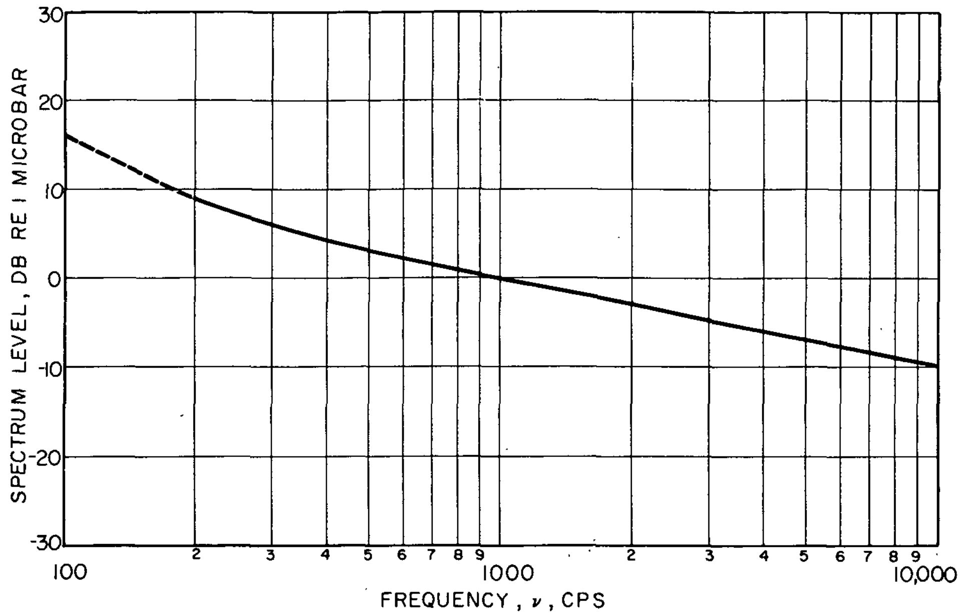

Fig. 7 - Calculated average mean-square pressure at the excited plate. Levels are reduced considerably at locations removed from the plate (see text for conditions).

3. The greater the plate damping, the less the noise (within the limits noted in the foregoing).

4. The greater the distance from the excited plate, the less the noise.

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DISCUSSION

H. S. Ribner (University of Toronto)

Dyer's formulation in his Eq. 25 is, I think, a plausible space-time correlation function for the turbulence pressures. With his numbers, it is almost a frozen, convected pattern: convection contributes about 50 times as much as fluctuation to the observed microphone frequencies. A different view is that an eddy is convected about 50 times its length before it has been destroyed by fluctuation. The ratio of 50, for convection-to-fluctuation, can be approximated as infinity in two ways: the time scale can be made infinite (frozen pattern, treated by Ribner and by Kraichnan), or the length scale can be set equal to zero. Dyer does the latter—he effectively reduces the eddy size to zero—by inserting delta-functions in his correlation. The entire later analysis utilizes this modified correlation, Eq. 32.

The assumption of zero eddy size appears to be related to Dyer's finding that the plate modes are statistically independent. It seems unlikely that such an independence can hold in reality for modes of wavelength less than the eddy size (the high frequency modes).

Further, zero eddy size implies that the cutoff frequency (ω_0) goes to infinity: the analysis implies excitation of the plate by "white" noise. For a given mean-square pressure, the "white" noise spectrum robs from the low frequencies to add to the high frequencies. The analysis, therefore, presumably leads to underexcitation of the low modes, and overexcitation (failure to show a cutoff) of the high modes. In this matter of the correlation function I cannot resist the conclusion that the frozen convected pattern would give more realistic results, since it preserves the eddy size and exhibits a realistic frequency spectrum.

Dyer discusses the role of "coincidence" of moving pressure waves with possible free-running, sinusoidal, flexural waves in the plate. The speed of these free waves is a function of the wavelength. Thus only certain Fourier components of a moving, fluctuating pressure pattern will have the correct wave length and speed for coincidence. The assumption of what amounts to zero "eddy" size in the pattern will grossly change the Fourier components and therefore the degree of coincidence.

I. Dyer

In comment to Dr. Ribner's statement I wish to note that the eddy size, in my estimation, has not been taken out of the problem. While the delta function appears, the eddy size is still retained in the quantity a. Of course the delta function was used as a matter of analytical simplicity. Th :onsequence of this is that the modes one obtains are statistically independent. I ..ink that in practice this is probably a very good approximation. While there are probably some modes that would be poorly approximated, they would have to be fairly close together in frequency space in order for any kind of correlation.

G. M. Corcos (University of California, Berkeley)

I wish to compliment Dr. Dyer for the clarity of his presentation. I would like him to comment on three points.

First, I fail to see the origin of the "viscous damping" term in the differential equation. It was assumed that the boundary layer was undisturbed by the plate, and I don't see how a plate oscillating normally to itself will dissipate energy if it doesn't modify the boundary layer.

Second, there have been at least two detailed attempts at solution of essentially the same problem by normal mode methods, one by Dr. Kraichnan and the other by Dr. Powell, and there have been other attempts using slightly different methods. Since the analysis is on the heavy side, it would be informative to indicate what new information, beyond that already available, has arisen out of the work presented here.

My third point is in regard to the use of normal modes. Is it possible to offer a simpler solution of the differential equation by using a fundamental solution which, in effect, ignores the effect of the boundaries? An answer is provided, in my opinion, by some work, to be published soon, by Dr. Paul Weyers of the California Institute of Technology, who has studied the generation of sound on the outside of a very thin plastic tube, inside of which is a fully developed turbulent flow. The point is that in the problem you consider, the scale of the turbulent phenomenon is small compared to the size of the plate. In Weyers' case, even though they are of the same order, the transmitted sound spectrum exhibits no peaks for frequencies higher than the very lowest range. Most of the energy is contained in the continuous portion, and I wonder why the use of a fundamental solution, ignoring all the details of effective boundaries, has not been considered.

I. Dyer

Dr. Kurtze will answer the question about how one can get viscous damping without modifying the external field.

As far as what new results are obtained, I think the primary result of this work is that it shows in a clear way the influence of the decay of the turbulence on the design of damping treatments for the reduction of boundary layer noise. I would say that this is the prime result and that the solutions at high frequencies ought to merge together and be independent of the particular details of the modes. In effect the formula is a manifestation of this.

Skudrzyk has also made some calculations along these lines and you might be interested in seeing what he has done on that basis. The use of that theory is not unique; it's just a matter of taste and convenience on my part.

K. U. Ingard (Massachusetts Institute of Technology)

It might be of interest to mention work by Dr. Lyon on the problem of excitation of a membrane caused by a flowing turbulence. It is interesting in the sense that in Dr. Dyer's treatment it was assumed that the convective velocity of the turbulence was small compared to the flexural wave speed and hence the resonance effects obtained in that manner were completely ignored. On the other hand, if one uses the membrane, the tension of which can be varied as you please, he can very easily adjust the flexural wave speed of the membrane to be coincident with the convective speed of the turbulent flow and, in that way, both theoretically and experimentally, show the resonance caused by this coincidence. Such experiments have been performed and have been shown to be in agreement with the calculations.

E. Mollo-Christensen (California Institute of Technology)

Figure D1 shows the results obtained by Weyers for the sound radiation from his pipe at the high frequencies only. This is the high-frequency part of his result. One finds this spectrum is practically continuous, in agreement with Corcos' comment. At the low frequency Weyers finds that he gets a spectrum with peaks in it, which then descends into a continuous spectrum for higher frequencies.

One may ask why the author did not find the stream directionality of the radiated flow field to free space. Listening to the bend of a garden hose lying on the lawn, one is impressed by the strong directionality of the emitted sound field.

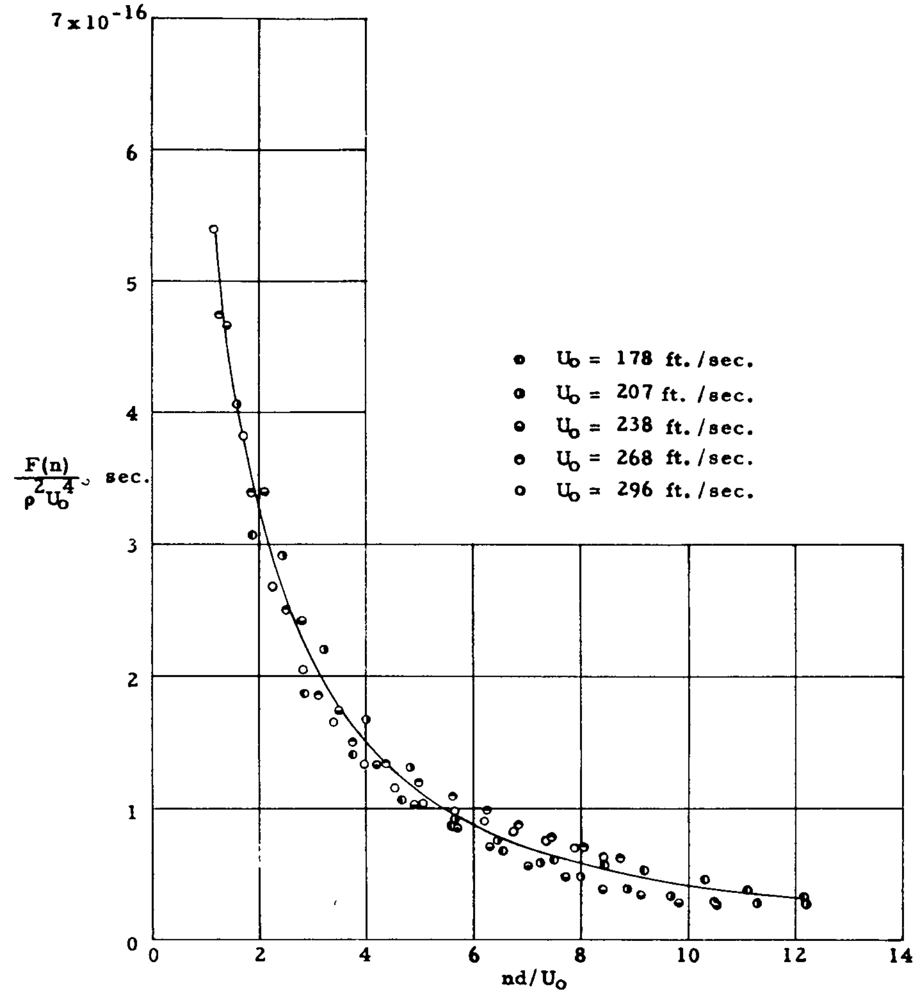

Fig. D1 - Similarity of power spectra of p(t). $\tilde{t}$ = 0.0015 in., d = 1 in., r/d = 3/4 in.

I. Dyer

The spectrum, of course, that would be predicted on the basis of the theory would also exhibit these peaks corresponding to the modes, I think, and what I plotted in the last slide was essentially this portion of it. There should be no directionality to the emitted sound field as long as essentially the flexural wave speed on the plate is less than the wave speed in the water. It is only above that so-called coincidence frequency that such directionality should occur.

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