WALL EFFECTS IN CAVITATING FLOWS

Hirsh Cohen and R. C. DiPrima
Rensselaer Polytechnic Institute

INTRODUCTION

In this paper we would like to review some recent work on cavity flows in bounded regions. In particular we shall consider the effects of solid walls and free surfaces on finite cavity flows. While our discussion will not, of course, be all inclusive, we hope it will be sufficiently comprehensive to make certain general conclusions about boundary effects.

There are three major points of distinction to be made in the general problem we have chosen to discuss. The first is the geometry of the flow. It is natural to distinguish between symmetric flows past symmetric bodies and unsymmetric flows past symmetric or unsymmetric bodies. The second point concerns the types of boundaries to be considered. The boundaries which are of interest are those which confront the experimentalist in testing devices — water tunnel walls or free jet surfaces — and those which provide interference in actual applications. These would include free surface effects on cavitating bodies traveling near the surface and the interference of solid bodies such as hulls near cavitating bodies. Finally, we shall emphasize in this discussion finite cavity flows. There is a famous and abundant literature dealing with cavities or wakes of infinite length. Obviously, infinite cavities will seldom be encountered in testing procedures or anywhere else. The infinite cavity theory does form, however, an interesting and often useful limiting case and it will be pertinent to make reference to many of the results. The need for a finite cavity model is obvious for physical reasons and has been met in several ways mathematically. All of these finite cavity flow models have been employed in discussing boundary effects and their use will be related here.

Most of the discussion to follow deals with two-dimensional flows. Few studies have been made of three-dimensional boundary effects. The three-dimensional cavity flow theories that have been available to the present time have been extremely difficult to use. The results for bounded flows that are available, which are referred to later in detail, are obtained by an indirect method.

For the cases reviewed and discussed in this paper, one of the more interesting conclusions seems to be that the presence of boundaries does not greatly alter the force coefficients acting on cavitating bodies, providing the same cavitation conditions can be attained with and without boundaries.* This statement must be carefully

Note: The research presented in this paper was sponsored by the Office of Naval Research.

*This conclusion seems to confirm in part the statement of the Principle of the Stability of the Pressure Coefficient suggested by Birkhoff, Plesset, and Simmons (1).

understood; for bodies placed within solid wall boundaries, blockage effects are often very large and do not allow for great freedom in modeling the cavitation conditions in an unbounded medium.

Our discussion will follow the following outline: A general formulation of the bounded cavity flow problem is given. The need for, and the particular characteristics of, the various finite cavity flow models are discussed and the general linearized problem is set out. Also, a study of the effects of a free surface on a wedge traveling parallel to it, given recently by DiPrima and Tu, is mentioned briefly (2). The effects of solid walls are then taken up in detail. The various theoretical and experimental results for both symmetric and unsymmetric flows are compared. A similar discussion of free jets is given, followed by some comparisons of flows in solid channels, jets, and in an unbounded stream for the two-dimensional case. Finally, a brief discussion is given of bounded, axially symmetric flows.

FORMULATION OF THE TWO-DIMENSIONAL PROBLEM

The general problem under consideration is that presented in rather compact form in Fig. 1. The solid body B is located in the bounded region D, which contains an incompressible, nonviscous fluid whose velocity at upstream infinity ($x \to -\infty$) is U_∞. Attached to B at the known points S_1 and S_2 is a constant-pressure region C, the cavity. The pressure in the cavity, p_c, the pressure, p_∞, corresponding to U_∞, and the constant fluid density, ρ, define the cavitation number $\sigma = (p_\infty - p_c)/(1/2)\rho U_\infty^2$. The boundary of the combination B and C is a streamline. A distance in the x direction may be associated with each of B and C (a chord, c, and a cavity length, ℓ). The boundaries of D are the streamlines ψ_u and ψ_ℓ. The leading edge of B is at distances h_u and h_ℓ from ψ_u and ψ_ℓ, respectively. Besides being streamlines, a second condition is required on ψ_u and ψ_ℓ. In the free jet case the pressure is constant and known; in the solid wall case the lines themselves are given.

Unfortunately, as is well known, a satisfactory solution to the problem just sketched does not exist except under special conditions, even for h_u and h_ℓ both infinitely large. If one is willing to accept cavity shapes concave to the flow with $p_c > p_\infty$ or to consider cavities of infinite length, a solution may be found within classical potential theory. This is essentially the content of a "non-existence" theorem of Serrin (3). This theorem, then, implies that finite cavities of the shape required are not available from the classical theory. The bounded infinite cavity case, the Kirchhoff-Helmholtz theory, has been taken up at some length by various authors. The work of Réthy (4), Valcovici (5), and von Mises (6) is discussed by Birkhoff, Plesset, and Simmons (1). Actually, a certain amount of prior credit seems to fall to Joukowsky (7) who considered virtually all of the cases taken up by the other

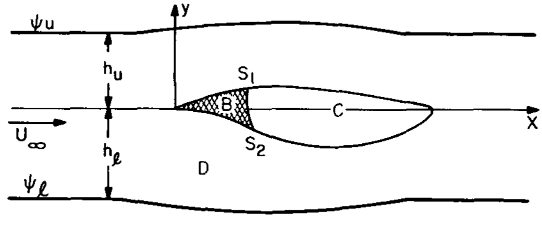

Fig. 1 - General two-dimensional cavity flow problem

authors plus several not discussed by them. It is important to observe that as the cavity becomes infinitely long the cavitation number σ may or may not approach zero. If the streamlines ψ_u and ψ_ℓ represent solid walls at downstream infinity, then σ approaches a limiting value σ_∞, and the water tunnel is effectively blocked by the body and its infinite downstream appendage. On the other hand, if either ψ_u or ψ_ℓ, or both of them, represent free surfaces at downstream infinity, σ approaches zero as the cavity length becomes infinitely long.

We turn now to a discussion of the finite cavity models. The fundamental element supplied by the exact finite cavity flow models is a mechanism for explaining dissipation near the downstream end of the cavity. The Riabouchinsky model achieves this by setting an image body symmetrically located downstream from the forebody. The re-entrant jet theory allows the free streamline to reverse direction at the rear of the cavity and flow toward the forebody. The transition flow model provides a gradual "dissipation" downstream of a constant-pressure region along streamlines whose direction is prescribed. These models all prove to be useful and to give comparable results. An excellent description of them and a comparison of their usefulness is given in a paper by Wu (8). Further details will not be related here. A common difficulty of these models is that they are not very flexible as to the shape of the forebodies which can be handled with enough facility to produce computable data. The cases which have been discussed, even for unbounded flows, are virtually limited to straight-line boundaries. Furthermore, the Riabouchinsky and re-entrant jet theories are additionally complicated for unsymmetrical problems; the transition flow model does not have this difficulty. All of the flow models mentioned have been used to describe bounded flows; the results of these analyses will be related in subsequent sections.

Because of the difficulties associated with these models it would appear useful to consider a non-exact model. Tulin (9) has applied the ideas of thin airfoil theory to finite cavity flows to obtain a linear cavity flow theory. The linearization is in terms of a slenderness ratio or a small angle of attack and involves the usual introduction of perturbation velocities. The conditions along solid contours and constant-pressure surfaces are expressed in terms of these velocities. Instead of trying to satisfy conditions along the body or the unknown constant-pressure surfaces, the boundary conditions are satisfied on a known mean chord line. A condition which insures that the body-cavity combination is closed and another condition, either providing a smooth body-cavity juncture or a smooth flow at the trailing edge of the foil, seem to be sufficient to yield a useful flow theory.

If the linearized theory is applied to the problem of Fig. 1, the problem becomes a mixed analytic function theory problem defined over the strip bounded by ψ_u and ψ_ℓ and exterior to the cut along the x-axis. (See Fig. 2.) The cut will represent the

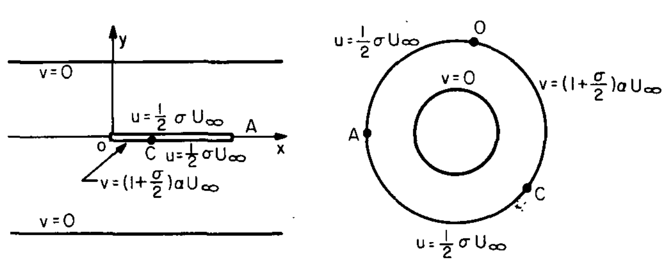

Fig. 2 - Linearized cavity flow for a lifting foil between solid walls

distance from the leading edge to the end of the cavity for the case of fully cavitating flow. For partially cavitating flow (the cavity closing on the body), the cut will be the mean chord length of the body. It is clear that unless there is symmetry about the x-axis or the cavity is infinitely long we are faced with solving a mixed boundary value problem in a multiply connected domain.

The general problem can be represented as one defined over the annulus shown in Fig. 2 with either the perturbation velocities u or v prescribed on the inner circle* and u and v prescribed in alternate segments on the outer circle. Furthermore, certain continuity and singularity conditions are imposed on the complex velocity function on the outer circle. It would seem as though this is just the type of problem to be handled in very general form by the methods of Muskhelishvili (10). As it turns out, however, there are difficulties in finding the analytic continuations of the analytic functions needed in the Muskhelishvili method. This is also pointed out by Woods (11) and in fact, Woods has developed an alternative approach to such boundary value problems. The original cut strip of Fig. 2 may be mapped to a periodic array of rectangles, periodic in both vertical and horizontal directions, by elliptic mapping functions. Then, using a Cauchy integral method, Woods (12,13) has given a formulation for fully wetted foils between porous walls which can be easily extended to the cavity flow problem. In the limiting cases, the porous walls of the Woods theory become solid or constant-pressure surfaces. An examination of this method, now underway at R.P.I., for both fully cavitating and partially cavitating flows, shows, as Woods has warned, that the expressions obtained for such required relations as those between σ and ℓ and for C_L and σ are very difficult to handle. However, if the walls are far from the body it appears possible to achieve some results.

Using the same mapping to the periodically repeated rectangles, Timman (14) has considered the problem of an oscillating airfoil between solid walls by finding appropriate Green's functions. In adopting his method for the bounded cavity flow problems, we find that the partial cavity flow past a lifting flat plate between solid walls requires the solution of the following rather interesting integral equation:

$$\int_{2K}^{2K+\xi_\ell} \gamma(\xi)\,\mathrm{sn}(\xi)\,H(\xi,\eta)\,d\xi = -\left\{\frac{2K\sigma}{1+\sigma/2}\,\mathrm{sn}\eta + \int_{2K+\xi_\ell}^{4K} \alpha(\xi)\,\mathrm{sn}(\xi)\,H(\xi,\eta)\,d\xi + \int_0^{2K} \alpha(\xi)\,\mathrm{sn}(\xi)\,H(\xi,\eta)\,d\xi\right\},$$

where $K(k)$ is the complete elliptic function of the first kind, α is the local inclination of the forebody, and ξ_ℓ represents a cavity dimension. The kernel $H(\xi,\eta)$ is a singular kernel with an analytic term and has the form

$$H(\xi,\eta) = \cot\frac{\pi(\xi-\eta)}{4K} + 4K\sum_1^\infty \frac{q^m}{1-q^m}\sin\frac{m\pi(\xi-\eta)}{2K}.$$

*If ψ_u and ψ_ℓ refer to different types of boundaries, u and v will be specified on separate portions of the inner circle.

Here q is the elliptic integral nome, $q = \exp(-\pi K'/K)$. The modulus k is a function of channel height and the body and cavity dimensions. The unknown function, $\gamma(\xi)$, is related to the cavity slope in the constant-pressure region. This singular integral equation may be reduced to a Fredholm equation with regular kernel by the methods of Carleman and Muskhelishvili but this is not a very attractive procedure. One may also do a perturbation solution starting from the case $q = 0$, which corresponds to the walls-at-infinity, as the zero-order solution. This is, in fact, the method which has been adopted and is now being calculated. It can be seen that, although analytic methods are available and some are being used, the general linearized boundary value problem set forth above has not been solved. Furthermore, it can be anticipated that its general solution will be of such complexity that usable data will be hard to obtain. The bounded finite cavity problems that have been treated successfully to date by the use of the linearized method are those which can be reduced to a potential problem in a simply connected domain, either by the use of symmetry or the linearization of a transition flow model. (See for instance 15, 16, 17, 18 & 19.) The results obtained in these cases will be discussed later.

In this connection, DiPrima and Tu have recently used the linearized theory to consider the problem of a fully cavitating symmetric wedge placed under a free surface or a solid wall (2). Since the problem is unsymmetric, it is necessary to use a distribution of vortices, as well as sources and sinks to represent the flow potential. The distributions are coupled by two simultaneous integral equations. By perturbing in terms of a parameter, δ = chord length over twice the distance from the free surface to the vortex of the wedge, it is possible to find a first approximation for C_L. Further, it was observed that for small δ the effect of the wall or free surface on the $\sigma - \ell$, and $C_D - \sigma$ relations was $O(\delta^2)$. It is interesting to note that in this problem, if the approximate body boundary condition introduced by Tulin is used, the term of order δ in the expansion of C_L is identically zero. It was found that the $C_D - \sigma$, and $\sigma - \ell$ relations were insensitive to the approximate body boundary conditions, but that C_L was very sensitive to any changes in the approximate conditions imposed on the body. The results of this analysis are more fully discussed in Ref. 2.

SOLID WALL CHANNELS

Earlier in this paper, reference has been made to solutions for two-dimensional cavity flows in solid wall tunnels. We would now like to consider these in detail. In the following discussion, we adopt the notation: T is the maximum width of the body, c is the chord length, ℓ is the cavity length, h is the width of the jet or channel, a is half the maximum cavity width, γ is the half wedge angle, and $-\alpha$ is the angle of attack for lifting foils. Further, if D is the drag per unit span, the drag coefficient C_D is defined as $D/(1/2)\rho U_\infty^2 T$, and if L is the lift per unit span, the lift coefficient C_L is defined as $L/(1/2)\rho U_\infty^2 c$.

The finite cavity problems which have been considered theoretically are the flat plate set perpendicular to the walls and the flow, slender wedges placed symmetrically in the channel, and lifting foils at small angles of attack placed anywhere between the walls. The first of these problems were considered by Gurevich (20), who made use of the re-entrant jet model; and by Birkhoff, Plesset, and Simmons (21), who employed the Riabouchinsky model. Slender symmetric bodies have been studied by Cohen and Gilbert (22) using the linearized theory, and lifting foils have been considered by a linearized transition flow model in a paper by Cohen, Sutherland, and Tu (19). For all of these cases except the last, there are infinite cavity solutions given by the exact Kirchhoff-Helmholtz theory.

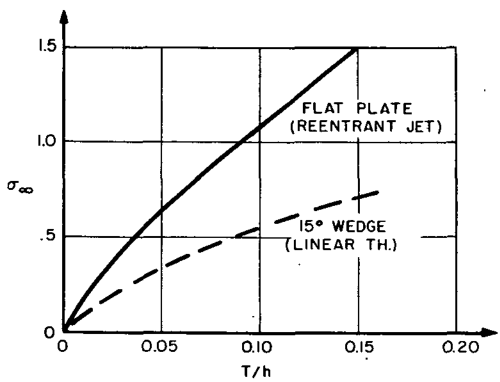

Fig. 3 - The dependence of blockage cavitation number on T/h

One of the central problems in water tunnel testing of cavity flows is the blockage problem. As the cavitation number decreases, the cavity lengthens and eventually a large portion of the channel downstream from the body becomes a deadwater region. For a given ratio of body height to channel width, T/h, cavitation numbers below σ_∞, the cavitation number at which an infinite cavity would result, are not attainable. Thus, not all cavity flow conditions can be modeled in a water tunnel.

In Fig. 3, σ_∞ has been plotted against T/h for the flat plate and for a 15-degree half-angle wedge. These curves give an indication of how drastic the blockage is, and one can expect that the wall effects on cavity length and width will be similarly large. This is borne out in the calculations made by Gurevich and by Cohen and Gilbert as shown in Figs. 4, 5, 6, and 7.

The other consideration of importance is the effect of the tunnel width on the relation between drag coefficient and cavitation number. For the flat plate, Gurevich has calculated C_D vs σ for small values of T/h and finds that all of the values lie on virtually one curve, independent of T/h (Fig. 9).* Now, for each value of T/h, there will be a minimum cavitation number, σ_∞, corresponding to an infinite cavity. If one plots C_D for these values of σ_∞ (T/h increases as σ_∞ increases) even these minimum C_D values fall on the single curve mentioned above.

There are at present no exact model results for the symmetric wedge in a solid wall channel. The linear theory results of Cohen and Gilbert are shown in Fig. 8 and will be discussed presently in connection with the experimental data.

There are experimental results for both of the symmetric flows mentioned above. Waid (23) has carried out a set of tests with flat plates and small angle wedges. For the flat plate with a T/h ratio of 0.027 the experimental points agree well with the curve given by Plesset and Shaffer (24) who used the Riabouchinsky model for a flat plate in an unbounded stream (Fig. 9). Also shown is the curve given by Gurevich. The re-entrant jet theory seems to give a slightly better fit than the unbounded flow results in the partial cavitation† region.

The Plesset and Shaffer results also give an excellent fit to the experimental data for the 15-degree wedge, with T/h = 0.027 (Fig. 8). As can be seen, the linear theory data lie consistently above the exact theory and, in this case, the experimental data. This overestimation seems to occur generally, as has been pointed out by Wu (25). In the present case, we have again plotted a curve showing the value of C_D

*We have replotted Gurevich's curve using data from the figure in his paper; hence it is possible that the curve in Fig. 9 may be very slightly inaccurate.

†Waid uses the term partial cavitation to mean that the cavities are filled with a rapidly moving combination of bubbles and water. In the figures such cavities are denoted by a solid circle.

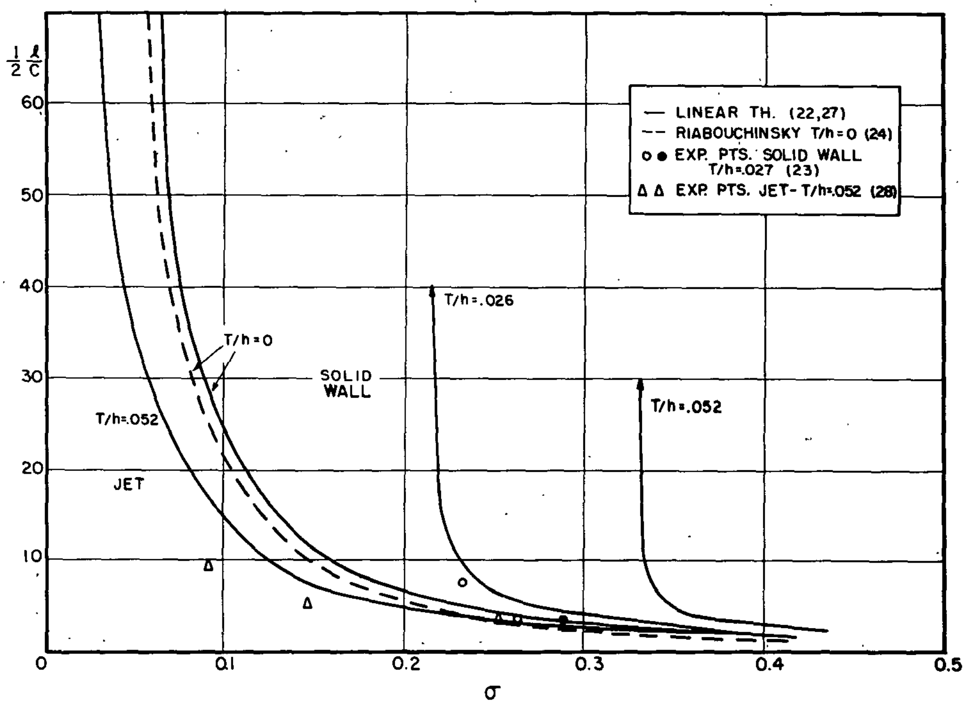

Fig. 4 - The dependence of cavity length on cavitation number for a 15-degree wedge

at the σ_∞ corresponding to any given value of T/h, the infinite cavity curve. These values of C_D will be the minimum attainable for the given value of σ. We have also plotted C_D as a function of σ for T/h = 0, the unbounded stream result. If one takes σ constant, then only those values of C_D lying between the two curves can be obtained by changing T/h. If one takes T/h constant, a curve can be drawn beginning at the σ_∞ value corresponding to T/h and lying always between the curves for T/h = 0 and for the infinite cavity condition. Thus, although the linear result does overestimate the drag at any given value of σ, it gives a qualitative confirmation to the idea that the change in force coefficient is indeed small for fixed cavitation number.

One must keep in mind, however, that the boundaries do invoke limitations on the cavitation numbers. Waid has measured values for C_D just below $\sigma = 0.2$ for the 15-degree wedge and these must be very long cavities since Cohen and Gilbert obtain $\sigma_\infty = 0.212$ as the blockage cavitation number for the 15-degree wedge with T/h = 0.026.

For cavity width, Gurevich's results obviously follow the data more faithfully than the Plesset and Shaffer result for the unbounded flow (Fig. 7). Gurevich has computed widths and lengths for rather high values of cavitation number ($\sigma > 0.5$) which is strange, but it appears that his results give the proper correction for Waid's data. For the 15-degree wedge, the Cohen and Gilbert data also seem to give the proper correction (Fig. 6).

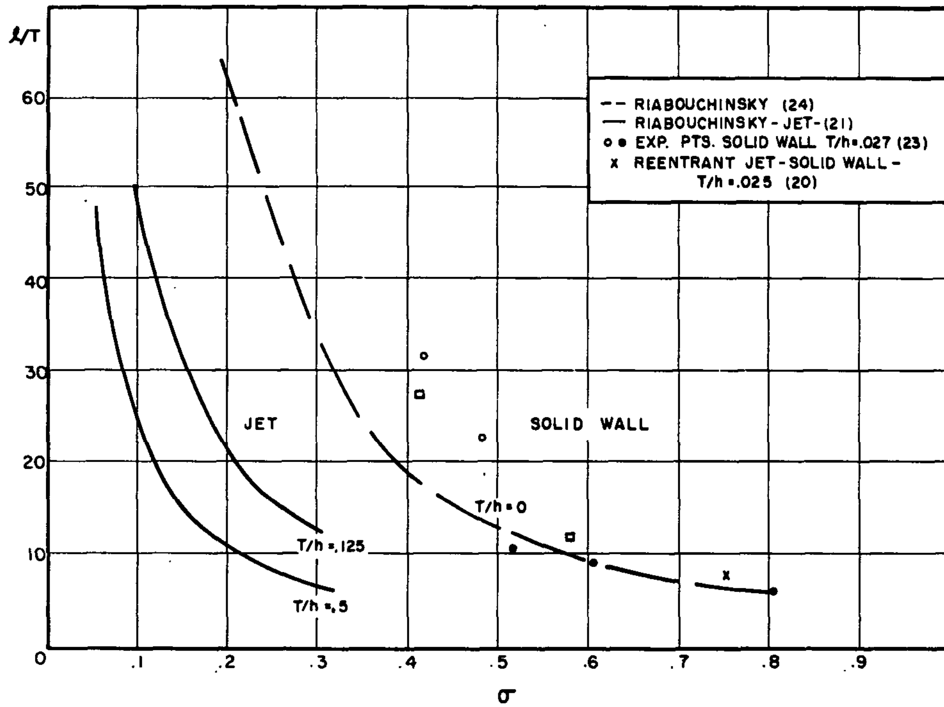

Fig. 5 - The dependence of cavity length on cavitation number for a flat plate

Unfortunately, Gurevich's calculations are not very useful for cavity length comparisons. The values of σ used are so large that they fall in the partial cavitation range (Fig. 5). Gurevich's results are given in explicit form and may be extended with some labor. The Plesset and Shaffer unbounded flow results are compared with Waid's data in Fig. 5. For the 15-degree wedge, the wall effect curves given by Cohen and Gilbert again seem to give the right correction (Fig. 4). The magnitude of the correction is large here; for $\sigma = 0.3$, $T/h = 0.026$, the correction to the Plesset and Shaffer unbounded flow results is about 50 percent and becomes larger as σ decreases.

Thus far, we have considered only symmetrical flows. When a thin foil is placed inside a water tunnel at an angle of attack, the flow is obviously no longer symmetrical. In order to avoid the mathematical difficulties which arise for the multiply-connected domain involved in this finite cavity flow, a linearization of the transition flow model was used by Cohen, Sutherland, and Tu (19).

The general solution was obtained for any ratio of h_u/h_ℓ (see Fig. 1). Three special cases were considered in detail for the flat plate foil. In the first, the foil is located midway between walls ($h_u = h_\ell$). The lift coefficient is obtained as a function of cavitation number as shown in Fig. 10 for the particular case, $\alpha = -6$ degrees; curves were plotted for several values of c/h. On the same graph we have shown the result for $h_\ell = h_u = h/2 \to \infty$, and the results for the case of the infinite cavity. For the range of cavitation numbers shown, the wall effects which lie between the two bounding curves are, indeed, small. If a choice of c/h is made, one begins at

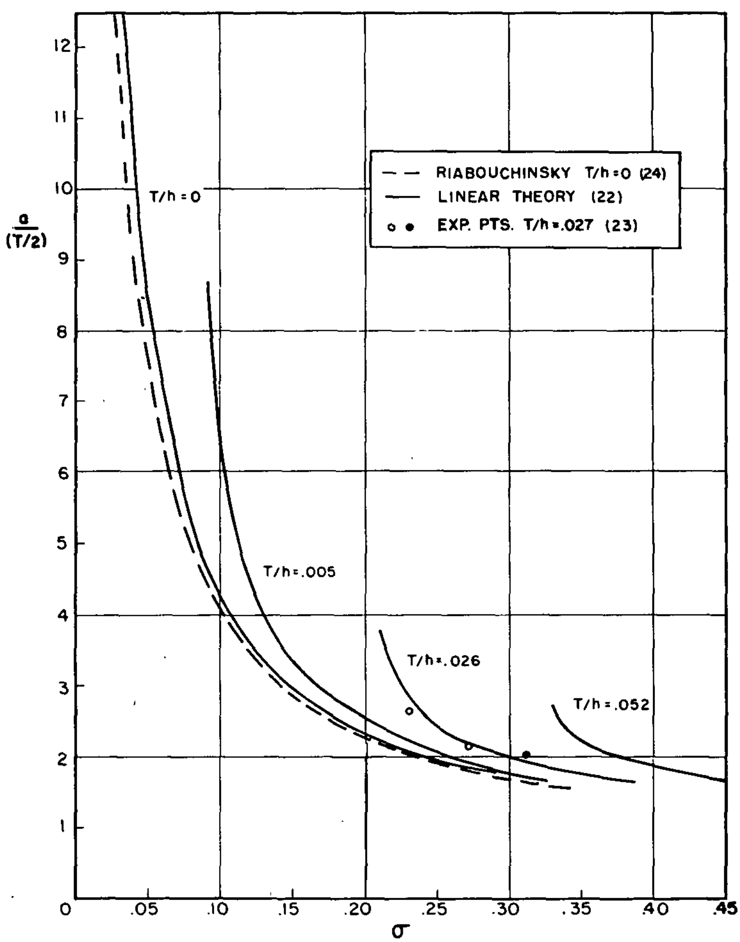

Fig. 6 - The dependence of maximum cavity width on cavitation number for a 15-degree wedge in a channel

the proper value of σ_∞ on the infinite cavity curve and proceeds along the C_L vs σ curve as is shown in Fig. 10. Also shown in Fig. 10 is a curve of σ_∞ vs c/h for $h_\ell = h_u = h/2$.

Parkin (26) has considered lifting flat plates experimentally. For the 12-degree flat plate, his data are given in Fig. 11 along with Wu's transition model theory results (8) for the unbounded stream and the results for the bounded flow using the linearized transition model (19). Here again the linear theory overestimates the force coefficient; however, it does predict only a small change in C_L for fixed σ with varying T/h.

The second case considered in Ref. 19 describes the flow behavior when $h_\ell \to \infty$ or $h_u \to \infty$; i.e., the lifting foil near the upper or lower wall. For infinite cavities,

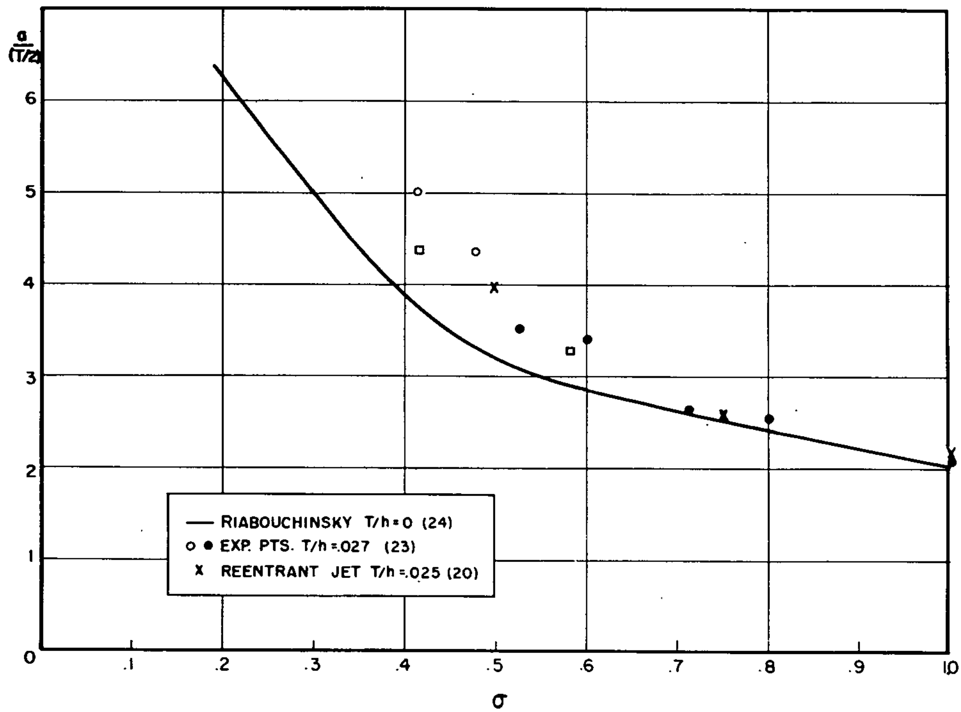

Fig. 7 - The dependence of maximum cavity width on cavitation number for a flat plate in a channel

the effect of the nearness to one wall is shown in Fig. 12. The lift coefficient for both h_ℓ, $h_u \to \infty$ in the linear theory is $(-\alpha)\pi/2$. In the case where the foil is close to the upper wall, the lift coefficient decreases as the distance from the wall decreases. On the other hand, when the foil is close to a lower wall, the lift coefficient increases as the distance from the wall decreases. It should be kept in mind, however, that these results are for the infinite cavity case with $\sigma = 0$ (since for either h_u or h_ℓ infinite there is no blockage cavitation number). The relation between C_L and σ for values of σ greater than zero has been obtained (19) for this case but has not yet been computed.

It would seem that the latter results may be of interest in observing the effect of larger bodies, such as ships' hulls, on hydrofoil characteristics. To our knowledge no experiments have been carried out on such problems.

THE FREE JET

Finite cavity flow in a jet seems to present a more difficult mathematical problem than the solid wall case just discussed. The preliminary results from the method of L. C. Woods mentioned previously lead to complicated expressions for the desired quantities. The linear transition flow model, although applicable, leads to hyperelliptic integrals which are cumbersome to use and evaluate. The cases which have

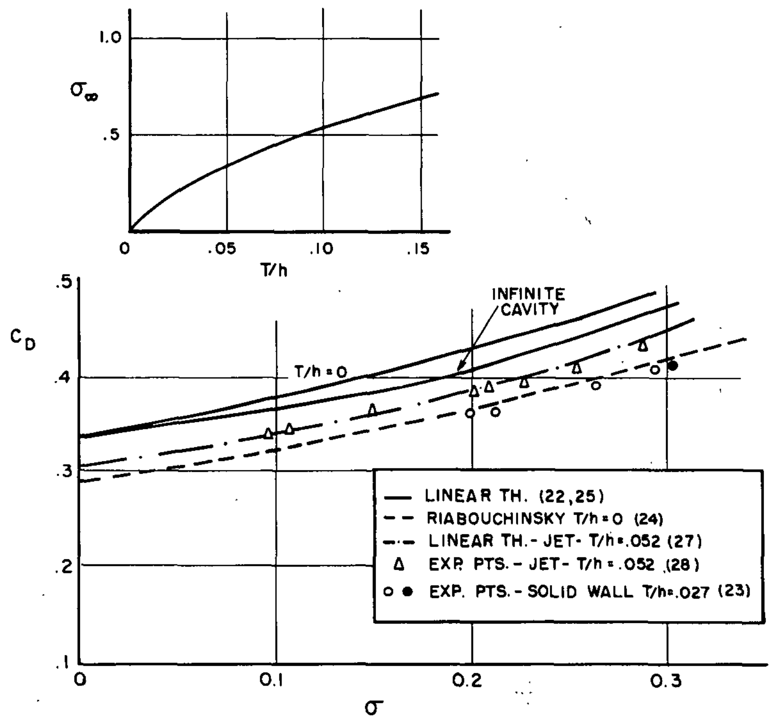

Fig. 8 - The dependence of the drag coefficient on cavitation number for a 15-degree wedge

been studied theoretically are the flat plate perpendicular to the flow direction in terms of the Riabouchinsky model by Birkhoff, Plesset, and Simmons (21), and the slender symmetrical body by Cohen and Tu (27) using the linearized theory.

The exact expressions obtained by Birkhoff, Plesset, and Simmons are very involved and, as far as we know, no calculations have been made from them. As a matter of fact, no curves or data of any kind are given in the paper. Using the approximate expressions appropriate for small cavitation numbers also derived in the paper, we have calculated the relations between C_D, cavity length, and the cavitation number. These are shown in Figs. 5 and 9 and for several values of T/h. As can be seen, a boundary correction is predicted here not only in the case of the changing cavity length, but also for the force coefficient.

Cohen and Tu have used the linearized theory to carry out calculations for the case of a 15-degree wedge. The effect on C_D of changing σ in a jet with $T/h = 0.052$ at infinity is shown in Fig. 8. These results are compared with unbounded flow results for the wedge obtained by Plesset and Shaffer (24) and also with unbounded flow results from the linear theory. In addition, the experimental results of Silberman (28) are shown. The linear theory in this case seems to give the right correction and the agreement with experiment is good. Silberman was able to obtain data at $\sigma = 0$ by venting the cavity to the upstream pressure. Here the linear theory fails, but the Kirchhoff-Helmholtz theory results of Siao and Hubbard (29) are not far off.

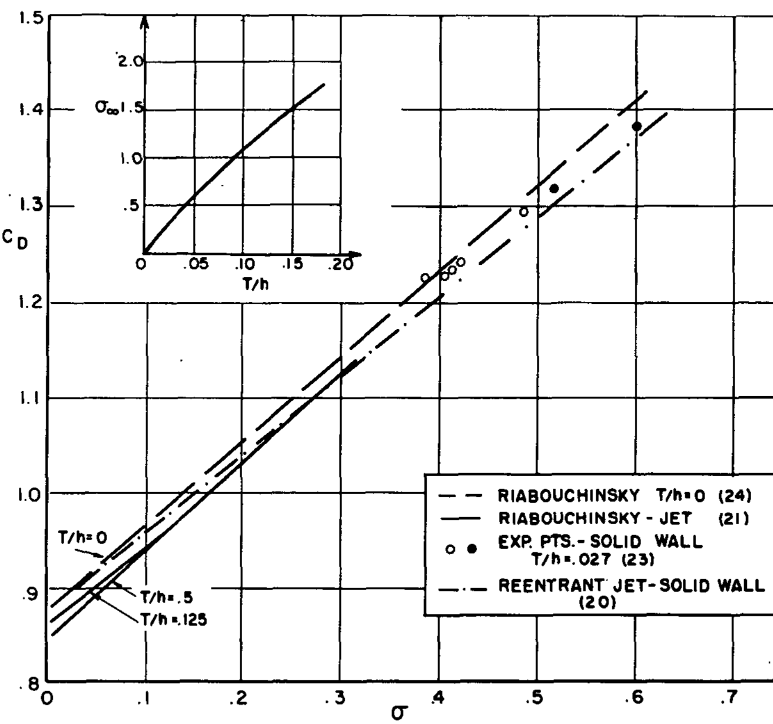

Fig. 9 - The dependence of the drag coefficient on cavitation number for a flat plate

For the cup (or scoop channel) and the circular cylinder with a ratio of body width to jet width of 0.0375, the experimental results for C_D are in good agreement with the theoretical predictions for an unbounded flow (30). For this value of T/h, it appears that the boundary effects on C_D for these bodies are negligible.

In the case of the lifting flat plate in the jet, experimental data have been compared with unbounded flow solutions. The agreement between the experimental results and the theoretical values obtained by Wu (8) using the transition flow model are quite good for cavitation numbers above 0.15. The values of C_L predicted by Wu (16) using the linearized theory lie consistently above the experimental data. The results for the drag coefficient are not in such good agreement. It is impossible to say whether this can be attributed to a boundary effect, since there are no computations for a lifting plate in a jet.

As for cavity dimensions, the jet boundary corrections given for the decrease of cavity length with diminishing jet width by Cohen and Tu for a symmetric wedge are confirmed by the experimental results (Fig. 4). The fit is not as close as for the force coefficient data but shows clearly the direction of the correction. Silberman has also compared cavity lengths for a lifting foil with those obtained theoretically for the unbounded stream flow by Wu (8) using the transition model and Tulin (15) using the linear theory. Both theories give reasonable agreement for σ greater than 0.1 but are not consistent with the experimental data for σ less than this value.

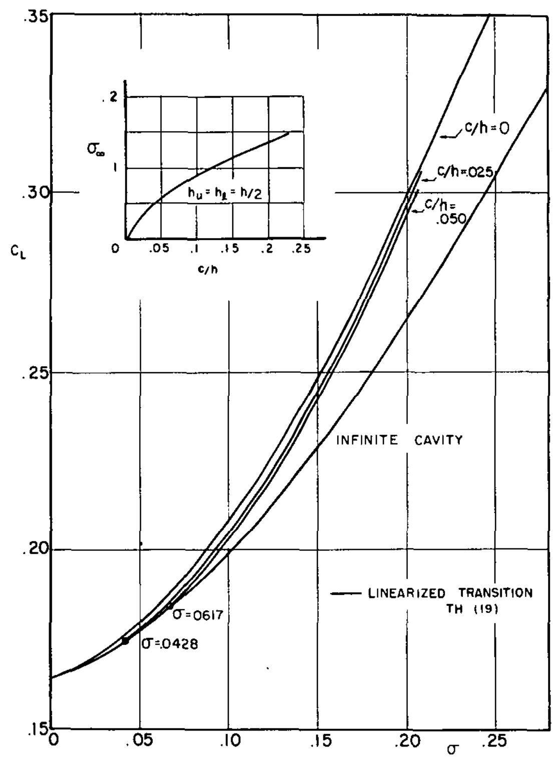

Fig. 10 - The dependence of the lift coefficient for a flat plate at angle of attack 6 degrees in a channel on cavitation number ($h_u = h_\ell = h/2$)

SOME COMPARISONS AND OBSERVATIONS

We have given separately the details of the effects of solid walls and of free jet boundaries or force coefficients and cavity dimensions in some particular bounded flow problems. In the present section we would like to compare these effects. We will then discuss the theoretical and experimental data presented thus far on two-dimensional flows and offer some observations based on the examples considered.

For the purpose of comparison we have chosen the 15-degree wedge. If one begins with an unbounded stream flowing past the wedge, the cavity length can increase from zero to infinity as the cavitation number decreases to zero as shown in Fig. 4. For a jet bounded by constant-pressure streamlines, the same is true, although the cavity length is always smaller at any given value of σ. On the other hand, if the flow is bounded by solid walls, the cavity length increases at a more rapid rate than in the unbounded stream and attains infinite length at the blockage value σ_∞. For a given cavitation number, say for example $\sigma = 0.35$, there may be as much as 100 percent difference in cavity lengths for the same size wedge in a jet and a tunnel of the same height ($T/h = 0.052$). More important, however, is the fact that cavities of very long lengths may be modeled in a solid wall tunnel at high cavitation numbers if desired and at low cavitation numbers in a jet if this is desirable.

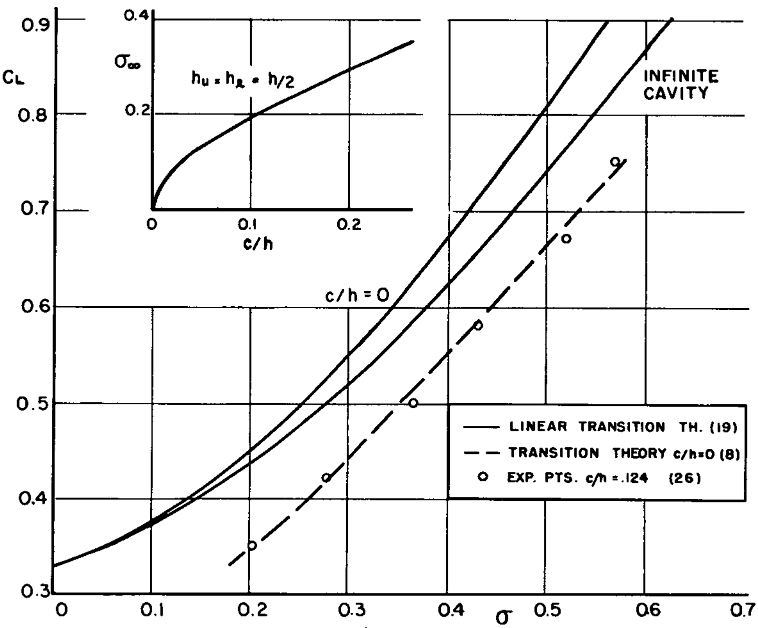

Fig. 11 - The dependence of the lift coefficient for a flat plate at angle of attack 12 degrees in a channel on cavitation number ($h_u = h_\ell = h/2$)

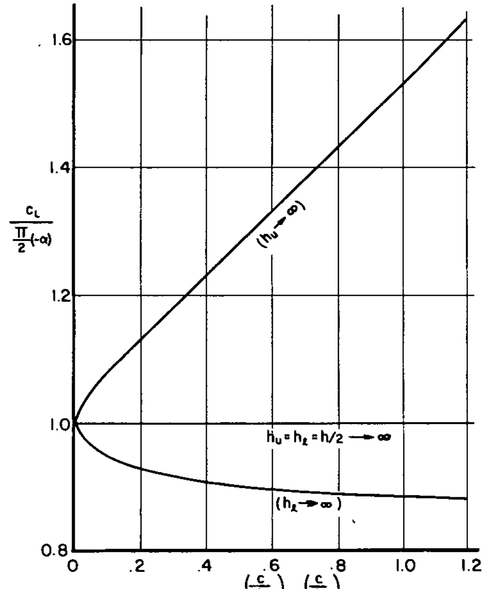

Fig. 12 - The dependence of the lift coefficient of an inclined flat plate in a channel on chord-wall height ratio for the case of one wall at infinity and cavitation number zero

Cavity width behavior in the jet and solid wall channel is similar to that of cavity length. In channels at blockage cavitation numbers, the cavity width has its maximum value. These, of course, are strongly limited by the tunnel height so that continuity of flow may be maintained. In the jet, the cavity may have any width, becoming infinite as σ approaches zero.

In comparing the drag coefficient for the wedge in the solid wall channel and in a jet, it must be kept in mind that the linear theory seems consistently to overestimate the force coefficient. The value of C_D predicted by the linear theory for the unbounded flow lies above that predicted by the Riabouchinsky model. If one remains within the framework of the linear theory, however, for a given value of σ, the value of C_D is lower in the jet than in the solid wall channel and both of these lie below the value predicted by the linear theory for the unbounded stream (Fig. 8). Actually, the experimental values for the solid wall channel lie below these linear results and, it will be recalled, coincide with the exact Riabouchinsky model results for the unbounded stream. On the other hand, the experimental results for the wedge in the jet coincide with those predicted by the linear theory. It would almost appear as though the solid walls do not affect the drag; whereas the jet boundaries do. The effect of the jet boundaries for the flat plate case (Fig. 9) is the same, although there is experimental verification for the wedge only. It is very important to keep in mind, however, that the solid wall channel does not allow the modeling of all cavitation conditions. For the experimental case cited in this report where $T/h = 0.027$, the minimum cavity number for the 15-degree wedge is 0.212 and for the flat plate it is 0.3. Thus there is a rather severe limitation on the use of solid wall tunnels for low cavitation number testing.

These comparisons serve to point up the more interesting results of the present survey. First of all, the experimental results of Waid (23), together with the theoretical analyses of Gurevich (20) and Plesset and Shaffer (24) for the symmetrical flows past flat plates and wedges, confirm the idea that the drag coefficient is insensitive to boundary changes in solid wall channels, providing σ is held constant. The linear theory provides the same result qualitatively for the wedge flow and also for the unsymmetrical flow past a flat plate foil in a solid wall channel. The linear theory results for these cases seem characteristically to overestimate the force coefficient when compared with the experiments. For the lifting flat plate foil the unbounded stream results given by Wu (8) using the non-linear transition flow model provide excellent agreement with the water tunnel data, further strengthening the remarks made on the insensitivity of the force coefficient. This observation seems to confirm at least a portion of Birkhoff's Principle of the Stability of the Pressure Coefficient (1).

The theoretical and experimental results examined do not yield quite such consistent results in the case of the free jet. Small boundary effects on C_D are predicted by the theory for the wedge and the flat plate and, in the case of the 15-degree wedge, are observed. On the other hand, fairly good agreement results when unbounded stream values of C_L are compared with the experimental values for a lifting flat plate in the jet.

Thus, in only one case do the experiments show the need for boundary corrections for C_D. It might be pointed out, however, that these experiments were run at the highest value of T/h of all those reviewed. Furthermore, and even more important, these C_D comparisons are made on the basis of constant cavitation number. The blockage computations show how difficult it is to obtain the cavitation conditions necessary to reproduce the C_D values in an unbounded stream. If one is tempted to discard wall effect computations because of the excellent agreement for C_D values with unbounded stream results, it is important to remember that one must also measure blockage effects. Blockage formulas may be obtained which involve C_D, σ, and T/h by rather elementary means (see Ref. 1) but it is more convenient to have σ and T/h related independently from C_D. Such a relation requires the solution of the bounded flow problem.

Besides the flows bounded by two free streamlines or two walls, flows near a single boundary have been discussed. There appears to be no experimental data for these cases, although it would be useful. When the lifting foil is near a solid wall above the foil, the lift coefficient decreases as the foil nears the wall and increases when the foil nears a solid wall below the foil. Further theoretical work on these single wall cases is probably necessary.

Note: After this paper had been completed, the authors obtained a copy of the original Simmons report (35) from which much of the Birkhoff, Plesset, and Simmons paper (21) has been derived. The original report gives the results of a limited number of numerical computations for the flat plate in a channel or a jet which are not included in the published paper (21). In particular, for the case of the flat plate in a channel, the dependence of σ_∞ on T/h and C_D on σ given by Simmons verifies the results of Gurevich (20) mentioned earlier. For this same case, in order to compare the theoretical results with the experimental results of Waid (23), the following points have been computed from Simmons' data: $T/h = 0.025$, $\sigma = 0.416$, $\ell/T = 27.5$, $a/T = 4.35$; and $T/h = 0.0295$, $\sigma = 0.578$, $\ell/T = 12$, $a/T = 3.25$. (Due to the complexity of the computations it is impractical to determine the complete dependence of a/T and ℓ/T on σ for non-zero values of T/h.) These points are indicated in Figs. 5 and 7 by small squares; they agree well with the experimental results for $T/h = 0.027$.

AXIALLY SYMMETRIC BODIES

The literature on boundary effects on axially symmetric cavity flows is extremely limited. In contrast to the recent analytic and numerical studies that have been made of the unbounded flow past a disk,* the theory of bounded, axially symmetric cavity flows rests on a combination of plausible assumptions based on physical observation.

Reichardt (33,34) observed that for an unbounded stream, certain cavity characteristics were approximately functions of σ only; i.e., they were essentially independent of body shape. In particular, the fineness ratio, $\ell/2a$, (a now refers to maximum cavity radius) and the drag coefficient based on maximum cavity radius, $K_D = 2D/\rho U_\infty^2 \pi a^2$, depend on σ only; and further, the drag coefficient based on wetted frontal area, πb^2, can be written as $C_D = C_D(0)(1 + \sigma)$ where $C_D(0)$ is the drag coefficient at $\sigma = 0$. Thus if $C_D(0)$, the $\ell/a - \sigma$ relation, and the $K_D - \sigma$ relation are known, one can compute a/b and ℓ/b for given values of σ.

The $\ell/a - \sigma$ relation, and the $K_D - \sigma$ relation can be obtained analytically by assuming that the flow around a cavity can be represented by an axial source-sink distribution. The constant pressure condition on the closed stream surface will, of course, be only approximately satisfied. That such a method will give meaningful results is a consequence of the "non-dependence" of ℓ/a and K_D on changes in the body shape. For an unbounded flow, Simmons (35) has used a source-sink distribution of linearly varying strength to determine the above relations. The value of $C_D(0)$ can be taken as the experimental value, or may be computed by assuming that the pressure distribution on the axially symmetric body is the same as on the equivalent two-dimensional body.†

*In particular, the work of Garabedian (31,32) should be mentioned.

† The latter method, while inaccurate, does give values of $C_D(0)$ which are not too much in error for the case of a circular disk.

The methods discussed above have been extended by Campbell and Thomas (36) to the axially symmetric cavity flow in a solid wall tunnel of radius R. In order to compute a/b and ℓ/b for this case, it is necessary to make the additional assumption that $C_D/(1 + \sigma)$ is insensitive not only to changes in σ but also to the presence of boundaries. The results for the cavity dimensions are similar to the two-dimensional case; for a given value of $\sigma > \sigma_\infty$ both a/b and ℓ/b increase as b/R increases.

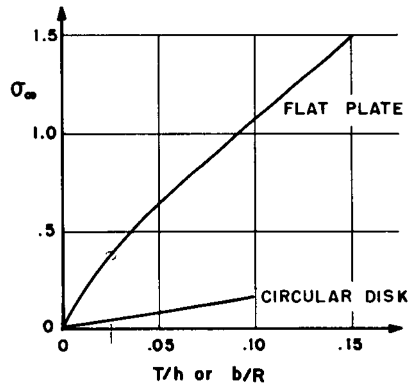

Fig. 13 - The dependence of blockage cavitation number on b/R for a circular disk

A relation between σ_∞ and b/R can be obtained by simple continuity and momentum considerations, and the above assumption on C_D. This relation is shown in Fig. 13 for the case of a circular disk. For a given value of T/h = b/R it is clear from Fig. 13 that the limiting cavitation number is much less for axially symmetric flow than for two-dimensional flow. However, a better measure of blockage for this case is the ratio of body cross-section area to tunnel cross-section area $(b/R)^2$. If σ_∞ is plotted against $(b/R)^2$, the resulting curve is closer to the σ_∞ vs T/h curve for the two-dimensional case. Even so, σ_∞ for the axially symmetric case is less than for the two-dimensional case.

The case of an axially symmetric cavity in a circular free jet or tunnel has been considered by Armstrong and Tadman (37) who also used the source-sink methods mentioned earlier. One of their conclusions is that for small values of σ and ℓ/2R the fractional decrease, based on the unbounded flow, in cavity length and width in a jet and the fractional increase in a solid wall tunnel due to the boundaries are proportional to $(\ell/2R)^3$. The corrections in a tunnel are roughly four times as large as the correction in a free jet.

Since these analyses are based on assumptions on the role of C_D, they do not, of course, yield any information about the drag coefficient.

As far as we could ascertain, there is no experimental work on boundary effects on cavity dimensions which could be used to check these theories, with the exception of the study of Self and Ripken (38) in a free jet tunnel. However, their conclusions are that the cavity dimensions are insensitive to changes in the position of the boundary. This is somewhat surprising in view of the above discussion, and the known large effect that boundaries have on cavity dimensions in two-dimensional flow.

REFERENCES

1. Birkhoff, G., Plesset, M., and Simmons, N., "Wall Effects in Cavity Flow-I," Quar. Appl. Math., 2, 8 (1950), pp. 151-168

2. DiPrima, R.C. and Tu, Yih-O, "Surface Effects on the Supercavitating Flow Past a Symmetric Wedge," R.P.I. Math. Rep. 17, Rensselaer Polytechnic Institute, Troy, July 1958

3. Serrin, J.B., "Existence Theorems for Some Hydrodynamical Free Boundary Value Problems," Jour. Rat. Mech. and Anal. 1, 1 (1952)

4. Réthy, M., "Strahlformen Incompressibler Reibungsloser Flüssigkeiten," Math. Annalen 46 (1895), pp. 249-272

5. Valcovici, V., "Uber Discontinuierliche Flüssigheitsbewegungen mit Freien Strahlen," Thesis Göettingen (1913)

6. Von Mises, R., "Berechnung von Ausfluss und Uberfallzahlen," Zeits. VDI 61 (1917), 447-452, 467-473, 493-497

7. Joukowsky, N.E., "A Modification of Kirchhoff's Method of Determining a Two-Dimensional Motion of a Fluid, Given a Constant Velocity Along an Unknown Streamline," Collected Papers, Vol. III, Moscow, 1936, 195-365

8. Wu, T.Y., "A Free Streamline Theory for Two-Dimensional Fully Cavitated Hydrofoils," Jour. Math. and Phys., 3, 35 (1956), pp. 236-265

9. Tulin, M.P., "Steady Two-Dimensional Cavity Flows About Slender Bodies," David Taylor Model Basin Report 834, May 1952

10. Muskhelishvili, N.I., Singular Integral Equations, P. Noordhoff, N.V. Groningen-Holland, 1953

11. Woods, L.C., "On Harmonic Functions Satisfying a Mixed Condition with an Application to the Flow Past a Porous Wall," Appl. Sci. Res., Ser. A, 6, No. 5-6 (1957), pp. 351-364

12. Woods, L.C., "Subsonic Plane Flow in an Annulus or a Channel with Spacewise Periodic Boundary Conditions," Proc. Roy. Soc. of London, A, 229 (1955), pp. 63-85

13. Woods, L.C., "On the Lifting Aerofoil in a Tunnel with Porous Walls," Proc. Roy. Soc. of London A, 242 (1957), pp. 341-254

14. Timman, R., "The Aerodynamic Forces on an Oscillating Aerofoil Between Two Parallel Walls," App. Sci. Res., No. 1, Vol. A3 (1951), pp. 31-58

15. Tulin, M., "Supercavitating Flow Past Foils and Struts," Symposium on Cavitation in Hydrodynamics, Nat. Phys. Lab., Teddington, England, 1955

16. Wu, T.Y., "A Note on the Linear and Non-Linear Theories for Fully Cavitated Hydrofoils," Hydrodynamics Laboratory Report No. 21-22, Cal. Inst. of Tech., August 1956

17. Acosta, A.S., "A Note on Partial Cavitation of Flat Plate Hydrofoils," Report No. E-19.9, Hydrodynamics Laboratory, Cal. Inst. of Tech., October 1955

18. Geurst, A.J. and Timman, R., "Linearized Theory of Flow with Finite Cavities About a Wing," Rapport 7, Technische Hogeschool, Institute voor Toegepaste Wiskunde, Delft

19. Cohen, H., Sutherland, C.D., and Tu, Yih-O, "Wall Effects in Cavitating Hydrofoil Flow, Jour. Ship Res., 3, 1 (1957), pp. 31-40

20. Gurevich, M.I., "Symmetrical Cavitation Flow Around a Flat Plate Situated Between Parallel Walls," Bull. of Acad. of Sci. of USSR, No. 4 (1946)

21. Birkhoff, G., Plesset, M., and Simmons, N., "Wall Effects in Cavity Flow-II," Quar. Appl. Math., 4, 9 (1952), pp. 413-421

22. Cohen, H. and Gilbert, R., "Two-Dimensional, Steady, Cavity Flow About Slender Bodies in Channels of Finite Breadth," Jour. Appl. Mech. Trans. ASME, 79 (1957), pp. 170-176

23. Waid, R.L., "Water Tunnel Investigations of Two-Dimensional Cavities," Hydrodynamics Laboratory Report No. E-73.6, Cal. Inst. of Tech., Sept. 1957

24. Plesset, M.S. and Shaffer, P.A., Jr., "Cavity Drag in Two and Three Dimensions," Nav. Ord. Report 1014, NOTS 131, October 1949

25. Wu, T.Y., "A Simple Method for Calculating Drag in the Linear Theory of Cavity Flows," Engineering Division Report No. 85-5, Cal. Inst. of Tech., August 1957

26. Parkin, B.R., "Experiments on Circular-Arc and Flat-Plate Hydrofoils," Jour. Ship Research, 4, 1 (1958), pp. 34-57

27. Cohen, H. and Tu, Yih-O, "A Comparison of Wall Effects on Supercavitating Flows Past Symmetric Bodies in Solid Wall Channels and Jets," Proc. of Ninth Internal Cong. Appl. Mech., Brussels, 2 (1956), pp. 359-370

28. Silberman, E., "Experimental Studies of Supercavitating Flow About Simple Two-Dimensional Bodies in a Jet," St. Anthony's Falls Hydraulic Laboratory Report No. 59, Univ. of Minnesota, April 1958

29. Siao, T.T. and Hubbard, P.G., "Deflection of Jets. I. Symmetrically Placed V-Shaped Obstacle," Free Streamline Analyses of Transition Flow and Jet Deflection. State Univ. of Iowa, Bull. No. 35, pp. 33-44, 1953

30. Birkhoff, G. and Zarantonello, E.H., Jets, Wakes and Cavities, Acad, Press Inc., New York, N.Y., p. 148, 1957

31. Garabedian, P.R., "The Mathematical Theory of Three-Dimensional Cavities and Jets," Bull. Amer. Math. Soc., 62, No. 2 (1956), pp. 219-236

32. Garabedian, P.R., "The Calculation of Axially Symmetric Cavities and Jets," Pacific Jour. of Math., 6 (1956), pp. 611-689

33. Reichardt, H., "The Laws of Cavitation Bubbles at Axially Symmetric Bodies in a Flow," Ministry of Aircraft Production Reports and Translations No. 766 (1946)

34. Reichardt, H., "The Physical Laws Governing the Cavitation Bubbles Produced Behind Solids of Revolution in a Fluid Flow," Ministry of Supply TPA3/TIB Translation G.D.C. No. 10/5678T (1945)

35. Simmons, N., "The Geometry of Liquid Cavities with Especial Reference to the Effects of Finite Extent of the Stream," Ministry of Supply, A.D.E. Report 17/48 (1948)

36. Campbell, I.J. and Thomas, G.E., "Water Tunnel Boundary Effects on Axially Symmetric Fully Developed Cavities," Admiralty Res. Lab./R1/G/HY/18/1 (1956)

37. Armstrong, A.H. and Tadman, K.G., "Wall Corrections to Axially Symmetric Cavities in Circular Tunnels and Jets," Ministry of Supply A.R.E. Report No. 7/52 (1953)

38. Self, M.W. and Ripken, J.F., "Steady-State Cavity Studies in a Free-Jet Water Tunnel," St. Anthony's Falls Hydraulic Laboratory Report No. 47, Univ. of Minnesota, July 1955

* * * * *

DISCUSSION

E. Silberman (University of Minnesota)

I have had the pleasure of carrying on a stimulating correspondence with one of the authors, Dr. Cohen, in connection with our two-dimensional free-jet cavity studies at the St. Anthony Falls Hydraulic Laboratory of the University of Minnesota. The comments in the paper are in agreement with our own experience and ideas, on the whole. However, one point could stand amplification.

The statement by the authors that force coefficients appear to be insensitive to boundary conditions is apparently correct for blunt bodies at all cavitation numbers. The following figures comparing experimental data obtained in a free jet at $T/h = 0.0375$ with results from unbounded fluid-flow theory and closed-tunnel data illustrate this point (these are data referred to, but not shown by the authors). Figure D1 is for the cup (or scoop) channel and compares data with theory. Figure D2a is for the circular cylinder at small cavitation numbers, comparing data with theory also. Figure D2b is for the cylinder at larger cavitation numbers, comparing free-jet data with closed-tunnel data obtained by Martyrer.*

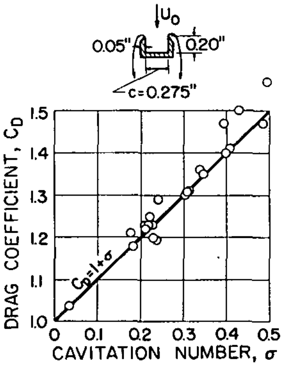

Fig. D1 - Effect of cup channel on drag coefficient

On the other hand, bodies with sharp leading edges, when operated in a free jet at small cavitation numbers (that is, at cavitation numbers generally below those obtainable in a closed tunnel), experience significantly smaller force coefficients than in unbounded fluid. Theoretical results at zero cavitation number, illustrated by Fig. D3 for the flat plate, confirm this statement.

The following three figures show pertinent experimental data: Fig. D4 presents drag coefficient data for the 15-degree semiangle wedge in a free jet; it contains much of the information shown in the authors' Fig. 8, but an experimental point obtained at zero cavitation number has been included, as well as the theoretical result at zero cavitation number (indicated by an arrow). The solid line in the figure represents the linear theory

*E. Martyrer, "Kraftsmessungen an Widerstands Körpern und Flügelprofilen in Wasserstrom beiz Kavitation," pp. 268-286 in "Hydromechanische Probleme des Schiffsentriebs," Ed. by G. Kempf and E. Foerster, Hamburg (1932).

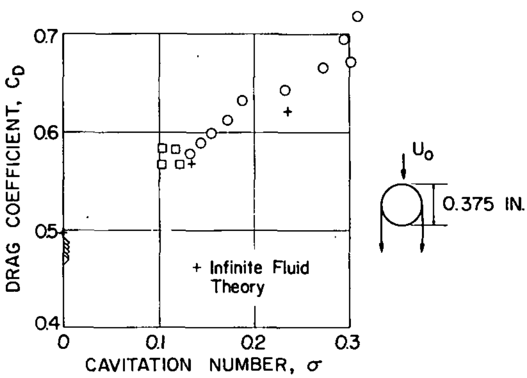

Fig. D2(a) - Effect of circular cylinder (for small σ) on drag coefficient

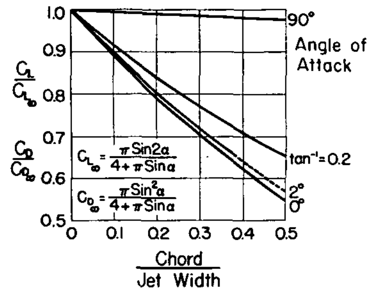

Fig. D3 - Effect of flat plate (at $\sigma \approx 0$) on drag coefficient (C_D) and lift coefficient (C_L) at various angles of attack

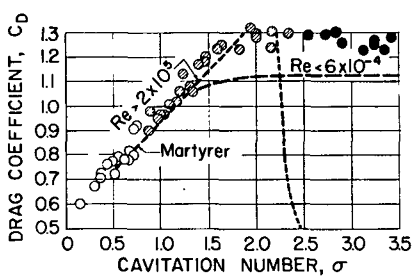

Fig. D2(b) - Effect of circular cylinder (large σ) on drag coefficient

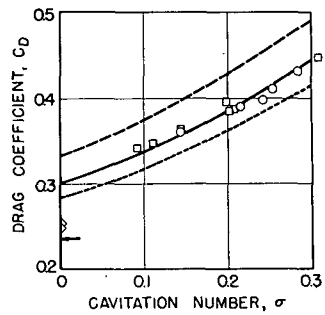

Fig. D4 - Effect of 15-degree semiangle wedge on drag coefficient

for the free jet while the broken line represents Wu's linear theory for infinite fluid and the dotted line represents the infinite fluid solution by the Riabouchinsky model. Figure D5 gives similar information for a 12.5-degree semiangle wedge. The free-jet linear theory is not available for this case. Figure D6 contains data for two inclined flat plates in a free jet. The unbounded fluid result is represented by the solid line taken from Wu's nonlinear theory. This solid line also represents closed-tunnel data obtained by Parkin at cavitation numbers exceeding the minimum for the tunnel (as shown in the authors' Fig. 11). The arrows again represent the theoretical results for the free-jet flow at zero cavitation number. It appears that the free-jet coefficients approach the unbounded-flow coefficients only at some finite cavitation number well in excess of zero.

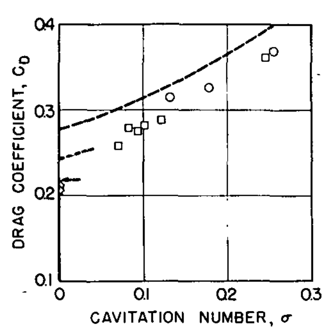

Fig. D5 - Effect of 12.5-degree semiangle wedge on drag coefficient

A comment also appears in order regarding the authors' final statement referring to experimental data for three-dimensional cavities. It should not

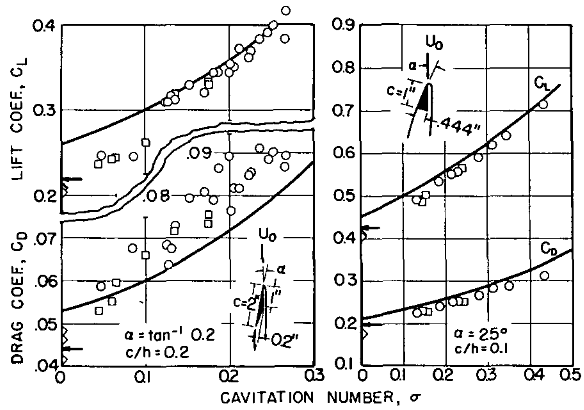

Fig. D6 - Effect of flat plates on drag coefficient (C_D) and lift coefficient (C_L) compared with results of Wu and Parkin (solid line)

seem surprising that shape effects on three-dimensional cavities in a free jet are very small or even nonmeasurable as compared to those on two-dimensional cavities. Nevertheless, it should be noted that the experimental work by Self and Ripken was performed in a free-jet tunnel with a contact window touching the jet for its full length and closing an eighth of the jet circumference (45 degrees). The tunnel might thus be considered slotted wall rather than free jet. Self and Ripken performed a limited experiment using two contact windows opposite each other and enclosing a quarter of the circumference. Close inspection of their data shows a small but systematic increase in cavity length for the more closed section. There are, incidentally, some interesting problems associated with cavity flows in slotted-wall tunnels, both two- and three-dimensional.

A. Silverleaf (National Physical Laboratory)

In addition to the work described here, I am particularly interested in several cases which are not mentioned and do not appear to have been studied. In hydrofoil work in the Lithgow Tunnel at N.P.L. we have been using slotted-wall test sections, and I feel that these configurations have advantages outweighing the difficulties they introduce. We are interested in three cases: first, the two-dimensional foil in a two-dimensional slotted-wall section having solid sides and slotted top and bottom; second, three-dimensional foils, or wings, in slotted-wall and in closed-throat sections; third, two-dimensional foils in a section with solid sides, slotted bottom, and a free surface, such as might be incorporated in a free-surface water tunnel or circulating water channel.

At N.P.L. we hope to complete within the next eighteen months some comparative tests with the same foils in a number of different test sections in a water tunnel and in an open towing tank. Using forced or artificial cavitation I think we can obtain better experimental data on wall correction coefficients than if the work is carried out only in a tunnel.

W. A. Clayden

As Professor DiPrima suggested, A. H. Armstrong at ARDE extended the work of Simmons and computed the cavity size and drag of a flat plate as a function of cavitation number and blockage ratio for a free jet. In our work we measured the cavity length for various two-dimensional flat plates in our free-jet tunnel. Armstrong's boundary corrections were then applied. This had the effect of collapsing the results onto one curve which is in close agreement with the theoretical curve obtained by Perry, Plesset and Shaffer. A similar result was obtained for the widths of the cavities as well, although, since the blockage correction is considerably less, the effect is not so striking.

M. P. Tulin (Office of Naval Research)

Professor DiPrima has made a general remark about the validity of linearized theory. It should be stated in defense of such theory that questions of agreement between linear and non-linear theory must certainly depend upon the angle of attack, the camber, and the thickness of the bodies producing the flow, as well as upon their shape. For example, the force and moment predictions made by a linearized theory for the case of a flat plate hydrofoil become asymptotically exact as the angle of attack approaches zero. At the same time the linearized theory becomes increasingly awry as the angle of attack is made larger. As is well known then, the case for linearized theory may thus be made as bright or as gloomy as one wishes, all depending on the angle-of-attack ranges chosen for discussion.

The important fact exists that linearized theory is proving quite satisfactory in an increasing number of important applications to very practical engineering problems.

R. Timman (Technische Hogeschool, Delft)

Dr. DiPrima mentioned the Winterhall correction we had been doing for some years previously. We got the exact solution in the integrals. It turned out, however, that if we wanted to have numerical results it was somewhat more convenient to work with integral equations and do the thing numerically.

Of course, one can go back one step and introduce integral equations along the walls. I think that would be sufficient unless the walls are too close to the hydrofoil.

Hirsh Cohen and R. C. DiPrima

In reply to Marshall Tulin, we would like to say that we are in agreement with his remarks. What we were trying to point out was simply that the linearized theory that has been used to estimate boundary effects does give the correct qualitative effects, but does not, at least in the cases considered here (the 15-degree, half angle wedge, and the flat plate at angle of attack of 12-degree), give the accuracy of a non-linear theory.

The question of slotted-wall tunnels has been brought up by Dr. Silverleaf and by Prof. Silberman. It seems to us that judging from the results presented in this paper, little effect on force coefficients is to be expected. It seems reasonable to expect the results for force coefficients to lie between those for solid wall and free-jet test sections. On the other hand, blockage effects on cavity dimensions are of some interest. For a given model and tunnel size, lower cavitation numbers may be obtained with a slotted-wall tunnel. It certainly seems worthwhile to make blockage studies for the slotted-wall case. There seems to be need of some caution in using such test sections as proven by the instability problems experienced at A.R.L. The possibility of cavity-slotted-wall instability interactions should be looked into.

Professor Silberman has raised a point which has, indeed, troubled us. He remarks that for very low cavitation numbers the drag on the 15-degree and 12.5-degree wedges is lower in the free jet than the drag predicted in an infinite stream by the exact theory. But this is borne out in his experiments only by the single point, $\sigma = 0$. The linear theory predicts that the drag in a free jet will be lower than in the infinite stream at all cavitation numbers. One feels that this should also appear in the experimental results. If there is an abrupt rise in the drag as the cavitation number increases from zero, it certainly should not become greater than the infinite stream value.

* * * * *