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NOGAPS Semi-Lagrangian

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LONG-TERM GOALS

The long-term goal of this project is to develop new dynamical core that would enable the Navy Operational Global Atmospheric Prediction System (NOGAPS) to achieve higher horizontal and vertical resolution, higher vertical extent, increase meteorological skill, and additional predictive constituents, all without an appreciable increase in run time efficiency.

OBJECTIVES

The objective of this project is to develop and transition semi-Lagrangian/semi-implicit (SL/SI) integration algorithm into the NOGAPS. The SL/SI numerical algorithm is the only numerical integration technique that removes the CFL constraint on the system's time step and thus is the only numerical technique that will allow for increased resolution, more dynamical variables, and higher accuracy, without a significant decrease in computational efficiency.

APPROACH

The work in FY 2010 was to prepare the NOGAPS SL/SI dynamical core with full physics and coupled to the NAVDAS/AR data assimilation system for transition to the 6.4 Large-Scale Atmospheric Models Project.

WORK COMPLETED

- A two-time-level NOGAPS SL/SI was developed and tested against the three-time-level NOGAPS SL/SI. While computationally more efficient, the results indicated that the two-time- level was less skillful than the three-time-level version of the SL/SI code. Since the completion of the two-time-level tests, several significant changes have occurred in the NOGAPS SL/SI dynamical core (see next two bullets) and further testing of the two-time-level SL/SI will take place in the 6.4 program.

- A further modification for vector momentum equation was developed, which enabled a true time-centered and therefore more consistent time discretization of the momentum equation. Tests indicated that this fix had a positive effect on the tropical forecasts and reduced the 120-hour tropical cyclone track error by 80 nautical miles.

- Several modifications vertical interpolation methods were developed and tested in an effort to reduce the height forecast errors in the upper atmosphere. These included a correction to the Hermite vertical interpolation in the upper atmosphere and a non-interpolating algorithm in the vertical. Forecast tests over the month of August 2009 indicated that both methods reduced the errors in the upper atmosphere, but the correction method to the Hermite vertical interpolation showed more overall forecast skill.

Hortal, M. and A. J. Simmons, 2002: The development and testing of a new two-time-level semi-Lagrangian scheme (SETTLS) in the ECMWF forecast model. *Q. J. R. Meteorol. Soc*., **128**, 1671-1687.

Ritchie, H., 1987: Semi-Lagrangian advection on a Gaussian grid. *Mon. Wea. Rev*., **115**, 608-619.

Ritchie, H., 1991: Application of the semi-Lagrangian method to a multilevel spectral primitive-equation model. *Q. J. R. Meteorol. Soc*., **117**, 91-106.

Staniforth, A. and J. Cote, 1991: Semi-Lagrangian integration schemes for atmospheric models – a review. *Mon. Wea. Rev*., **119**, 2206-2223.

RESULTS

The explicit vector momentum equation in the hybrid coordinate system is given by

$$\frac{d\vec{V}}{dt} = -\vec{f} \times \vec{V} - \vec{\nabla}\Phi - \left[c_p \Theta \frac{\partial P}{\partial \pi} \right] \vec{\nabla}\pi \,, \tag{1}$$

where $\vec{V}$ is the horizontal wind vector, $\vec{f}$ is the Coriolis vector, Φ is the geopotential, π is the terrain pressure, c_p is the specific heat, Θ is the virtual potential temperature, and P is the Exner function:

$$P = \left(\frac{p}{p_0} \right)^{R/c_p} . \tag{2}$$

The last term in (1) arises because of the terrain following coordinates and is zero when the vertical coordinate switches from sigma to pure pressure (at approximately 100 hPa in NOGAPS). The three-time-level, semi-Lagrangian, semi-implicit treatment equation (1) is given by

$$\frac{\vec{V}_k^{n+1} - \vec{V}_k^{n-1}}{2\Delta t} = -\vec{f} \times \vec{V}_k^n \; - \{\vec{\nabla}\Phi_k^n\} \; - c_p \{\Theta_k^n \frac{\partial P_k^n}{\partial \pi}\} \vec{\nabla}\pi^n \;\; , \tag{3}$$

where the subscript term k indicates the vertical level and the superscript n indicates the time level- the (n+1) term on the computational grid and the (n) and (n-1) on the interpolated trajectory. The two bracketed terms on the right hand side are treated in a semi-implicit fashion - that is these terms are

expanded to obtain those terms responsible for the fast gravity wave modes. After some algebraic manipulations of the definitions of the geopotential and Exner function (3) can be expressed as:

$$\frac{\vec{V}_k^{n+1} - \vec{V}_k^{n-1}}{2\Delta t} = -\vec{f} \times \vec{V}_k^n - \vec{\nabla}\Phi_k^n - c_p \Theta_k^n \frac{\partial P_k^n}{\partial \pi} \vec{\nabla}\pi^n$$
$$-\sum_{l=1}^{LM} C_{kl}^1 \left(\beta \vec{\nabla}\Theta_l^{n+1} + [1-\beta]\vec{\nabla}\Theta_l^{n-1} - \vec{\nabla}\Theta_l^n \right) - C_k^2 \left(\beta \vec{\nabla}\pi^{n+1} + [1-\beta]\vec{\nabla}\pi^{n-1} - \vec{\nabla}\pi^n \right) \qquad (4)$$

where The C's are constant matrices and vectors and β is any number between 0 and 1. $\beta = 1/2$ gives the classic centered difference approach to the semi-implicit algorithm, but a stability analysis of the non semi-Lagrangian Eulerian advection in NOGAPS indicates that this choice is unstable. Stability is achieved for $\beta \geq 0.8$. Tests of the SL/SI code indicate that this criterion is also valid and so in NOGAPS SL/SI:

$$\beta = 0.8 \ . \qquad (5)$$

In a pure three-time-level semi-Lagrangian, the first three terms on the right of (4) require their own separate spatial interpolation (see Figure 1).

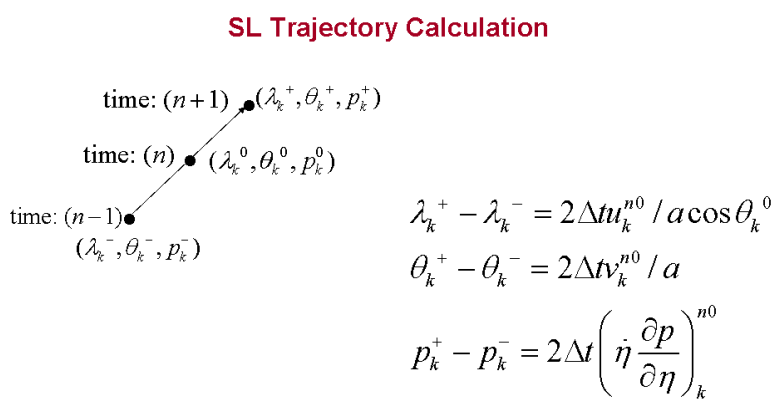

Figure 1. The three-time-level interpolation in the semi-Lagrangian algorithm. The (+) point lies on the computational grid and the (-) point is the $2\Delta t$ interpolation back in time, indication where the parcel was at $t - 2\Delta t$. The (0) point is the location of the trajectory at $t - \Delta t$.

However, in the original three-time-level version of NOGAPS SL/SI these terms are given by a linear interpolation in space of the departure point (denoted by superscript -) and the arrival point (denoted by a +):

$$(\vec{f} \times \vec{V}_k^n)^0 = \alpha(\vec{f} \times \vec{V}_k^n)^- + (1-\alpha)(\vec{f} \times \vec{V}_k^n)^+, \tag{6}$$

where $\alpha = 0.8$. It was discovered that this choice was not as optimal as first appeared and lead to significant errors in the winds. Figure (2) shows a comparison of the tropical cyclone tracks forecasts for the month of August 2009 (forecasts starting at 0Z only) of the T319L42 NOGAPS SL/SI system with $\alpha = 0.8$ and with $\alpha = 0.5$, together with the non semi-Lagrangian results. The track error is substantially improved with $\alpha = 0.5$, which is now used in all subsequent version of NOGAPS SL/SI.

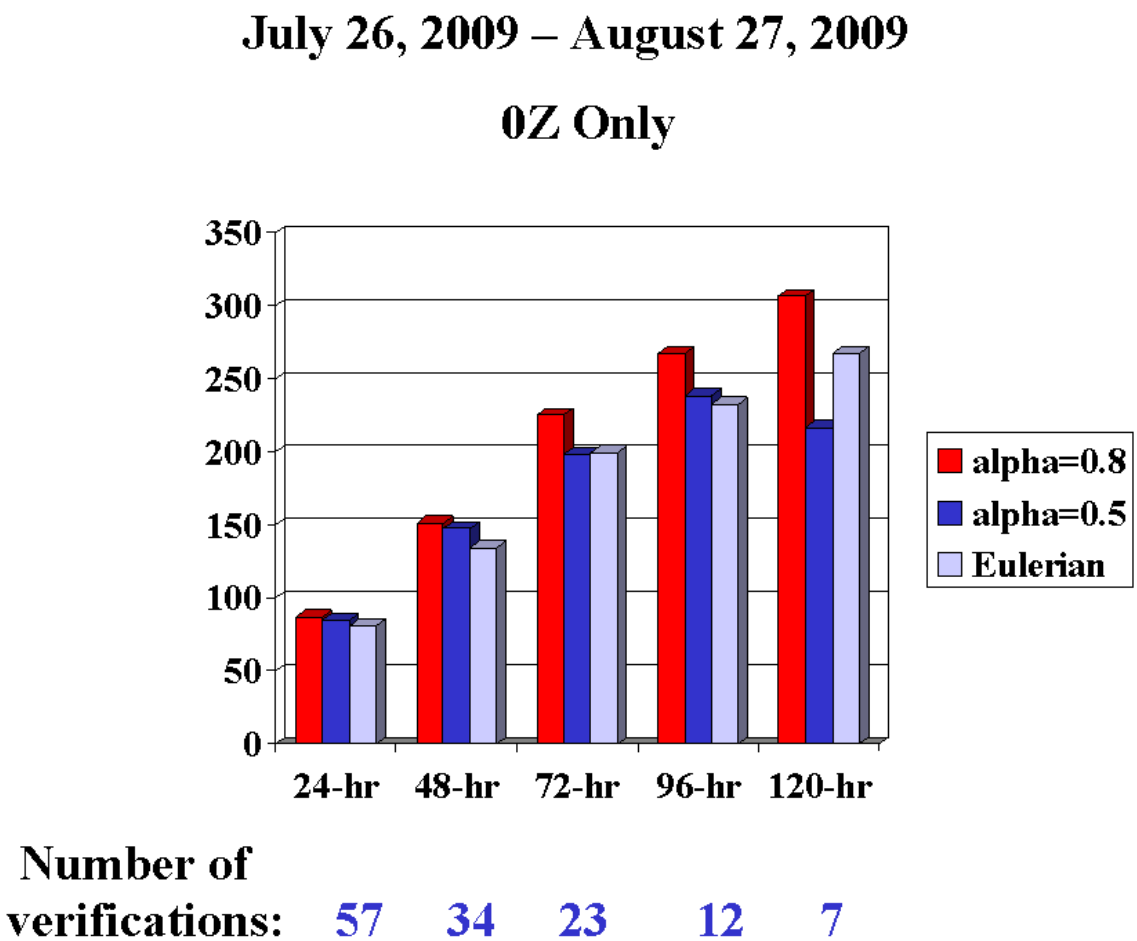

Figure 2. A comparison of the tropical cyclone track error for NOGAPS SL/SI for 2 different values of alpha, together with the results from the non semi-Lagrangian (Eulerian). The resolution of the forecast model is T319L42.

A persistent problem in the NOGAPS SL/SI has been the large temperature and height forecast errors in the lower stratosphere (100 hPa – 10 hPa). In FY 2010 two approaches were tried to reduce this error: (1) add an adjustment to the vertical interpolation in the lower stratosphere, and (2) employ a non-interpolating scheme in the vertical.

If we let η be the generalized vertical coordinate, then the vertical departure point η_D^{n-1} is given by

$$\eta_D^{n-1} = \eta_A^{n+1} - 2\Delta t \dot{\eta}^n , \tag{7}$$

where $\dot{\eta}^n$ is the interpolated (Hermite interpolation) generalized vertical motion valid at time t. In the first approach for the region above 100 hPa the departure position is adjusted by a small amount f :

$$\eta_D^{n-1} = \eta_A^{n+1} - f 2\Delta t \dot{\eta}^n , \tag{8}$$

where

$$f = \begin{cases} > f_+ & \text{if } \dot{\eta} > 0 \\ < f_- & \text{if } \dot{\eta} < 0 \end{cases} . \tag{9}$$

It was found by experimentation that the for the L42 configuration of NOGAPS that $f_+ = f_- = 0.07$ was an optimal fit. Equation (8) is clearly an engineering fix, trying to account for a systematic error of the interpolation in the vertical. Tests with other interpolating schemes without any adjustment (linear, cubic, and spline) gave even poorer results than the Hermite, so it was decided to test this approach with the Hermite interpolation.

The second approach is to rewrite the advection equations, adding to both sides a pseudo vertical motion that forces the departure point (see Equation 8) to be on a grid point. So if we write the advection equation for any field F as

$$\frac{dF}{dt} = \frac{d_H F}{dt} + \dot{\eta}\frac{\partial F}{\partial \eta} = R , \tag{10}$$

where $\frac{d_H F}{dt}$ represents the horizontal advection and R represents all the remaining non-advection terms, then rewrite the equation with the vertical motion on the right hand side and add in to both sides a pseudo vertical motion term $\dot{\sigma}\frac{\partial F}{\partial \eta}$, we obtain:

$$\frac{dF}{dt} = \frac{d_H F}{dt} + \dot{\sigma}\frac{\partial F}{\partial \eta} = R - (\dot{\eta} - \dot{\sigma})\frac{\partial F}{\partial \eta} \tag{11}$$

The specification of the pseudo vertical velocity is that the departure point given by is the

$$\sigma_D^{\,n-1} = \eta_A^{\,n+1} - 2\dot{\sigma}^n \Delta t^n \tag{12}$$

computational vertical grid point, closest to the point given by (7). This procedure does remove the vertical interpolation of the field F, but does require an extra interpolation of the new term on the right hand side of (11). The overall results does require less overall interpolations and is more computational efficient than the full 3-d semi-Lagrangian.

Figure 3 is a time mean plot of the 1000 hPa and the 500 hPa Northern Hemisphere anomaly correlation (AC) for T319L42 forecast tests (August 2009) of the NOGAPS SL/SI without any vertical interpolation correction (SL CONTROL), NOGAPS SL/SI with the vertical interpolation adjustment given by Equation 8 (SL OMT), NOGAPS SL/SI with non-interpolation in the vertical (SL NIV), and the non semi-Lagrangian NOGAPS (Euler). While the control, adjusted interpolation, and non semi-Lagrangian all show similar skill as measured by the AC, the SL with a non interpolating in the vertical shows less skill throughout the troposphere.

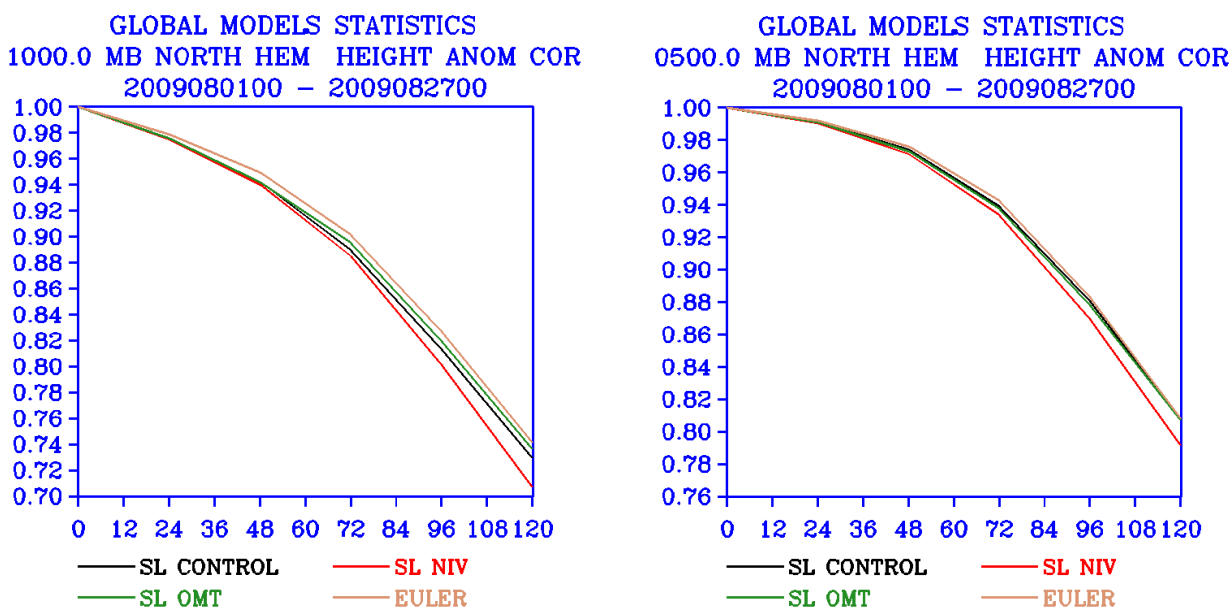

Figure 3. The Northern Hemisphere 1000 hPa and 500 hPa geopotential height anomaly correlation for forecasts for the month of August 2009 comparing the control NOGAPS SL/SI with the adjusted vertical interpolation scheme (SL OMT), the non-interpolating scheme in the vertical (SL NIV) and the non semi-Lagrangian NOGAPS (EULER).

Figures 4 and 5 are the 1 hPa (47.8 km) height and temperature errors for the 4 forecast test runs. It should not be too surprising that the adjusted vertical interpolation scheme performs well, since it has been tuned to remove the cold bias seen in the control NOGAPS SL/SI. The problem with the non

interpolation scheme appears to be that it has introduced a positive temperature bias in place of the un-adjusted cold bias. More work will be pursued on different approaches, including increase vertical resolution, before a final decision is made on the best approach to go forward into operations.

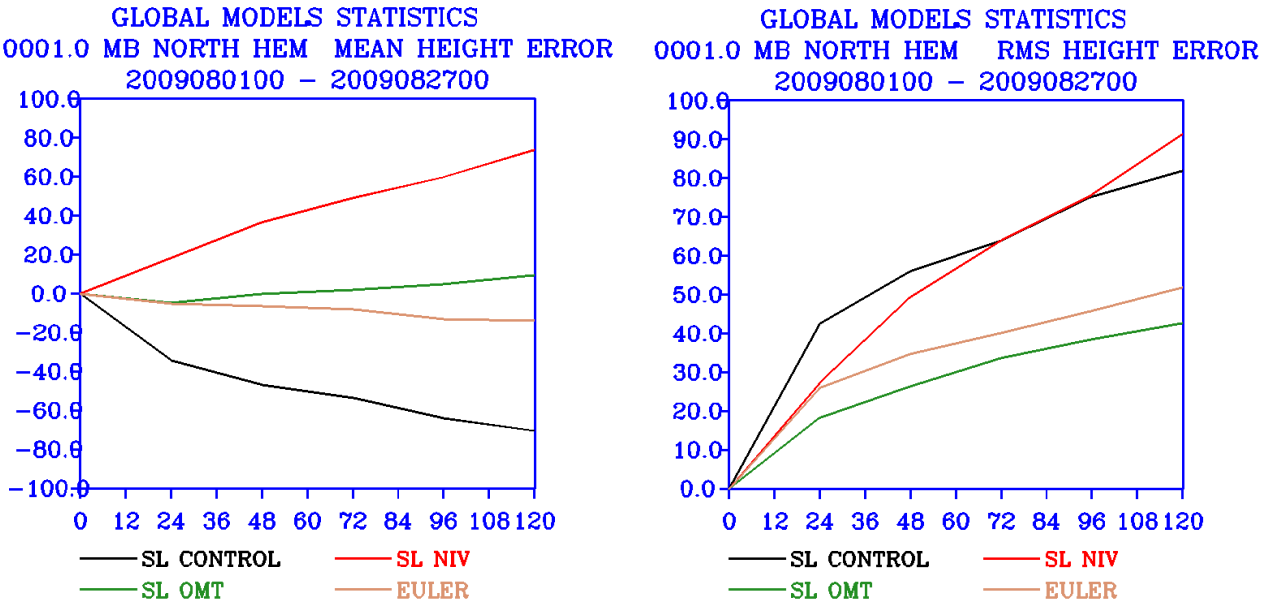

Figure 4. The Northern Hemisphere 1 hPa geopotential height errors (mean and root mean square) for forecasts for the month of August 2009 comparing the control NOGAPS SL/SI with the adjusted vertical interpolation scheme (SL OMT), the non-interpolating scheme in the vertical (SL NIV) and the non semi-Lagrangian NOGAPS (EULER).

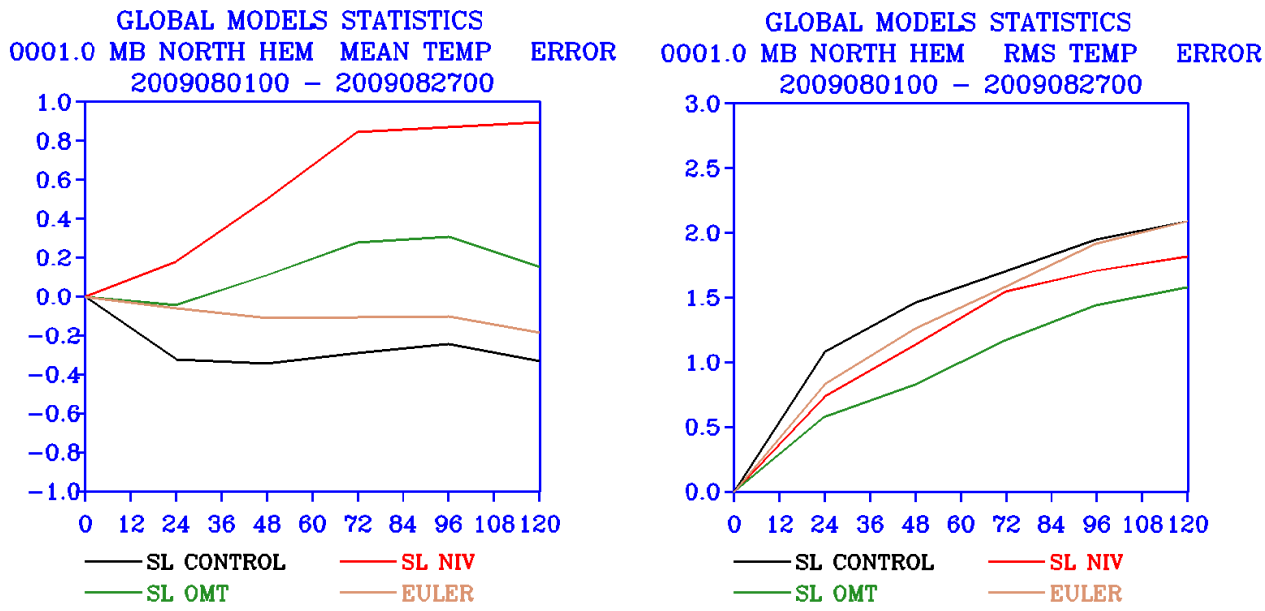

Figure 5. The Northern Hemisphere 1 hPa temperature errors (mean and root mean square) for forecasts for the month of August 2009 comparing the control NOGAPS SL/SI with the adjusted vertical interpolation scheme (SL OMT), the non-interpolating scheme in the vertical (SL NIV) and the non semi-Lagrangian NOGAPS (EULER).

IMPACT/APPLICATIONS

The successful completion of this project will enabled an significant increase in the NOGAPS horizontal and vertical resolution, an increase in meteorological skill as measured by both the standard statistical (NOGAPS scorecard) and synoptic measures, and the addition of more constituents (ozone, cloud liquid water, dust, and aerosols) without a substantial increase in run time.

TRANSITIONS

No transitions to the operational code at this point.

RELATED PROJECTS

This project is related to the 6.4 Large-Scale Atmospheric Models work-unit, sponsored by PMW-120, which is responsible for transitions to the operational large-scale atmospheric models run at FNMOC.

PUBLICATIONS

None

PRESENTATIONS

6.2/6.4NRL/MRY Review, March 9, 2010.