Development and application of a three dimensional coupled-mode model

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LONG-TERM GOALS

The over-all goal of this research is the development of an accurate and reliable propagation model applicable to environments which exhibit strong range dependence in all three spatial dimensions.

OBJECTIVES

The objective of this current work is to gain an understanding the physics of propagation in continental shelf areas, specifically horizontal refraction and mode coupling induced by three-dimensional inhomogeneities in the waveguide. Of particular interest is the environment of the South Florida Test Facility (SFTF) for which significant horizontal multipath arrivals have been observed [Heaney and Murray (2009)]. A coupled-mode approach is chosen for this work because the field decomposition into modal eigenvalues and amplitudes can provide insight into the effects of environmental inhomogeneities on the acoustic field.

APPROACH

This work is based on coupled-mode theory. In this approach, the solution for pressure is given by

$$\mathbf{P}(x,y,z) = \sum_n R_n(x,y)\phi_n(z;x,y), \tag{1}$$

where $\phi_n(z;x,y)$ are the depth dependent eigenfunctions and $R_n(x,y)$ are the modal amplitudes. $R_n(x,y)$ satisfies

$$\begin{aligned}\frac{\partial^2 R_m}{\partial x^2} + \frac{\partial^2 R_m}{\partial y^2} + k_m^2(x,y)R_m = &- \sum_n (A_{mn} + \vec{B}_{mn}\cdot\nabla_t)R_n \\ &- 4\pi S(\omega)\frac{\phi_m(z_0;x,y)}{\rho(z_0,x,y)}\delta(x-x_0)\delta(y-y_0)\end{aligned} \tag{2}$$

where k_m^2 are modal eigenvalues, A_{mn} and B_{mn} are the coupling coefficients, and $S(\omega)$ is the amplitude of the source located at (x_0, y_0, z_0).

The main task is to identify an efficient and accurate solution to equation (2). Multiple approaches have been attempted including an integral equation method and a perturbative technique. These approaches are briefly described below. Development of coupled-mode model for three-dimensional environments is ongoing work.

Theoretically, extension of the two-way coupled-mode model based on an integral equation formalism developed by Knobles et. al. [Knobles et al. (2001); Knobles (1994)] to three dimensions is straightforward. According to this technique, the basic integral equations are reformulated to permit a Lanczos expansion to converge. The solution of the original set of integral equations is then recovered from the solution of the modified equations. However, application of this technique in three dimensions has proven to be computationally prohibitive. The limiting factor is computation of the two-dimensional Green's function which must be calculated to and from every point in the x-y plane.

The second approach to solving equation (2) is based on a perturbative technique described by Rutherford [Rutherford (1979)]. According to this method, the right side of equation (2) is estimated using the mode amplitudes of the adiabatic solution. This creates a correction term which is a function of x and y and of the mode number. For the one-dimensional case, Numerov's method is then applied to calculate an approximate solution for the (coupled) mode amplitudes. These estimates of the mode amplitudes are used to update the correction term. After a few iterations, this perturbative technique agrees well with the integral equation method described above. Efforts were made to apply this method for calculating mode amplitudes in two dimensions using standard solution techniques [Collins (1993); Weinberg and Burridge (1974)] for the homogeneous version of equation (2). However, because the correction term takes the form of a source function distributed over the entire x-y plane, correctly incorporating it into the solution involves a computational demand equivalent to that of calculating the two-dimensional Green's function.

As an alternative to these approaches, a method has been implemented for solving the horizontal refraction equation that focuses on range-dependent environments for which the lateral variations occur in one dimension only. To apply coupled-mode theory to problems of this sort, the y-dependence of equation (2) is removed by taking its Fourier transform [Rutherford (1979)]:

$$R_m(x,\gamma) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} R_m(x,y) e^{i\gamma y} dy. \tag{3}$$

The transformed equation is

$$\begin{aligned} \frac{\partial^2 R_m(x,\gamma)}{\partial x^2} + [k_m^2 - \gamma^2] R_m(x,\gamma) = & - \sum_n (A_{mn} + \vec{B}_{mn} \cdot \nabla_t) R_n \\ & - 4\pi S(\omega) \frac{\phi_m(z_0;x,y)}{\rho(z_0,x,y)} \delta(x - x_0). \end{aligned} \tag{4}$$

Equation (4) is solved over a sufficient range of γ such that the inverse transform of $R_m(x,\gamma)$ can be calculated using

$$R_m(x,y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} R_m(x,\gamma) e^{-i\gamma y} d\gamma. \tag{5}$$

WORK COMPLETED

The translationally invariant coupled-mode solution described above was applied to a penetrable wedge. The geometry of the wedge is based on the SFTF environment. Figure 1 shows bathymetry of the SFTF environment with the location of the array denoted by the black cross. The slope of bathymetry in the cross-slope (east-west) direction is shown in figure 2. The slope of the wedge for the model was set to the maximum observed for the SFTF environment (0.1 rad. $\approx 5.7^o$), which is about twice the slope of the wedge considered by the ASA benchmark problems [Collins (1990)]. The source is located at $(x,y,z) = (1800,0,20)$ meters where the water depth is 250 meters. The environment properties are sound speed and density of 1500 m/s and 1.0 g/cm^3 in the water column and sound speed, density, and attenuation of 1700 m/s, 1.5 g/cm^3 and 0.5 dB/λ in the seabed.

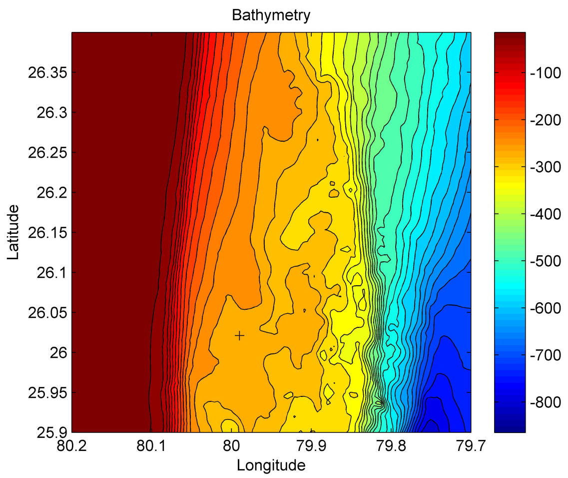

Figure 1: Bathymetry of the South Florida Test Facility.

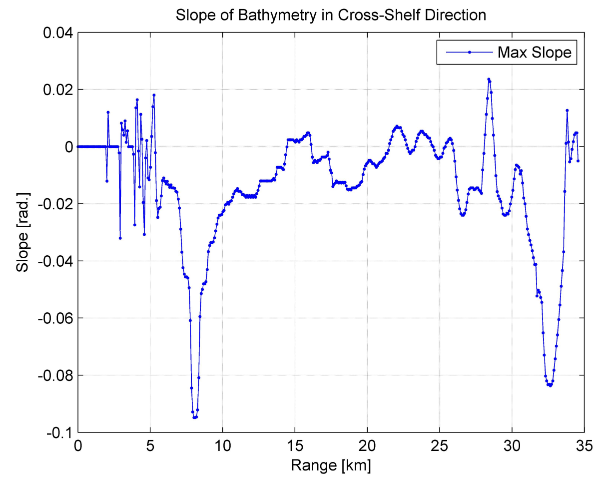

Figure 2: Slope of bathymetry shown in figure 1 in the cross-slope direction.

Modal eigenvalues and eigenfunctions for a frequency of 10 Hz were calculated using the normal mode code ORCA [Westwood et al. (1996); Westwood and Koch (1999)]. Only modes 1 and 2 propagate at the source location. Modes 3, 4, and 5 are leaky modes which contribute to the coupled-mode solution.

The three-dimensional coupled-mode solution for the pressure field was calculated using the integral equation coupled-mode approach to solve the transformed horizontal refraction equation (equation (4)). The adiabatic solution was calculated using the same method but neglecting the coupling terms. These solutions are shown as a function of x and y for a receiver depth of 20 meters in the first two panels of figure 3. The effects of horizontal refraction are identifiable as curved interference patterns. The difference between the adiabatic and coupled-mode solutions, shown in the third panel of figure 3, are two orders of magnitude lower in amplitude.

Figure 4 shows transmission loss on a radial from the source along the 250 meter isobath. Also shown in figure 4 is the range-independent solution which was calculated with the same water column and seabed parameter values but for a flat bottom. Because the range-independent solution does not account for horizontal refraction, the two mode interference pattern persists out to 20 km. On the other hand, the effects of horizontal refraction can be clearly observed from three dimensional solutions: mode two is refracted down the shelf after 5 km and mode one is refracted down the shelf after 15 km. Conversely,

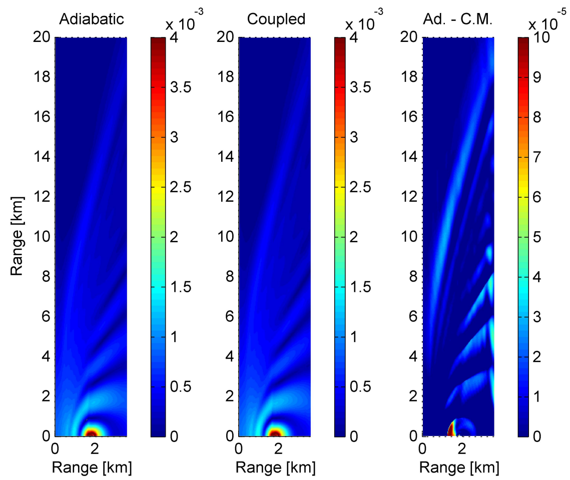

Figure 3: Pressure [Pa] at a depth of 20 meters. The left panel shows the adiabatic solution, the center panel shows the coupled-mode solution, and the right panel is the difference between the adiabatic and coupled-mode solutions.

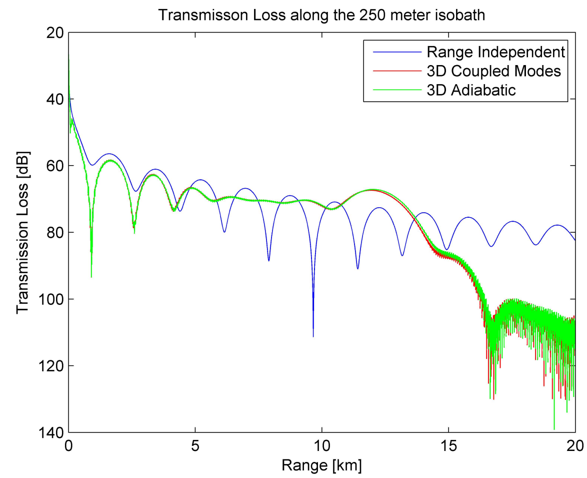

Figure 4: Transmission Loss [dB] for the coupled-mode and adiabatic solutions shown in the first two panels of figure 3 as a function of y for x=1.8 km with the range independent solution for comparison.

the effects of mode coupling appear less important as the transmission curves computed with and without mode coupling differ by only a fraction of a decibel.

This example indicates that significant mode coupling is not caused by the 5.7 degree slope in bathymetry. Therefore, in this initial work, a faithful representation of the SFTF has been modeled with an adiabatic mode description. The analysis presented here is concerned with a single source tow heading north from the array location along the 250 meter isobath. The source was towed at a depth of 100 meters while repeatedly broadcasting a two minute sequence consisting of several tones. This analysis will concentrate on the 206 Hz signal which had a source level of 171.6 dB/μPa. The data were recorded on a horizontal line array which contained 120 elements with 1.75 m spacing and were sampled at 1 kHz. The array line of bearing was perpendicular to the slope, providing a broadside look along the 250 meter isobath.

The array data were processed using standard techniques to create a narrowband bearing-time-record (BTR). The BTR for the northbound source tow is shown in figure 5 where the time axis has been replaced by distance north of the array location. The towed source signal is recognizable on the BTR from its repeated on/off sequence. The direct (i.e., non-refrected) path can be observed near the array at an angle of 5 degrees. Its signal level drops off after 35 km. At a range of 45 km, the refracted path is first visible at angle of -20 degrees. The refracted path is predicted to contribute at shorter ranges, but on the BRT the signal is masked by an interfering ship.

The adiabatic mode propagation was obtained by calculating the modal amplitudes in the *x*-*y* plane with a parabolic equation model. The environment for the model was based on measurements taken at the SFTF site. The range-dependent water column sound speed profile was constructed from XBT measurements taken during the experiment. These measurements indicate the water column sound speed profile varies significantly in the cross-shelf direction, having a deeper thermocline further from the shore. Piston cores from the SFTF site provide sound speed estimates for the seabed [McNeill

(2001)]. These data suggest sound speed in the top layer of sediment decreases with increasing water depth. The seabed attenuation in the model is higher (0.9 dB/λ) on the flat part of the shelf, i.e. the area around the 250 meter isobath where the array was located, and lower (0.1 dB/λ) on the incline inshore of the array.

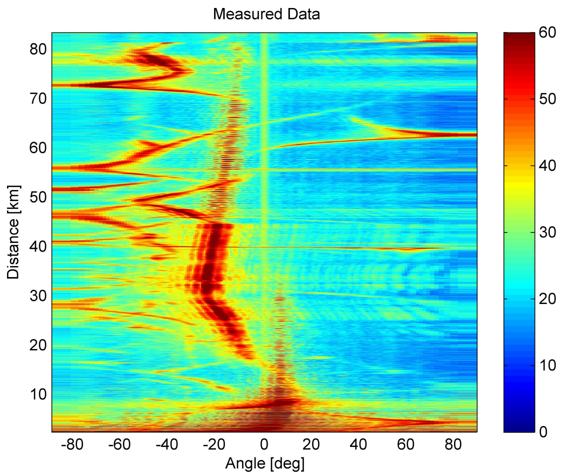

Figure 5: Narrowband BTR of the measured data. The time axis of the BTR has been replaced by distance north of the array location.

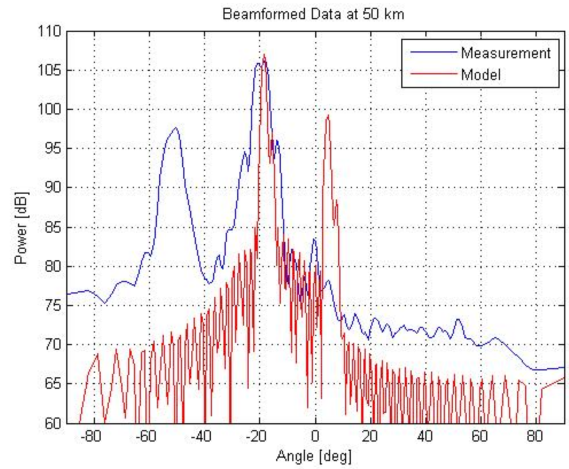

Figure 6: Beam power from measured (blue) and modeled (red) data for a source 50 km north of the array location.

For a frequency of 206 Hz, transmission loss was computed from 24 modes for a source located 50 km north of the array. The modeled data were then beamformed for comparison with the measured data. Figure 6 shows the beam power from the measured and modeled data at a distance 50 km north of the array location. The measured data contain an interfering ship at -55 degrees. The refracted path is clearly visible at -20 degrees in both the measured and modeled data. The model accurately predicts both the arrival angle and the amplitude of the refracted path. However, the direct path is also clearly visible in the modeled data at an angle of 5 degrees. This is in contrast to the measured data for which the direct path has dropped below the noise floor at this range. It is expected that the model could be brought into agreement with the measured data by adjusting the attenuation along the 250 isobath.

RESULTS

Based on the preliminary work completed, mode coupling appears to be a secondary effect to horizontal refraction for the environment of the SFTF. This is shown by comparing coupled-mode and adiabatic solutions for the translationally invariant penetrable wedge. Although the example considered a very low frequency, the conclusions about the importance of mode coupling due to bathymetric changes are in agreement with previous work [Graves et al. (1975)]. On this basis, a propagation model of the SFTF was constructed with adiabatic modes. This model accurately predicts both the arrival angle and the amplitude of the refracted signal.

IMPACT/APPLICATIONS

The impact of this work will be an increased understanding of acoustic propagation through complicated coastal environments for which the bathymetry, seabed properties, and oceanography can vary in three dimensions.

TRANSITIONS

The primary transition for this project is an accurate and reliable model for acoustic propagation in environments with strong three-dimensional range dependence.

RELATED PROJECTS

An on-going project titled "Application of an acoustic inversion technique to infer the location of an oil plume from sound speed inhomogeneities" benefits from this work. The goal this project is to estimate water column sound speed in a three dimensional volume using modal travel time measurements from multiple source-receiver pairs. The presence of oil will then be inferred from inhomogeneities in the sound speed field. A thorough understanding of the forward problem, including the effects of horizontal refraction and mode coupling, is necessary to successfully estimate environment parameters. This project is funded by ARL:UT's IR&D program.

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PUBLICATIONS

Ballard, M. S. and K. M. Becker, "Propagation of low-frequency sound through densely sampled internal waves (A)," *Journal of the Acoustical Society America*, **127**., 1973 (2010). [published, non-refereed]