Development and application of a three-dimensional coupled-mode model

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Award Number: N00014-101-0649

LONG-TERM GOALS

The over-all goal of this research is the development of an accurate and reliable propagation model applicable to environments which exhibit strong range dependence in all three spatial dimensions.

OBJECTIVES

The objective of this work is to gain an understanding of the physics of propagation in continental shelf areas, specifically horizontal refraction and mode coupling induced by three-dimensional inhomogeneities in the waveguide. A coupled-mode approach has been applied for this purpose. The coupled-mode approach is attractive for solving problems involving three-dimensional propagation for several reasons. First, this technique provides intuitive results for understanding the features responsible for observed propagation effects in range-dependent environments. For example, upslope propagation is characterized by acoustic energy radiated into the bottom at discrete depths associated with mode cut-off. This phenomenon is illustrated on the cover of the well-known text Computational Ocean Acoustics [Jensen et al. (1994)]. The modal decomposition of the acoustic field has also been used to describe horizontal refraction in a wedge-shaped ocean, for which the single-mode interference pattern associated with rays launched up and across the shelf has been well documented [Weinberg and Burridge (1974)]. Furthermore, coupled-mode solutions are highly accurate and have been used for benchmarking solutions to range-dependent problems [Jensen and Ferla (1990)]. In order to appreciate the limitations of existing three-dimensional models, it is necessary to have methods which can provide reference solutions for comparison.

APPROACH

A three-dimensional acoustic propagation model based on the stepwise coupled-mode approach [Evans (1983)] implemented using the single-scatter solution has been developed. The theory for this modeling technique is briefly described in this section.

The inhomogeneous Helmholtz equation for pressure $P(r, \theta, z)$ due to a point continuous wave source of
amplitude $S(\omega)$ located at $(r_0, z_0)$ is

$$
\rho(r, \theta, z) \nabla \cdot \left[ \frac{1}{\rho(r, \theta, z)} \nabla P(r, \theta, z) \right] + k^2(r, \theta, z)P(r, \theta, z) = -4\pi S(\omega) \frac{\delta(r)}{r} \delta(z - z_0), \tag{1}
$$

where $k = \omega/c(r, \theta, z)$, $\omega = 2\pi f$, $f$ is the acoustic frequency, and $c(r, \theta, z)$ is sound speed.

The solution for pressure can be found from normal mode theory

$$
P(r, \theta, z) = \sum_{m=1}^{M} A_m(r, \theta) \phi_m(z; r, \theta), \tag{2}
$$

where $\phi_m(z; r, \theta)$ are the depth-dependent eigenfunctions and $A_m(r, \theta)$ are the modal amplitudes.

The modal eigenfunctions $\phi_m(z; r, \theta)$ satisfy

$$
\rho(r, \theta, z) \frac{\partial}{\partial z} \left[ \frac{1}{\rho(r, \theta, z)} \frac{\partial \phi_m(z; r, \theta)}{\partial z} \right] + \left[ k^2(r, \theta, z) - k_m^2(r, \theta) \right] \phi_m(z; r, \theta) = 0, \tag{3}
$$

with boundary conditions defined by the plane wave reflection coefficient at the sea surface and above the lower halfspace.

In the absence of mode-coupling, the adiabatic approximation for a modal amplitude $\tilde{A}_m(r, \theta)$ satisfies the horizontal refraction equation,

$$
\frac{\partial^2 \tilde{A}_m}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \tilde{A}_m}{\partial \theta^2} + k_m^2(r, \theta) \tilde{A}_m = -4\pi S(\omega) \frac{\delta(r)}{r} \frac{\phi_m(r_0, z_0)}{\rho(r, \theta, z_0)}, \tag{4}
$$

where $k_m(r, \theta)$ is the complex horizontal wavenumber of the $m^{th}$ mode. Eq. (4) must be solved for each mode with the horizontal refraction determined by the modal phase speed $c_{phm}(r, \theta) = \omega/\text{Re}\{k_m(r, \theta)\}$ and modal attenuation $\alpha_m(r, \theta) = \text{Im}\{k_m(r, \theta)\}$. Eq. (4) is a two-dimensional Helmholtz equation, and the solution is obtained using a parabolic equation solution [Collins (1994)] in cylindrical coordinates [Napolitano (1985)],

$$
\frac{\partial \tilde{A}_m}{\partial r} = ik_0 \sqrt{1 + k_0^{-2} \left( \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + k^2 - k_m^2 \right)} \tilde{A}_m. \tag{5}
$$

Mode-coupling is included into the PE solution using the stepwise coupled-mode technique [Evans (1983)]. This approach, originally derived for the two-dimensional Helmholtz equation ($P = P(r, z)$), discretizes a range-dependent environment into a series of range-independent segments with continuity of pressure and particle velocity satisfied at the vertical boundaries between segments. Application of the boundary conditions results in a set of linear equations from which the coupling matrix $R_{m,n}(r, \theta)$ is obtained. The coupling matrix describes the scattering of acoustic energy at each vertical interface. The single-scatter approximation is applied because this type of matching condition lends itself to a simple marching solution. The single-scatter solution is obtained by treating each pair of segments as an independent problem, thus neglecting the higher-order terms resulting from multiple scattering at other interfaces. The mode coupling matrix $R_{m,n}(r, \theta)$ resulting from the single-scatter approximation is given by

$$
R_{m,n}(r, \theta) = C_{m,n}^{RL}(r, \theta) + C_{m,n}^{LR}(r, \theta) \frac{k_n(r_L, \theta)}{k_n(r_R, \theta)}, \tag{6}
$$
In Eqs. 6 and 7, the suffixes \( L \) and \( R \) denote the properties to the left and right of the vertical interface. In the three-dimensional stepwise coupled-mode model, the range-independent segments are defined in the radial direction, forming angular sectors in the \( r - \theta \) plane. By incorporating the mode-coupling matrix \( R_{m,n}(r, \theta) \) into Eq. 5, the solution for the modal amplitudes is obtained from

\[
\frac{\partial A_m}{\partial r} = \sum_{n=1}^{N} R_{m,n}(r, \theta) A_n + ik_0 1 + k_0^{-2} \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + k^2 - k_0^2 A_m. \tag{8}
\]

Thus, at each range, mode coupling is included in the radial direction, and horizontal refraction is accounted for in the azimuthal direction.

**WORK COMPLETED**

The main accomplishments of 2011 include: (1) acoustic propagation modeling of out-plane-arrivals recorded off the southeast coast of Florida, (2) modeling of horizontal refraction of propagating sound due to seafloor scours over a range-dependent layered bottom on the New Jersey shelf, and (3) development of a three-dimensional stepwise coupled-mode model applicable to environments with fluid seabeds.

**Analysis of Southeast Florida data**

An acoustic propagation model was applied to predict measurements of three-dimensional effects recorded off the southeast coast of Florida. The measured signal was produced by a low frequency source which was towed north parallel to the shelf from a fixed receiving array. The acoustic data show the direct path arrival at the bearing of the tow ship and a second refracted path arrival as much as 30 degrees inshore of the direct arrival. Notably, the refracted arrival has a received level (RL) more than 25 dB greater than that of the direct arrival. A geoacoustic model of the environment was created to explain the data, and the measured signals were modeled using three-dimensional adiabatic modes. A modal decomposition of the field provided insight into the variability in the arrival angle and RL of the measured signal.

**Analysis of New Jersey shelf data**

Three-dimensional propagation effects of low frequency sound from 100 to 400 Hz caused by seafloor topography and range-dependent bottom structure over a 20 km range along the New Jersey shelf were investigated using a three-dimensional adiabatic-mode model. Examination of modal amplitudes demonstrated the effect of environmental range dependence on modes trapped in the water column, modes interacting with the bottom, and modes trapped in the bottom. Using normal mode ray tracing, topographic features responsible for three-dimensional effects of horizontal refraction and focusing were identified. The modeled effects were observed in the measurements from the Shallow Water 2006 experiment. Specifically, measured signals from a pair of fixed sources recorded on a horizontal line...
array sitting on the seafloor showed an intensification that the model showed to be caused by horizontal focusing due to the seabed topography of nearly 4 dB along the array.

Development of three-dimensional coupled-mode model

A three-dimensional coupled-mode model has been developed. The modeling approach is based on the stepwise coupled-mode technique implemented using the single-scatter solution [Evans (1983)] as described in the Approach section. Results calculated using the three-dimensional stepwise coupled-mode model show good agreement to a published solution involving propagation over a corrugated bottom [Abawi et al. (1997)]. Calculations of sound propagation over a penetrable wedge have also been compared to results calculated using a translationally invariant solution technique [Rutherford (1979)]. Overall, good agreement in the solutions from the two methods is observed, with differences attributed to assumptions made in calculating the modal eigenvalues and in solving the coupled-mode equations.

RESULTS

Analysis of Southeast Florida data

An adiabatic-mode model was applied to predict measurements of out-of-plane propagation recorded on the southeast Florida shelf. The modeling results are shown in Fig. 1(a) and provide good agreement with the measured data, shown in Fig. 1(b). The local shoaling of the seafloor was demonstrated to be responsible for horizontal refraction of sound. The higher RL of the refracted path observed in the measured data is explained by the range-dependent sediment properties. The arrival along the direct path which propagated over the limestone bottom is highly attenuated due to shear conversion, while the amplitude of the arrival along the refracted path, which propagated over the sandy slope, is

Figure 1: (a) Modeled data with beam angle maxima (asterisks), (b) modeled beam angle maxima (asterisks) overlaid on measured data, and (c) modeled beam angle maxima from mode one (circles), mode two (diamonds), and mode three (squares) overlaid on measured data.

Figure 2: Measured RL as a function of source distance from the array (dots) with (a) model prediction (asterisks) and (b) modeled modal decomposition from mode one (circles), mode two (diamonds), and mode three (squares).
Figure 3: The modeled TL for the incoherent mode sum for a 300 Hz source at (a,b) the location of the NRL source and (c,d) the location of the MSM source. In (a) and (c), TL from the source to receiver is shown, (b) and (d) show a close up of TL at the HLA, whose two ends are denoted by circles.

Figure 4: Distribution of measured RL from the (a) the NRL300 signal, (b) the MSM200 signal, (c) the MSM100 signal, and the RL difference between (d) NRL300 and MSM200, and (e) NRL300 and MSM100. Distance is referenced to the Shark VLA. The lines are from the three-dimensional adiabatic model.

preserved. The RL of the modeled and measured data, shown together in Fig. 2(a), show good agreement for the direct path arrival, including the region between 20 and 30 km where there is a reduced rate of transmission loss (TL) due to the presence of a pool sand covering the limestone bottom. The model overestimates the RL of the refracted path arrival for ranges from 44 to 48 km, but the modeled and measured data are in good agreement between 54 and 74 km.

The modal decomposition of the acoustic field illustrated individual modal contributions to the refracted path arrival. As shown in Fig. 2(b), between ranges 45 to 55 km, the refracted path arrival is dominated by modes two and three, while mode one makes the largest contribution to the field for ranges from 55 to 75 km. This results in the curved shape of the refracted path arrival, shown in Fig. 1(b), as mode one is associated with higher arrival angles than modes two and three as demonstrated by the modal decomposition in Fig. 1(c). Additionally, through examination of the individual modal amplitudes, it was shown that the uneven face of the slope is responsible for horizontal focusing of the refracted sound. Horizontal ray tracing showed that modes turn down the shelf at a location that is controlled by their cut-off depth, and this produces distinct modal arrival angles at the HLA.

Analysis of New Jersey shelf data

Three-dimensional propagation effects caused by seafloor scours and range-dependent layered bottom structure over a 20 km range along the New Jersey shelf were investigated. Signals from the Miami Sound Machine (MSM) and Naval Research Laboratory (NRL) sources are compared, and the refraction effects are observed differ despite a relatively small separation in their positions. TL contours calculated from the incoherent mode sum for a frequency of 300 Hz from the MSM and NRL sources to
the Shark HLA are shown in Fig. 3. The range-dependent propagation environment includes measured seafloor topography and range- and depth-dependent seabed properties based on previous inversion results [Ballard et al. (2010)]. Solutions from both source locations result in horizontal focusing as shown in Fig. 3(a,c). However, differences in the horizontal refraction patterns can be observed. Furthermore, the close-up views near the Shark HLA, Fig. 3(b,d), show that the field from the NRL source position results in a focused beam passing over the center of the array, whereas the MSM beam just misses the array. A modal ray trace was used to identify the topographic feature responsible for the observed focusing effect, which is a localized topographic depression near the sources.

Twenty-two days of data recorded on the Shark HLA were processed. Long averaging times were necessary to isolate the three-dimensional effects of the bottom from the temporal variability of sound propagation due to water column fluctuations. The distribution of the RL for each of the three signals over twenty-two days is shown in Fig. 4(a-c) with the modeled RL calculated from the incoherent mode sum from each source. The horizontal focusing for the NRL source is evident in both modeled and measured data from the intensification over the center of the array. On the other hand, the somewhat flat RL along the array for the MSM is consistent with the model’s prediction of the focused beams missing the array. Distributions of the RL difference of these three signals at each transmission period are shown in Fig. 4(d,e) along with differences in the modeled curves.

**Development of three-dimensional coupled-mode model**

An initial test of the three-dimensional stepwise coupled-mode model has been completed. The model was applied to calculate sound propagation over a sloping bottom. The test environment consists of a flat, deep section from \( x = 0 \) m to \( x = 300 \) m, a linear slope with angle \( \beta = 0.1 \) radians from \( x = 300 \) m to \( x = 3300 \) m, and a flat, shallow section from \( x = 3300 \) m to \( x = 3600 \) m. The maximum and minimum depths of the waveguide are 401.3 m and 100.3 m, respectively. The water column is isovelocity with sound speed \( c = 1500 \) m/s, density \( \rho = 1.0 \) g/cm\(^3\), and attenuation \( \alpha = 0.0 \) db/λ, and the seabed is an
acoustic halfspace with sound speed \( c = 1700 \text{ m/s}, \) density \( \rho = 1.5 \text{ g/cm}^3, \) and attenuation \( \alpha = 0.5 \text{ db/}\lambda. \) Acoustic propagation was modeled for a 10 Hz source located at \( x_0 = 1800 \text{ m} \) and \( z_0 = 100 \text{ m}. \)

The adiabatic and coupled-mode solutions calculated using the three-dimensional stepwise coupled-mode model are shown in the first two panels of Fig. 5. The solution is computed as a sum over five modes; however, only mode one has significant amplitude beyond a range of 6 km. The horizontal refraction of sound down the slope is evident in both solutions from the single mode interference pattern of mode one. The difference between the adiabatic and coupled-mode solutions is shown in the third panel of Fig. 5. The oscillatory pattern observed in this figure occurs because the mode coupling causes the modal interference pattern to be shifted farther down the shelf. Figure 6 shows the solution to the same problem calculated by exploiting the translational invariance of the two-dimensional non-axisymmetric environment [Rutherford (1979)] implemented with the integral equation coupled-mode (IECM) technique [Knobles (1994)]. Comparing Figs. 5 and 6, similar features are observed in the results, including the shifted modal interference pattern evidenced by the striations in the third panel. However, differences in the solutions can be observed. In particular, the third panels of Figs. 5 and 6 which show some disagreement in the region near the source for \( y < 2 \text{ km}. \) Also, in the farfield the third panel of Fig. 6 shows a striation peak centered at \( x = 1 \text{ km}, y = 10 \text{ km} \) which is lacking in the third panel of Fig. 5. This mismatch is a matter of continued investigation and is possibly caused by the different methods used to calculate the modal eigenvalues and mode coupling.

**IMPACT/APPLICATIONS**

The impact of this work will be an increased understanding of acoustic propagation through complicated coastal environments for which the bathymetry, seabed properties, and oceanography can vary in three dimensions.

**TRANSITIONS**

The primary transition for this project is an accurate and reliable model for acoustic propagation in environments with strong three-dimensional range dependence. Because coupled-mode approaches are computationally intensive, they have historically been used to benchmark faster techniques which approximate the solution to the wave equation.

**RELATED PROJECTS**

An on-going project concerned with geoacoustic inversion in range-dependent environments benefits from this work. The goal of this project is to estimate water column sound speed in a three-dimensional volume using modal travel time measurements from multiple source-receiver pairs. A thorough understanding of the forward problem, including the effects of horizontal refraction and mode coupling, is necessary to successfully estimate environment parameters environments with three-dimensional inhomogeneities. This project is funded by ARL:UT’s IR&D program.
REFERENCES


PUBLICATIONS

*Refereed Journal Articles*


*Presentations*


M. S. Ballard and K. M. Becker, Inversion for water column and sediment sound speed profiles in range-dependent environments with application to data from the SW06 Experiment, Maritime Rapid Environmental Assessment Conference, Lerici, Italy, October, 2010.

**HONORS/AWARDS/PRIZES**

Best Presenter Award, Maritime Rapid Environmental Assessment Conference, NATO Undersea Research Centre, October 2010.