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Development of Improved Algorithms and Multiscale Modeling Capability with SUNTANS

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LONG-TERM GOALS

The long-term goal is to develop a parallel ocean simulation tool that is capable of simulating processes on a wide range of scales by coupling two vastly different codes, namely the Regional Ocean Modeling System (ROMS, Shchepetkin and McWilliams (2005)), and the Stanford Unstructured Nonhydrostatic Terrain-following Adaptive Navier-Stokes Simulator (SUNTANS, Fringer *et al.* (2006)). The tool will adaptively nest SUNTANS, an unstructured-grid, coastal-scale code, into ROMS, a curvilinear grid, regional-scale code, in regions where the motions are small-scale and so nonhydrostatic. The nested tool will be applied to study highly nonlinear internal waves in the South China Sea in order to develop an improved understanding of mechanisms that govern their generation, propagation, and dissipation.

OBJECTIVES

The primary objective is to enhance the capabilities of the SUNTANS model through development of algorithms to study multiscale processes in estuaries and the coastal ocean. This involves development of 1) improved momentum and scalar advection on unstructured, staggered grids, 2) accurate and efficient algorithms for solution of the nonhydrostatic pressure, and 3) adaptive grid capabiliites with adaptive mesh refinement and model nesting.

APPROACH

This work focuses on the continued development of SUNTANS (Stanford Unstructured Nonhydrostatic Terrain-following Adaptive Navier-Stokes Simulator), a free-surface, nonhydrostatic, unstructured-grid, parallel coastal ocean and estuary simulation tool that solves the Navier-Stokes equations under the Boussinesq approximation (Fringer et al.,2006). The formulation is based on the method outlined by Casulli and Walters (2000), in which the free-surface and vertical diffusion are discretized with the theta method, which eliminates the Courant condition associated with fast free-surface waves and the elevated friction term associated with small vertical grid spacing at the free-surface and bottom boundary. For flows with extensive wetting and drying, advection of momentum is

accomplished with the semi-Lagrangian advection scheme (Wang et al. 2011a), which ensures stability in the presence of cells that fill and empty with the tides. Scalar advection is accomplished semi-implicitly and continuity of volume and mass are guaranteed for the hydrostatic solver, following Gross et al. (2002). The theta method for the free-surface yields a two-dimensional Poisson equation, and the nonhydrostatic pressure is governed by a three-dimensional Poisson equation. These are both solved with the preconditioned conjugate gradient algorithm with Jacobi and block-Jacobi preconditioning, respectively. Because the nonhydrostatic component of SUNTANS is essentially a correction to the hydrostatic component, SUNTANS can be run seamlessly in nonhydrostatic or hydrostatic modes. SUNTANS is written in the C programming language, and the message-passing interface (MPI) is employed for use in a distributed-memory parallel computing environment. SUNTANS employs the generalized length scale approach to Reynolds-averaged turbulence modeling (Wang et al. 2011b). The SUNTANS grid employs z-levels in the vertical and is unstructured in plan, which enables the resolution of complex coastlines and topographic features. Unstructured grids also enable the use of high grid resolution in regions of interest while coarsening the grid in regions where grid resolution is not required, thereby significantly reducing computational overhead.

WORK COMPLETED

We have developed a criterion to determine grid resolution requirements to resolve nonhydrostatic internal waves. This is important for developing strategies for adaptive mesh refinement for nonhydrostatic processes.

RESULTS

Vitousek and Fringer (2011) show that nonhydrostatic dispersion is the basic physical feature that differentiates hydrostatic from nonhydrostatic models. Dispersion is essential for the simulation of solitary-like waves because it balances nonlinear steepening, the process responsible for the wave speed increasing with wave height. Although hydrostatic models do not possess physical dispersion, most ocean models are second-order accurate in space, and so they possess numerical dispersion which can mimic the physical effects of nonhydrostatic dispersion. Vitousek and Fringer have demonstrated that, contrary to common belief, it is not the advection terms but rather the pressure gradient terms that are the dominant cause of numerical dispersion in internal wave simulations. In nonhydrostatic models, accurate simulation of solitary waves requires that the grid resolution be fine enough to ensure that the numerical dispersion is much smaller than the physical nonhydrostatic dispersion.

Vitousek and Fringer (2011) show for accurate simulation that this leads to the constraint $\lambda = \Delta x/h_1 < 1$, where λ is the grid lepticity (or "thinness", from Scotti and Mitran 2009), Δx is the horizontal grid spacing, and h_1 is the depth of the wave guide. Figures 1 and 2 compare the result of an evolving solitary-like wave when computed with SUNTANS, in both hydrostatic and nonhydrostatic modes. All conditions are the same except for λ. As shown in Figure 1, for large values of λ, nonhydrostatic effects are not resolved and the hydrostatic and nonhydrostatic results are essentially identical. Figure 1 also shows how the hydrostatic model produces solitary waves, but this is purely a numerical effect since the nonlinear steepening is in balance with the *numerical* dispersion. As shown in Figure 2, for small values of λ, nonhydrostatic effects are now resolved and there are significant differences between the hydrostatic and nonhydrostatic model results with a well defined and realistic wave train evolving in the nonhydrostatic case.

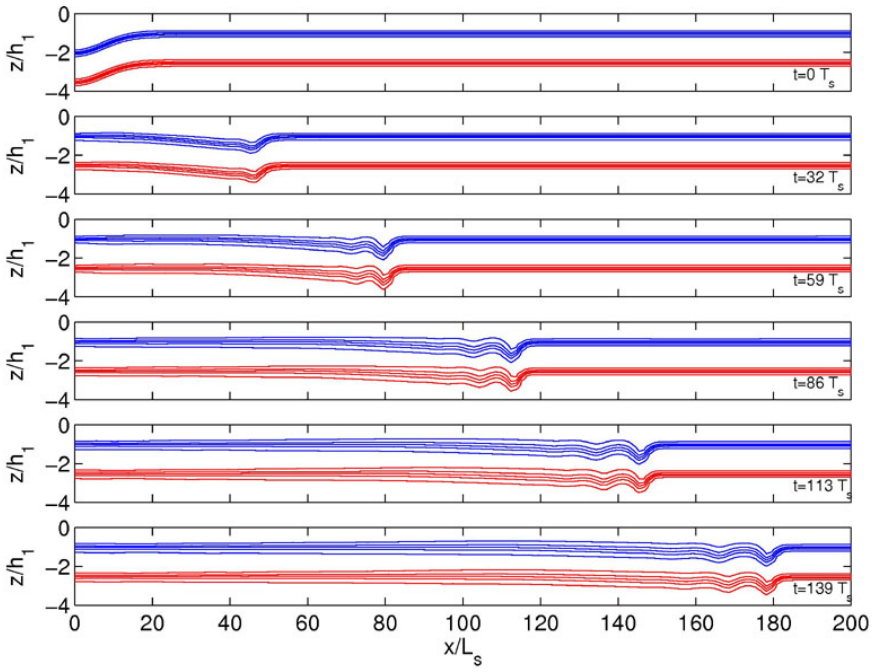

Figure 1: Comparison of the evolution of a solitary-like wave using hydrostatic SUNTANS (red) and nonhydrostatic SUNTANS (blue) with $\lambda = \Delta x/h_1 = 8$ (from Vitousek and Fringer 2011).

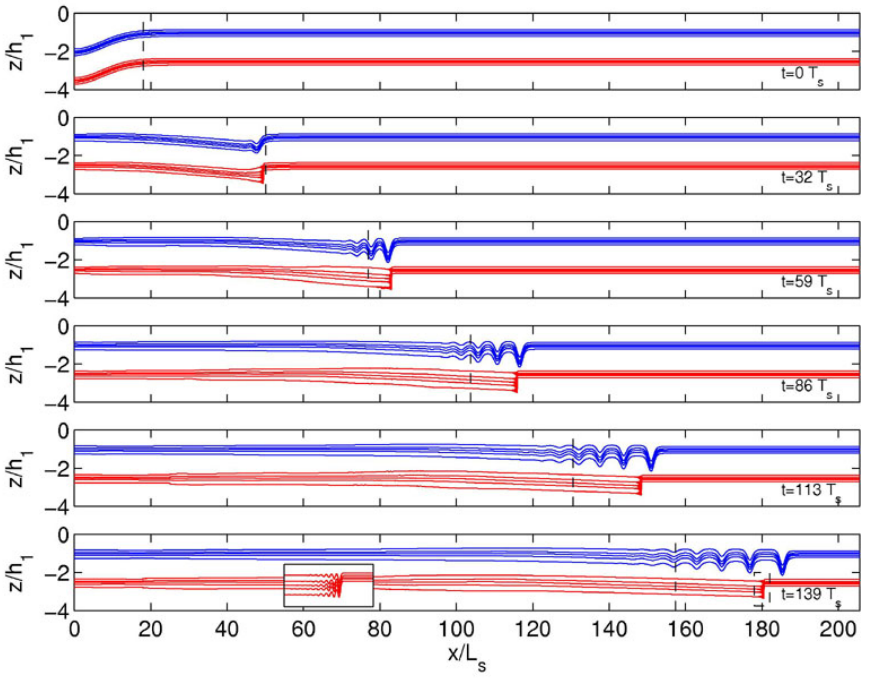

Figure 2: Comparison of the evolution of a solitary-like wave using hydrostatic SUNTANS (red) with nonhydrostatic SUNTANS (blue) with $\lambda = \Delta x/h_1 = 0.25$ (from Vitousek and Fringer 2011).

The criterion of Vitousek and Fringer (2011) has important ramifications for nonhydrostatic modeling of internal waves. Consider, for example, numerical simulation of nonhydrostatic waves in a coastal shelf domain. Since the typical mixed-layer depth is O(10 m), then the horizontal grid resolution must be less than 10 m, implying that billions of grid cells are needed to simulate internal waves in a 100 km coastal shelf domain. With modern computing resources, simulations with 100 million grid cells require 100s of processors running for weeks at a time to simulate a month of real time. A coarser grid would not sufficiently resolve the physical nonhydrostatic dispersion and the resulting waves would be numerical solitons. Depending on the resolution, computation of the nonhydrostatic pressure may even degrade the results because the physical nonhydrostatic dispersion would be in addition to the (incorrect) numerical dispersion. As an example, Figure 3 compares internal waves in the South China Sea (Zhang et al. 2011) computed with and without the nonhydrostatic pressure. These simulations employ a horizontal grid resolution of 1400 m. With a mixed-layer depth of 200 m, this implies a lepticity of λ=(1400 m)/(200 m)=7. Therefore, the numerical dispersion overwhelms the physical dispersion by a factor of 3.6 (following Vitousek and Fringer 2011), and so the nonhydrostatic result in Figure 3 is overly dispersive because it includes the effects of both physical and numerical dispersion.

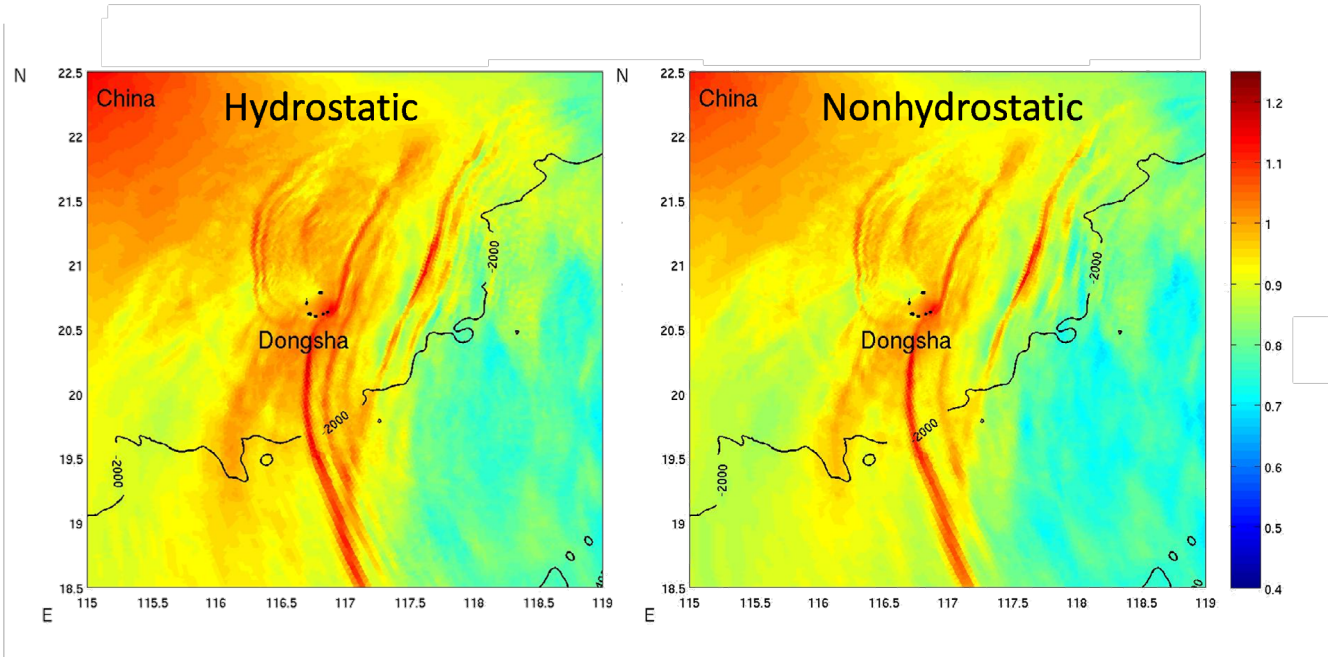

Figure 3: Free-surface displacement (in m above mean sea level) and the associated internal wave signatures at 12:30am on 25 June 2005 around Dongsha Atoll as computed by the hydrostatic (left) and nonhydrostatic (right) versions of SUNTANS (from Zhang et al. 2011).

The grid-resolution criterion of Vitousek and Fringer (2011) imposes extensive computational requirements on numerical models of internal waves in coastal domains. The computational overhead is large not only because of the number of grid points required, but also because of the overhead arising from the three-dimensional elliptic equation associated with the nonhydrostatic pressure. Fortunately, however, as shown in Figure 4, computational overhead associated with the nonhydrostatic pressure solver is only high when the grid leptic ratio is low. This behavior of the computational overhead arises with use of suitable preconditioners for the nonhydrostatic pressure solver (Fringer et al. 2006) that eliminate computational overhead when nonhydrostatic processes are not being resolved, or when

λ>1. Therefore, unstructured-grid models like SUNTANS can be designed to employ low-lepticity grids only where physical nonhydrostatic dispersion is expected, such as on continental shelves. The computational overhead associated with the nonhydrostatic pressure can thus be incurred only where it is necessary.

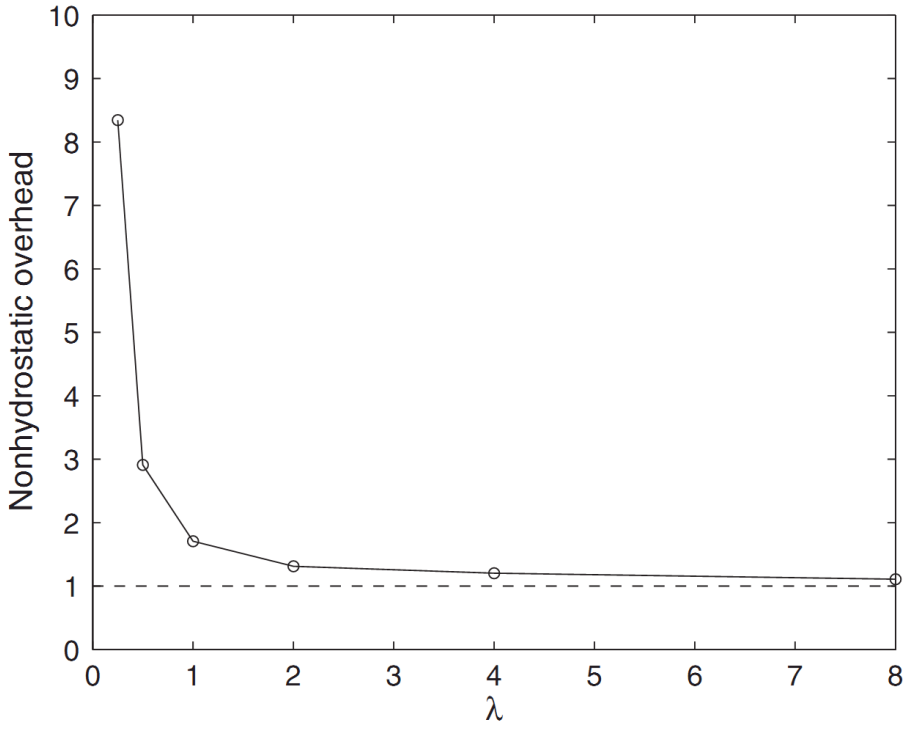

Figure 4: Relative workload associated with computing the nonhydrostatic pressure as a function of the grid lepticity $\lambda = \Delta x/h_1$. The overhead is computed as the ratio of the total wallclock time for the nonhydrostatic simulation to that of the hydrostatic simulation to obtain the results shown in Figures 1 and 2 (from Vitousek and Fringer 2011).

IMPACT/APPLICATIONS

High-resolution simulations using nonhydrostatic models like SUNTANS are crucial for understanding multiscale processes that are unresolved, and hence parameterized, in larger-scale ocean models.

REFERENCES

Casulli, V. and R.A. Walters, 2000, An unstructured, three-dimensional model based on the shallow water equations, Int. J. Numer. Meth. Fluids., 32, 331-348.

Fringer, O. B., Gerritsen, M., and R. L. Street, 2006, An unstructured-grid, finite-volume, nonhydrostatic, parallel coastal ocean simulator, Ocean Modelling, 14 (3-4), 139-278.

Gross, E. S., Bonaventura, L., and R. Giorgio, 2002, Consistency with continuity in conservative advection schemes for free-surface models, 38 (4), 307-328.

Scotti, A., and S. Mitran, 2008, An approximated method for the solution of elliptic problems in thin domains: Application to nonlinear internal waves, Ocean Modelling, 25, 144-153.

Shchepetkin, A., and J. C. McWilliams, 2005, The Regional Oceanic Modeling System: A split-explicit, free-surface, topography-following-coordinate ocean model. Ocean Modell., 9, 347–404.

Vitousek, S., and O. B. Fringer, 2011, Physical vs. numerical dispersion in nonhydrostatic ocean modeling, Ocean Modelling, 40 (1), 72-86.

Wang, B., Zhao, G., and O. B. Fringer, 2011a, Reconstruction of vector fields for semi-Lagrangian advection on unstructured, staggered grids, Ocean Modelling, 40 (1), 52-71.

Wang, B., Giddings, S. N., Fringer O. B., Gross, E. S., Fong, D. A., and S. G. Monismith, 2011b, Modeling and understanding turbulent mixing in a macrotidal salt wedge estuary, J. Geophys. Res., 116, C02036.

Zhang, Z., Fringer, O. B., and S. R. Ramp, 2011, Three-dimensional, nonhydrostatic numerical simulation of nonlinear internal wave generation and propagation in the South China Sea, J. Geophys. Res., 116, C05022.

PUBLICATIONS

Fringer, O. B., 2011, Grid resolution requirements and computational overhead in nonhydrostatic coastal ocean modeling, presentation at minisymposium, "Recent advances in coastal ocean modeling", SIAM Conference on Mathematical & Computational Issues in the Geosciences (invited), Long Beach, CA, USA.

Fringer, O. B., and S. Vitousek, 2011, Physical vs numerical dispersion in nonhydrostatic internal wave modeling, Conference on Geophysical and Astrophysical Internal Waves, Les Houches, France.

Gil, G. T. C., and O. B. Fringer, 2010, On the potential for transport via internal tides in the coastal ocean, 63rd Annual Meeting of the APS Division of Fluid Dynamics, 55 (16), Long Beach, CA, USA.

Vitousek, S., and O. B. Fringer, 2011, Physical vs. numerical dispersion in nonhydrostatic ocean modeling, Ocean Modelling, 40 (1), 72-86.