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State-Space Analysis Of Model Error: A Probabilistic Parameter Estimation Framework With Spatial Analysis Of Variance

PI: Joshua P. Hacker
Department of Meteorology
Naval Postgraduate School
589 Dyer Road, Root Hall Rm 254
Monterey, CA 93943
phone: (831) 656-2722 fax: (831) 656-3061 email: jphacker@nps.edu

Co-PI: Cari G. Kaufman
Department of Statistics
University of California, Berkeley
367 Evans Hall
Berkeley, CA 94720
phone: (510) 643-0915 fax: (510) 642-7892 email: cgk@stat.berkeley.edu

Co-I: James Hansen
Naval Research Laboratory
Bldg 702, Rm 204
7 Grace Hopper Ave.
Monterey, CA 93943
phone: (831) 656-4741 fax: (831) 656-4769 email: jim.hansen@nrlmry.navy.mil

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LONG-TERM GOALS

An over-arching goal in prediction science is to objectively improve numerical models of nature. Meeting that goal requires objective quantification of deficiencies in our models. The structural differences between a numerical model and a true system are difficult to ascertain in the presence of multiple sources of error. Numerical weather prediction (NWP) is subject to temporally and spatially varying error, resulting from both imperfect atmospheric models and the chaotic growth of initial-condition (IC) error. The aim of our work is to provide a method that begins to systematically disentangle the model inadequacy signal from the initial condition error signal.

OBJECTIVES

We are engaging a comprehensive effort that uses state-of-the-science estimation methods in data assimilation (DA) and statistical modeling, including: (1) the characterization of existing model-to-model differences via novel spatial Analysis of Variance (ANOVA) methods; (2) the development of a flexible representation for the various spatial and temporal scales of model error; (3) the estimation of parameters in representing those scales using a probabilistic approach to DA, namely the Ensemble Kalman Filter; and (4) the determination of whether incorporation of estimated error structure in

improves short-term forecasts, again using spatial ANOVA methods, this time within a formal testing framework. Research focus is on near-surface winds over both the ocean and land. The method under development are sufficiently general and can apply to a wide range of battlespace environments.

APPROACH

The technical approach includes numerical weather prediction and state estimation efforts at NPS, and statistical modeling efforts at University of California Berkeley (UCB) under sub-contract. At NPS PI Hacker is implementing the Navy's Operational Global Atmospheric Prediction (NOGAPS), and two limited-area mesoscale models: the Navy's Coupled Ocean-Atmosphere Mesoscale Prediction System (COAMPS) model, and the open-source Weather Research and Forecast (WRF) model, within a state-of-the-science ensemble Data Assimilation Research Testbed (DART). The NOGAPS-DART provides global ensemble prediction capability that can be consistently applied to the COAMPS and WRF as lateral boundary conditions. Scientific objectives will be me by systematically choosing the WRF or COAMPS as the "truth," which can provide observations for assimilating into the other model. Under this approach, spatio-temporal distributions of uncertainty (error in this context) are available for analysis with special attention to second-order moments. Eventually, we will use the same framework to objectively estimate parameters in statistical models, of NWP model error, developed at UCB. Hypotheses will be formed and formally tested. This work is benefitting from collaboration Co-I James Hansen, Justin McLay, and others NRL staff. A budgeted post-doc has not been hired due to delays in funding distributions, and partial increments.

UCB PI Cari Kaufman is working to advance the statistical methods needed to provide an objective space-time characterization of the error distributions. We approach the characterization of the uncertainty via fitting a hierarchical Bayesian model that captures the important features and variability in the data. The implied distribution from the model will be a valid stochastic spatial process under probability theory. Ideally, fitting the statistical model to different datasets should allow us to capture the significant differences between the different underlying data generating distributions. Moreover, a realistic statistical model can also simulate realistic wind fields quickly which can be beneficial for studying other processes that require surface winds as an input. Graduate student Wayne Lee (unfunded) is contributing substantially to this work. Postdoc Benjamin Shaby began work in September 2011.

WORK COMPLETED

Work in FY2011 was toward various components of tasks 1 and 2. At NPS we focused on the technical implementations of the tools required to complete the research. This included implementation of NOGAPS, COAMPS, and WRF with DART. NOGAPS-DART and WRF-DART is completed. COAMPS-DART has been implemented at NRL but not yet at NPS. NOGAPS-DART was tested for the period of Oct 2009 and results compared against the operational global data assimilation system used at FNMOC to ensure quality. In the WRF-DART implementation, simple model error terms have been added within the WRF code in preparation for estimating the coefficients of the model-error process estimated as described below.

At UCB focus was on addressing challenges associated with applying heirarchical Bayesian techniques to large, multivariate, and non-stationary datasets typical of NWP. Progress is documented in the following results section.

RESULTS

Primary results are the development of a viable statistical model for spatial variability of near-surface surface winds. This represents an important step toward a model capable of characterizing the comlexity of model errors in those winds. Here we describe how we successfully addressed three primary challenges associated with this model. Results represent a significant step toward objectively characterizing the time-space structures of model inadequacy.

Results are focused on the surface-wind forecasts from the WRF. A dataset was prepared at NPS from existing ensemble prediction runs (no ensemble-filter data assimilation) to minimize delays on UCB progress. In this case, the WRF lateral boundary conditions were provided by the Global Forecast System (GFS) operational model from the National Centers for Environmental Prediction. Although we anticipate the primary research will be completed over the Korean Peninsula and surrounding seas, this example data set is over CONUS. An example wind field is shown in Fig. 1.

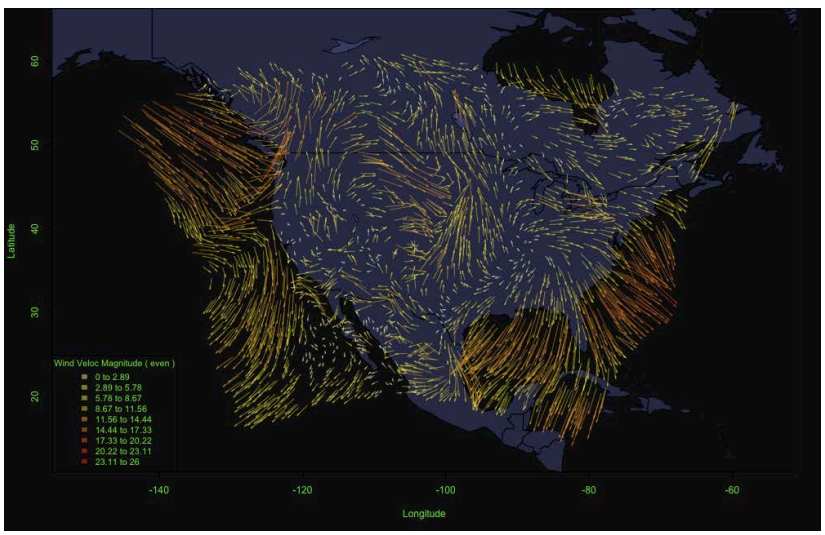

Figure 1: Example surface wind field. Data are thinned to 20% of the original resolution.

We approach the characterization of the uncertainty via fitting a hierarchical Bayesian model that captures the important features and variability in the data. The implied distribution from the model will be a valid stochastic spatial process under probability theory. Ideally, fitting the statistical model to different datasets should allow us to capture the significant differences between the different underlying data generating distributions. Moreover, a realistic statistical model can also simulate realistic wind fields quickly which can be beneficial for studying other processes that require surface winds as an input.

We face three statistical challenges with surface wind fields: (1) with roughly 12,000 spatial locations over North America over two months observed every 2 days (32 forecasts), it is a large data set and poses a computational problem for classical geostatistical methods, (2) winds are represented as a sum of 3 components U , V , and W, introducing a multivariate spatial process problem (currently we ignore vertical velocity W in our study since its magnitude and variability is negligible); (3) surface wind fields exhibit great nonstationarity and some discontinuity depending on the underlying topography.

Classical stationary methods tend to model the first-order trends well, but fail to capture the second order uncertainty which is the primary quantity of interest.

We first tackle the multivariate issue by leveraging the geostrophic wind relationship linking the pressure gradient to U and V under frictionless environments.

$$fV = \frac{1}{\rho}\frac{\partial p}{\partial x}; \qquad fU = -\frac{1}{\rho}\frac{\partial p}{\partial y},$$

where f is the Coriolis parameter, ρ is air density, x is the longitude, y is latitude, and p is pressure. This basic relationship is a linear relationship between pressure gradient and wind speeds that can simplify the joint model between U and V, so that it can be explained by a single stationary pressure field. After some exploration, we decided on the form:

$$U(s) = \beta_{u,0}(s) + \beta_{u,x}(s)\frac{dp}{dx}(s) + \beta_{u,y}\frac{dp}{dy}(s) + \varepsilon_u(s)$$
$$V(s) = \beta_{v,0}(s) + \beta_{v,x}(s)\frac{dp}{dx}(s) + \beta_{v,y}\frac{dp}{dy}(s) + \varepsilon_v(s)$$

where coefficients β are functions of location s, and ε are residuals. Royle et al. (1999) modeled ocean wind fields as above with spatially constant coefficents β. However, the interpretation of β based on the geostrophic relationship suggests that β should vary spatially by latitude and the local ratio of moist *vs*. dry air instead of treated as a fixed global quantity. The varying coefficient model was advocated by Gelfand et al. (2003) to efficiently model first order nonstationary features for the β. To quickly and naively evaluate this model, we run a linear regression using the 32 days for each location ignoring all spatial dependency and examine the estimated β and R-squared to evaluate the fit (R-squared = 1 is a perfect fit). A great majority of the R-squared values are above 0.8 which means more than 80% of the variability in the data can be explained by the simple geostrophic relationship without borrowing any information from neighboring locations. Figure 2 shows estimates of the first-order coefficients. It is clear that the underlying topography, which is not smooth, dominates the estimates. We can expect this result, and more generally that the departures from geostrophy are strongly modulated by lower-boundary forcing on the atmosphere.

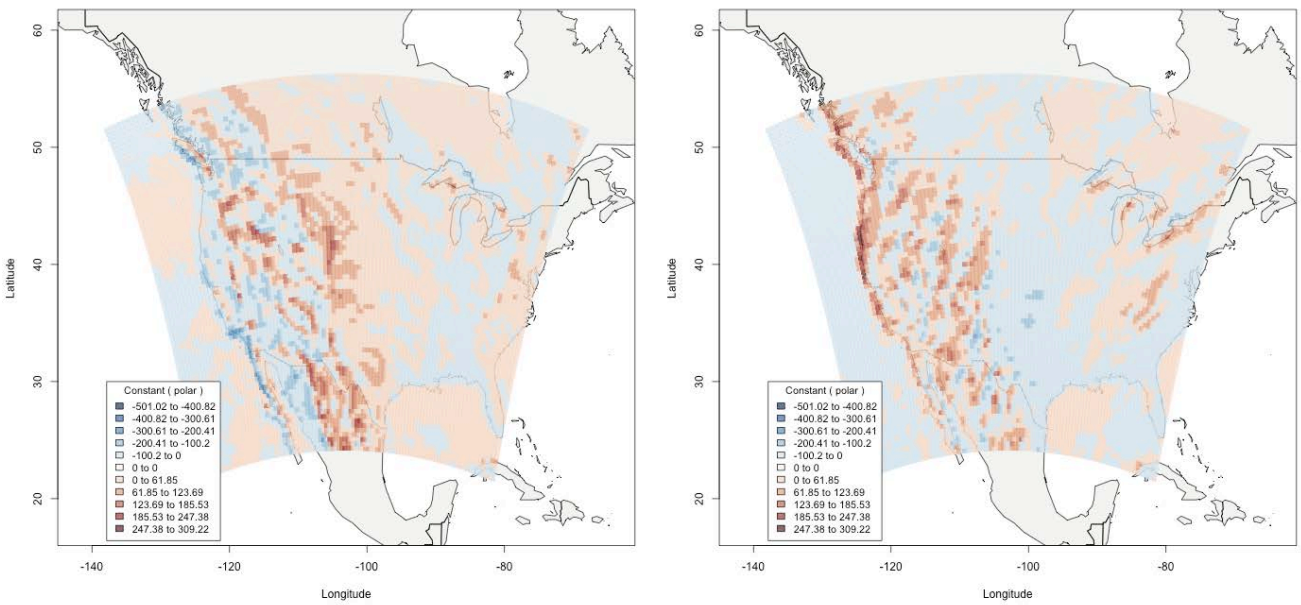

Figure 2: Estimates for first-order constants (left) $\beta_{u,0}$ and (right) $\beta_{v,0}$.

After the first-order nonstationarity is handled, we look at the second-order nonstationarity in the distribution of the β fields. To handle it, we propose to decompose each β_i term above into a linear combination of three independent stationary spatial processes:

$$\beta_i(s) = \beta_{i,0}(s) + [Land(s)][\beta_{i,1}(s)] + [Elev(s)][\beta_{i,2}(s)],$$

where *Land(s)* is an indicator and *Elev(s)* is the elevation for location *s*. Using matrix notation where $Land_{ii} = Land(s_i)$ and $Elev_{ii} = Elev(s_i)$,

$$Cov[\beta_i(s)] = \Sigma_{\beta_{i,0}} + [Land]\,\Sigma_{\beta_{i,1}}\,[Land]^T + [Elev]\,\Sigma_{\beta_{i,2}}\,[Elev]^T.$$

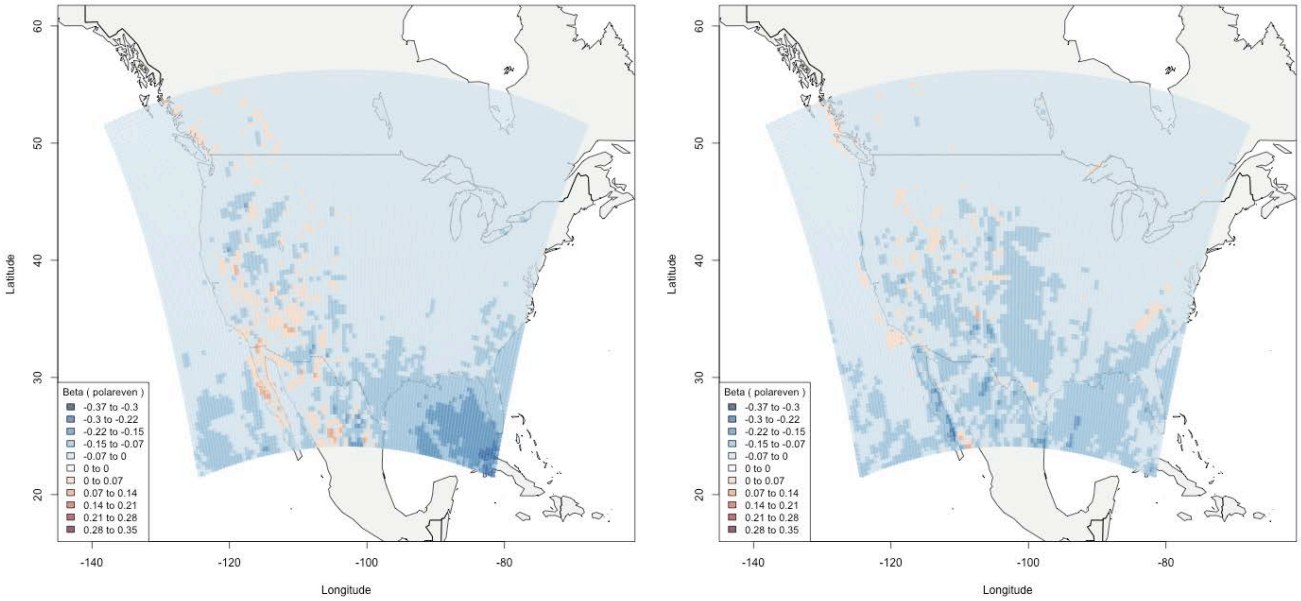

Figure 3: Estimates for second-order coefficients (left) $\beta_{u,x}$ and (right) $\beta_{v,y}$.

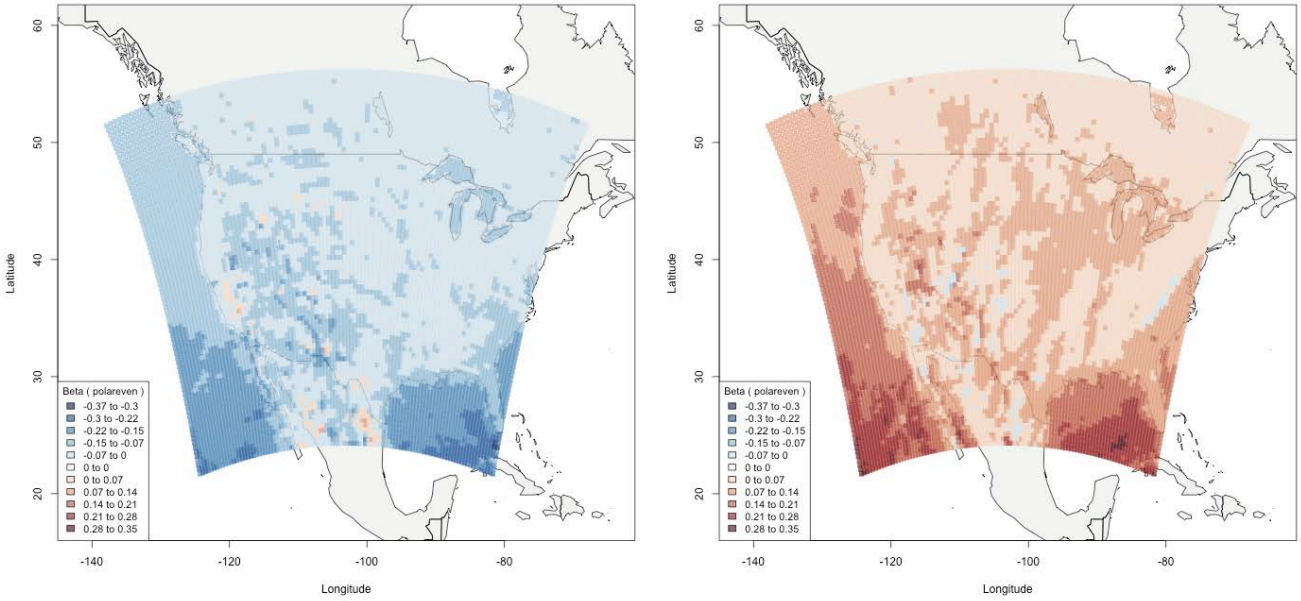

Figure 4: Estimates for second-order cross-coefficients (left) $\beta_{u,y}$ and (right) $\beta_{v,x}$.

This naturally creates a valid covariance that reflects the different topography, and the decomposition can be extended to other surface properties affecting the winds. We tested multiple simulations to ensure that the different processes can be identified with sufficient data, approximating the posterior distribution for each simulated dataset using a Gibbs sampler. To summarize the above model, we present a schematic of the Bayesian varying coefficient hierarchical model in Fig.4. Here *i* indexes the terms in the varying coefficient model: intercept and pressure gradient in the N-S and E-W directions, and *j* indexes the three components used in constructing the nonstationary model for the coefficients, corresponding to an intercept, land, and elevation components.

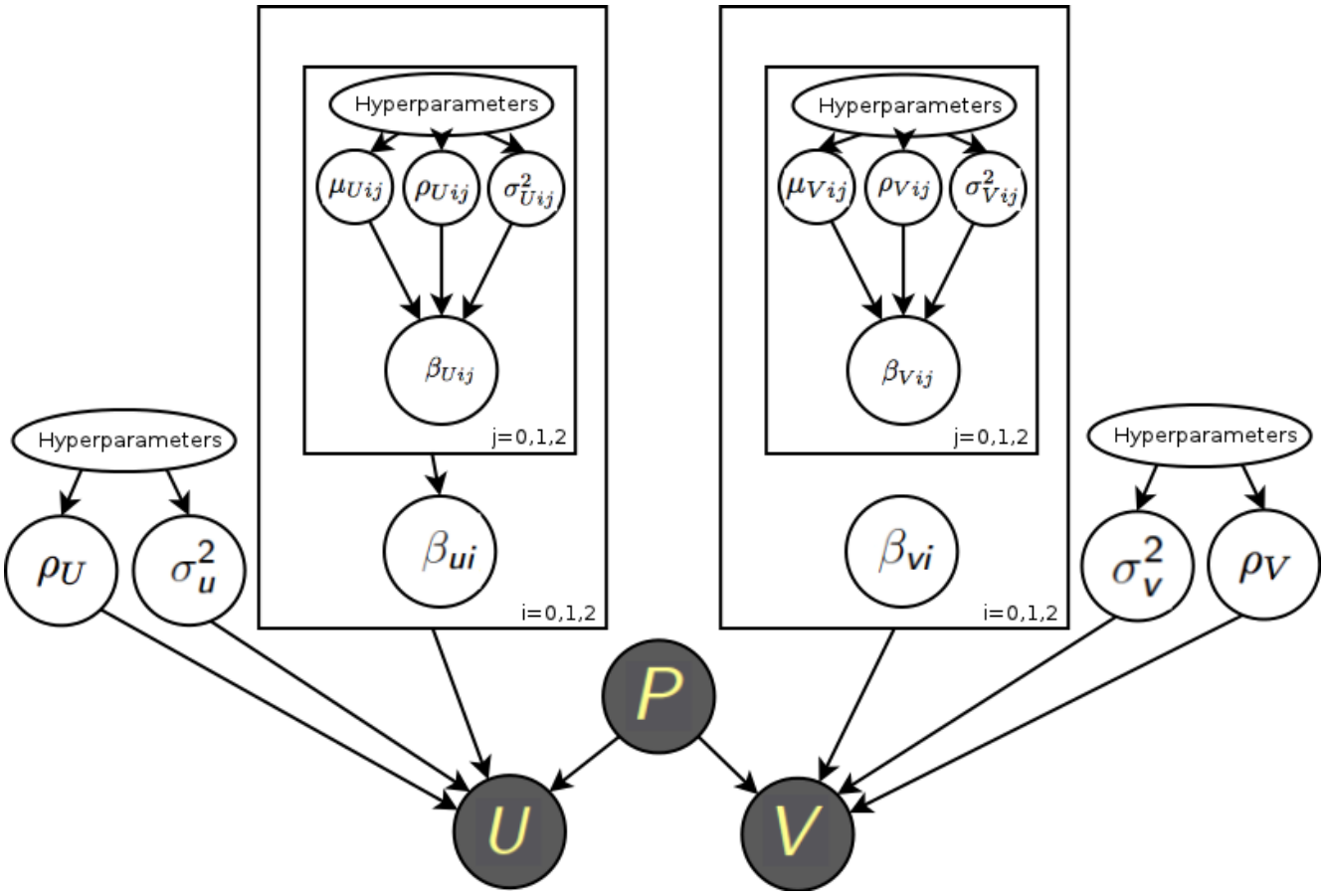

Figure 5: Schematic of hierarchical Bayesian model. Hyperparameters are a priori independent.

The remaining challenge is the computational burden of the hierarchical model. Lindgren et al. (2011) found the Gaussian Markov Random Field (GMRF) precision representation for a variety of Matérn covariance functions, a popular covariance family among the spatial statisticians for certain theoretical properties. GMRF methods are known for their extraordinary computational efficiency due to its sparse precision structure but suffer from the inability to easily model long range dependencies (Wall 2004) and its requirement on specific boundary conditions. We commented on the published method's bias due to different boundary conditions and its implications for parameter estimation (Lindgren et al. 2011). Their approximation method provides estimates that consistently underestimate the parameters that are consistently estimable under classical methods. The authors responded with the reassurance that the bias may be shrunk if the finite element mesh density was increased at the cost of higher computational requirements (Lee and Kaufman, 2011; Lindgren et al. 2011).

For future directions, we are exploring the optimal MCMC algorithm to run this hierarchical model. Traditional Gibb sampling produces parameters with high autocorrelation from the MCMC chain. We are currently exploring slice sampling and potentially hybrid MCMC. Once this runs, then we will run a computer experiment to see if our stochastic model can detect the changes in the dynamic equations within the WRF. In other words, we will introduce a synthetic NWP model inadequacy and see if fitting the bayesian model above before and after the model discrepancy is introduced will produce detectable and informative differences.

Upon successfully modeling the differences between NWP models, we will try to estimate he descrepancy via the parameter estimation framework under development at NPS. In preparation, we have experimented with a simple 960-variable model with dynamics analogous to atmospheric waves to determine the potential to estimate parameters with weak correlations to the model state.

Estimation of satellite radiance bias parameters is a particular problem displaying weak correlations with the model state. It is typically performed sub-optimally in variational estimation systems; ensemble estimation systems offer a more optimal solution but have only been implemented assuming no correlation between state and bias. This is a limiting assumption, and we proved that the estimation procedure itself produces correlations which should not be ignored. Further, we demonstrated that ensemble filters are cabable of estimating the bias parameters without invoking that assumption.

IMPACT/APPLICATIONS

The bulk of DoN day-to-day operations rely on accurate predictions of winds, seas, ceiling, and visibility. The focus of the proposed work is to identify inadequacies associated with the modeled atmospheric boundary layer. Any discoveries that enable the improvement of boundary layer modeling will ultimately have a positive impact on Navy warfighters.

The proposed methods have the potential to enable essential improvement in modeling capability. Instead of tuning models based on intuition, we are forming a foundation for objective identification of model errors. Those errors could immediately be accounted for in probablistic forecast systems, and also be subject to physical interpretation by subject experts.

RELATED PROJECTS

The MATERHORN project (http://www.nd.edu/~dynamics/materhorn/index.html), funded by ONR, seeks to improve atmospheric predictability over complex terrain. It is similarly focused on predictions in the atmospheric boundary layer. Rather than a focus on model inadequacy, MATERHORN focuses on field programs aimed at improving models via direct comparison to observations, and quantifying optimal observing strategies for improving predictions. PI Hacker is using some of the technical developments here to aid that effort, and vice versa.

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PUBLICATIONS

Lee, W. and Kaufman, C. (2011), Comment on "An Explicit Link between Gaussian Fields and Gaussian Markov Random Fields: The SPDE Approach" by Lindgren, F., Rue, H., and Lindstrom, J., Journal of the Royal Statistical Society, Series B, 73: 479–480.