

## **Report: Physics Constrained Stochastic Statistical Models for Extended Range Environmental Prediction**

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### ***I. Findings: Scientific Results***

The PI, Andrew Majda, and Co-PI's, Sam Stechmann (University of Wisconsin), John Harlim (North Carolina State University), Duane Waliser (UCLA/Cal Tech) just received the MURI funding in early August 2012 with a one-day MURI kickoff meeting July 13, 2012 so this report is necessarily brief. Nevertheless, there are some exciting initial MURI developments to report on two MURI topics.

#### **A) Limits of Predictability in the North Pacific Sector of a Comprehensive Climate Model**

Majda and new Courant Institute Assistant Professor and MURI collaborator, Dimitri Giannakis, have just submitted a paper to GRL (1). We study limits of interannual to decadal predictability of sea surface temperature (SST) in the North Pacific sector of the Community Climate System Model version 3 (CCSM3). Using a set of low-frequency and intermittent spatiotemporal SST modes acquired through nonlinear Laplacian spectral analysis (a nonlinear data manifold generalization of singular spectrum analysis), we build a hierarchy of regression models with external factors to determine which modes govern the dynamic evolution and predictability of prominent large-scale patterns, namely the Pacific Decadal Oscillation (PDO) and North Pacific Gyre Oscillation (NPGO). Retaining key triple correlations between prognostic variables and external factors, as well as the seasonality of the data, we find that the PDO and NPGO modes of CCSM3 can be described with remarkably high fidelity as an outcome of forcing by the intermittent modes (with phase demodulation by the seasonal cycle) and cubic interactions between the low-frequency modes. Our results differ from the classical picture of ENSO-driven autoregressive models for North Pacific SST variability, providing evidence that intermittent processes, such as variability of the Kuroshio current, limit long-range predictability in this climate model.

This use Nonlinear Laplacian Spectral Analysis (NLSA) for a comprehensive climate model opens the way for further applications of these techniques to reveal significant intermittent behavior which limits predictability. Majda and Giannakis intend to work with Wen-Wen Tung of Purdue University, a MURI collaborator on applications of NLSA to CLAUSS data for tropics and with various CAOS PhD

students at Courant Institute for midlatitude and tropical/extra tropical data. This is preliminary to using the above approach for limits of predictability for these problems.

## B) Physics Constrained Nonlinear Regression Models for Time Series

Majda and co-PI John Harlim have submitted a paper to *Nonlinearity* on this important topic (2). A central issue in contemporary science is the development of data driven statistical nonlinear dynamical models for time series of partial observations of nature or a complex physical model. It has been established recently that ad-hoc quadratic multi-level regression models can have finite-time blow up of statistical solutions and/or pathological behavior of their invariant measure. Here a new class of physics constrained multi-level quadratic regression models are introduced, analyzed, and applied to build reduced stochastic models from data of nonlinear systems. These models have the advantages of incorporating memory effects in time as well as the nonlinear noise from energy conserving nonlinear interactions. The mathematical guidelines for the performance and behavior of these physics constrained multi-level regression models as well as filtering algorithms for their implementation are developed here. Data driven applications of these new multi-level nonlinear regression models are developed for test models involving a nonlinear oscillator with memory effects and the difficult test case of the truncated Burgers-Hopf (TBH) model. These new physics constrained quadratic multi-level regression models are proposed here as process models for Bayesian estimation through Markov Chain Monte Carlo algorithms of low frequency behavior in complex physical data.

As more practical applications of this technique are developed by the MURI team, we plan to keep developing the theory as needed.

## *II. Contributions to Discipline*

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### ***III. Contributions to Outside Disciplines***

Majda and new Courant Institute Assistant Professor and MURI collaborator, Dimitri Giannakis, have just submitted a paper to *GRL* (1). We study limits of interannual to decadal predictability of sea surface temperature (SST) in the North Pacific sector of the Community Climate System Model version 3 (CCSM3). Using a set of low-frequency and intermittent spatiotemporal SST modes acquired through nonlinear Laplacian spectral analysis (a nonlinear data manifold generalization of singular spectrum analysis), we build a hierarchy of regression models with external factors to determine which modes govern the dynamic evolution and predictability of prominent large-scale patterns, namely the Pacific Decadal Oscillation (PDO) and North Pacific Gyre Oscillation (NPGO). Retaining key triple correlations between prognostic variables and external factors, as well as the seasonality of the data, we find that the PDO and NPGO modes of CCSM3 can be described with remarkably high fidelity as an outcome of forcing by the intermittent modes (with phase demodulation by the seasonal cycle) and cubic interactions between the low-frequency modes. Our results differ from the classical picture of ENSO-driven autoregressive models for North Pacific SST variability, providing evidence that intermittent processes, such as variability of the Kuroshio current, limit long-range predictability in this climate model.

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### **PUBLICATIONS**

1. (A. Majda, D. Giannakis) "Limits of predictability in the North Pacific sector of a comprehensive climate model", submitted, *GRL*, July 12, 2012
2. (A. Majda, J. Harlim) "Physics Constrained Nonlinear Regression Models for Time Series", submitted, *Nonlinearity*, February 23, 2012