# Multiscale Models of Melting Arctic Sea Ice

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## LONG-TERM GOALS

Sea ice reflectance or albedo, a key parameter in climate modeling, is primarily determined by melt pond and ice floe configurations. Ice-albedo feedback has played a major role in the recent declines of the summer Arctic sea ice pack. However, understanding the evolution of melt ponds and sea ice albedo remains a significant challenge to improving climate models. Our research is focused on obtaining extensive imagery of melt pond evolution, and developing mathematical models of the melting process that can help us better understand the role of sea ice in the climate system, and represent sea ice more rigorously in climate models.

## OBJECTIVES

Viewed from high above, the melting sea ice surface can be thought of as a two phase composite of ice and melt water. The boundaries between the two phases evolve with increasing complexity and a rapid onset of large scale connectivity, or percolation of the melt phase. We plan to document this phenomenon with photographic imagery and to develop percolation and other models to quantitatively describe the evolution.

## APPROACH

### Key Particpants:

Ken Golden (Professor of Mathematics, U. of Utah)

Don Perovich (Research Geophysicist, ERDC-CRREL)

Tolga Tasdizen (Associate Professor, Department of Electrical and Computer Engineering, Scientific Computing and Imaging Institute, U. of Utah)

Court Strong (Assistant Professor, Department of Atmospheric Sciences, U. of Utah)

Chris Polashenski (Research Geophysicist, ERDC-CRREL)

Ivan Sudakov (Lorenz Postdoc in Mathematics of Climate, U. of Utah)

Bacim Alali (former Lorenz Postdoc, now a postdoc in Scientific Computing with at Florida State)

Ben Murphy (post-doc at U. of Utah, finished Ph.D. in 2012, now starting a post-doc at UC Irvine)

Brady Bowen (sophomore in Mathematics and Physics, U.of Utah)

Boya Song (sophomore in Computer Science, U. of Utah)

Rajath Thekkedath (junior in Applied Math and Mechanical Engineering, U.of Utah)

Rebecca Nickerson (West High School student, Salt Lake City, currently beginning college at Yale)

Sarah Silcox (West High School student, Salt Lake City)


We describe here various facets of our approach to characterizing and modeling Arctic melt pond evolution. Much of what we report is on "work in progress," since we are still only partially through the first year of a three year grant.

**Acquisition of melt pond imagery** (Don Perovich and Chris Polashenski): We are developing a library of melt pond imagery that will be analyzed. This library includes aerial photographs giving complete time series of melt pond evolution during the 1997-1998 Surface Heat Budget of the Arctic Ocean field campaign plus a trans-Arctic set of photographs from August-September 2005 showing spatial variability of ponds. In addition, we are archiving satellite images from Quikbird and National Technical Means satellites.

**Mapping melt pond images onto discrete networks** (Tolga Tasdizen, Boya Song, Ken Golden): Given a photograph of a complex melt pond, we are developing methods for mapping the melt pond onto a resistor network so that we can study the evolution of connectivity and look for a percolation threshold.

**Modeling projects**
We are developing mathematical characterizations of melt pond evolution using models of composite materials and statistical physics. In earlier work done just prior to this grant, we discovered that, as melt ponds grow, coalesce, and become more complex, they exhibit a transition in fractal dimension from about 1 to 2, near a critical length scale of 100 square meters. We have been pursuing a number of avenues to model the evolution of melt pond configurations and better understand the transition in complexity that real ponds display.

**Continuum percolation model** (Brady Bowen and Ken Golden): We have developed a continuum percolation model of melt pond evolution. The sets created by the intersection of a plane (water level) with a surface generated by a Fourier series (representing the snow and ice topography) look very similar to melt ponds, as the plane is moved up and down.

**Ising model** (Rajath Thekkedath, Bacim Alali, Court Strong, Ivan Sudakov, and Ken Golden): The Ising model has been perhaps the most widely studied model of a phase transition in statistical physics. The model describes a lattice of magnetic spins that interact with an applied magnetic field as well as with each other. We are working to adapt this model to describe melt pond evolution. The key magnetic states, spin up and spin down, are replaced by melt water and ice.

**Network model of horizontal fluid permeability** (Boya Song and Ken Golden): Drainage of melt ponds can occur not only through porous sea ice below the pond, but at a distance though seal holes, cracks, etc. Horizontal flow through fluid pathways facilitates this type of drainage. We have been working on creating resistor network models of horizontal fluid flow, and computing the horizontal permeability of melt pond configurations.

**Spectral Measures for Melt Pond Configurations** (Ben Murphy and Ken Golden): The analytic continuation method provides rigorous integral representations for the effective transport coefficients of composite media. The key mathematical object in an integral representation is a so-called "spectral measure" which depends only on the composite geometry. In the previous sub-project the importance of horizontal fluid transport is described, and computation of these spectral measures provides a general technique for computing horizontal flow properties, as well as other parameters of interest.

**Voter model and the fractal geometry of melt ponds** (Rebecca Nickerson and Ken Golden): Interacting particle systems have been used to model the evolution of a broad range of systems. We have developed a simple voter-type model to study how melt ponds form. The key step is to mimic the assumption in the voter model that a voter may be more likely to vote for a candidate if his or her neighbors have. For melt ponds, a square is more likely to melt if more of its neighbors have melted.

**Nonlinear PDE models for melt pond evolution** (Ivan Sudakov and Ken Golden): The transition from simple Euclidean shapes to chaotic structures in melt pond geometry can potentially be described by the Kuramoto–Sivashinsky equation (KSE). This equation is a nonlinear PDE that has been used as a model for complex spatio-temporal dynamics in extended systems driven far from equilibrium by intrinsic instabilities.

**Nonlinear PDE model for melt pond water depth** (Court Strong and Ken Golden): In order to provide a physically based mathematical framework to complement our statistical physics analyses of melt pond fractal dimension, we have been working with a partial differential equation governing melt pond water depth $h$ (Lüthje et al., 2006).

## WORK COMPLETED

**Image Acquisition.** We are building a library of aerial and satellite imagery. So far we have compiled a few thousand aerial photographs of ponded sea ice plus approximately 50 satellite images.

**Fractal dimension computation.** In discovering that there is a transition in the fractal dimension of melt ponds, we needed to come up with a way of computing the fractal dimension from area-perimeter data on melt ponds. We developed a deterministic algorithm to do this. However, a statistical approach that creates a "best fit" fractal dimension curve for the given data set eluded us. Court Strong found such a method just recently. It creates an optimal fit of a hyperbolic tangent model for the fractal dimension as a function of log $A$, where $A$ is the area of the melt ponds. This algorithm now makes possible the fractal analysis of melt pond configurations produced by mathematical models, as well as raw melt pond data that we will be obtaining in the future.

**Melt pond evolution models.** The pond configurations in the continuum percolation model are for a surface produced by a Fourier series with random amplitudes and phases. These configurations look quite similar to actual melt ponds. Similarly, the voter model produces configurations that look like

melt ponds, as does the Ising model. The challenge now is to better understand the basic physics of melt ponds and how the parameters of these models are related to melt pond characteristics.

**Network model of horizontal fluid permeability.** We have worked through an example where a complex melt pond is mapped onto a resistor network. This network contains not only connectivity information about the different components of the pond, but the widths of narrow channels through which melt water must flow, for example, to be drained through a nearby seal hole. The permeability calculation depends on computing the effective conductivity of the network. We have implemented an exact formula for the conductivity of a random graph – based on an enumeration of trees of the graph via the determinants of adjacency matrices - in order to compute the horizontal fluid permeability.

**Spectral Measures for Melt Pond Configurations.** We have computed examples of spectral measures for discretizations of actual melt pond images.

**Nonlinear PDE model for melt pond water depth.** We developed code to numerically integrate the melt pond water depth PDE on grids with various horizontal resolutions, and initialized the model with topography adapted from LiDAR observations (e.g., Polashenski et al., 2010). This framework developed melt ponds with realistic fractal properties, and we are investigating how these properties depend on the specification of the initial topography.

**Nonlinear PDE model for melt pond evolution.** Our findings on the critical length scale for a transition in fractal dimension indicate that if a critical radius for simple ponds is exceeded, then the shapes become more complex. The Kuramoto-Sivashinsky equation allows us to explore this in a theoretical model, in the sense that the melting front of the pond becomes unstable with respect to perturbations along the front beyond a critical radius.

## RESULTS

All three types of models that we have examined in some detail – continuum percolation, voter, and Ising - produce configurations that display a transition in the fractal dimension with length scale, around some critical value of pond area. Since the models we consider are so general, this finding indicates that the transitional behavior displayed by Arctic melt ponds as they grow and coalesce is a general mathematical feature exhibited by a range of statistical physics models. We are currently looking into this very interesting mathematical phenomenon.

Applying homogenization techniques such as resistor networks, percolation models, and analytic continuation to melt ponds represents a novel use of methods from materials science and condensed matter physics in polar climate science.

## IMPACT/APPLICATIONS

The simplicity of some of our melt pond models holds out the possibility of efficiently accounting for melt ponds and sea ice albedo in climate models.

## RELATED PROJECTS

NSF DMS Math Climate Research Network Grant (http://www.mathclimate.org/). This grant is partially funding our melt pond work and research on related sea ice properties.

# REFERENCES

M. Lüthje, D. L. Feltham, P. D. Taylor & M. G. Worster (2006) Modeling the summertime evolution of sea-ice melt ponds. *J. Geophys. Res.,* **111**, doi:10.1029/2004JC002818.

C. Polashenski, Z. Courville, D. K. Perovich, and D. Finnegan (2010) *Monitoring Melt Pond Evolution with LiDAR*, Presented at: International Glaciological Society Symposium on Sea Ice: May 31 - June 4, Tromso, Norway.

# HONORS/AWARDS/PRIZES

## Kenneth M. Golden, University of Utah

2013 Fellow of the American Mathematical Society (AMS), member of the inaugural class.

2012 Distinguished Scholarly & Creative Research Award, University of Utah, $10,000. First math professor since 1992 to receive the University's highest faculty award for research.

2012 Myriad Faculty Award for Research Excellence, University of Utah, College of Science, $20,000. The award recognizes efforts to foster undergraduate research and provide learning experiences for students.

Golden has been honored with numerous invitations to give high profile public lectures. We list a few during the reporting period (and just beyond):

2013 MAA-AMS-SIAM Gerald and Judith Porter Public Lecture, Joint Mathematics Meetings, San Diego (An official kick-off lecture for worldwide activities on Mathematics of Planet Earth 2013)

2013 IMA Public Lecture, Institute for Mathematics and its Applications, University of Minnesota, Minneapolis

2013 Distinguished Lecturer Series, Inaugural Lecture, Department of Mathematics, University of Memphis, TN

2013 Christie Public Lecture, co-hosted by the Department of Mathematics and the Bowdoin College Museum of Art, coinciding with a museum exhibition April 4, 2013 - June 2, 2013, entitled, ``Sense of Scale, Measure by Color: Art, Science, and Mathematics of Planet Earth,'' featuring paintings and sculpture by Per Kirkeby, images of rocks, some of my Antarctic sea ice photos, and other ice images, Bowdoin College, Maine

2013 First SIAM Public Lecture, Department of Mathematical Sciences, Florida Institute of Technology, Melbourne, FL

2013 Inaugural Bernoulli Society Public Lecture, 36th Conference on Stochastic Processes and their Applications, Boulder

2013 G. Milton Wing Lectures, University of Rochester, Rochester, NY

2013 Guest of Honor and Presenter, Institut des Hautes Etudes Scientifiques (IHES) Charity Gala, Theme of Mathematics of Planet Earth 2013, New York City

2014 Math Encounters, Public Presentation Series (sponsored by the Simons Foundation), National Museum of Mathematics, New York City

Golden's research has been covered extensively in the media, with 15 media encounters (TV, radio, magazine and newspaper articles, etc.) since Fall of 2012. These include requests by NSF to appear on Science Nation, and to be the "ice guy" for an NBC News / NBC Learn segment in a series on the Science of the Winter Olympic Games for grades 6-12.

**Donald K. Perovich, ERDC - CRREL**

2013 Fellow of the American Geophysical Union

Commander's Award for Civilian Service, Partnership in Education Award, 2013

NASA Group Achievement Award